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The phonon mass and the Hawking temperature in the two-dimensional acoustic black hole model

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ABSTRACT

In this paper, the acoustic metric and effective action are derived by using the shift transformation and functional integrals method, this metric can describe the fluctuation of the phonon fields in the quantum fluid. When the phonon fields are coupled with different types of 2-D gravity, i.e., Jackiw-Teitelboim model and Almheiri-Polchinski model, then the effective mass of the phonon fields and the Hawking temperature can be calculated in the analytical form. In addition, the influence of fluctuation intensity and deformation parameter to the phonon mass and the Hawking temperature are also analyzed and discussed.

1. Introduction

In 1981, Unruh first proposed the concept of the acoustic black hole [1], which is a specific representative model of analogy gravity, this work has opened a new way for using classical or quantum fluids to simulate the properties of the black holes in the experiments [2]. Unruh studied the fluctuation equations of the fluid density and pressure in the classical hydrodynamics, he found that the velocity potential equation is equivalent to a Klein-Gordon type equation in the curved space-time background, whose background metric is called the acoustic black hole metric, this analogy black hole also exist the Hawking radiation of the phonon fields. Many researchers extended Unruh's idea to BEC, He-3 and other quantum fluids, they trying to simulate the acoustic black holes in the lower temperature environment, so as to realize the experimental observation of the phonon Hawking radiation on the condensed matter physical platform [3–16].

In the 1990s, the analogy black hole and the analogy gravity have been studied by Jacobson, Vissev and Volovik et al. in the different physical model [10,11]. Garay et al. suggested to simulate acoustic black hole by using the BEC matter as working fluid [5]. Steinhauer conducted the BEC simulation experiment of the acoustic black hole in the laboratory [12]. Moreover, Cadoni studied the two-dimensional acoustic black holes in the classical fluid system, he calculated the effective mass and the Hawking temperature of the black holes [13]. Generally, there are two methods to deduce the acoustic metric: the classical fluid mechanics and the quantum fluid mechanics. Ilinsik and Stepanenko used the field theory approach to study the fluctuation fields of the BEC fluid, they regained correct form of the acoustic metric [14–16].

In this paper, the physical properties of two-dimensional acoustic black holes are studied based on the BEC fluid model, the acoustic black hole metric and the effective action of the phonon fields are derived using the shift transformations and the functional integrals method [17,19]. When the phonon fields are coupled with different types of two-dimensional gravity models, then the acoustic horizon, the Hawking temperature and effective mass of the phonon fields can be calculated from the analytical solutions of the field equations, these two-dimensional gravity models include Jackiw-Teitelboim (J-T) model and Almheiri-Polchinski (A-P) model. In addition, we will discuss the influence of the phonon field intensity and the deformation parameter on the effective mass and the Hawking temperature.

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2. Acoustic metric and phonon field equation in the quantum fluid mechanics

The reduced form action in two-dimensional BEC system is [14–16]

$$S=\int d\tau\int dx\left\{i\psi^{+}\frac{\partial\psi}{\partial\tau}-\left[\frac{\partial}{\partial x}\psi^{+}\frac{\partial}{\partial x}\psi+(f-\mu)\psi^{+}\psi+l\left(\psi^{+}\psi\right)^{2}\right]\right\},\tag{1}$$

where ψ^+ and ψ are the wave functions of the BEC fluid, μ is the chemical potential, f is the external potential, l is the coupling coefficient, the wave functions can be decomposed as

$$\begin{cases}\psi(x,\tau)=\sqrt{\rho(x,\tau)}e^{-i\varphi(x,\tau)}\\ \psi^{+}(x,\tau)=\sqrt{\rho(x,\tau)}e^{i\varphi(x,\tau)}\end{cases},\tag{2}$$

where $\rho=\psi^+\psi$ is the condensation density, then there is following reduced action

$$S=\int d\tau\int dx\left[\rho\frac{\partial\psi}{\partial\tau}-\left(\frac{\partial\sqrt{\rho}}{\partial x}\right)^{2}-\rho\left(\frac{\partial\varphi}{\partial x}\right)^{2}-(f-\mu)\rho-l\rho^{2}\right].\tag{3}$$

From this action we obtain the Euler equation and the conservation equation, that is

$$\frac{\partial\varphi}{\partial\tau}-\left(\frac{\partial\varphi}{\partial x}\right)^{2}-(f-\mu)-2l\rho+\frac{1}{\sqrt{\rho}}\frac{\partial^{2}}{\partial x^{2}}\sqrt{\rho}=0,\tag{4}$$

$$-\frac{\partial\rho}{\partial\tau}+2\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\varphi\cdot\rho\right)=0,\tag{5}$$

by redefining the fluid velocity $\vec{V}=-2\nabla\varphi$, the Euler equation and the conservation equation can be obtained from expressions (4)–(5), they are

$$\begin{cases}\dfrac{\partial\vec{V}}{\partial\tau}+\dfrac{\partial}{\partial x}\left(\dfrac{\vec{V}^{2}}{2}+2f-2\mu+4l\rho-\dfrac{2}{\sqrt{\rho}}\dfrac{\partial^{2}}{\partial x^{2}}\sqrt{\rho}\right)=0\\ \dfrac{\partial\rho}{\partial\tau}+\dfrac{\partial}{\partial x}(\vec{V}\cdot\rho)=0\end{cases}.\tag{6}$$

Let us introduce the following shift transformations

$$\psi(x,\tau)\to\psi(x,\tau)+\chi(x,\tau),\ \psi^{+}(x,\tau)\to\psi^{+}(x,\tau)+\chi^{+}(x,\tau),\tag{7}$$

here $\chi(x,\tau)$ and $\chi^+(x,\tau)$ describe the quantum fluctuation effect of the wave functions. From these transformations we can obtain the action of fluctuation part

$$\begin{aligned}S_f=&\int d\tau\int dx\left\{i\chi^{+}\tfrac{\partial\chi}{\partial\tau}-\tfrac{\partial}{\partial x}\chi^{+}\tfrac{\partial}{\partial x}\chi+(f-\mu)\chi^{+}\chi\right.\\&\left.+l\left(\psi^{+}\psi^{+}\chi\chi+\psi\psi\chi^{+}\chi^{+}+4\chi^{+}\chi\psi^{+}\psi\right)\right\}.\end{aligned}\tag{8}$$

A standard first approximation in solving Euler equation (4) is to ignore the kinetic energy term $\partial^2/\partial x^2\sqrt{\rho}/\sqrt{\rho}$ [18], so we can get the following equation

$$f-\mu\approx\partial\varphi/\partial\tau-2l\rho-(\partial\varphi/\partial x)^{2},\tag{9}$$

then the Hamiltonian of fluctuation part becomes

$$H=\int dx\left[\frac{\partial}{\partial x}\hat{\chi}^{+}\frac{\partial}{\partial x}\hat{\chi}+\left(\dot{\varphi}-\varphi'^{2}+2l\rho\right)\hat{\chi}^{+}\hat{\chi}-l\rho e^{2i\varphi}\hat{\chi}^{+}\hat{\chi}^{+}-l\rho e^{-2i\varphi}\hat{\chi}\hat{\chi}\right].\tag{10}$$

Now we introduce the canonical transformations

$$\begin{cases}\hat{\chi}=\sqrt{\rho}e^{i\varphi}\left(\frac{\hat{\sigma}}{2\rho}+i\hat{\phi}\right)\\ \hat{\chi}^{+}=\sqrt{\rho}e^{-i\varphi}\left(\frac{\hat{\sigma}}{2\rho}-i\hat{\phi}\right)\end{cases},\tag{11}$$

where

$$2l\rho\hat{\chi}^{+}\hat{\chi}-l\rho e^{2i\varphi}\hat{\chi}^{+}\hat{\chi}^{+}-l\rho e^{-2i\varphi}\hat{\chi}\hat{\chi}=l\hat{\sigma}^{2},\tag{12}$$

$$\dot{\varphi}\hat{\chi}^{+}\hat{\chi}=\rho\dot{\varphi}\hat{\phi}^{2}+\frac{\dot{\varphi}}{4\rho}\hat{\sigma}^{2}.\tag{13}$$

In order to introduce the potential energy of the phonon fields, we can make a transformation

$$\hat{\sigma}^2 = \hat{\sigma}^2 + \frac{U(\hat{\phi})}{l}, \tag{14}$$

this change of variables is equivalent to a shift transformation $l = l + U(\hat{\phi})/\hat{\sigma}^2$, l is a constant parameter, $U(\hat{\phi})$ is the potential energy of the phonon fields, it is a function of $\hat{\phi}$, thus we get the following functional action of the fluctuation fields

$$\begin{aligned} S_f = & \int d\tau \int dx \{ \tfrac{\partial \phi}{\partial \tau} \sigma - [\rho \left(\tfrac{\partial}{\partial x} \phi \right)^2 + \rho \dot{\varphi} \phi^2 \\ & + \left(l + \tfrac{\dot{\varphi}}{4\rho} \right) \sigma^2 + U(\phi) + 2\sigma \tfrac{\partial}{\partial x} \varphi \tfrac{\partial}{\partial x} \phi] \}, \end{aligned} \tag{15}$$

we consider $\sigma = \widetilde{\sigma} \ /(l + \frac{\dot{\varphi}}{4\rho})^{1/2}$, according to the functional integral of the field $\widetilde{\sigma}$,

$$\int d\widetilde{\sigma} \mathrm{e}^{\int d\tau \int dx \left[-\widetilde{\sigma}^2 - \frac{(2\varphi' \phi' - \phi)\widetilde{\sigma}}{\sqrt{l + \dot{\varphi}/4\rho}} \right]} = \mathrm{e}^{\int d\tau \int dx [\frac{(2\varphi' \phi' - \dot{\phi})}{4\sqrt{l + \dot{\varphi}/4\rho}}]^2}. \tag{16}$$

For the static case $\dot{\varphi} = 0$, so following effective action can be derived by using some algebraic operation

$$S_{eff} \sim \int dt \int dx \sqrt{-g} [\phi \Delta \phi + U(\phi)], \tag{17}$$

where, $dt = 4l\rho_0 d\tau, dx = \sqrt{4l\rho_0} dx$, the potential energy of the phonon field ϕ is rescaled to $U(\phi) = U(\phi)/4l\rho_0^2$, and the acoustic metric can be expressed as

$$g_{\mu\nu} = \rho^{1/2} \begin{pmatrix} -\rho + v^2 & v \\ v & 1 \end{pmatrix}, \tag{18}$$

where, $\rho^{1/2} = c$ is the sound velocity of the fluid, v is the fluid velocity and it is only related to the coordinates x. The components of Ricci tensor are

$$\begin{aligned} & R_{00} = \frac{(vv'' + v'^2)(v^2 - c^2)}{c^2}, R_{11} = \frac{(vv'' + v'^2)}{c^2} \\ & R_{01} = \frac{(vv'' + v'^2)v}{c^2}, R_{10} = \frac{(vv'' + v'^2)v}{c^2} \end{aligned}, \tag{19}$$

and the scalar curvature is

$$R = -\alpha''(x) = \frac{2\left(vv'' + v'^2\right)}{c^3}, \tag{20}$$

where $\alpha(x) = g^{11}$ is the metric factor. In addition, the energy momentum tensor can be expressed as

$$T_{\mu\nu}(\phi) = \phi \partial_\mu \partial_\nu \phi - \frac{1}{2} g_{\mu\nu} (\phi \Delta \phi + U(\phi)) + \frac{\partial}{\partial g^{\mu\nu}} \left(\phi \frac{g^{\mu\nu}}{\sqrt{-g}} \frac{\partial \sqrt{-g}}{\partial x^\mu} \frac{\partial \phi}{\partial x^\nu} \right), \tag{21}$$

and the phonon field equation can be derived from the Lagrangian $L_\phi = \phi \Delta \phi + U(\phi)$, which is

$$\frac{\partial}{\partial x^\mu} \left[\phi . (\frac{\partial g^{\mu\nu}}{\partial x^\nu})(\sqrt{-g}) + \phi . g^{\mu\nu} \frac{\partial \sqrt{-g}}{\partial x^\mu} \right] = \sqrt{-g} \left[\Delta \phi + \frac{\partial U(\phi)}{\partial \phi} \right], \tag{22}$$

where $\sqrt{-g} = c^2$ is a constant. The second term in left-hand-side is zero in the case of static acoustic metric, meanwhile this phonon field equation becomes

$$\frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\mu} [\phi . (\frac{\partial g^{\mu\nu}}{\partial x^\nu})(\sqrt{-g})] = \Delta \phi + \frac{\partial U(\phi)}{\partial \phi}. \tag{23}$$

The left-hand side of the Eq. (23) is not zero, this equation is different from the standard motion equation of a free field with a potential, because the Lagrangian of scalar field is $L_s = g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi + U(\phi)$, and the phonon field Lagrangian is $L_\phi = \phi \Delta \phi + U(\phi)$, thus the field equations derived from two Lagrangian have the different forms.

3. Phonon mass and Hawking temperature in the Jackiw-Teitelboim model

Now we consider the coupling of a two-dimensional gravity model with the phonon fields ϕ, the action of extended Jackiw-Teitelboim model coupled to phonon fields is [20–22]

$$S = \int d^2x \sqrt{-g} \left[-\frac{1}{2} \nabla^\mu \psi \nabla_\mu \psi + \psi R + \kappa (\phi \Delta \phi + U(\phi)) \right], \tag{24}$$

where g is the acoustic metric, R is the scalar curvature, κ is the coupling constant, $U(\phi)$ is the potential energy of the phonon field ϕ. In equation (24), ψ is an auxiliary field, the purpose of introducing this real Bose field is to ensure that Einstein's equation holds [20–22].

In equation (1), ψ is the complex wave function of BCE fluid, $\rho = \psi^{+}\psi$ represents the number density of particles, and the derivative of phase angle is related to the velocity of fluid [18], so the physical meanings of the two ψ are different. From the action (24) we can obtain the following auxiliary field equation and gravitational field equation

$$\nabla^2\psi - R = 0, \tag{25}$$

$$\frac{1}{2}g^{\mu\nu}\nabla_\mu\psi\nabla_\nu\psi - \frac{1}{2}(\nabla\psi)^2 + 2\nabla^2\psi - g^{\mu\nu}\nabla_\mu\nabla_\nu\psi = \kappa g^{\mu\nu}T_{\mu\nu}(\phi), \tag{26}$$

where $T_{\mu\nu}(\phi)$ is the energy momentum tensor of the phonon fields. Almheiri and Polchinski developed models of 1+1 dimensional dilaton gravity by add matter fields action $\Omega(\Phi)(\nabla f)$ [23], when the coupling factor $\Omega(\Phi) = 1$, this model exhibits nice properties and it can give the vacuum solution and deformed black hole solution [25]. In addition, Frolov and Zelnikov studied the Liouville 2D dilaton gravity models with sinh-Gordon matter [27], by using an action without the coupling of matter fields and scalar fields, they found some exact black hole solutions in the J-T gravity model. In this paper, we will ignore the coupling action of phonon fields and auxiliary Bose fields, the direct addition action (24) can be used to seek the analytic solutions of the field equations.

According to acoustic metric (18) and scalar curvature (20), when $\kappa = 1$, we further derive three field equations

$$\alpha\psi' = -\alpha', \tag{27}$$

$$(\phi\alpha')' = (\alpha\phi')' + \frac{\partial U(\phi)}{\partial\phi}, \tag{28}$$

$$(\alpha\psi')' = \alpha\phi\phi'' - \phi\left(\alpha\phi'\right)' - U(\phi). \tag{29}$$

In order to describe the physical properties of acoustic black hole and phonon fields, we can vary over the metric in the 2-d action (24), the field equations (23) and (25) can be derived from the variation of the action with respect to ϕ and ψ, the energy momentum tensors (21) and (26) can be deduced from the variation of the action with respect to $g^{\mu\nu}$. From the solutions of these field equations, we can get the analytic expressions of phonon mass and Hawking temperature, and the auxiliary field ψ can also help us to achieve this goal. For the phonon fields with large fluctuations, the analytic solutions of above fields equations are

$$\phi = \phi_0 x^p, \tag{30}$$

$$\alpha(x) = C_1 x^{m_q} + C_2 x^{n_q}, \ \psi = \ln\alpha, \tag{31}$$

$$U(\phi) = -\phi_0^2\left(\frac{q+1}{2}\right)\left[C_1 m_q\left(\frac{\phi}{\phi_0}\right)^{\frac{2(m_q+q-1)}{q+1}} + C_2 n_q\left(\frac{\phi}{\phi_0}\right)^{\frac{2(n_q+q-1)}{q+1}}\right], \tag{32}$$

where ϕ_0 is the fluctuation coefficient, p is the fluctuation intensity, $q = 2p - 1$, C_1 and C_2 are metric coefficients, two exponents in the metric are

$$m_q = \frac{2 - q + \sqrt{2\left(q^2 - 1\right) + (q-2)^2}}{4}, \tag{33}$$

$$n_q = \frac{2 - q - \sqrt{2\left(q^2 - 1\right) + (q-2)^2}}{4}. \tag{34}$$

Moreover, the phonon potential $U(\phi)$ in Eq. (32) can be obtained from the field equations (27)-(29), its expression has been derived by using the analytic solutions of the metric factor and the phonon fields. From the derivative of potential energy $\partial U(\phi)/\partial\phi = 0$, we obtain the following critical point for the phonon fields

$$\phi_c = \phi_0\left[-\frac{C_2}{C_1}\cdot\frac{n_q\left(n_q+q-1\right)}{m_q\left(m_q+q-1\right)}\right]^{\frac{q+1}{2(m_q-n_q)}}, \tag{35}$$

so the effective mass of the phonon becomes

$$m_{eff}^2 = \left.\frac{\partial^2 U(\phi)}{\partial\phi^2}\right|_{\phi=\phi_c} = C_2\left(\frac{2}{q+1}\right)n_q\left(n_q+q-1\right)\left(m_q-n_q\right)\left(\frac{\phi_c}{\phi_0}\right)^{\frac{2(n_q+q-1)}{q+1}-2}. \tag{36}$$

When $\partial^2 U(\phi)/\partial^2\phi\big|_{\phi=\phi_c} > 0$, the potential energy $U(\phi)$ is taken as a minimum value, the phonon field can reach a stable state. The horizon coordinates of the acoustic black hole can be determined by the relation $\alpha(x_0) = 0$, that is

$$x_0 = \left(-\frac{C_2}{C_1}\right)^{\frac{1}{m_q-n_q}}, \tag{37}$$

here the coefficients C_1 and C_2 have opposite signs. From Eq. (35) we get the coordinate of the critical point, it is

$$x_c = \left[-\frac{C_2}{C_1}\cdot\frac{n_q\left(n_q+q-1\right)}{m_q\left(m_q+q-1\right)}\right]^{\frac{1}{(m_q-n_q)}}, \tag{38}$$

the corresponding metric factor is

$$\alpha\left(x_c\right)=C_2 x_c^{n_q}\left[1-\frac{n_q\left(n_q+q-1\right)}{m_q\left(m_q+q-1\right)}\right], \tag{39}$$

the Hawking temperature at the event horizon is defined as

$$T_H=\frac{1}{4\pi c}\left.\frac{\partial\left(c^2-v^2\right)}{\partial x}\right|_{v=c}=\frac{c^2}{4\pi}\alpha'\left(x_0\right), \tag{40}$$

Substituting Eq. (31) and Eq. (37) into Eq. (40), we obtain the following Hawking temperature of the acoustic black hole

$$T_H=\frac{c^2}{4\pi}\left[C_2\left(-\frac{C_2}{C_1}\right)^{\frac{n_q-1}{m_q-n_q}}\left(n_q-m_q\right)\right]. \tag{41}$$

So the fluctuations of phonon fields become stronger in the larger q case, when $q=1, p=1$, then the horizon coordinate is located at $x_0=\left(-C_2/C_1\right)$. In addition, the coordinate of the vacuum point is $x_c=0$. From the fluid velocity relation $v(x_0)=v(x_c)$, we know that the acoustic black hole can form two horizons. In this case, the effective mass of phonons becomes $m_{eff}=0$, and Hawking temperature is

$$T_H=\frac{c^2}{4\pi}\left[C_2\left(-\frac{C_2}{C_1}\right)^{-2}\left(-\frac{1}{2}\right)\right], \tag{42}$$

when $q=0$, $p=1/2$, since $x_0=x_c=\left(-C_2/C_1\right)^{\sqrt{2}}$, the two horizon coordinates of the black hole coincide, the effective mass of phonons in this extreme black hole becomes $m_{eff}=(-C_2)^{1/2}.2^{-5/2}$, and the Hawking temperature is

$$T_H=\frac{c^2}{4\pi}\left[C_2\left(-\frac{C_2}{C_1}\right)^{-\frac{1+\sqrt{2}}{2}}\left(-\frac{\sqrt{2}}{2}\right)\right]. \tag{43}$$

With the change of field index from $q=0$ to $q=1$, then $m_{q=0}>m_{q=1}$, $T_{q=0}>T_{q=1}$, the phonon effective mass and Hawking temperature decrease. Therefore, in the J-T gravity model, the fluctuation intensity of the phonon fields will affect the phonon mass and Hawking temperature.

4. Phonon mass and Hawking temperature in the Almheiri-Polchinski model

Now we will study the Almheiri-Polchinski (A-P) model by add the phonon fields action, the direct addition action of this extension model is [23]

$$S=\int d^2x\sqrt{-g}\left[\psi R+W(\psi)+\kappa\left(\phi\Delta\phi+U(\phi)\right)\right], \tag{44}$$

the field equations of the auxiliary field ψ and the phonon fields ϕ are

$$\alpha''=\frac{\partial W(\psi)}{\partial\psi}, \tag{45}$$

$$\left(\phi\alpha'\right)'=\left(\alpha\phi'\right)'+\frac{\partial U(\phi)}{\partial\phi}, \tag{46}$$

$$-W(\psi)+\left(\alpha\psi'\right)'=\alpha\phi\phi''-\phi\left(\alpha\phi'\right)'-U(\phi), \tag{47}$$

where the potential energy of auxiliary field ψ is $W(\psi)=D-\Lambda\psi$, D and Λ are constants, scalar curvature is $R=\Lambda$, when cosmological constant $\Lambda<0$, the negative constant curvature can represent Ads space. Moreover, by integrating the equation $\alpha''=-\Lambda$, we get the following metric factor

$$\alpha(x)=-\frac{\Lambda}{2}x^2+B, \tag{48}$$

where B is an integral constant. Let us consider $k=1$, $\psi=\psi_0 x$, then field equations (46)–(47) also give an analytic solution of the phonon fields, namely

$$\frac{\phi^2}{2}=-\frac{\phi_1}{\Lambda^2 x}+\phi_2. \tag{49}$$

In the J-T model, the phonon fields are proportional to the coordinate x if $q>0$. However, in the A-P model, the phonon fields are inversely proportional to the coordinate x and they asymptotically tend to a constant ϕ_2. The potential energy of the phonon fields is

$$U(\phi)=D-\frac{\Lambda}{2}\phi^2+\Lambda.\phi_2, \tag{50}$$

thus the effective mass of the phonons becomes

$$m_{eff}^2 = \frac{\partial^2 U(\phi)}{\partial \phi^2}\bigg|_{\phi_c=0} = -\Lambda. \tag{51}$$

The horizon coordinate of the black hole is

$$x_0 = \sqrt{\frac{2B}{\Lambda}}, \tag{52}$$

where integral constant $B < 0$, the cosmological constant $\Lambda < 0$ represents an anti-de Sitter space, the corresponding expression of the Hawking temperature is

$$T_H = \frac{c^2}{4\pi}\alpha'(x_0) = \frac{c^2}{4\pi}(-\Lambda)\sqrt{\frac{2B}{\Lambda}}. \tag{53}$$

Recently, Kyono et al. study deformations of the A-P model by using the Yang-Baxter deformation method [24–26], they found the deformed black hole solutions can be derived from the deformed AP model, and the Bekenstein-Hawking entropy is also modified by the deformed parameter, then a hyperbolic function-type deformed potential is expressed as

$$W(\psi) = D - \frac{\Lambda}{\eta}\sinh(\eta\psi), \tag{54}$$

when the deformed parameter $\eta \to 0$, the deformed potential (46) reduces to the linear potential in the A-P model. From the auxiliary field equation $\alpha'' = \partial W(\psi)/\partial\psi$, we obtain the following metric factor

$$\alpha(x) = -\frac{\Lambda}{(\eta\psi_0)^2}\cosh(\eta\psi_0 x) + B. \tag{55}$$

According to the field Eqs. (38), (39), we derived two analytic solutions of the phonon fields and potential energy

$$\frac{\phi^2}{2} = -\left(\frac{\eta\psi_0}{\Lambda^2}\right)\phi_1\coth(\eta\psi_0 x) + \phi_{2,} \tag{56}$$

$$U(\phi) = D - \phi_1\sqrt{a\left(\frac{\phi^2}{2} - \phi_2\right)^2 - b}, \tag{57}$$

where $a = (\Lambda/\phi_1)^2, b = (\eta\psi_0/\Lambda)^2$, hence deformed phonon mass can be derived from the extreme value of the potential energy, that is

$$m_{eff}^2(\eta) = \frac{\partial^2 U(\phi)}{\partial\phi^2}\bigg|_{\phi_c=0} = -\Lambda . f(\eta), \tag{58}$$

here the corrected factor has a "Lorentz transformation" form

$$f(\eta) = \frac{1}{\sqrt{1 - \frac{\eta^2}{l^2}}}, \tag{59}$$

where $l = \phi_2\Lambda^2/\phi_1\psi_0$, the constraint condition of the deformed parameter is $\eta^2 < l^2$. It is clear that the horizon position of the black hole will also change, the shift coordinate is

$$x_0(\eta) = \frac{1}{(\eta\psi_0)}\operatorname{arccos}h\frac{B(\eta\psi_0)^2}{\Lambda}, \tag{60}$$

meanwhile the Hawking temperature can be expressed as the following form

$$T_H(\eta) = \frac{c^2}{4\pi}\alpha'(x_0(\eta)) = \frac{c^2}{4\pi}\left[-\frac{\Lambda}{(\eta\psi_0)}\sqrt{\left(\frac{B(\eta\psi_0)^2}{\Lambda}\right)^2 - 1}\right]. \tag{61}$$

So the Hawking temperature also produces a deformation, this result is different from the conclusion of Kyono et al. [24–26]. In particular, they used a hyperbolic function-type to obtain a deformed black hole solution, the Hawking temperature is not deformed but Bekenstein-Hawking entropy is changed due to the deformation.

In the deformed A-P model, we take the values of the physical quantities as follows: cosmological constant is $\Lambda = -1$, auxiliary field coefficient is $\psi_0 = 1$, ratio of metric integral constant to cosmological constant is $B/\Lambda = 10^2$, ratio of potential energy coefficient is $\phi_2/\phi_1 = 1$, so the range of deformation parameter becomes $0.1 < \eta < 1$. When $\eta = 0.2$, the calculation results are $x_0(\eta) \approx 0.73x_0$, $m_{eff}(\eta) \approx 1.01m_{eff}, T_H(\eta) \approx 1.37T_H$, at this time the horizon of black hole becomes smaller, Hawking temperature and effective mass of phonon increase. When $\eta = 0.9$, the calculated results change into $x_0(\eta) \approx 0.40x_0$, $m_{eff}(\eta) \approx 1.52m_{eff}, T_H(\eta) \approx 6.36T_H$. Therefore, when the deformation parameter increases from $n = 0.2$ to $n = 0.9$, the horizon of the black hole will become smaller, and the Hawking temperature and effective mass of the phonon fields will become larger.

Balbinot et al. pointed out that an atomic Bose–Einstein condensate can be used to test the radiation effect of the acoustic black hole [7–9], they found that density correlations involve a signature of Hawking radiation from the phonon pairs, so it is possible to finish a direct detection of this radiation in the BEC experiment. Moreover, the round-trip time τ_{RT} for excitations to travel from the black-hole horizon to the inner horizon is related to the mixing coefficient $\beta(\omega)$ of the Hawking radiation, τ_{RT} can also be determined by fluid velocity and speed of sound. From Eq. (61) we know that the informations of potential energy function $W(\psi)$ are included in Hawking temperature, once the potential energy $W(\psi)$ and the phonon field ϕ are coupled through the metric factor α, then we can model the interactions induced by the J-T mode using the original acoustic black hole.

5. Conclusion and discussion

In this letter, the effective action of the phonon fields and the metric of 2-D acoustic black hole are derived by using the shift transformation and the functional integrals technique, when two-dimensional gravity models are coupled with the phonon fields, then Hawking temperature and the effective mass of the phonon fields are calculated from the analytical solutions of the field equations.

In the J-T gravity model, the influence of phonon field fluctuation intensity on different physical quantities is discussed. With the increase of the fluctuation intensity of the phonon fields, the position of the event horizon increases, the Hawking temperature decreases, and the effective mass of the phonon fields also decreases.

In the A-P gravity model, the influence of the deformation parameters on the same physical quantities is further analyzed. With the increase of the deformation parameter, the position of the black hole horizon first increases and then decreases, the Hawking temperature increases, and the effective mass of the phonon fields increases. The effective mass of the deformed phonon is greater than that of the undeformed one, the position of the event horizon and the Hawking temperature are all smaller than the undeformed values.

The results in this paper may be useful for the simulation of the phonon vacuum and acoustic black holes in different types of two-dimensional gravity models, our research methods and thinking can be used to study similar problems in hyperbolic potential model and Liouville gravity model [28–30], this will be the main task in the next work.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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