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SYNCHRONOUS TRANSFER OF BEAM FROM MAIN RING TO ENERGY DOUBLER

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We investigate, here, the rf requirements for the synchronous transfer of beam from the main ring (MR) to the energy doubler (ED). To begin with we assume

(1) The acceleration rate in the ED is 20 GeV/sec which gives 40 sec for 800 GeV (200 GeV to 1000 GeV) or 45 sec for 900 GeV (100 GeV to 1000 GeV).

(2) The ED pulse has a flat-bottom and the MR pulse has a flat-top, hence the beam is transferred from stationary bucket to stationary bucket.

(3) The invariant longitudinal phase-space area of the beam is 0.1 eVsec per bunch, as observed from the MR beam at 300 GeV.

(4) Whenever it is a good approximation we shall assume that the ED has the same parameters as the MR. e.g. $R = 1000$ m, $\gamma_t = 18.6$.

Now we examine different possibilities.

A. Voltage requirement in ED

For the ED $\beta = \frac{v}{c} \cong 1$ the energy gain per turn corresponding to 20 GeV/sec is

$$eV\sin\phi_s = \frac{2\pi R}{c}\,(20\ \text{GeV/sec}) = 419\ \text{keV}.$$

For various synchronous phase angle ϕ_s we get

ϕ_s	V(kV)
45°	593
50°	547

55°	512
60°	484
65°	462

It is undesirable to go beyond 65°, especially since further reduction in the required voltage is relatively minor. For the following studies we will assume

$$V = 500 \text{ kV} \qquad \phi_s = 57° \; .$$

B. Range of frequency modulation

At various transfer energies we have

transfer energy (GeV)	$\beta = \frac{v}{c}$	% frequency modulation
100	0.9999568	.00428%
150	0.9999807	.00189%
200	0.9999891	.00105%
1000	0.9999996	-

These are all small modulation ranges. Perhaps untuned rf cavities can be used.

C. Phase-space matching at same harmonic number

If the ED rf has the same frequency (harmonic number) as the MR rf at transfer the rf buckets are identical. The MR rf voltage should be adiabatically reduced to 500 kV on the flat-top. This reduction can easily be done adiabatically since there is no difficulty providing a flat-top at 100 GeV - 200 GeV of adequate duration. The half widths of the stationary rf bucket in the (ϕ, w) space are given by

$$\begin{cases} \Delta\phi = \pi \\ \Delta W = \left[\frac{2}{\pi}\frac{1}{\Lambda}\frac{E(eV)}{h^3\omega^2}\right]^{1/2} \end{cases}$$

where

$$\begin{cases} E = mc^2\gamma = \text{total energy} \\ \Lambda = \frac{1}{\gamma_t^{\,2}} - \frac{1}{\gamma^2} \quad , \quad \omega = \frac{c}{R} \end{cases}$$

and the beam bunch occupies a small and roughly elliptical area with semi-axes given by

$$\begin{cases} \delta\phi = \sqrt{\frac{A}{\pi}\frac{2}{\Delta W}} \\ \delta W = \sqrt{\frac{A}{\pi}\frac{\Delta W}{2}} \end{cases}$$

where

A = phase-space area of bunch = 0.1 eVsec.

At various transfer energies and at V = 500 kV we get

transfer energy (GeV)	bucket		beam bunch	
	$\Delta\phi$	ΔW(eVsec)	$\delta\phi$	δW(eVsec)
100	180°	0.304	26.2°	0.0696
150	180°	0.369	23.8°	0.0766
200	180°	0.424	22.2°	0.0822

These beam bunches when transferred to the ED r.f. buckets operating at 500 kV will be automatically matched in shape. It remains to check that the phase oscillation frequencies for these stationary buckets are sufficiently high for adiabatic changes in rf parameters. The phase oscillation frequency F is given by

$$F = \frac{h\omega}{2\pi}\left[\frac{1}{2\pi h}\,\frac{eV}{E}\,\Lambda\right]^{1/2}$$

For various transfer energies we get

transfer energy (GeV)	F(Hz)
100	74.8
150	61.7
200	53.7

These frequencies are certainly high enough.

D. Going to a higher harmonic number in ED

To keep the ED rf cavities small one may contemplate a higher frequency rf system. To find the scaling law in h and V we have to go to physical variables: length ($\delta \ell$) and momentum spread (δp) of the bunch. They are related to $\delta\phi$ and δw by

$$\delta \ell = \frac{R}{h}\,\delta\phi \qquad \text{and} \qquad \delta p = \frac{h}{R}\,\delta w.$$

When everything except h and V are constant we have

$$\begin{cases} \delta \ell \propto \dfrac{1}{h\sqrt{\Delta w}} \propto (hV)^{-\frac{1}{4}} \\[2ex] \delta p \propto h\sqrt{\Delta w} \propto (hV)^{\frac{1}{4}} \end{cases}$$

For matching we should have

$$(hV)_{MR} = (hV)_{ED}$$

or

$$\frac{V_{MR}}{V_{ED}} = \frac{h_{ED}}{h_{MR}} \equiv n$$

Since from (C) above we see that the beam bunch length is $< \frac{30°}{180°} = \frac{1}{6}$ of the MR bucket length it is safe to have n as high as 6. For n = 6 we must have $V_{MR} = 6\ V_{ED} = 3$ MV which can easily be supplied by the MR rf system. The scaling of the semi-dimensions of the stationary rf bucket with respect to h and V are

$$\Delta \ell = \frac{R}{h} \pi \propto h^{-1}$$

$$\Delta p = \frac{h}{R} \Delta w \propto \left(\frac{V}{h}\right)^{1/2} .$$

For various transfer energies and various n values we get (see Table 1). In this table we have assumed that R_{ED} is slightly smaller than R_{MR} such that

$$\frac{R_{ED}}{R_{MR}} = \frac{1112}{1113} .$$

Thus, at the same rf frequency the harmonic number in the ED is 1112.

Take the case of transfer energy = 100 GeV and n = 6, the transfer process is as follows:

"On the 100 GeV flat-top of the MR the rf voltage is adiabatically adjusted to 3.0 MV. The ED should be flat-bottomed. The ED rf should have a voltage of 0.5 MV and a frequency 6 times that of the MR and properly phased. When synchronously transferred to the ED to fill every 6th of the rf buckets the beam bunches will be properly matched in phase-space shape although fitting more tightly in the smaller ED rf buckets."

It is tantalizing to think of the possibility of filling all the ED buckets with 6 pulses from the MR before acceleration in the ED. But this requires an rf injection kicker which produces a train of pulsed kicks at the MR frequency of 53.1028 MHz. The kick-pulses should be square and very short, just long enough and properly timed to kick only the newly injected beam bunches without affecting any previously injected beam bunches. In view of the magnitude of the kicks required this is very difficult indeed.

Several discussions with S. Ohnuma were very helpful.

Table 1

	MR					ED						
	physical			bucket		physical			bucket		beam bunch	
n	h	f(MHz)	V(MV)	$\Delta \ell$(m)	Δp(eVsec/m)	h	f(MHz)	V(MV)	$\Delta \ell$(m)	Δp(eVsec/m)	$\delta \ell$(m)	δp(eVsec/m)
100 GeV = transfer energy												
1	1113	53.1028	0.5	2.823	0.338	1112	53.1028	0.5	2.823	0.338	0.411	0.077
2	1113	53.1028	1.0	2.823	0.479	2224	106.2056	0.5	1.411	0.239	0.346	0.092
3	1113	53.1028	1.5	2.823	0.586	3336	159.3084	0.5	0.941	0.195	0.312	0.102
4	1113	53.1028	2.0	2.823	0.677	4448	212.4111	0.5	0.706	0.169	0.291	0.110
5	1113	53.1028	2.5	2.823	0.757	5560	265.5139	0.5	0.565	0.151	0.275	0.116
6	1113	53.1028	3.0	2.823	0.829	6672	318.6167	0.5	0.470	0.138	0.263	0.121
150 GeV = transfer energy												
1	1113	53.1041	0.5	2.823	0.410	1112	53.1041	0.5	2.823	0.410	0.373	0.085
2	1113	53.1041	1.0	2.823	0.580	2224	106.2081	0.5	1.411	0.290	0.314	0.101
3	1113	53.1041	1.5	2.823	0.711	3336	159.3122	0.5	0.941	0.237	0.284	0.112
4	1113	53.1041	2.0	2.823	0.821	4448	212.4162	0.5	0.706	0.205	0.264	0.121
5	1113	53.1041	2.5	2.823	0.918	5560	265.5203	0.5	0.565	0.184	0.250	0.127
6	1113	53.1041	3.0	2.823	1.005	6672	318.6243	0.5	0.470	0.168	0.239	0.133
200 GeV = transfer energy												
1	1113	53.1045	0.5	2.823	0.472	1112	53.1045	0.5	2.823	0.472	0.348	0.091
2	1113	53.1045	1.0	2.823	0.668	2224	106.2090	0.5	1.411	0.334	0.293	0.109
3	1113	53.1045	1.5	2.823	0.818	3336	159.3135	0.5	0.941	0.273	0.264	0.120
4	1113	53.1045	2.0	2.823	0.944	4448	212.4180	0.5	0.706	0.236	0.246	0.129
5	1113	53.1045	2.5	2.823	1.056	5560	265.5225	0.5	0.565	0.211	0.233	0.137
6	1113	53.1045	3.0	2.823	1.156	6672	318.6270	0.5	0.470	0.193	0.222	0.143