

RECEIVED: February 9, 2024

REVISED: June 14, 2024

ACCEPTED: June 24, 2024

PUBLISHED: August 5, 2024

$SU(2)_L$ deconstruction and flavour (non)-universality

Bernat Capdevila ^{a,b,c} Andreas Crivellin ^{d,e} Javier M. Lizana ^{e,f}
 and Stefan Pokorski ^g

^a*Università di Torino and INFN Sezione di Torino,
 Via P. Giuria 1, Torino I-10125, Italy*

^b*DAMTP, University of Cambridge,
 Wilberforce Road, Cambridge, CB3 0WA, U.K.*

^c*Department of Physics and IFAE, Universitat Autònoma de Barcelona,
 Edifici Ciències, Bellaterra, 08193, Barcelona, Spain*

^d*Paul Scherrer Institut,
 CH-5232 Villigen PSI, Switzerland*

^e*Physik-Institut, Universität Zürich,
 Winterthurerstrasse 190, CH-8057 Zürich, Switzerland*

^f*Instituto de Física Teórica UAM/CSIC,
 Nicolas Cabrera 13-15, Madrid 28049, Spain*

^g*Institute of Theoretical Physics, Faculty of Physics, University of Warsaw,
 Pasteura 5, PL-02-093 Warsaw, Poland*

*E-mail: bernat.capdevila.soler@gmail.com, andreas.crivellin@psi.ch,
jmlizana@ift.csic.es, Stefan.Pokorski@fuw.edu.pl*

ABSTRACT: We study two-site deconstructions of the $SU(2)_L$ gauge group factor of the SM. Models based on this approach can explain the hierarchies of the quark masses and CKM mixing between third and light families if these fields are localised on different sites by the presence of hierarchical new physics scales. The model leads to an accidental global $U(2)_q \times U(3)_u \times U(3)_d$ flavour symmetry which prevents dangerously large effects in flavour observables, making a TeV extension of the SM possible. Given the structure of the PMNS matrix in the neutrino sector, we explore different possibilities for the arrangement of the leptons on the two sites, and consider different models with $U(2)_\ell$ or $U(3)_\ell$ flavour symmetries. The phenomenology of the models is mostly governed by a massive vector triplet of $SU(2)_L$. We study the interesting interplay between LHC searches and precision observables. In particular, one of the models can give a sizeable lepton flavour universal effect in the Wilson coefficient C_9 while naturally suppressing contributions to C_{10} , as suggested by current $b \rightarrow s\ell^+\ell^-$ data, predicting simultaneously a mild positive shift in the W boson mass.

KEYWORDS: New Gauge Interactions, Specific BSM Phenomenology, Theories of Flavour, Flavour Symmetries

ARXIV EPRINT: [2401.00848](https://arxiv.org/abs/2401.00848)

Contents

1	Introduction	2
2	2-site model	3
2.1	Quark sector	4
2.2	Lepton sector	6
2.3	$Z - Z'$ and $W - W'$ mixing	8
3	Phenomenology: U(2)-preserving observables	9
3.1	LHC constraints	9
3.2	Electroweak precision observables	9
3.3	LEP-II $e^+e^- \rightarrow \ell^+\ell^-$ data	10
3.4	Lepton flavour universality tests	10
3.5	CKM elements	10
3.6	Results	11
4	Phenomenology: U(2)-breaking observables	12
4.1	$\Delta F = 2$ observables	12
4.2	$b \rightarrow s\ell^+\ell^+$ data	13
4.3	$B \rightarrow K^{(*)}\nu\nu$ and $K \rightarrow \pi\nu\nu$	15
4.4	$R_{D^{(*)}}$	15
4.5	LFV processes	16
4.6	Results	16
5	Future prospects	18
5.1	LFV: Mu3e, Belle II, COMET and Mu2e	18
5.2	High-luminosity phase of LHC	18
5.3	Future circular collider	19
5.4	Results	19
6	Conclusions	19
A	Matching to SMEFT	21
B	Vector-like-lepton phenomenology	22
C	Electroweak fit	24
D	Lepton flavour universality violation in charge currents	25
E	Cabibbo angle anomaly	26
F	$B_{d,s} - \bar{B}_{d,s}$ mixing	26
G	A protection for $B_s \rightarrow \ell^+\ell^-$	27
H	LFV processes	27

1 Introduction

The flavour sector of the Standard Model (SM) seems ad-hoc; quarks and charged lepton masses as well as the CKM matrix elements display a strong hierarchy. Many dynamical explanations have been proposed to explain these features, e.g. based on adding new horizontal symmetries such as Froggatt-Nielsen models [1] or gauged flavour symmetries [2, 3], or new strong dynamics such as anarchic partial compositeness [4–6]. However, stringent bounds from flavour physics, in particular kaon and $D^0 - \bar{D}^0$ mixing or electric dipole moments [7, 8], typically impose stringent lower limits on the scale of new physics (NP) that realises these mechanisms. A very interesting possibility is that the SM flavour structure is generated at different scales [9–12], that could happen due to strongly coupled sectors developing several condensates [8], or, as we consider here, because the SM gauge group is extended to a non-universal larger group. In this setup, an approximate global $U(2)$ flavour symmetry minimally broken can emerge, weakening these stringent flavour bounds [13–18]. Then, the lowest scale can be as low as a few TeV, providing interesting phenomenological connections between low-energy precision experiments and LHC searches as well as possible relations with solutions to the Higgs hierarchy problem. For models establishing this relation, see for instance [8, 19, 20].

Beyond the SM theories containing a product of identical gauge groups, sometimes referred to as *moose* or *quiver* structures, have a long history [21, 22]. An additional motivation for them emerged with the idea of deconstructing (latticising) extra spacial dimensions which leads to dual four-dimensional gauge theories [23, 24], referred to as multi-site models. In such models the SM fermions can be assigned to different sites [25], in analogy with their localisations in the extra dimension, such that the flavour universality is in general broken.

There have been many proposals to address flavour hierarchies following these ideas based on the deconstruction of the SM gauge group or its UV completions: deconstructions of $SU(3)_c$ [26, 27], including quark-lepton unification of the third family [28–30], theories with a deconstructed Pati-Salam gauge symmetry (totally or partially) [31–33], deconstructions of hypercharge [34, 35], $SU(5)$ GUT [36] or $U(1)$ SM extensions [37].

On the other hand, it has been suggested that the $U(2)_q$ flavour-symmetry for the light left-handed (LH) quark doublets is sufficient to explain partially the flavour hierarchies of the quark sector [38] as it only allows for third-family-quark Yukawas and forbids light-family Yukawas and light-heavy CKM mixing angles. We propose a model that realises this symmetry as an accidental one due to a moose gauge structure: a two-site model for $SU(2)_L$ (i.e. $SU(2)_1 \times SU(2)_2 \rightarrow SU(2)_L$), with first and second family quarks on one site, and third-family quarks on the other site. While similar extensions of the SM gauge group have been proposed and studied [39–45], we put our emphasis on the use of this gauge symmetry to (partially) address flavour hierarchies while having interesting phenomenological consequences at the same time. By charging the Higgs under the same $SU(2)$ group as the third-family quarks, only third-generation quark Yukawa couplings are allowed at the renormalisable level. Heavier NP, well above the TeV scale, can generate the remaining light-quark Yukawa couplings and mixing angles in a suppressed way. This explains (partially) the hierarchy in the quark sector.

However, the flavour pattern in the lepton sector, in particular the PMNS matrix, is not that well addressed by the analogous $U(2)_\ell$ symmetry [46]. Therefore, we consider several

models with different arrangements for the leptons on the two sites: we compare the model that realises $U(2)_\ell$ with models that preserve the full $U(3)_\ell$ symmetry, without addressing flavour hierarchies in the lepton sector (whose explanation is relegated to higher scales). This is achieved by situating all leptons in the same site. Possible gauge anomalies can be cancelled by adding new fermionic degrees of freedom.

The breaking of $SU(2)_1 \times SU(2)_2 \rightarrow SU(2)_L$ results in a heavy $SU(2)_L$ triplet vector field containing a Z' and W'^{\pm} bosons [47, 48]. As we will see, their masses can be as low as a few TeV, even though they have non-universal couplings to LH SM fermions which introduce new sources of flavour breaking. The resulting rich phenomenology includes Flavour Changing Neutral Currents (FCNC) processes such as $b \rightarrow s\ell^+\ell^-$ transitions or meson mixing. Notice that these FCNCs are restricted to the LH sector, as within the SM ones, which allows to keep them under control. Still, they already appear at the tree-level, compensating for the mass suppression w.r.t. the SM contribution. The Higgs, which is charged under one of the $SU(2)$ factors, interacts with the massive triplet generating a mixing between the SM electroweak (EW) gauge bosons and the Z' , W' that affects the EW precision observables (EWPO), featuring a nice complementarity between flavour, electroweak, and collider physics.

During the completion of the first version of this manuscript, ref. [49] appeared on the arXiv where a full deconstruction of $SU(2)_L$ to $SU(2)_1 \times SU(2)_2 \times SU(2)_3$, with one factor for each family was proposed. One of the models discussed in our work corresponds to the infrared limit of ref. [49]. We have checked that the phenomenological results we obtain for this model are quantitatively similar to the ones of ref. [49].

The details of the proposed models are presented in section 2. section 3 is devoted to the phenomenology in the $U(2)$ conserving limit, i.e. without FCNCs. In section 4 we include in our study the relevant flavour observables sensitive to the $U(2)$ breaking, with $b \rightarrow s$ transitions being the most important ones, and discuss the current status of the B anomalies [50] in the context of our models. section 5 is dedicated to discussing the prospects of future measurements which can further explore the relevant parameter space. Finally, we conclude in section 6.

2 2-site model

We consider the gauge group $SU(3)_c \times SU(2)_1 \times SU(2)_2 \times U(1)_Y$, where $SU(2)_1 \times SU(2)_2$ is broken at the (multi-)TeV scale to $SU(2)_L$. The gauge couplings corresponding to $SU(2)_1$ and $SU(2)_2$ are g_1 and g_2 respectively. For the breaking $SU(2)_1 \times SU(2)_2 \rightarrow SU(2)_L$ we introduce the link field Φ_{ij} , a doublet under $SU(2)_1$ and $SU(2)_2$, i.e. $\Phi \rightarrow U_1 \Phi U_2^\dagger$, with $U_{1,2}$ being rotations in $SU(2)_{1,2}$ space. This link field could be an elementary scalar, but also a condensate originating from strong dynamics of a composite sector [19]. In any case, if it develops a vacuum expectation value (VEV), $\Phi_{ij} = \Lambda \delta_{ij}$, it generates a gauge boson mass matrix, which in the (W_μ^1, W_μ^2) basis is given by

$$M_W^2 = \frac{\Lambda^2}{4} \begin{pmatrix} g_1^2 & -g_1 g_2 \\ -g_1 g_2 & g_2^2 \end{pmatrix}. \quad (2.1)$$

The massless eigenstate corresponds to the SM gauge boson $W_\mu^{(0)}$, with a universal coupling

$$g_L = \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}}, \quad (2.2)$$

to SM $SU(2)_L$ doublets, independently if they are doublets of $SU(2)_1$ or $SU(2)_2$ prior to the breaking.¹ In addition there is a massive $SU(2)_L$ triplet $W_\mu^{(0)} \sim (\mathbf{1}, \mathbf{3})_0$, with squared mass $M_{W'}^2 = \frac{\Lambda^2}{4}(g_1^2 + g_2^2)$ and couplings

$$g'_1 = -\sqrt{g_1^2 - g_L^2}, \quad g'_2 = \frac{g_L^2}{\sqrt{g_1^2 - g_L^2}}, \quad (2.3)$$

where $g'_{1(2)}$ is the coupling to fields located in the first (second) site prior to the breaking. As a convention, we fix the sign of the coupling of g'_2 to be positive, which fixes g'_1 to be negative. Interestingly, $g'_1 g'_2 = -g_L^2$, making impossible to decouple the massive triplet simultaneously from both sites.

The interactions between the massive vector triplet and the SM fields can be parametrised by the Lagrangian

$$\mathcal{L} \supset -\frac{1}{2} \left[g^q \sum_{i=1,2} \bar{q}_L^i \gamma^\mu \sigma_a q_L^i + g_{33}^q \bar{q}_L^3 \gamma^\mu \sigma_a q_L^3 + \sum_{i=1,2,3} g_{ii}^\ell \bar{\ell}_L^i \gamma^\mu \sigma_a \ell_L^i + g^H H^\dagger \sigma_a i \overleftrightarrow{D}^\mu H \right] W_\mu^{(0)a}, \quad (2.4)$$

where σ_a are the Pauli matrices and we have assumed a $U(2)_q$ flavour symmetry for the light LH quarks to evade the stringent flavour bounds in the light families. The different couplings depend on how we arrange the families among the two sites that we discuss next.

2.1 Quark sector

To realise the desired $U(2)_q$ symmetry for the light-family quarks, which avoids dangerously large effects in kaon and $D^0 - \bar{D}^0$ physics and, at the same time, explains (part of) the hierarchical structures of the quark masses, we localise the first two quark generations on one site (which we choose to be site 1, the “light-quark site”) and the third generation on the other site (site 2 or “top site”). This means that the first two generations are charged under $SU(2)_1$ and the third generation under $SU(2)_2$. This setup endows the model with a $U(2)_q \times U(3)_u \times U(3)_d$ accidental symmetry. If we charge the Higgs doublet under $SU(2)_2$ one can only write third generation Yukawa couplings²

$$-\mathcal{L} \supset y_i^{(t)} \bar{q}_L^3 H^c u_R^i + y_i^{(b)} \bar{q}_L^3 H d_R^i. \quad (2.5)$$

Here we can use the freedom to perform rotations between the right-handed (RH) quarks, i.e. redefining $y_i^{(t)} u_R^i \rightarrow y_t t_R$, and $y_i^{(b)} d_R^i \rightarrow y_b b_R$. We will thus work in the interaction basis where $u_R^3 = t_R$ and $d_R^3 = b_R$. Note that these Yukawa couplings have further broken the $U(3)_{u,d}$ flavour symmetries to $U(2)_{u,d}$. This fixes the quark and Higgs couplings of the vector triplet in eq. (2.4) to be

$$g^q = -\frac{g_L^2}{g'_2}, \quad g_{33}^q = g^H = g'_2. \quad (2.6)$$

¹To get fermions which are doublets of $SU(2)_L$, they must be charged under only one of the two $SU(2)_i$ before the breaking.

²While this model does not address the top-bottom or charm-strange hierarchy (as we discuss below), one could think of UV completions with some approximate (effective) Z_2 symmetry under which d_R^i are odd, which suppresses any Yukawa term in the Lagrangian involving RH down quarks.

Higher dimension operators can generate the complete Yukawa couplings of light families and CKM mixing after the $SU(2)_1 \times SU(2)_2 \rightarrow SU(2)_L$ breaking:

$$-\mathcal{L} \supset \frac{1}{\Lambda'} \sum_{\substack{i=1,2 \\ j=1,2,3}} \left(y_{ij}^{(u)} \bar{q}_L^i \Phi H^c u_R^j + y_{ij}^{(d)} \bar{q}_L^i \Phi H d_R^j \right), \quad (2.7)$$

where Λ' is some NP scale above Λ . After breaking $SU(2)_1 \times SU(2)_2$ to the SM gauge symmetry, these terms generate Yukawa couplings suppressed by the ratio Λ/Λ' . The flavour hierarchies between third and light-family quarks, therefore, emerge due to the existence of separated scales with $\Lambda \ll \Lambda'$. Dynamical explanations of flavour hierarchies between first and second families can be postponed to scales $\sim \Lambda'$ and above. The largest breaking of $U(2)_q$ corresponds to the CKM mixing angle between second and third-generation quarks, i.e. $V_{cb,ts}$. Assuming $y_{23}^{(u)} \sim y_t$ and $y_{23}^{(d)} \sim y_b$, this implies that a first layer of NP beyond our model should appear at $\Lambda' \sim \Lambda/V_{cb}$.

Several UV completions could generate these effective operators:

- (1) **Extra Higgses:** we may add a scalar doublet of $SU(2)_1$, with hypercharge $1/2$, H_1 . We can write in the Lagrangian terms like,

$$-\mathcal{L} \supset m_{H_1}^2 |H_1^2| + \mu H_1^\dagger \Phi H_2 + \sum_{\substack{i=1,2 \\ j=1,2,3}} \left(y_{ij}^{(u)} \bar{q}_L^i H_1^c u_R^j + y_{ij}^{(d)} \bar{q}_L^i H_1 d_R^j \right), \quad (2.8)$$

where now $H_2 \equiv H$ is the Higgs located in the top-site used before. When H_1 is integrated out, the effective Yukawa couplings of eq. (2.7) with $\Lambda' \sim m_{H_1}^2/\mu$ are generated.³

- (2) **Vector-like fermions:** by adding heavy vector-like quarks, $Q_{L,R}$ with the same quantum numbers as q_L^3 and mass m_Q , we can then write the terms

$$-\mathcal{L} \supset \sum_{j=1,2,3} \left(y_j^{(u)} \bar{Q}_L H^c u_R^j + y_j^{(d)} \bar{Q}_L H d_R^j \right) + \sum_{i=1,2} \lambda_i \bar{q}_L^i \Phi Q_R. \quad (2.9)$$

Integrating out $Q_{L,R}$ generates the effective Yukawa couplings of eq. (2.7) with $\Lambda' \sim m_Q$.

Thus, the SM Yukawa couplings can be written as

$$Y_u = y_t \begin{pmatrix} \Delta_u & \epsilon_t V_q \\ 0 & 1 \end{pmatrix}, \quad Y_d = y_b \begin{pmatrix} \Delta_d & \epsilon_b V_q \\ 0 & 1 \end{pmatrix}, \quad (2.10)$$

where we fix $\epsilon_t - \epsilon_b = 1$ and V_q and $\Delta_{u,d}$ are naturally suppressed by Λ/Λ' .⁴ It is convenient to write the rotation matrices which transform the interaction basis to the mass eigenbasis, i.e. which diagonalize the up-quark or down-quark Yukawa matrices:

$$Y_u = L_u^\dagger \hat{Y}_u R_u, \quad Y_d = L_d^\dagger \hat{Y}_d R_d, \quad (2.11)$$

³This UV completion corresponds to a two-Higgs doublet model in the limit of large mass for H_1 and small mixing angle [51], so the SM Higgs is mostly the top-site Higgs H_2 .

⁴For simplicity we assume that V_q is the same in the up and down sector and that $\epsilon_{t,b}$ are real. This, in fact, is the case if $y_{23}^{u,d}$ are generated by one extra vector-like quark.

where $\hat{Y}_{u,d}$ are diagonal matrices. Using the freedom of the accidental $U(2)$ flavour symmetries, we choose the interaction basis to be aligned with the down basis for the light LH quarks and identical to the mass basis for the light right-handed (RH) quarks. Then, $V_q = (V_{td}^*, V_{ts}^*)^T$ and the rotation matrices are

$$L_u = \begin{pmatrix} V_{ud} & V_{us} & \epsilon_t V_{ub} \\ V_{cd} & V_{cs} & \epsilon_t V_{cb} \\ \epsilon_t V_{td} & \epsilon_t V_{ts} & 1 \end{pmatrix} + O(V_{us}^4), \quad (2.12)$$

$$L_d = \begin{pmatrix} 1 & 0 & -\epsilon_b V_{td}^* \\ 0 & 1 & -\epsilon_b V_{ts}^* \\ \epsilon_b V_{td} & \epsilon_b V_{ts} & 1 \end{pmatrix} + O(V_{us}^4), \quad (2.13)$$

$$R_u \approx \begin{pmatrix} 1 & 0 & \epsilon_t \frac{m_u}{m_t} V_{ub} \\ 0 & 1 & \epsilon_t \frac{m_c}{m_t} V_{cb} \\ -\epsilon_t \frac{m_u}{m_t} V_{ub}^* & -\epsilon_t \frac{m_c}{m_t} V_{cb}^* & 1 \end{pmatrix}, \quad (2.14)$$

$$R_d \approx \begin{pmatrix} 1 & 0 & -\epsilon_b \frac{m_d}{m_b} V_{td}^* \\ 0 & 1 & -\epsilon_b \frac{m_s}{m_b} V_{ts}^* \\ \epsilon_b \frac{m_d}{m_b} V_{td} & \epsilon_b \frac{m_s}{m_b} V_{ts} & 1 \end{pmatrix}. \quad (2.15)$$

2.2 Lepton sector

To ensure anomaly freedom with minimal particle content, we have to locate two families of leptons in the first site, and the third one in the second site. This implements an accidental symmetry $U(2)_\ell \times U(3)_e$, which can explain the hierarchy of the charged lepton masses. Because $y_\tau \sim y_b$ and $y_\mu \sim y_s$, the natural way to split leptons is choosing $s_1 = \{\ell_1, \ell_2\}$ and $s_2 = \{\ell_3\}$, where we denote $s_{1(2)}$ the set of lepton fields in site 1 (2) and the indices of ℓ_i are ordered accordingly to the mass of the corresponding charge lepton. The hierarchies between the τ and light lepton Yukawa couplings is explained in the same way as the hierarchy between third-family and light-family quark Yukawas:

$$-\mathcal{L} \supset y_\tau \bar{\ell}_L^3 H \tau_R + \frac{1}{\Lambda'} \sum_{\substack{i=1,2 \\ j=1,2,3}} \left(y_{ij}^{(\ell)} \bar{\ell}_L^i \Phi H e_R^j \right), \quad (2.16)$$

where we have chosen a basis for e_R^i such that $e_R^3 \equiv \tau_R$ is the only RH charged lepton appearing in the first term. The dimension-5 terms of eq. (2.16) can be generated by the same extra Higgs considered before, or by vector-like leptons with the same quantum numbers as ℓ_L^3 . We will call to this arrangement model 0. Other anomaly free assignments of the leptons to the sites are strongly constrained by lepton flavour universality (LFU) tests between electrons and muons and can generate potentially dangerous $\mu \rightarrow e$ flavour violation.

Naive expectations within model 0 however suggest also a hierarchical PMNS matrix, contrary to observations [38, 46]. Indeed, assuming we do not include more degrees of freedom at the TeV scale, the neutrino sector is described by the effective operators

$$-\mathcal{L} \supset \frac{1}{\Lambda_\nu} \left[y_{\nu_3} (\bar{\ell}_L^3 H^c) (H^\dagger \ell_L^{3c}) + \frac{1}{\Lambda'} \sum_{i=1,2} y_{3i}^\nu (\bar{\ell}_L^3 H^c) (H^\dagger \Phi^\dagger \ell_L^{ic}) + \frac{1}{\Lambda'^2} \sum_{i,j=1,2} y_{ij}^\nu (\bar{\ell}_L^j \Phi H^c) (H^\dagger \Phi^\dagger \ell_L^{ic}) \right]. \quad (2.17)$$

Field	SU(3) _c	U(1) _Y	Model 0		Model 1		Model 2	
			SU(2) ₁	SU(2) ₂	SU(2) ₁	SU(2) ₂	SU(2) ₁	SU(2) ₂
$q_L^{1,2}$	3	1/6	2	1	2	1	2	1
q_L^3	3	1/6	1	2	1	2	1	2
$\ell_L^{1,2}$	1	−1/2	2	1	2	1	1	2
ℓ_L^3	1	−1/2	1	2	2	1	1	2
L_L^r	1	−1/2	—	—	1	2	2	1
L_R^r	1	−1/2	—	—	2	1	1	2
H	1	1/2	1	2	1	2	1	2
Φ	1	0	2	$\bar{\mathbf{2}}$	2	$\bar{\mathbf{2}}$	2	$\bar{\mathbf{2}}$

Table 1. Representations of the fermion and scalar fields under the extended gauge group for the three models. The index r runs over the number of will-be vector-like leptons $L_{L,R}$, which is 0, 1 and 2 for models 0, 1 and 2 respectively. Beyond-the-SM fields are shown in grey. We only show the fields charged under SU(2)_{1,2}. Fields not shown have the same charge assignments as in the SM.

Denoting by m_{ν_i} the masses of the neutrinos, and θ_{ij} the mixing angle between i -th and j -th family of neutrinos in the PMNS matrix, this suggests a hierarchy $m_{\nu_2}/m_{\nu_3} \ll \theta_{23} \ll 1$, which is not observed in nature (in particular, $\theta_{ij} \sim O(1)$). Still, it is interesting to consider this model at the TeV because particular UV completions may present mechanisms that undo this hierarchy (see for instance refs. [38, 52]).

However, an orthogonal possibility is to preserve the $U(3)_\ell \times U(3)_e$ and postpone a dynamical explanation capable to address simultaneously the charged lepton and neutrino hierarchies to a higher scale. For instance, it has been suggested that a $U(2)_e$ symmetry helps to explain the charge-lepton mass hierarchies while allowing large PMNS angles [46]. UV completions realising this symmetry while preserving $U(3)_\ell$ would be interesting to explore. Therefore, we will also consider the possibility of locating all leptons on the same site. The resulting gauge anomalies can be cancelled introducing new degrees of freedom, or, in case the breaking to the SM group is triggered by the condensate of a composite sector, via this sector [53]. Here, for concreteness, we introduce heavy leptons L_L and L_R , doublets of one SU(2) _{i} with hypercharge $-1/2$, which are chiral under SU(2)₁ \times SU(2)₂ but vector-like under the SM group after breaking of SU(2)₁ \times SU(2)₂ \rightarrow SU(2) _{L} . There are two possible models:

- Model 1: all SM leptons are in the light-quark site. We have $s_1 = \{\ell_L^i, L_R\}$ and $s_2 = \{L_L\}$. The mass of the vector-like lepton is generated via the term

$$-\mathcal{L} \supset \lambda_L \bar{L}_L \Phi^\dagger L_R. \quad (2.18)$$

All the lepton Yukawa couplings originate from effective operators, which explains their overall suppression compared to the top quark. Notice that $y_\tau \sim y_c$.

- Model 2: all SM leptons are in the top site. We have $s_1 = \{L_L^{1,2}\}$ and $s_2 = \{\ell_L^a, L_R^{1,2}\}$. The mass of the vector-like lepton is generated from the term

$$-\mathcal{L} \supset (\lambda_L)_{rs} \bar{L}_L^r \Phi L_R^s. \quad (2.19)$$

All lepton Yukawas are allowed at the renormalisable level in this case.

In both cases, we need to assume a suppression of mass terms coupling ℓ_i to L_R , for which we choose a Z_2 parity under which the fields $L_{L,R}$ are odd.

The three models are defined by table 1, and the couplings of the simplified Lagrangian of eq. (2.4) are given by eq. (2.6) together with

- Model 0:

$$g_{11}^\ell = g_{22}^\ell = -\frac{g_L^2}{g_2'}, \quad g_{33}^\ell = g_2'. \quad (2.20)$$

- Model 1:

$$g_{11}^\ell = g_{22}^\ell = g_{33}^\ell = -\frac{g_L^2}{g_2'}, \quad (2.21)$$

- Model 2:

$$g_{11}^\ell = g_{22}^\ell = g_{33}^\ell = g_2'. \quad (2.22)$$

Integrating out the vector triplet at tree level generates the Wilson coefficients in the SM effective field theory (SMEFT) written in appendix A.

In models 1 and 2, for large enough values of the Yukawa couplings λ_L of eqs. (2.18) and (2.19), the phenomenology of the triplet vector boson can be studied independently of the vector-like leptons, as we do in the following. We thus leave open the possibility of other UV completions that can fix the gauge anomalies. In any case, for completeness, the phenomenology of the vector-like leptons is briefly discussed in appendix B.

2.3 $Z - Z'$ and $W - W'$ mixing

Once the SM Higgs acquires a VEV, the EW symmetry is broken. The coupling between the massive-vector triplet and the Higgs current in eq. (2.4) generates a mass mixing between the SM gauge bosons and the W' and Z' bosons. The mass eigenstates are thus given by

$$\begin{aligned} W^\pm &= \cos \alpha_{WW'} W^{(0)\pm} + \sin \alpha_{WW'} W'^{(0)\pm}, \\ W'^\pm &= \cos \alpha_{WW'} W'^{(0)\pm} - \sin \alpha_{WW'} W^{(0)\pm}, \end{aligned} \quad (2.23)$$

$$\begin{aligned} Z &= \cos \alpha_{ZZ'} Z^{(0)} + \sin \alpha_{ZZ'} W_3'^{(0)}, \\ Z' &= \cos \alpha_{ZZ'} W_3'^{(0)} - \sin \alpha_{ZZ'} Z^{(0)}, \end{aligned} \quad (2.24)$$

where $Z^{(0)} = c_W W_3^{(0)} + s_W B$ and the mixing angles are

$$\sin \alpha_{WW'} = -\frac{g^H}{g_L} \frac{m_W^2}{M_{W'}^2}, \quad (2.25)$$

$$\sin \alpha_{ZZ'} = \frac{\sin \alpha_{WW'}}{c_W}, \quad (2.26)$$

where $m_W = g_L v/2$. These mixings will lead to corrections of the couplings of the SM Z and W to fermions, affecting the EWPO. When the triplet is integrated out, these corrections are described in the SMEFT by the Wilson coefficients $C_{Hq}^{(3)}$ and $C_{H\ell}^{(3)}$ (see appendix A).

3 Phenomenology: U(2)-preserving observables

We first analyse the phenomenology of the three models without taking into account observables related to the breaking of the U(2) flavour symmetries. In particular, U(2)_q-breaking observables are sensitive to the misalignment between the interaction and the mass basis, parametrised by $\epsilon_{t,b}$ in the quark sector. Excluding them for now, we can analyse the phenomenology of the other observables as functions of only two parameters: g'_2 and $M_{W'}$.

3.1 LHC constraints

CMS and ATLAS searched for sequential Z' and W' bosons, massive vector bosons with the same couplings as the SM ones. We rescale the branching fractions and production cross sections, taking into account the LHC luminosities that we obtain from **ManeParse** [54], to obtain the limits on $M_{W'}$ in our model. The total widths for the branching fractions are calculated taking into account both fermionic and bosonic decays [48].⁵ We use $pp \rightarrow W', Z'$ with $W' \rightarrow \ell\nu$ [55], $Z' \rightarrow \ell^+\ell^-$ [56], $W' \rightarrow \tau\nu$ [57], $Z' \rightarrow b\bar{b}$ [58], and $Z' \rightarrow \tau^+\tau^-$ [59] searches, with integrated luminosities of $139 - 140 \text{ fb}^{-1}$, except the last one which uses 36 fb^{-1} . To correct for this difference in luminosities, we do a naive rescaling of $\sqrt{36/140}$ to the cross-section limits to estimate the limit at 140 fb^{-1} .⁶ We then show the strongest limit we obtain out of these different channels. Limits on the new bosons in these searches are only given for masses up to $4 - 7 \text{ TeV}$. For heavier masses, we use non-resonant di-lepton and mono-lepton searches to further constrain the parameter space. For this, we implemented a χ^2 as a function of the dimension-6 semileptonic Wilson coefficients (see appendix A) using **HighPT** [60]. For large triplet masses, we also include di-jet+photon searches using 79.8 fb^{-1} [61] to constrain four-quark operators of light families, taking the limit of the Wilson coefficient from [62],

$$-(5.3 \text{ TeV})^{-2} < C_{qq}^{(3)} < (4.3 \text{ TeV})^{-2}, \quad (95\% \text{ C.L.}). \quad (3.1)$$

3.2 Electroweak precision observables

For the electroweak fit, we employ the $\{\alpha_{EM}, m_Z, G_F\}$ input scheme and implement the χ^2 function provided in ref. [63] into our analysis. In ref. [63], the χ^2 for the EWPO is given as a function of the parameters $\{\delta g_a\}$, defined in appendix C, which parametrise the modification of the fermion couplings to EW bosons. In appendix C we also provide the relation between these parameters and the SMEFT Wilson coefficients. The fit is dominated by the tree-level effect in appendix A, but we also include the leading logarithms due to the running from the matching scale Λ ($\sim 10 \text{ TeV}$) to the EW scale. Regarding the W mass, the PDG average

⁵Here, for the models 1 and 2, we assume that the vector-like leptons $L_{L,R}$ are heavy enough to forbid the decay of the massive vector triplet into them. Notice that if this were not the case, the branching fractions of the triplet to SM fields would decrease, weakening the bounds from direct searches.

⁶Both signal and background scales linearly in the luminosity L . Since statistical relative fluctuations decrease like $1/\sqrt{L}$, the limits on the NP signal cross section typically scale like \sqrt{L} .

$m_W^{\text{Exp}} = (80377 \pm 12) \text{ MeV}$ [64] and the CDF-II result of $m_W^{\text{Exp}} = (80434 \pm 9) \text{ MeV}$ [65] are not well compatible.⁷ Both results imply an enhancement w.r.t. the SM prediction, $m_W^{\text{SM}} = (80361 \pm 7) \text{ MeV}$ [67]. However, while the first result is compatible within errors, the second one displays a 7σ tension. Instead of combining both values, we choose to perform the EW fit excluding the W mass as an EW observable, and show instead contours with the predicted change in m_W . We have checked that for models 0 and 1, that predict a negative shift in m_W , including the CDF experimental value of m_W can make the EW limit $\sim 2 - 3 \text{ TeV}$ stronger in the region dominated by the EW fit.

3.3 LEP-II $e^+e^- \rightarrow \ell^+\ell^-$ data

Four-leptons operators can be constrained by $e^+e^- \rightarrow \ell^+\ell^-$ data of LEP-II obtained with center-of-mass energies above the Z -pole. Note that the Wilson coefficients of the Higgs-lepton operators $O_{H\ell}$ also enter in these observables, but are much better constrained by the EW fit. We build our own χ^2 function using the Wilson coefficients of appendix A and following the procedure of ref. [68] using the formulas of ref. [69], and the data published in ref. [70]. Similar methods have been used in ref. [71].

3.4 Lepton flavour universality tests

LFU is only violated in model 0 for τ versus light leptons. There, the ratios involving $\tau \rightarrow \ell\nu\nu$ and $\tau \rightarrow K(\pi)\nu$ of eqs. (D.2)–(D.4) of appendix D potentially constrain the parameter space. eqs. (D.2)–(D.4) show the contribution from the Wilson coefficients of the SMEFT evaluated at the EW scale. In our fit, we include the leading log running from the matching to the EW scale, which is small because there is no QCD running, and the potential contribution from y_t to the running of the relevant Wilson coefficients,

$$\mu \frac{d}{d\mu} [C_{H\ell}^{(3)}]_{ii} \propto y_t^2 (g_{33}^q - g^H), \quad (3.2)$$

cancel out due to the arrangement of the third-family-quark doublet and Higgs field on the sites.

3.5 CKM elements

Our vector triplet, with the couplings given in eq. (2.4), potentially yields NP contributions at the tree level to beta, pion and kaon decays, affecting the extraction of the involved CKM elements [72, 73]:

$$V_{ud}^{\text{exp}} \equiv \frac{[C_{vedu}^{V,LL}]_{1111}}{[C_{ve}^{V,LL}]_{1221}} = V_{ud}(1 + \delta V_{ud}), \quad (3.3)$$

$$V_{us}^{\text{exp}} \equiv \frac{[C_{vedu}^{V,LL}]_{1121}}{[C_{ve}^{V,LL}]_{1221}} = V_{us}(1 + \delta V_{us}). \quad (3.4)$$

Here V_{ud} and V_{us} are the actual CKM elements within our model, and $C_{vedu}^{V,LL}$, $C_{ve}^{V,LL}$ are the Wilson coefficients of the LEFT operators

$$O_{vedu}^{V,LL} = (\bar{\nu}_L \gamma_\mu e_L)(\bar{d}_L \gamma^\mu u_L), \quad (3.5)$$

$$O_{ve}^{V,LL} = (\bar{\nu}_L \gamma_\mu \nu_L)(\bar{e}_L \gamma^\mu e_L). \quad (3.6)$$

⁷There is also a new result by ATLAS not included in the PDG average $(80360 \pm 16) \text{ MeV}$ [66].

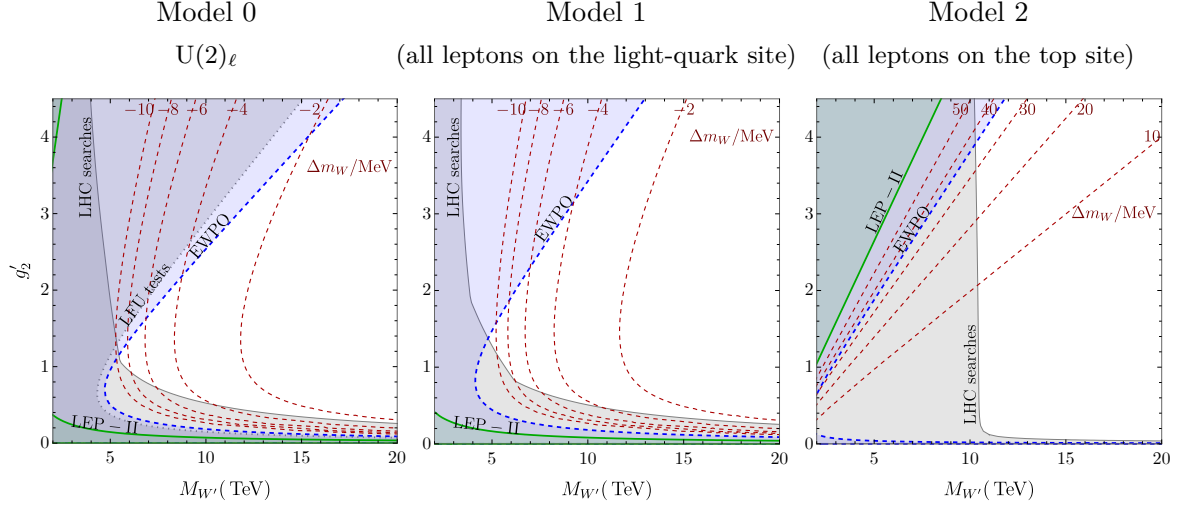


Figure 1. Exclusion limits for models 0, 1 and 2 (from left to right) as discussed in section 3. We include LHC searches (grey solid line), EWPO (blue dashed line) and LEP-II $e^+e^- \rightarrow \ell^+\ell^-$ bounds (green solid line). For model 0 (τ) LFU tests are affected (grey dotted line). Coloured regions are excluded at the 95% C.L. for 2 d.o.f., except for LFU tests that we use 1 d.o.f. because its χ^2 only depends on one particular combination of the parameters (see eqs. (D.2)–(D.4)). The red dashed contour lines corresponding to $\Delta m_W / \text{MeV} = (m_W - m_W^{\text{SM}}) / \text{MeV}$.

Neglecting V_{ts} and V_{td} suppressed contributions, we get

$$\begin{aligned}
 \delta V_{ud} &= \delta V_{us} \\
 &= v^2 \left[[C_{Hq}^{(3)}]_{ii} - [C_{\ell q}^{(3)}]_{11ii} - [C_{Hl}^{(3)}]_{22} + \frac{1}{2} [C_{\ell\ell}]_{1221} \right] \\
 &= \frac{(g^H - g_{11}^\ell)(g_{22}^\ell - g^q)}{g_L^2} \frac{m_W^2}{M_{W'}^2},
 \end{aligned} \tag{3.7}$$

with $i = 1, 2$ and we have neglected the small effects from the running of the Wilson coefficients. We can see that for the three models considered here, the various contributions cancel such that $\delta V_{ud} = \delta V_{us} = 0$. Possible extensions of the model to address experimental deviations of unitarity in the first row of the CKM matrix, known as the Cabibbo Angle Anomaly (CAA), are discussed in appendix E.

3.6 Results

We show the 95% C.L. exclusion regions of the relevant bounds for our three models in figure 1. For small g'_2 , the most relevant bound comes from LHC searches, because a small g'_2 implies large couplings to valence quarks, $g^q = g'_1 = -g_L^2/g'_2$, and thus large production cross sections for the triplet at LHC. The leading constraint comes from $pp \rightarrow \ell\nu$ searches, and, for model 2 and very small g'_2 , di-jet searches. It is remarkable that for model 2 most LHC limits are approximately independent of g'_2 . The reason for this is that $pp \rightarrow W' \rightarrow \ell\nu$ and $pp \rightarrow Z' \rightarrow \ell^+\ell^-$ have cross sections $\propto g^q g_{ii}^\ell = -g_L^2$, so that the dependence on the coupling cancels out.

For large g'_2 ,⁸ the most relevant bound comes from EWPO, although for model 0, also τ -LFU tests are relevant, and for model 2, EWPO only become stronger than LHC searches for very large g'_2 . The key to understanding the EW fit in this region of the parameter space is the NP effect on the very constrained modifications of Z couplings to leptons and b_L , which at tree level read

$$\delta g_{Lii}^{Ze} = \frac{g^H g_{ii}^\ell m_W^2}{2g_L^2 M_{W'}^2} + \frac{\delta G_F}{4(1-2s_W^2)}, \quad (3.8)$$

$$\delta g_{L33}^{Zd} = \frac{g^H g_{33}^q m_W^2}{2g_L^2 M_{W'}^2} + \frac{3-4s_W^2}{12(1-2s_W^2)} \delta G_F, \quad (3.9)$$

$$\delta g_{Rii}^{Ze} = -\delta m_W = \frac{s_W^2 \delta G_F}{2(1-2s_W^2)}, \quad (3.10)$$

where the precise definition of δx is given in appendix C and

$$\delta G_F = \frac{g_{11}^\ell g_{22}^\ell - g^H (g_{11}^\ell + g_{22}^\ell) m_W^2}{g_L^2 M_{W'}^2}. \quad (3.11)$$

For model 0, the EW fit is dominated, in the region with sizeable g'_2 , by the constraints on the $Z \rightarrow \tau_L \tau_L$ and $Z \rightarrow b_L b_L$ corrections. This is why within model 1 the limit from EWPO is weaker: the tension in $Z \rightarrow \tau_L \tau_L$ observed in model 0 is removed while the $Z \rightarrow b_L b_L$ one remains, weakening the limit. Interestingly, contrary to the naive expectation, model 2 further weakens the EW limit. This is because of an accidental approximate cancellation in $Z \rightarrow e_L^i e_L^i$ between the universal contribution from δG_F and the one proportional to g_{ii}^ℓ occurs, due to the Weinberg angle being close to $\pi/6$, $s_W \approx 1/2$. Also, $Z \rightarrow b_L b_L$ becomes smaller. The leading bound originates from $Z \rightarrow e_R^i e_R^i$, which is affected by the universal contribution from δG_F .

Contours of $\Delta m_W = m_W - m_W^{\text{SM}}$ show how in models 0 and 1 the W mass is reduced by at most ~ 10 MeV, originating from the shift in G_F (see eq. (3.10)). It is interesting that for model 2, we get an enhancement that can be of about $\Delta m_W \approx 30$ MeV, going in the direction preferred by the W mass measurements.

4 Phenomenology: U(2)-breaking observables

Our models necessarily violates quark flavour since $U(2)_q$ is broken (to generate light-heavy CKM elements) through the V_q spurion. This breaking generates FCNCs, in particular, $B_s - \bar{B}_s$ mixing and $b \rightarrow s$ transitions play an important role. Concerning the latter, it is interesting to see if the observed anomalies in $b \rightarrow s \ell^+ \ell^-$ data and other B decays can be explained in our setup. We also study possible violations of $U(2)_\ell$ that appear naturally in model 0.

4.1 $\Delta F = 2$ observables

Meson oscillations are one of the most constraining flavour observables. Integrating out our vector triplet at tree-level gives contributions to the four-quark operators in LEFT

$$\mathcal{L}_{\text{LEFT}} \supset -C_{B_s} (\bar{s}_L \gamma_\mu b_L)^2 - C_{B_d} (\bar{d}_L \gamma_\mu b_L)^2 - C_K (\bar{d}_L \gamma_\mu s_L)^2 - C_D (\bar{u}_L \gamma_\mu c_L)^2. \quad (4.1)$$

⁸Large values of g'_2 can lead to fast proton decay through non-perturbative effects in this model [74, 75]. However, as pointed out in ref. [74], these calculations assume small quartic coupling of the link field Φ . Models with large quartic coupling or realising Φ as a condensate of a composite sector may avoid this bound.

The leading NP contribution to these Wilson coefficients is

$$C_{B_s}^{\text{NP}} = V_{tb}^2 V_{ts}^{*2} \frac{(1 - \epsilon_t)^2}{8M_{W'}^2} (g_{33}^q - g^q)^2, \quad (4.2)$$

$$C_{B_d}^{\text{NP}} = V_{tb}^2 V_{td}^{*2} \frac{(1 - \epsilon_t)^2}{8M_{W'}^2} (g_{33}^q - g^q)^2, \quad (4.3)$$

$$C_K^{\text{NP}} = V_{ts}^2 V_{td}^{*2} \frac{(1 - \epsilon_t)^4}{8M_{W'}^2} (g_3^q - g_q)^2, \quad (4.4)$$

$$C_D^{\text{NP}} = V_{ub}^2 V_{cb}^{*2} \frac{\epsilon_t^4}{8M_{W'}^2} (g_3^q - g_q)^2. \quad (4.5)$$

For $B_{d,s} - \bar{B}_{d,s}$ mixing, assuming that the phases originate only from the CKM elements, we obtain the 95% C.L. limits (see appendix F for details)

$$-(27.7 \text{ TeV})^{-2} < \frac{C_{B_s}^{\text{NP}}}{V_{tb}^2 V_{ts}^{*2}} < (8.4 \text{ TeV})^{-2}, \quad (4.6)$$

$$-(13.1 \text{ TeV})^{-2} < \frac{C_{B_d}^{\text{NP}}}{V_{tb}^2 V_{td}^{*2}} < (7.5 \text{ TeV})^{-2}. \quad (4.7)$$

For kaon and $D^0 - \bar{D}^0$ mixing, we take the 95% C.L. limits from refs. [76, 77]:

$$-(2.89 \times 10^4 \text{ TeV})^{-2} < \text{Im} C_K^{\text{NP}} < (2.04 \times 10^4 \text{ TeV})^{-2}, \quad (4.8)$$

$$-(1.03 \times 10^4 \text{ TeV})^{-2} < \text{Im} C_D^{\text{NP}} < (1.06 \times 10^4 \text{ TeV})^{-2}. \quad (4.9)$$

These observables only depend on the couplings of the new states to the quark sector and are thus the same for the three models, being a test of the main idea behind: a dynamical explanation of the quark hierarchies, without relying on any assumption made for the leptons.

4.2 $b \rightarrow s\ell^+\ell^-$ data

Our class of models naturally gives contributions to $b \rightarrow s\ell^+\ell^-$ transitions via the effective operators $O_{9,10}^\ell$,

$$\mathcal{L} \supset \frac{2}{v^2} V_{ts}^* V_{tb} \frac{\alpha_{\text{EM}}}{4\pi} \sum_{a,i} C_a^{\ell_i} O_a^{\ell_i}, \quad (4.10)$$

where $a = 9, 10$, $i = 1, 2, 3$, and

$$O_9^{\ell_i} = (\bar{s}_L \gamma_\mu b_L)(\bar{\ell}_i \gamma^\mu \ell_i), \quad (4.11)$$

$$O_{10}^{\ell_i} = (\bar{s}_L \gamma_\mu b_L)(\bar{\ell}_i \gamma^\mu \gamma^5 \ell_i). \quad (4.12)$$

These Wilson coefficients do not renormalise under QCD, and the QED running (or the one due to other SM couplings) is negligible. The structure of the NP effect is more clearly seen in the basis given by $Z^{(0)}$ and $W_3'^{(0)}$, with flavour diagonal couplings. The tree-level exchange of $W_3'^{(0)}$ gives contributions proportional to g_{ii}^ℓ while the $Z^{(0)} - W_3'^{(0)}$ mixing, with $W_3'^{(0)}$

coupled to the bs vertex, give universal contributions proportional to g^H :

$$C_9^{\ell_i, \text{NP}} = -\frac{\pi(1-\epsilon_t)}{\alpha_{\text{EM}}} \frac{(g_{33}^q - g^q)[g_{ii}^\ell - (1-4s_W^2)g^H]}{g_L^2} \frac{m_W^2}{M_{W'}^2}, \quad (4.13)$$

$$C_{10}^{\ell_i, \text{NP}} = \frac{\pi(1-\epsilon_t)}{\alpha_{\text{EM}}} \frac{(g_{33}^q - g^q)(g_{ii}^\ell - g^H)}{g_L^2} \frac{m_W^2}{M_{W'}^2}. \quad (4.14)$$

It is interesting to note that $C_{10}^{\ell_i, \text{NP}}$ vanishes if ℓ_i is located in the same site as the Higgs because the $W_3'^{(0)}$ contribution cancels the one from the $Z^{(0)} - W_3'^{(0)}$ mixing.⁹

Among the $b \rightarrow s\ell^+\ell^-$ observables, $B_s \rightarrow \mu^+\mu^-$ is very constraining due to its precise SM prediction [78, 79]. It receives contributions from $C_{10}^{\mu, \text{NP}}$:

$$\frac{\mathcal{B}(B_s \rightarrow \mu^+\mu^-)}{\mathcal{B}(B_s \rightarrow \mu^+\mu^-)_{\text{SM}}} = \left| 1 + \frac{C_{10}^{\mu, \text{NP}}}{C_{10}^{\mu, \text{SM}}} \right|^2, \quad (4.15)$$

where $C_{10}^{\mu, \text{SM}} = 4.188$ and we take [80, 81]

$$\mathcal{B}(B_s \rightarrow \mu^+\mu^-)_{\text{Exp}} = (3.52_{-0.30}^{+0.32}) \times 10^{-9}, \quad (4.16)$$

$$\mathcal{B}(B_s \rightarrow \mu^+\mu^-)_{\text{SM}} = (3.67 \pm 0.15) \times 10^{-9}. \quad (4.17)$$

After the latest measurement of $R_{K^{(*)}}$ by LHCb [82, 83], $b \rightarrow s\ell^+\ell^-$ data do not show indications of lepton flavour universality violation between electrons and muons and are thus compatible with $C_{9e}^{\text{NP}} = C_{9\mu}^{\text{NP}} \equiv C_9^{\text{U}}$ and $C_{10e}^{\text{NP}} = C_{10\mu}^{\text{NP}} \equiv C_{10}^{\text{U}}$. A global fit of $b \rightarrow s\ell^+\ell^-$ data within the NP scenario ($C_9^{\text{U}}, C_{10}^{\text{U}}$) yields [84, 85]

$$C_9^{\text{U}} = -1.18_{-0.17}^{+0.18}, \quad C_{10}^{\text{U}} = 0.10_{-0.14}^{+0.13}, \quad (4.18)$$

where C_{10}^{U} is mainly constrained by $B_s \rightarrow \mu^+\mu^-$, which we discuss above, and C_9^{U} shows a sizeable negative shift of $\sim 25\%$ w.r.t. its SM prediction. This effect primarily arises from persistent tensions observed in the angular distribution of $B^{(0,+)} \rightarrow K^{*(0,+)}\mu^+\mu^-$ [84], with particular emphasis in two anomalous bins of the so-called P_5' observable [86]. Additionally, we have recently seen the exacerbation of the deviations in the branching ratio of $B^{(0,+)} \rightarrow K^{(0,+)}\mu^+\mu^-$ channels, as a consequence of the increased precision in theoretical predictions due to newly available lattice determinations for the relevant form factors across the entire q^2 region [84, 87]. Other collaborations involved in $b \rightarrow s\ell^+\ell^-$ global fits obtain different results depending on the experimental data input considered and theoretical assumptions regarding the non-perturbative effects that enter into the calculations [88–90]. However, all analyses that employ available theoretical calculations, based on light-cone sum rules (LCSRs) [91, 92], for the determination of the relevant non-perturbative form factors, coincide in finding a $C_9^{\text{U}} \sim -1$ and $C_{10}^{\text{U}} \sim 0$.

⁹We have checked that the cancellation of $C_{10}^{\ell_i, \text{NP}}$ is a general property of Z' models when the Z' is associated to a $U(1)'$ that charges the chiralities of the leptons and the down component of the Higgs such that the ℓ_i -Yukawa term can be written at the renormalisable level (see appendix G). In our case, $U(1)' \subset \text{SU}(2)_1 \times \text{SU}(2)_2$ and it is generated by the combination of the generators $(T_3)_1$ and $(T_3)_2$ associated to $W_3'^{(0)}$.

4.3 $B \rightarrow K^{(*)}\nu\nu$ and $K \rightarrow \pi\nu\nu$

For the $\Delta F = 1$ processes $B \rightarrow K^{(*)}\nu\nu$, $K_L \rightarrow \pi^0\nu\nu$ and $K^+ \rightarrow \pi^+\nu\nu$ the relevant effective Lagrangian is

$$\begin{aligned} \mathcal{L}_{\text{LEFT}} \supset & -\frac{2}{v^2} V_{ts}^* V_{tb} \frac{\alpha_W}{2\pi} \sum_i C_{sb,i} (\bar{s}_L \gamma_\mu b_L) (\bar{\nu}_L^i \gamma^\mu \nu_L^i) \\ & -\frac{2}{v^2} V_{td}^* V_{ts} \frac{\alpha_W}{2\pi} \sum_i C_{ds,i} (\bar{d}_L \gamma_\mu s_L) (\bar{\nu}_L^i \gamma^\mu \nu_L^i), \end{aligned} \quad (4.19)$$

with the NP contributions

$$C_{sb,i}^{\text{NP}} = \pi^2 (1 - \epsilon_t) \frac{(g_{33}^q - g^q)(g^H - g_{ii}^\ell)}{g_L^2} \frac{v^2}{M_{W'}^2}, \quad (4.20)$$

$$C_{ds,i}^{\text{NP}} = (1 - \epsilon_t) C_{sb,i}^{\text{NP}}, \quad (4.21)$$

where the running is negligible. Notice that in our models, $\tilde{C}_{sb,i}^{\text{NP}} = -2\tilde{C}_{10}^{\ell_i, \text{NP}}$, where \tilde{C} are the Wilson coefficients including their respective normalization factors of eqs. (4.10) and (4.19). Likewise, the effect is vanishing if ℓ_i is located in the same site as the Higgs. Thus, only models with leptons in the first site (models 0 and 1) will give a contribution. The resulting branching fractions are [93, 94]:

$$\frac{\mathcal{B}(B \rightarrow K^{(*)}\nu\bar{\nu})}{\mathcal{B}(B \rightarrow K^{(*)}\nu\bar{\nu})_{\text{SM}}} = \frac{1}{3} \sum_{i=1}^3 \left| 1 + \frac{C_{sb,i}^{\text{NP}}}{X_t} \right|^2, \quad (4.22)$$

$$\frac{\mathcal{B}(K_L \rightarrow \pi^0\nu\bar{\nu})}{\mathcal{B}(K_L \rightarrow \pi^0\nu\bar{\nu})_{\text{SM}}} = \frac{1}{3} \sum_{i=1}^3 \left| 1 + \frac{\text{Im}(V_{td}^* V_{ts} C_{ds,i}^{\text{NP}})}{\text{Im}(V_{td}^* V_{ts}) X_t} \right|^2, \quad (4.23)$$

$$\frac{\mathcal{B}(K^+ \rightarrow \pi^+\nu\bar{\nu})}{\mathcal{B}(K^+ \rightarrow \pi^+\nu\bar{\nu})_{\text{SM}}} = \frac{1}{3} \sum_{i=1}^3 \left| 1 + \frac{C_{ds,i}^{\text{NP}}}{X_t + \frac{V_{us}^4 \text{Re}(V_{cd}^* V_{cs})}{V_{td}^* V_{ts}} P_c^{(i)}} \right|^2, \quad (4.24)$$

where $X_t = 1.48$, $P_c^{(1)} = P_c^{(2)} = 0.45$ and $P_c^{(3)} = 0.31$.

We will see that model 0, and to a larger extent model 1, can give an enhancement of up to $\sim 10\%$ w.r.t. the SM prediction, which is however still far from the current experimental limit: among the three, $\mathcal{B}(K^+ \rightarrow \pi^+\nu\bar{\nu})$ is measured most precisely [95] but still with an error of $\sim 35\%$, to be increased to 15% by 2025 [96]. Potential future experiments [96] can measure $K^+ \rightarrow \pi^+\nu\bar{\nu}$ (HIKE) [97] with a 5% error of its SM value, and $K_L \rightarrow \pi^0\nu\bar{\nu}$ (KOTO-II) [98] with 20%. Similarly, Belle II aims to measure $B \rightarrow K^{(*)}\nu\bar{\nu}$ within 10% [99] of its SM value. Note that the current excess observed in $B \rightarrow K\nu\nu$ by Belle II [100] (2.2σ), which combined with other measurements suggests an enhancement of about 180% with respect to the SM, is not explainable within our model.¹⁰

4.4 $R_{D^{(*)}}$

For completeness, we also discuss

$$R_{D^{(*)}} = \frac{\mathcal{B}(B \rightarrow D^{(*)}\tau\nu)}{\mathcal{B}(B \rightarrow D^{(*)}\ell\nu)}, \quad (4.25)$$

¹⁰The differential distributions measured by Belle II are better compatible with the assumption of light NP [101].

with ℓ representing light leptons, within our model. The current combined measurements suggests an excess of 10 – 20% w.r.t the SM prediction ($\sim 3\sigma$) [102]. Since these observables are a measure of LFU violation, only model 0 can give a contribution. The relevant effective Lagrangian is

$$\mathcal{L}_{\text{LEFT}} \supset -\frac{2V_{cb}}{v^2} \sum_{i=1}^3 C_{\ell_i} (\bar{\ell}_L^i \gamma_\mu \nu_L^i) (\bar{c}_L \gamma^\mu b_L), \quad (4.26)$$

resulting in

$$\begin{aligned} \frac{R_{D^{(*)}}}{R_{D^{(*)}}^{\text{SM}}} &= 1 + 2(C_\tau^{\text{NP}} - C_{e,\mu}^{\text{NP}}) \\ &= 1 + 2 \frac{(g_{33}^\ell - g_{ii}^\ell)(g_{33}^q \epsilon_t + g^q(1 - \epsilon_t) - g^H)}{g_L^2} \frac{m_W^2}{M_{W'}^2}, \end{aligned} \quad (4.27)$$

with $i = 1, 2$, and the running is negligible. We observe that the effect in model 0, in the region not excluded by other observables, is below 0.1%.

4.5 LFV processes

Model 0 breaks the $U(3)_\ell$ symmetry of the gauge sector of the SM to $U(2)_\ell \times U(1)_\tau$. Then, it is natural to check contributions to lepton flavour violating (LFV) decays triggered by a possible breaking of $U(2)_\ell$. From eq. (2.16), we see that, after $SU(2)_1 \times SU(2)_2 \rightarrow SU(2)_L$ breaking, model 0 naturally has the Yukawa couplings

$$\mathcal{L} \supset \frac{\Lambda}{\Lambda'} y_{i3}^{(\ell)} \bar{\ell}_L^i H \tau_R, \quad (4.28)$$

with $i = 1, 2$. Assuming that $y_{i3}^{(\ell)} \sim y_{ii}^{(\ell)}$, they generate mixing angles between the interaction and the diagonal Yukawa basis, $\theta_{\tau\ell_i} \sim m_{\ell_i}/m_\tau$, which induce LFV couplings of the Z' :

$$g_{i3}^\ell \sim \frac{m_{\ell_i}}{m_\tau} (g_{33}^\ell - g_{ii}^\ell), \quad g_{12}^\ell \sim \frac{m_\mu m_e}{m_\tau^2} (g_{33}^\ell - g_{ii}^\ell). \quad (4.29)$$

When the triplet is integrated out, these couplings generate the Wilson coefficients of the LEFT operators in eqs. (H.10)–(H.14), giving rise to LFV processes with stringent experimental bounds (see appendix H). Assuming that eq. (4.29) is exact, these bounds are dominated by $\tau^- \rightarrow \mu^- \ell^+ \ell^-$ in eqs. (H.3) and (H.4) and $\mu \rightarrow e$ conversion in eq. (H.8). We use these observables to build a χ^2 -function.

Other LFV decays $\ell_k \rightarrow \ell_j \gamma$ receive contributions from the UV matching and from $[C_{H\ell}^{(3)}]_{jk}$ in the SMEFT-LEFT matching, both at one loop level. The first contribution depends on the UV completion realising eq. (2.16), which we do not specify here. However, it can be estimated from appendix A.1 of ref. [73]. We have checked that the bounds from these processes are significantly weaker than the ones discussed above.

4.6 Results

All FCNC observables in the quark sector depend crucially on ϵ_t . In the down-alignment limit $\epsilon_t = 1$, the effects in the down sector disappear and only $D^0 - \bar{D}^0$ mixing survives. However,

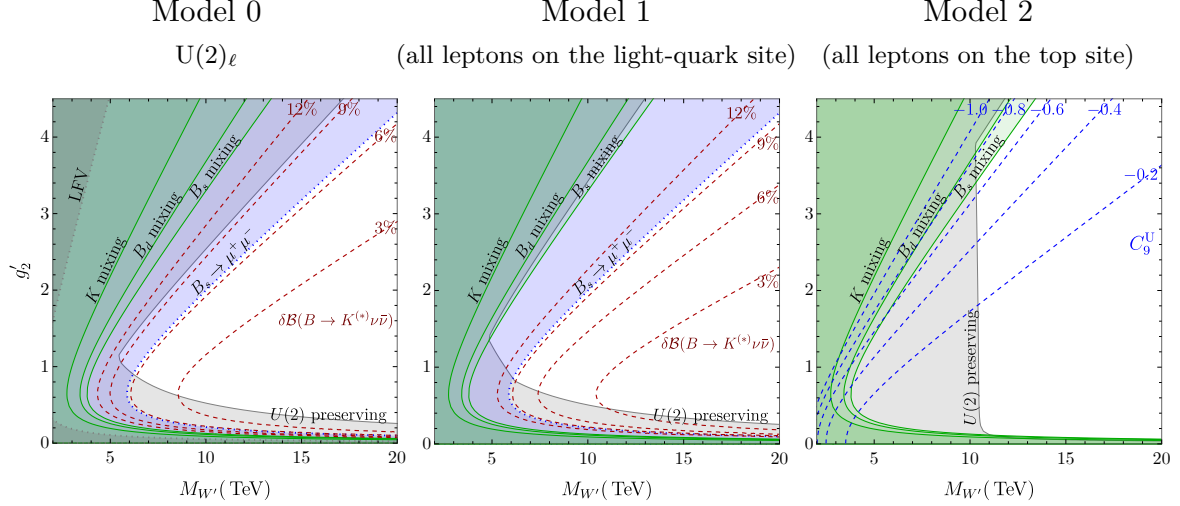


Figure 2. Exclusion limits for models 0, 1 and 2 (from left to right) for $\epsilon_t = 0$, discussed in section 4. We include the strongest U(2)-preserving limit from previous section (grey solid line), $\Delta F = 2$ processes (green solid lines), $B_s \rightarrow \mu^+\mu^-$ (blue dotted line), and LFV processes for model 0 assuming eq. (4.29) is exact (grey dotted line). Coloured regions are excluded at the 95% C.L. for 2 d.o.f. except for mesons mixing and $B_s \rightarrow \mu\mu$ that we take 1 d.o.f. because their χ^2 only depends on a particular combination of the parameters, C_X^{NP} and $C_{10}^{\mu,\text{NP}}$ respectively. For model 0 and 1, red dashed contour lines depict $\delta\mathcal{B}(B \rightarrow K^{(*)}\nu\bar{\nu})$. The same contour lines give $\delta\mathcal{B}(K_L \rightarrow \pi^0\nu\bar{\nu})$ for the same values (3%, 6%, 9% and 12%) and $\delta\mathcal{B}(K^+ \rightarrow \pi^+\nu\bar{\nu})$ for an enhancement of 2%, 4%, 6% and 8% respectively. For model 2, the blue dashed lines show contours of constant value of C_9^{U} .

the resulting limit is substantially weaker than the ones from U(2)-preserving observables, in particular EWPO. In the following, we will therefore study the limit of alignment in the up-quark sector, $\epsilon_t = 0$, resulting in FCNC in the down sector. In figure 2 we show the 95% C.L. exclusion regions of $B_s \rightarrow \mu^+\mu^-$ and meson mixing for the three models, together with the strongest bounds from the U(2)-preserving observables discussed in the previous section and the limits from LFV decays for model 0 assuming eq. (4.29) is exact. We also show the contour lines for C_9^{U} in model 2, and for $\delta\mathcal{B}(B \rightarrow K^{(*)}\nu\bar{\nu})$ in models 0 and 1, where

$$\delta\mathcal{B}(A) = \mathcal{B}(A)/\mathcal{B}(A)_{\text{SM}} - 1. \quad (4.30)$$

The same contour lines also predict the effect in $K_L \rightarrow \pi^0\nu\bar{\nu}$ and $K^+ \rightarrow \pi^+\nu\bar{\nu}$ (see caption of figure 2). Note that $B \rightarrow K^{(*)}\nu\bar{\nu}$, and $K \rightarrow \pi\nu\bar{\nu}$ is exactly 0 for model 2 (see eqs. (4.20) and (4.21)). Also, the value of C_9^{U} for models 0 and 1 is strictly positive and at most ~ 0.1 in the non-excluded region (see eq. (4.13)).

We see that among $\Delta F = 2$ processes, $B_s - \bar{B}_s$ mixing is the most constraining one, but that all three of them give similar bounds. This is a general property of models with Minimal Flavour Violation (MFV) [103, 104] or minimally-broken U(2) [8, 18, 35, 105] as ours. Interestingly, $\Delta F = 2$ processes give same order-of-magnitude bounds as the other limits, that, unlike meson mixing, depend on the assumptions made for the leptons in each model.

For the three models the strongest limit for small g'_2 is again from the U(2) preserving observables (in particular LHC searches) and $\Delta F = 2$ processes for very small g'_2 for model

2. For models 0 and 1, the strongest limit for large g'_2 is $B_s \rightarrow \mu^+ \mu^-$. However, in model 2, $B_s \rightarrow \mu^+ \mu^-$ remains at its SM value due to the cancellation in the NP contribution to C_{10}^μ observed in eq. (4.14). This opens the possibility of a large g'_2 coupling for a triplet mass of 10 TeV–12 TeV allowing for a sizeable shift in C_9^U of about -0.6 , in the line with the $b \rightarrow s \ell^+ \ell^-$ fit. Still, the amount of C_9^U is limited by $B_s - \bar{B}_s$ -mixing and the U(2) preserving bounds from EWPO, LHC and LEP-II data. This is similar to what is observed in models with an LFU Z' with bs couplings and vector couplings to leptons [88], which are limited by B_s -mixing and LEP-II data. It is interesting to note that in the region where the contribution to C_9^U is large we also get an enhancement of the W mass of ~ 30 MeV (see figure 1).

5 Future prospects

It is interesting to discuss how future measurements and experiments can explore the parameter space of our three models. For this, we estimate different future limits, assuming that no deviations from the SM prediction are observed. We will comment on the expected sensitivity of different experiments relevant to our models and build the corresponding χ^2 functions to set the projections of the limits in the parameters of the three models in section 5.4.

5.1 LFV: Mu3e, Belle II, COMET and Mu2e

In the coming years, we expect a significant improvement in limits on LFV processes, affecting the bounds for model 0. Phase-2 of the Mu3e experiment [106] aims to reach a sensitivity of 10^{-16} at 90% C.L. in $\mathcal{B}(\mu^+ \rightarrow e^+ e^- e^+)$ in 2028. Belle II will improve the bounds on τ LFV decays. For an integrated luminosity of 50 ab^{-1} , the projected limits are [99]

$$\mathcal{B}(\tau^- \rightarrow \mu^- \mu^+ \mu^-) < 4.6 \times 10^{-10}, \quad (5.1)$$

$$\mathcal{B}(\tau^- \rightarrow \mu^- e^+ e^-) < 3.0 \times 10^{-10}, \quad (5.2)$$

at 90% C.L.¹¹ The COMET [108] and Mu2e [109] experiments will test $\mu \rightarrow e$ conversion in aluminium with an expected sensitivity of $10^{-16} - 10^{-17}$. Conservatively, we take $\mathcal{B}(\mu Al^- \rightarrow e Al) < 10^{-16}$ at 90% C.L.

5.2 High-luminosity phase of LHC

We have seen in figure 1 that non-resonant searches currently result in the strongest bounds in case of small values of g'_2 . To estimate the limits from the high-luminosity phase of LHC (HL-LHC), we use HighPT [60] to extract the projection for the χ^2 for an integrated luminosity of 3 ab^{-1} for $pp \rightarrow Z' \rightarrow \ell^+ \ell^-$ and $pp \rightarrow W' \rightarrow \ell \nu$. We also include limits on the Wilson coefficient $C_{qq}^{(3)}$ related to the light families from di-jets+photon searches [62]. Assuming that the contribution of this operator to the cross section is dominated by its interference with the SM (see table 1 of ref. [62]), we rescale the limits of eq. (3.1) by $\sqrt{79.8/3000}$, as suggested in Footnote 6, to estimate the HL-LHC bound.

LHCb and CMS will also improve the measurement of $\mathcal{B}(B_s \rightarrow \mu\mu)$ [110]. In particular, the expected experimental error of LHCb is $\Delta\mathcal{B}(B_s \rightarrow \mu\mu) = 0.16 \times 10^{-9}$ for an integrated luminosity of 300 fb^{-1} [107]. The projected limit is then calculated assuming the theory error in eq. (4.17) for the SM prediction.

¹¹LHCb will also be able to probe branching ratios of $\mathcal{O}(10^{-9})$ of $\tau \rightarrow 3\mu$ [107].

5.3 Future circular collider

The Future Circular e^+e^- Collider (FCC-ee) [111, 112] and CEPC [113] have the potential to improve the limits on the scale of NP coupled to the Higgs boson to several tens of TeV [71]. We focus here on the prospects for FCC-ee.

We build a χ^2 with the EW observables of table 3 of ref. [114] (Z -pole observables and W boson mass and width) using the $\{\alpha_{EM}, m_Z, G_F\}$ input scheme. We add in quadrature the statistical and systematic errors and neglect possible correlations. Table 30 of the same reference also includes the estimated theory uncertainties for most of them. When provided, we also include them, adding in quadrature intrinsic and parametric errors. Theory uncertainties for the observables A_ℓ are extracted from $\Delta \sin^2 \theta_W$. We also neglect theory correlations except in the cases of A_ℓ and R_ℓ , where only one theory error is provided for the three observables associated with the different lepton families, so we assume a full correlation between them. To calculate the contributions of the NP Wilson coefficients to the EW observables, we use the expressions given in appendix C of ref. [30].

Table 27 of ref. [114] also shows the projected measurements of cross sections and forward-backward asymmetries of $e^-e^+ \rightarrow \ell^-\ell^+$ at two center-of-mass energies, $s = (240 \text{ GeV})^2$ and $s = (365 \text{ GeV})^2$. Similarly to section 3.3, we use them to build a χ^2 constraining 4-lepton operators.

Finally, FCC-ee would work as a τ factory and significantly improve the accuracy of τ decays [115], relevant for model 0. FCC-ee can reach a sensitivity of 10^{-10} for the LFV $\tau \rightarrow 3\mu$ decay. However, regarding τ -physics, the limits for model 0 are dominated by $\tau \rightarrow \ell\nu\bar{\nu}$, with $\ell = e, \mu$, affecting the LFU tests (see appendix D):

$$\Delta R \left[\frac{\tau \rightarrow \ell\nu\bar{\nu}}{\mu \rightarrow e\nu\bar{\nu}} \right] = \frac{\Delta \mathcal{B}(\tau \rightarrow \ell\nu\bar{\nu})}{2\mathcal{B}(\tau \rightarrow \ell\nu\bar{\nu})}, \quad (5.3)$$

where the expected error is $\Delta \mathcal{B}(\tau \rightarrow \ell\nu\bar{\nu}) = 3 \times 10^{-5}$ and we have neglected the error from $\mathcal{B}(\mu \rightarrow e\nu\bar{\nu})$.

5.4 Results

In figure 3 we show the 95% C.L. exclusion limits of the discussed projections for the three models. Despite the expected improvement in sensitivity on LFV decays from Mu3e and Belle II, we see that for model 0, these bounds will still be weaker than the current U(2)-preserving limits of figure 1. This is due to the strong suppression of the lepton mixing angles of eq. (4.29).¹² However, $\mu \rightarrow e$ conversion will be able to explore the parameter space beyond the current experimental limits. We can also see that, for small g'_2 , non-resonant searches at HL-LHC can improve the limit on the triplet mass by roughly a factor ~ 2 for the three models.

FCC-ee will significantly enhance the limits on the triplet mass for moderate and large values of g'_2 . This is especially the case for models 0 and 2, where also τ LFU tests and $e^+e^- \rightarrow \ell^+\ell^-$ observables respectively impose similar or stronger constraints than those coming from the Z -pole measurements.

6 Conclusions

A puzzling feature of the quark mass spectrum and the CKM elements is their hierarchy. While the top Yukawa coupling is of order one, all other Yukawa couplings are much smaller.

¹²Our results for Mu3e are substantially weaker than the ones obtained in ref. [49]. The reason is that they assume a CKM-like lepton mixing matrix which gives less suppressed mixing angles for electrons.

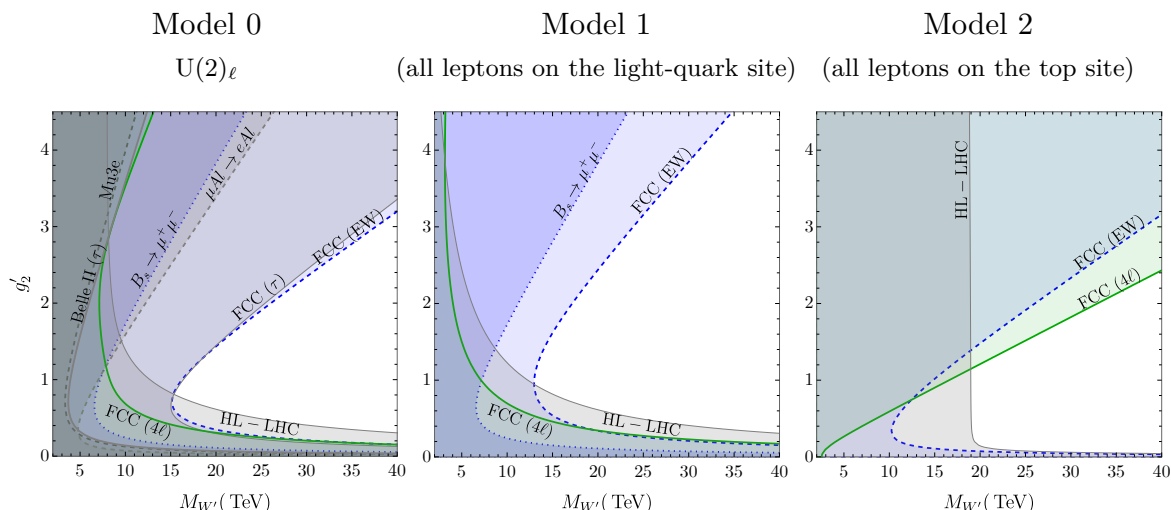


Figure 3. Projections of the exclusion limits from different future experiments for models 0, 1 and 2 (from left to right). We include the LFV limits assuming that eq. (4.29) is exact in $\mu^+ \rightarrow e^+e^-e^+$ from Mu3e, $\mu \rightarrow e$ conversion from COMET and Mu2e (gray dashed lines) and τ decays from Belle II (grey solid line), non-resonant searches in HL-LHC (grey solid line), $B_s \rightarrow \mu^+\mu^-$ limit from LHCb assuming $\epsilon_t = 0$, and EW observables (blue dashed line), $e^+e^- \rightarrow \ell^+\ell^-$ bounds (green solid line) and τ LFU tests (grey solid line) from FCC-ee. Coloured regions are excluded at the 95% C.L. for 2 d.o.f., except for Mu3e, Belle II (τ), $\mu Al \rightarrow e Al$, $B_s \rightarrow \mu^+\mu^-$ and τ LFU tests that we use 1 d.o.f. because their χ^2 functions only depend on one particular combination of the parameters.

An interesting possibility for understanding this pattern is that only the third-family quark Yukawas are allowed at the renormalizable level while all other couplings are suppressed by higher mass scales, realising a multi-scale explanation of the flavour hierarchies. We implement this idea in a two-site deconstructed model for the $SU(2)_L$ gauge factor, i.e. $SU(2)_1 \times SU(2)_2$ is broken to $SU(2)_L$. The first two generations of quarks are charged under one $SU(2)$ factor while the third family transforms as a doublet under the second factor, together with the Higgs doublet. This only allows third-family quark Yukawas at the tree-level while higher dimensional operators can generate the remaining quark Yukawas in a suppressed way. We discuss different possible UV completions to generate these effective operators, but stay agnostic about the specific realisation. In any case, the model possesses an accidental approximate $U(2)_q$ flavour symmetry which protects FCNC processes from dangerously large effects such that a TeV scale realisation is viable.

The lepton sector differs from the quark sector, in particular, the PMNS matrix is qualitatively different from the CKM matrix. A straightforward extension of the quark structure to the lepton sector predicts LFU violation and would require some particular UV completion to account for the anarchic mixing angles of the PMNS matrix. Another option considered in this paper is to assume that all leptons are charged under the same gauge factor, i.e. localised on the same site and realising a $U(3)_\ell$ symmetry. A dynamical explanation of the lepton hierarchies is then postponed to higher energies. We have then explored the phenomenology of three models with different distributions of the lepton doublets on the sites.

In the phenomenological analysis, we study the complementary bounds for EWPO, LHC searches and FCNC processes. Interestingly, all those sectors give in general comparable

limits on the couplings and masses of the heavy gauge bosons that appear in the models, W' and Z' , which are in the range of $(5 - 20)$ TeV. We observed several total or partial non-trivial cancellations in NP contributions that weaken or remove some limits, like the limits coming from the determination V_{ud} and V_{us} , or the EWPO for model 2. A particularly remarkable cancellation happens in $C_{10}^{\ell_i, \text{NP}}$, if ℓ_i is located on the same site as the Higgs. This allows a sizeable contribution to C_9^{U} as preferred by the current $b \rightarrow s\ell\ell$ fit, while avoiding the bound from $B_s \rightarrow \mu^+\mu^-$. This reveals that, regarding Z' bosons, not only those with vector couplings to leptons are the natural candidates to address the deviations observed in C_9^{U} [69, 88], but more generally any Z' where similar cancellations occur.¹³

Possible UV completions of our models to address the hierarchies between first and second families at higher scales could be further deconstructions of the light-family gauge factor [49], which in turn, could be UV-completed to models with gauge-flavour unification [116, 117], or realisations of another kind of horizontal gauge symmetries charging the light families and broken in a far UV.

Acknowledgments

The authors would like to thank Marco Ardu, Gino Isidori and Ben Stefanek for useful discussions. The authors also thank Lukas Allwicher for providing codes to implement the LEP-II $e^+e^- \rightarrow \ell^+\ell^-$ data likelihood. The work of B.C. is supported by the Margarita Salas postdoctoral program funded by the European Union-NextGenerationEU. The work of A.C. is supported by a Professorship Grant (PP00P2_176884) of the Swiss National Science Foundation. The work of J.M.L. has been supported by the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation program under grant agreement 833280 (FLAY) in the initial stages of the project, and by the grant CSIC-20223AT023 in the final ones. J.M.L. also acknowledges the support of the Spanish Agencia Estatal de Investigacion through the grant "IFT Centro de Excelencia Severo Ochoa CEX2020-001007-S". The work of S.P. is supported by the National Science Centre, Poland, grant DEC-2019/35/B/ST2/02008.

A Matching to SMEFT

We integrate out the heavy triplet at the tree level to match to dimension 6 SMEFT,

$$\mathcal{L} \supset \sum_n C_n O_n, \quad (\text{A.1})$$

and work in the Warsaw basis [118]. We have crosschecked our results with `Matchete` [119]. Neglecting all Yukawa couplings except the top one, and working in the interaction basis, we get at the matching scale the following non-vanishing Wilson coefficients:

$$[C_{\ell q}^{(3)}]_{iikk} = -\frac{g_{ii}^\ell g_{kk}^q}{4M_{W'(0)}^2}, \quad (\text{A.2})$$

$$[C_{\ell\ell}]_{iiii} = -\frac{(g_{ii}^\ell)^2}{8M_{W'(0)}^2}, \quad (\text{A.3})$$

¹³We have checked this is the case for Z' bosons with bs couplings and associated to a $U(1)'$ that allows for the lepton Yukawas at the renormalisable level (see appendix G).

$$[C_{\ell\ell}]_{iijj} = \frac{g_{ii}^\ell g_{jj}^\ell}{4M_{W'(0)}^2}, \quad (\text{A.4})$$

$$[C_{\ell\ell}]_{ijji} = -\frac{g_{ii}^\ell g_{jj}^\ell}{2M_{W'(0)}^2}, \quad (\text{A.5})$$

$$[C_{qq}^{(3)}]_{iiii} = -\frac{(g_{ii}^q)^2}{8M_{W'(0)}^2}, \quad (\text{A.6})$$

$$[C_{qq}^{(3)}]_{iijj} = -\frac{g_{ii}^q g_{jj}^q}{4M_{W'(0)}^2}, \quad (\text{A.7})$$

$$[C_{Hq}^{(3)}]_{ii} = -\frac{g_{ii}^q g^H}{4M_{W'(0)}^2}, \quad (\text{A.8})$$

$$[C_{H\ell}^{(3)}]_{ii} = -\frac{g_{ii}^\ell g^H}{4M_{W'(0)}^2}, \quad (\text{A.9})$$

$$C_{H\Box} = -\frac{3(g^H)^2}{8M_{W'(0)}^2}, \quad (\text{A.10})$$

$$[C_{uH}]_{33} = -\frac{y_t (g^H)^2}{4M_{W'(0)}^2}, \quad (\text{A.11})$$

$$C_H = -\frac{\lambda (g^H)^2}{2M_{W'(0)}^2}, \quad (\text{A.12})$$

where $i, k = 1, 2, 3$; $j = 2, 3$; $i < j$, repeated indices are not summed, and $g_{11}^q = g_{22}^q \equiv g^q$, g_{33}^q and g_{ii}^ℓ are the couplings of the fermions to the extra triplet depending on the model (see eqs. (2.6) and (2.20)–(2.22)).

B Vector-like-lepton phenomenology

Models 1 and 2 with only the SM fermion fields require of some UV completion to fix their gauge anomalies. One possibility is the inclusion of the vector-like leptons described in table 1. Up to dimension 5 interactions, for model 1 we can write

$$-\mathcal{L} \supset \lambda^L \bar{L}_L \Phi^\dagger L_R + m_i \bar{\ell}_L^i L_R + y_i^L \bar{L}_L H e_R^i + \text{h.c.}, \quad (\text{B.1})$$

and for model 2:

$$-\mathcal{L} \supset M_r \delta_{rs} \bar{L}_L^r L_R^s + m_{ir} \bar{\ell}_L^i L_R^r + y_{ri}^L \bar{L}_L^r H e_R^i + \text{h.c.}, \quad (\text{B.2})$$

where we assume we work in the basis that diagonalizes $\lambda_{rs}^L = \lambda_r \delta_{rs}$. It is necessary to assume a Z_2 parity charging $L_{L,R}$ that suppresses all terms except λ_L . After the $\text{SU}(2)_1 \times \text{SU}(2)_2 \rightarrow \text{SU}(2)_L$ breaking, we obtain for both models

$$-\mathcal{L} \supset M_r \delta_{rs} \bar{L}_L^r L_R^s + m_{ir} \bar{\ell}_L^i L_R^r + y_{ri}^L \bar{L}_L^r H e_R^i + \text{h.c.}, \quad (\text{B.3})$$

where r, s run over the number of vector-like leptons depending on the model, $M_r = \Lambda \lambda_r$, and $y_{ri}^L = y'_{ri} \Lambda / \Lambda'$ for model 2. Although suppressed by the Z_2 parity, the parameters m_{ir} and y_{ri}^L can trigger the decay of the vector-like leptons to SM fields through a small mixing with SM leptons.

Direct searches at LHC can put limits on the vector-like-lepton mass through double production. Assuming prompt decay through mixing with the tau, these limits are at the level of the TeV [120], still far from their expected mass $\sim M_{W'}$.

Their low-energy impact could be more dangerous, even for small values of m_{ir} and y_{ri}^L , mainly because of LFV processes (see appendix H). Here we derive the bounds these couplings need to satisfy to be phenomenologically acceptable.

The mass terms m_{ir} induce non-diagonal couplings of the Z' boson,

$$g_{ij}^\ell = \sum_r \frac{m_{ri} m_{rj}^*}{M_r^2} \left(g_2' + \frac{g_L^2}{g_2'} \right). \quad (\text{B.4})$$

They contribute to the LEFT Wilson coefficients given in eqs. (H.10)–(H.14) generating LFV processes. In our models, the most constraining observables are τ LFV decays, $\tau \rightarrow \mu \ell \ell$ and $\tau \rightarrow e \ell \ell$ (see eqs. (H.3)–(H.6)), and $\mu \rightarrow e$ conversion (see eq. (H.8)). Varying the value of value of $g_2' \in (g_L^2/3, 3)$, we obtain for models 1 and 2,

$$\frac{1}{M_{W'}^2} \left| \sum_r \frac{m_{r1}^* m_{r2}}{M_r^2} \right| < (0.2 - 2) \times 10^{-5} \text{ TeV}^{-2}, \quad (\text{B.5})$$

$$\frac{1}{M_{W'}^2} \left| \sum_r \frac{m_{r2}^* m_{r3}}{M_r^2} \right| < (0.004 - 0.1) \text{ TeV}^{-2}, \quad (\text{B.6})$$

$$\frac{1}{M_{W'}^2} \left| \sum_r \frac{m_{r1}^* m_{r3}}{M_r^2} \right| < (0.004 - 0.1) \text{ TeV}^{-2}. \quad (\text{B.7})$$

Furthermore, integrating out the vector-like leptons, the Yukawa terms y_L^{ai} generate contributions to the SMEFT Wilson coefficients

$$[C_{He}]_{ij} = \sum_r \frac{y_{ri}^* y_{rj}}{2M_r^2}. \quad (\text{B.8})$$

Flavour-diagonal Wilson coefficients are mostly constrained by EWPO [63]. For a detailed analysis of the impact of vector-like leptons on the EWPO, see [121]. Off-diagonal ones are constrained by the same LFV processes commented above (LFV τ decays and $\mu \rightarrow e$ conversion) through their contribution to the LEFT operators

$$[C_{ee}^{V,LR}]_{iijk} = (2s_W^2 - 1)[C_{He}]^{jk}, \quad (\text{B.9})$$

$$[C_{ee}^{V,RR}]_{iijk} = 2s_W^2 [C_{He}]_{jk}, \quad (\text{B.10})$$

$$[C_{ue}^{V,LR}]_{iijk} = \left(1 - \frac{4}{3}s_W^2 \right) [C_{He}]_{jk}, \quad (\text{B.11})$$

$$[C_{eu}^{V,RR}]_{jkii} = -\frac{4}{3}s_W^2 [C_{He}]_{jk}, \quad (\text{B.12})$$

$$[C_{de}^{V,LR}]_{iijk} = \left(\frac{2}{3}s_W^2 - 1 \right) [C_{He}]_{jk}, \quad (\text{B.13})$$

$$[C_{ed}^{V,RR}]_{jkii} = \frac{2}{3}s_W^2 [C_{He}]_{jk}, \quad (\text{B.14})$$

where $k > j \geq i$. We thus obtain

$$|\sum_r \frac{y_{ri}^* y_{ri}}{M_r^2}| < (0.01 - 0.1) \text{ TeV}^{-2}, \quad (\text{B.15})$$

$$|\sum_r \frac{y_{r1}^* y_{r2}}{M_r^2}| < 10^{-5} \text{ TeV}^{-2}, \quad (\text{B.16})$$

$$|\sum_r \frac{y_{r1}^* y_{r3}}{M_r^2}| < 0.03 \text{ TeV}^{-2}, \quad (\text{B.17})$$

$$|\sum_r \frac{y_{r2}^* y_{r3}}{M_r^2}| < 0.02 \text{ TeV}^{-2}. \quad (\text{B.18})$$

C Electroweak fit

We work in the $\{\alpha_{EM}, m_Z, G_F\}$ input scheme where the relevant terms of the effective Lagrangian for the EW fit are

$$\begin{aligned} \mathcal{L} \supset & -\frac{g_L}{\sqrt{2}} W^{+\mu} \left[\bar{u}_L^i \gamma_\mu (V_{ij} + \delta g_{ij}^{Wq}) d_L^j + \bar{\nu}_L^i \gamma_\mu (\delta_{ij} + \delta g_{ij}^{W\ell}) e_L^j \right] + \text{h.c.} \\ & -\frac{g_L}{c_W} Z^\mu \left[\bar{f}_L^i \gamma_\mu (g_L^{Zf} \delta_{ij} + \delta g_{Lij}^{Zf}) f_L^j + \bar{f}_R^i \gamma_\mu (g_R^{Zf} \delta_{ij} + \delta g_{Rij}^{Zf}) f_R^j \right] \\ & + \frac{g_L^2 v^2}{4} (1 + \delta m_W)^2 W^{+\mu} W_\mu^- + \frac{g_L^2 v^2}{8c_W^2} Z^\mu Z_\mu, \end{aligned} \quad (\text{C.1})$$

where

$$g_L^{Zf} = T_f^3 - s_W^2 Q_f, \quad g_R^{Zf} = -s_W^2 Q_f. \quad (\text{C.2})$$

The general relation between the parameters δg and δm_W with the SMEFT Wilson coefficients in the Warsaw basis can be found in ref. [63] and, approximating $V_{\text{CKM}} = \mathbb{1}$, is

$$\delta g_{Lij}^{Z\nu} = -\frac{v^2}{2} \left([C_{Hl}^{(1)}]_{ij} - [C_{Hl}^{(3)}]_{ij} \right) + \delta^U(1/2, 0) \delta_{ij}, \quad (\text{C.3})$$

$$\delta g_{Lij}^{Ze} = -\frac{v^2}{2} \left([C_{Hl}^{(1)}]_{ij} + [C_{Hl}^{(3)}]_{ij} \right) + \delta^U(-1/2, -1) \delta_{ij} \quad (\text{C.4})$$

$$\delta g_{Rij}^{Ze} = -\frac{v^2}{2} [C_{He}]_{ij} + \delta^U(0, -1) \delta_{ij}, \quad (\text{C.5})$$

$$\delta g_{Lij}^{Zu} = -\frac{v^2}{2} \left([C_{Hq}^{(1)}]_{ij} - [C_{Hq}^{(3)}]_{ij} \right) + \delta^U(1/2, 2/3) \delta_{ij}, \quad (\text{C.6})$$

$$\delta g_{Rij}^{Zu} = -\frac{v^2}{2} [C_{Hu}]_{ij} + \delta^U(0, 2/3) \delta_{ij}, \quad (\text{C.7})$$

$$\delta g_{Lij}^{Zd} = -\frac{v^2}{2} \left([C_{Hq}^{(1)}]_{ij} + [C_{Hq}^{(3)}]_{ij} \right) + \delta^U(-1/2, -1/3) \delta_{ij}, \quad (\text{C.8})$$

$$\delta g_{Rij}^{Zd} = -\frac{v^2}{2} [C_{Hd}]_{ij} + \delta^U(0, -1/3) \delta_{ij}, \quad (\text{C.9})$$

$$\delta g_{ij}^{W\ell} = \delta g_{Lij}^{Z\nu} - \delta g_{Lij}^{Ze}, \quad (\text{C.10})$$

$$\delta g_{ij}^{Wq} = \delta g_{Lij}^{Zu} - \delta g_{Lij}^{Zd}, \quad (\text{C.11})$$

Observable	Measurement
$R \left[\frac{\tau \rightarrow e \nu \bar{\nu}}{\mu \rightarrow e \nu \bar{\nu}} \right]$	1.0010 ± 0.0014
$R \left[\frac{\tau \rightarrow \pi \nu}{\pi \rightarrow \mu \bar{\nu}} \right]$	0.9958 ± 0.0026
$R \left[\frac{\tau \rightarrow K \nu}{K \rightarrow \mu \bar{\nu}} \right]$	0.9879 ± 0.0063
$R \left[\frac{\tau \rightarrow \mu \nu \bar{\nu}}{\mu \rightarrow e \nu \bar{\nu}} \right]$	1.0029 ± 0.0014

Table 2. Measurements ratios testing LFU violation in τ that appear in eqs. (D.2)–(D.4) from [122]. The correlations are given in the same reference.

$$\delta m_W = -\frac{v^2 g_L^2}{4(g_L^2 - g_Y^2)} C_{HD} - \frac{v^2 g_L g_Y}{g_L^2 - g_Y^2} C_{HWB} - \frac{g_Y^2}{2(g_L^2 - g_Y^2)} \delta G_F, \quad (\text{C.12})$$

where $\delta^U(T^3, Q)$ is a family-universal contribution that is given by

$$\delta^U(T^3, Q) = -\left(T^3 + Q \frac{g_Y^2}{g_L^2 - g_Y^2}\right) \left(\frac{v^2}{4} C_{HD} + \frac{1}{2} \delta G_F\right) - Q \frac{g_L g_Y}{g_L^2 - g_Y^2} v^2 C_{HWB}, \quad (\text{C.13})$$

and

$$\delta G_F = v^2 \left([C_{H\ell}^{(3)}]_{11} + [C_{H\ell}^{(3)}]_{22} - \frac{1}{2} [C_{\ell\ell}]_{1221} \right) \quad (\text{C.14})$$

is the NP contribution to G_F :

$$G_F^{\text{exp}} \equiv -\frac{[C_{\nu e}^{V,LL}]_{1221}}{2\sqrt{2}} = \frac{1}{\sqrt{2}v^2} (1 + \delta G_F), \quad (\text{C.15})$$

where $C_{\nu e}^{V,LL}$ is the Wilson coefficient of the LEFT operator $O_{\nu e}^{V,LL} = (\bar{\nu}_L \gamma_\mu \nu_L)(\bar{e}_L \gamma^\mu e_L)$.

D Lepton flavour universality violation in charge currents

In general, extensions of a vector-triplet with couplings like in eq. (2.4) will be constrained by tests of LFU in τ decays [122]. Defining

$$R \left[\frac{A}{B} \right] = \left[\frac{\mathcal{B}(A)/\mathcal{B}(A)_{\text{SM}}}{\mathcal{B}(B)/\mathcal{B}(B)_{\text{SM}}} \right]^{\frac{1}{2}}, \quad (\text{D.1})$$

we have that

$$\begin{aligned} R \left[\frac{\tau \rightarrow e \nu \nu}{\mu \rightarrow e \nu \nu} \right] &= 1 + v^2 \left([C_{H\ell}^{(3)}]_{33} - [C_{H\ell}^{(3)}]_{22} + \frac{1}{2} ([C_{\ell\ell}]_{1221} - [C_{\ell\ell}]_{1331}) \right) \\ &= 1 + \frac{(g_{11}^\ell - g^H)(g_{33}^\ell - g_{22}^\ell)}{g_L^2} \frac{m_W^2}{M_{W'}^2}, \end{aligned} \quad (\text{D.2})$$

$$\begin{aligned} R \left[\frac{\tau \rightarrow \mu \nu \nu}{\mu \rightarrow e \nu \nu} \right] &= 1 + v^2 \left([C_{H\ell}^{(3)}]_{33} - [C_{H\ell}^{(3)}]_{11} + \frac{1}{2} ([C_{\ell\ell}]_{1221} - [C_{\ell\ell}]_{2332}) \right) \end{aligned}$$

$$= 1 + \frac{(g_{22}^\ell - g^H)(g_{33}^\ell - g_{11}^\ell)}{g_L^2} \frac{m_W^2}{M_{W'}^2}, \quad (\text{D.3})$$

$$\begin{aligned} R \left[\frac{\tau \rightarrow K \nu}{K \rightarrow \mu \nu} \right] &= R \left[\frac{\tau \rightarrow \pi \nu}{\pi \rightarrow \mu \nu} \right] \\ &= 1 + v^2 \left([C_{H\ell}^{(3)}]_{33} - [C_{H\ell}^{(3)}]_{22} + [C_{\ell q}^{(3)}]_{22ii} - [C_{\ell q}^{(3)}]_{33ii} \right) \\ &= 1 + \frac{(g^q - g^H)(g_{33}^\ell - g_{22}^\ell)}{g_L^2} \frac{m_W^2}{M_{W'}^2}, \end{aligned} \quad (\text{D.4})$$

where $i = 1$ or 2 . We have included the NP contribution from SMEFT, which should be evaluated at the EW scale, and their value at tree level neglecting running. table 2 shows the experimental measurements.

E Cabibbo angle anomaly

The Cabibbo angle anomaly (CAA) (see ref. [123] for a review) is a deficit in the first-row CKM unitarity [124]

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9985(5). \quad (\text{E.1})$$

Here, V_{us} is most precisely determined from kaon, pion and τ decays and V_{ud} from beta decays, in particular super allowed and mirrored beta decays. The impact of V_{ub} and its uncertainty is completely negligible. Explaining the CAA requires $\delta V_{ud} = -0.0075 \pm 0.0025$. The simplified model of eq. (2.4) give the NP contributions at the tree level given in eq. (3.7). Assuming $g^q \leq g_{22}^\ell$, which is fixed by the quark charge assignment, the only possibility to achieve a sizable contribution with the right sign is $g^H < g_{11}^\ell$. Putting the Higgs on the first site and ℓ_1 on the second site would achieve this, but lead to a suppressed top Yukawa. Notice that more general realizations with two Higgs doublets, one in the first site and one in the second site, would allow us to find a compromise between the large top Yukawa and an explanation of the CAA. Indeed, the SM Higgs — the Higgs that gets a VEV v — could be a combination of the two Higgses, with θ_H the mixing angle. Then, $g^H = \cos^2 \theta_H g_2 + \sin^2 \theta_H g_1$, so an intermediate $g_1 \leq g^H \leq g_2$ is possible. At the same time, the VEV of the second-site Higgs would be $\langle H_2 \rangle = v \cos \theta_H$, which could allow for an $O(1)$ top Yukawa. However, the first-site Higgs would also develop an EW VEV for sizable mixing θ_H , $\langle H_1 \rangle = v \sin \theta_H$, that would spoil the explanation of the flavour hierarchies in the quark sector that our models have. As an example, we take model 2, and choose the triplet mass at the LHC searches limit $M_{W'} \sim 10$ TeV (see figure 1, and notice that LHC limits do not depend on g^H), and a large $g_2' \sim 4$ to maximize the effect. Then, explaining the deficit of the CAA at one sigma would imply $\sin \theta_H \sim 0.4 - 0.5$, far from the suppression of V_{cb} and second family Yukawas.

F $B_{d,s} - \bar{B}_{d,s}$ mixing

Our models generate the operators eq. (4.1), that at the matching scale receive the contribution given in eq. (4.5). They contribute to $B_{d,s}$ mixing like

$$\frac{\Delta m_{B_{d,s}}}{\Delta m_{B_{d,s}}^{\text{SM}}} = \left| 1 + \frac{C_{B_{d,s}}^{\text{NP}}}{C_{B_{d,s}}^{\text{SM}}} \right|, \quad (\text{F.1})$$

where, at the EW scale,

$$\begin{aligned} C_{B_d}^{\text{SM}}(\mu_{\text{EW}}) &= \frac{g_L^2}{32\pi^2 v^2} (V_{tb} V_{td}^*)^2 S_0, \\ C_{B_s}^{\text{SM}}(\mu_{\text{EW}}) &= \frac{g_L^2}{32\pi^2 v^2} (V_{tb} V_{ts}^*)^2 S_0, \end{aligned} \quad (\text{F.2})$$

and $S_0 \approx 2.49$ [93, 125]. We have that [77, 126]

$$\frac{\Delta m_{B_d}^{\text{Exp}}}{\Delta m_{B_d}^{\text{SM}}} = 1.09 \pm 0.09, \quad \frac{\Delta m_{B_s}^{\text{Exp}}}{\Delta m_{B_s}^{\text{SM}}} = 1.10 \pm 0.06. \quad (\text{F.3})$$

Including the running to the matching scale $\mu_{\text{UV}} \approx 10 \text{ TeV}$, $C_{B_s}^{\text{NP}}(\mu_{\text{EW}}) \approx 0.82 C_{B_s}^{\text{NP}}(\mu_{\text{UV}})$ [127], and assuming that the only phases come from the CKM elements, we get the limits reported in eqs. (4.6) and (4.7).

G A protection for $B_s \rightarrow \ell^+ \ell^-$

Let us assume a generic gauge extension of the SM giving a heavy Z' associated to some $\text{U}(1)'$. Working in components of the $\text{SU}(2)_L$ doublets, we can write the couplings

$$\begin{aligned} \mathcal{L} \supset & -Z'_\mu (g^{H_d} H_d^* D^\mu H_d + g_{23}^q \bar{s}_L \gamma^\mu b_L + \text{h.c.} \\ & + g_{ii}^{eL} \bar{e}_L^i \gamma^\mu e_L^i + g_{ii}^{eR} \bar{e}_R^i \gamma^\mu e_R^i), \end{aligned} \quad (\text{G.1})$$

where H_d is the down component of the Higgs getting the VEV, $H = (H_u, H_d)^T$, and the couplings g are proportional to the $\text{U}(1)'$ -charge of each field. Note that the Higgs coupling will generate a $Z - Z'$ mixing when the Higgs gets a VEV $H_d = v/\sqrt{2}$. When the Z' is integrated out, these couplings contribute to the Wilson coefficients of the $bs\ell\ell$ operators defined in eqs. (4.11) and (4.12),

$$\mathcal{L} \supset \tilde{C}_9^{\ell_i} O_9^{\ell_i} + \tilde{C}_{10}^{\ell_i} O_{10}^{\ell_i}, \quad (\text{G.2})$$

like

$$\tilde{C}_9^{\ell_i \text{ NP}} = -\frac{g_{23}^q}{2M_{Z'}^2} [g_{ii}^{eL} + g_{ii}^{eR} - (1 - 4s_W^2) g^{H_d}], \quad (\text{G.3})$$

$$\tilde{C}_{10}^{\ell_i \text{ NP}} = \frac{g_{23}^q}{2M_{Z'}^2} (g_{ii}^{eL} - g^{H_d} - g_{ii}^{eR}). \quad (\text{G.4})$$

We see that $C_{10}^{\ell_i \text{ NP}}$, and therefore the contribution to $\mathcal{B}(B_s \rightarrow \ell_i^+ \ell_i^-)$ (see eq. (4.15)), generically vanishes when the charges are such that the Yukawa $\bar{e}_L^i H_d e_R^i$ can be written respecting $\text{U}(1)'$. This mechanism was also observed in the model presented in ref. [35].

H LFV processes

The LEFT operators

$$\mathcal{L} \supset \sum_{\substack{f=e,u,d \\ X,Y=L,R}} C_{ef}^{V,XY} (\bar{e}_X \gamma_\mu e_X) (\bar{f}_Y \gamma^\mu f_Y), \quad (\text{H.1})$$

with non-diagonal flavour indices on the e field generate LFV processes. In particular Wilson coefficients $C_{ee}^{V,XY}$ generate LFV three-body decays with branching fractions [128]:

$$\mathcal{B}(\ell_k \rightarrow \ell_j \bar{\ell}_i \ell_i) = \frac{M_{\ell_k}^5}{1536\pi^3\Gamma_{\ell_k}(1+\delta_{ij})} \times \left(\left| [C_{ee}^{V,LL}]_{iijk} \right|^2 + \left| [C_{ee}^{V,LR}]_{jkii} \right|^2 + \left| [C_{ee}^{V,LR}]_{iijk} \right|^2 + \left| [C_{ee}^{V,RR}]_{iijk} \right|^2 \right), \quad (\text{H.2})$$

where $k > j \geq i$. The experimental bounds on these decays at 90% C.L. are [122, 129]:

$$\mathcal{B}(\tau^- \rightarrow \mu^- \mu^+ \mu^-) < 1.1 \times 10^{-8}, \quad (\text{H.3})$$

$$\mathcal{B}(\tau^- \rightarrow \mu^- e^+ e^-) < 1.1 \times 10^{-8}, \quad (\text{H.4})$$

$$\mathcal{B}(\tau^- \rightarrow e^- \mu^+ \mu^-) < 1.6 \times 10^{-8}, \quad (\text{H.5})$$

$$\mathcal{B}(\tau^- \rightarrow e^- e^+ e^-) < 1.4 \times 10^{-8}, \quad (\text{H.6})$$

$$\mathcal{B}(\mu^- \rightarrow e^- e^+ e^-) < 1.0 \times 10^{-12}. \quad (\text{H.7})$$

The Wilson coefficients $C_{eu}^{V,XY}$, $C_{ed}^{V,XY}$ generate $\mu \rightarrow e$ conversion processes [130]. To compute their contribution, we use the formulas given in ref. [131]. The current most constraining bound comes from SINDRUM II experiment with gold nuclei [132]:

$$\mathcal{B}(\mu \text{ Au} \rightarrow e \text{ Au}) < 7 \times 10^{-13}, \quad (\text{H.8})$$

at 90% C.L.

When a vector triplet with interaction terms of eq. (2.4) and LFV couplings,

$$-\mathcal{L} \supset \frac{1}{2} W_\mu^{\prime(0)a} \sum_{i \neq j} g_{ij}^\ell \bar{\ell}_i \gamma^\mu \sigma_a \ell_j, \quad (\text{H.9})$$

is integrated out, we get LFV contributions to $C_{\ell\ell}$, $C_{\ell q}^{(3)}$ and $C_{H\ell}^{(3)}$ Wilson coefficients in SMEFT, that generate contributions to the LEFT Wilson coefficients

$$[C_{ee}^{V,LL}]_{iijk} = -\frac{g_{jk}^\ell (g_{ii}^\ell + g^H(2s_W^2 - 1))}{4M_{W'}^2}, \quad (\text{H.10})$$

$$[C_{ee}^{V,LR}]_{jkii} = -\frac{g_{jk}^\ell g^H s_W^2}{2M_{W'}^2}, \quad (\text{H.11})$$

$$[C_{eu}^{V,LL}]_{jkii} = \frac{g_{jk}^\ell (g^q - g^H(1 - 4s_W^2/3))}{4M_{W'}^2}, \quad (\text{H.12})$$

$$[C_{ed}^{V,LL}]_{jkii} = -\frac{g_{jk}^\ell (g^q + g^H(2s_W^2/3 - 1))}{4M_{W'}^2}, \quad (\text{H.13})$$

$$[C_{eu}^{V,LR}]_{jkii} = -2[C_{ed}^{V,LR}]_{jkii} = \frac{g_{jk}^\ell g^H s_W^2}{3M_{W'}^2}, \quad (\text{H.14})$$

where $k > j \geq i$ and we have neglected the running, which at one loop is only due to QED.

Open Access. This article is distributed under the terms of the Creative Commons Attribution License ([CC-BY4.0](https://creativecommons.org/licenses/by/4.0/)), which permits any use, distribution and reproduction in any medium, provided the original author(s) and source are credited.

References

- [1] C.D. Froggatt and H.B. Nielsen, *Hierarchy of Quark Masses, Cabibbo Angles and CP Violation*, *Nucl. Phys. B* **147** (1979) 277 [[INSPIRE](#)].
- [2] S.F. King and G.G. Ross, *Fermion masses and mixing angles from $SU(3)$ family symmetry and unification*, *Phys. Lett. B* **574** (2003) 239 [[hep-ph/0307190](#)] [[INSPIRE](#)].
- [3] A.J. Buras, M.V. Carlucci, L. Merlo and E. Stamou, *Phenomenology of a Gauged $SU(3)^3$ Flavour Model*, *JHEP* **03** (2012) 088 [[arXiv:1112.4477](#)] [[INSPIRE](#)].
- [4] D.B. Kaplan, *Flavor at SSC energies: A new mechanism for dynamically generated fermion masses*, *Nucl. Phys. B* **365** (1991) 259 [[INSPIRE](#)].
- [5] Y. Grossman and M. Neubert, *Neutrino masses and mixings in nonfactorizable geometry*, *Phys. Lett. B* **474** (2000) 361 [[hep-ph/9912408](#)] [[INSPIRE](#)].
- [6] T. Gherghetta and A. Pomarol, *Bulk fields and supersymmetry in a slice of AdS*, *Nucl. Phys. B* **586** (2000) 141 [[hep-ph/0003129](#)] [[INSPIRE](#)].
- [7] B. Keren-Zur et al., *On partial Compositeness and the CP asymmetry in charm decays*, *Nucl. Phys. B* **867** (2013) 394 [[arXiv:1205.5803](#)] [[INSPIRE](#)].
- [8] G. Panico and A. Pomarol, *Flavor hierarchies from dynamical scales*, *JHEP* **07** (2016) 097 [[arXiv:1603.06609](#)] [[INSPIRE](#)].
- [9] Z.G. Berezhiani, *The Weak Mixing Angles in Gauge Models with Horizontal Symmetry: A New Approach to Quark and Lepton Masses*, *Phys. Lett. B* **129** (1983) 99 [[INSPIRE](#)].
- [10] Z.G. Berezhiani and R. Rattazzi, *Inverse hierarchy approach to fermion masses*, *Nucl. Phys. B* **407** (1993) 249 [[hep-ph/9212245](#)] [[INSPIRE](#)].
- [11] R. Barbieri, G.R. Dvali and A. Strumia, *Fermion masses and mixings in a flavor symmetric GUT*, *Nucl. Phys. B* **435** (1995) 102 [[hep-ph/9407239](#)] [[INSPIRE](#)].
- [12] G.R. Dvali and M.A. Shifman, *Families as neighbors in extra dimension*, *Phys. Lett. B* **475** (2000) 295 [[hep-ph/0001072](#)] [[INSPIRE](#)].
- [13] R. Barbieri, G.R. Dvali and L.J. Hall, *Predictions from a $U(2)$ flavor symmetry in supersymmetric theories*, *Phys. Lett. B* **377** (1996) 76 [[hep-ph/9512388](#)] [[INSPIRE](#)].
- [14] R. Barbieri et al., *$U(2)$ and Minimal Flavour Violation in Supersymmetry*, *Eur. Phys. J. C* **71** (2011) 1725 [[arXiv:1105.2296](#)] [[INSPIRE](#)].
- [15] A. Crivellin, L. Hofer and U. Nierste, *The MSSM with a Softly Broken $U(2)^3$ Flavor Symmetry*, *PoS EPS-HEP2011* (2011) 145 [[arXiv:1111.0246](#)] [[INSPIRE](#)].
- [16] R. Barbieri, D. Buttazzo, F. Sala and D.M. Straub, *Flavour physics from an approximate $U(2)^3$ symmetry*, *JHEP* **07** (2012) 181 [[arXiv:1203.4218](#)] [[INSPIRE](#)].
- [17] A.J. Buras and J. Girrbach, *On the Correlations between Flavour Observables in Minimal $U(2)^3$ Models*, *JHEP* **01** (2013) 007 [[arXiv:1206.3878](#)] [[INSPIRE](#)].
- [18] L. Calibbi et al., *Z' models with less-minimal flavour violation*, *Phys. Rev. D* **101** (2020) 095003 [[arXiv:1910.00014](#)] [[INSPIRE](#)].

- [19] J. Fuentes-Martín and P. Stangl, *Third-family quark-lepton unification with a fundamental composite Higgs*, *Phys. Lett. B* **811** (2020) 135953 [[arXiv:2004.11376](#)] [[INSPIRE](#)].
- [20] J. Fuentes-Martín et al., *Flavor hierarchies, flavor anomalies, and Higgs mass from a warped extra dimension*, *Phys. Lett. B* **834** (2022) 137382 [[arXiv:2203.01952](#)] [[INSPIRE](#)].
- [21] H. Georgi, *A Tool Kit for Builders of Composite Models*, *Nucl. Phys. B* **266** (1986) 274 [[INSPIRE](#)].
- [22] M.R. Douglas and G.W. Moore, *D-branes, quivers, and ALE instantons*, [hep-th/9603167](#) [[INSPIRE](#)].
- [23] N. Arkani-Hamed, A.G. Cohen and H. Georgi, *(De)constructing dimensions*, *Phys. Rev. Lett.* **86** (2001) 4757 [[hep-th/0104005](#)] [[INSPIRE](#)].
- [24] C.T. Hill, S. Pokorski and J. Wang, *Gauge Invariant Effective Lagrangian for Kaluza-Klein Modes*, *Phys. Rev. D* **64** (2001) 105005 [[hep-th/0104035](#)] [[INSPIRE](#)].
- [25] H.-C. Cheng, C.T. Hill, S. Pokorski and J. Wang, *The Standard Model in the Latticized Bulk*, *Phys. Rev. D* **64** (2001) 065007 [[hep-th/0104179](#)] [[INSPIRE](#)].
- [26] R.S. Chivukula, E.H. Simmons and N. Vignaroli, *A Flavorful Top-Coloron Model*, *Phys. Rev. D* **87** (2013) 075002 [[arXiv:1302.1069](#)] [[INSPIRE](#)].
- [27] A. Crivellin, G. D'Ambrosio and J. Heeck, *Addressing the LHC flavor anomalies with horizontal gauge symmetries*, *Phys. Rev. D* **91** (2015) 075006 [[arXiv:1503.03477](#)] [[INSPIRE](#)].
- [28] A. Greljo and B.A. Stefanek, *Third family quark-lepton unification at the TeV scale*, *Phys. Lett. B* **782** (2018) 131 [[arXiv:1802.04274](#)] [[INSPIRE](#)].
- [29] Ò.L. Crosas et al., *Flavor non-universal vector leptoquark imprints in $K \rightarrow \pi \nu \bar{\nu}$ and $\Delta F = 2$ transitions*, *Phys. Lett. B* **835** (2022) 137525 [[arXiv:2207.00018](#)] [[INSPIRE](#)].
- [30] L. Allwicher et al., *Third-family quark-lepton Unification and electroweak precision tests*, *JHEP* **05** (2023) 179 [[arXiv:2302.11584](#)] [[INSPIRE](#)].
- [31] M. Bordone, C. Cornella, J. Fuentes-Martín and G. Isidori, *A three-site gauge model for flavor hierarchies and flavor anomalies*, *Phys. Lett. B* **779** (2018) 317 [[arXiv:1712.01368](#)] [[INSPIRE](#)].
- [32] M. Blanke and A. Crivellin, *B Meson Anomalies in a Pati-Salam Model within the Randall-Sundrum Background*, *Phys. Rev. Lett.* **121** (2018) 011801 [[arXiv:1801.07256](#)] [[INSPIRE](#)].
- [33] J. Davighi and G. Isidori, *Non-universal gauge interactions addressing the inescapable link between Higgs and flavour*, *JHEP* **07** (2023) 147 [[arXiv:2303.01520](#)] [[INSPIRE](#)].
- [34] M. Fernández Navarro and S.F. King, *Tri-hypercharge: a separate gauged weak hypercharge for each fermion family as the origin of flavour*, *JHEP* **08** (2023) 020 [[arXiv:2305.07690](#)] [[INSPIRE](#)].
- [35] J. Davighi and B.A. Stefanek, *Deconstructed hypercharge: a natural model of flavour*, *JHEP* **11** (2023) 100 [[arXiv:2305.16280](#)] [[INSPIRE](#)].
- [36] M. Fernández Navarro, S.F. King and A. Vicente, *Tri-unification: a separate $SU(5)$ for each fermion family*, *JHEP* **05** (2024) 130 [[arXiv:2311.05683](#)] [[INSPIRE](#)].
- [37] R. Barbieri and G. Isidori, *Minimal flavour deconstruction*, *JHEP* **05** (2024) 033 [[arXiv:2312.14004](#)] [[INSPIRE](#)].
- [38] A. Greljo and A.E. Thomsen, *Rising through the ranks: flavor hierarchies from a gauged $SU(2)$ symmetry*, *Eur. Phys. J. C* **84** (2024) 213 [[arXiv:2309.11547](#)] [[INSPIRE](#)].

- [39] X. Li and E. Ma, *Gauge Model of Generation Nonuniversality*, *Phys. Rev. Lett.* **47** (1981) 1788 [[INSPIRE](#)].
- [40] D.J. Muller and S. Nandi, *Top flavor: A Separate $SU(2)$ for the third family*, *Phys. Lett. B* **383** (1996) 345 [[hep-ph/9602390](#)] [[INSPIRE](#)].
- [41] E. Malkawi, T.M.P. Tait and C.P. Yuan, *A model of strong flavor dynamics for the top quark*, *Phys. Lett. B* **385** (1996) 304 [[hep-ph/9603349](#)] [[INSPIRE](#)].
- [42] J. Shu, T.M.P. Tait and C.E.M. Wagner, *Baryogenesis from an Earlier Phase Transition*, *Phys. Rev. D* **75** (2007) 063510 [[hep-ph/0610375](#)] [[INSPIRE](#)].
- [43] C.-W. Chiang, N.G. Deshpande, X.-G. He and J. Jiang, *The family $SU(2)_l \times SU(2)_h \times U(1)$ Model*, *Phys. Rev. D* **81** (2010) 015006 [[arXiv:0911.1480](#)] [[INSPIRE](#)].
- [44] K. Hsieh, K. Schmitz, J.-H. Yu and C.-P. Yuan, *Global Analysis of General $SU(2) \times SU(2) \times U(1)$ Models with Precision Data*, *Phys. Rev. D* **82** (2010) 035011 [[arXiv:1003.3482](#)] [[INSPIRE](#)].
- [45] ATLAS collaboration, *Search for high-mass resonances in final states with a τ -lepton and missing transverse momentum with the ATLAS detector*, *Phys. Rev. D* **109** (2024) 112008 [[arXiv:2402.16576](#)] [[INSPIRE](#)].
- [46] S. Antusch, A. Greljo, B.A. Stefanek and A.E. Thomsen, *$U(2)$ Is Right for Leptons and Left for Quarks*, *Phys. Rev. Lett.* **132** (2024) 151802 [[arXiv:2311.09288](#)] [[INSPIRE](#)].
- [47] J. de Blas, J.M. Lizana and M. Perez-Victoria, *Combining searches of Z' and W' bosons*, *JHEP* **01** (2013) 166 [[arXiv:1211.2229](#)] [[INSPIRE](#)].
- [48] D. Pappadopulo, A. Thamm, R. Torre and A. Wulzer, *Heavy Vector Triplets: Bridging Theory and Data*, *JHEP* **09** (2014) 060 [[arXiv:1402.4431](#)] [[INSPIRE](#)].
- [49] J. Davighi, A. Gosnay, D.J. Miller and S. Renner, *Phenomenology of a Deconstructed Electroweak Force*, *JHEP* **05** (2024) 085 [[arXiv:2312.13346](#)] [[INSPIRE](#)].
- [50] B. Capdevila, A. Crivellin and J. Matias, *Review of semileptonic B anomalies*, *Eur. Phys. J. ST* **1** (2023) 20 [[arXiv:2309.01311](#)] [[INSPIRE](#)].
- [51] G.C. Branco et al., *Theory and phenomenology of two-Higgs-doublet models*, *Phys. Rept.* **516** (2012) 1 [[arXiv:1106.0034](#)] [[INSPIRE](#)].
- [52] J. Fuentes-Martín, G. Isidori, J. Pagès and B.A. Stefanek, *Flavor non-universal Pati-Salam unification and neutrino masses*, *Phys. Lett. B* **820** (2021) 136484 [[arXiv:2012.10492](#)] [[INSPIRE](#)].
- [53] J. Fuentes-Martín and J.M. Lizana, *Deconstructing flavor anomalously*, [arXiv:2402.09507](#) [[INSPIRE](#)].
- [54] D.B. Clark, E. Godat and F.I. Olness, *ManeParse: A Mathematica reader for Parton Distribution Functions*, *Comput. Phys. Commun.* **216** (2017) 126 [[arXiv:1605.08012](#)] [[INSPIRE](#)].
- [55] ATLAS collaboration, *Search for a heavy charged boson in events with a charged lepton and missing transverse momentum from pp collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector*, *Phys. Rev. D* **100** (2019) 052013 [[arXiv:1906.05609](#)] [[INSPIRE](#)].
- [56] CMS collaboration, *Search for resonant and nonresonant new phenomena in high-mass dilepton final states at $\sqrt{s} = 13$ TeV*, *JHEP* **07** (2021) 208 [[arXiv:2103.02708](#)] [[INSPIRE](#)].
- [57] ATLAS collaboration, *Search for high-mass resonances in final states with a tau lepton and missing transverse momentum with the ATLAS detector*, ATLAS-CONF-2021-025 (2021) [[INSPIRE](#)].

- [58] ATLAS collaboration, *Search for new resonances in mass distributions of jet pairs using 139 fb^{-1} of pp collisions at $\sqrt{s} = 13\text{ TeV}$ with the ATLAS detector*, *JHEP* **03** (2020) 145 [[arXiv:1910.08447](#)] [[INSPIRE](#)].
- [59] ATLAS collaboration, *Search for additional heavy neutral Higgs and gauge bosons in the ditau final state produced in 36 fb^{-1} of pp collisions at $\sqrt{s} = 13\text{ TeV}$ with the ATLAS detector*, *JHEP* **01** (2018) 055 [[arXiv:1709.07242](#)] [[INSPIRE](#)].
- [60] L. Allwicher et al., *HighPT: A tool for high- p_T Drell-Yan tails beyond the standard model*, *Comput. Phys. Commun.* **289** (2023) 108749 [[arXiv:2207.10756](#)] [[INSPIRE](#)].
- [61] ATLAS collaboration, *Search for low-mass resonances decaying into two jets and produced in association with a photon using pp collisions at $\sqrt{s} = 13\text{ TeV}$ with the ATLAS detector*, *Phys. Lett. B* **795** (2019) 56 [[arXiv:1901.10917](#)] [[INSPIRE](#)].
- [62] R. Bartocci, A. Biekötter and T. Hurth, *A global analysis of the SMEFT under the minimal MFV assumption*, *JHEP* **05** (2024) 074 [[arXiv:2311.04963](#)] [[INSPIRE](#)].
- [63] V. Bresó-Pla, A. Falkowski and M. González-Alonso, *A_{FB} in the SMEFT: precision Z physics at the LHC*, *JHEP* **08** (2021) 021 [[arXiv:2103.12074](#)] [[INSPIRE](#)].
- [64] PARTICLE DATA GROUP collaboration, *Review of Particle Physics*, *Phys. Rev. D* **98** (2018) 030001 [[INSPIRE](#)].
- [65] CDF collaboration, *High-precision measurement of the W boson mass with the CDF II detector*, *Science* **376** (2022) 170 [[INSPIRE](#)].
- [66] ATLAS collaboration, *Improved W boson Mass Measurement using 7 TeV Proton-Proton Collisions with the ATLAS Detector*, ATLAS-CONF-2023-004 (2023) [[INSPIRE](#)].
- [67] E. Bagnaschi et al., *SMEFT analysis of m_W* , *JHEP* **08** (2022) 308 [[arXiv:2204.05260](#)] [[INSPIRE](#)].
- [68] A. Falkowski and K. Mimouni, *Model independent constraints on four-lepton operators*, *JHEP* **02** (2016) 086 [[arXiv:1511.07434](#)] [[INSPIRE](#)].
- [69] B. Allanach and A. Mullin, *Plan B: new Z' models for $b \rightarrow s\ell^+\ell^-$ anomalies*, *JHEP* **09** (2023) 173 [[arXiv:2306.08669](#)] [[INSPIRE](#)].
- [70] ALEPH et al. collaborations, *Electroweak Measurements in Electron-Positron Collisions at W -Boson-Pair Energies at LEP*, *Phys. Rept.* **532** (2013) 119 [[arXiv:1302.3415](#)] [[INSPIRE](#)].
- [71] L. Allwicher, C. Cornella, G. Isidori and B.A. Stefanek, *New physics in the third generation. A comprehensive SMEFT analysis and future prospects*, *JHEP* **03** (2024) 049 [[arXiv:2311.00020](#)] [[INSPIRE](#)].
- [72] A. Crivellin and M. Hoferichter, *β Decays as Sensitive Probes of Lepton Flavor Universality*, *Phys. Rev. Lett.* **125** (2020) 111801 [[arXiv:2002.07184](#)] [[INSPIRE](#)].
- [73] B. Capdevila, A. Crivellin, C.A. Manzari and M. Montull, *Explaining $b \rightarrow s\ell^+\ell^-$ and the Cabibbo angle anomaly with a vector triplet*, *Phys. Rev. D* **103** (2021) 015032 [[arXiv:2005.13542](#)] [[INSPIRE](#)].
- [74] D.E. Morrissey, T.M.P. Tait and C.E.M. Wagner, *Proton lifetime and baryon number violating signatures at the CERN LHC in gauge extended models*, *Phys. Rev. D* **72** (2005) 095003 [[hep-ph/0508123](#)] [[INSPIRE](#)].
- [75] J. Fuentes-Martín, J. Portoles and P. Ruiz-Femenia, *Instanton-mediated baryon number violation in non-universal gauge extended models*, *JHEP* **01** (2015) 134 [[arXiv:1411.2471](#)] [[INSPIRE](#)].

- [76] Luca Silvestrini, *Flavor Constraints on New Physics*, in the proceedings of the XXXII Workshop in Particle Physics La Thuile, Italy, February 25 – 3 March (2018) [<https://agenda.infn.it/event/14377/contributions/24434/attachments/17481/19830/silvestriniLaThuile.pdf>].
- [77] UTFIT collaboration, *Model-independent constraints on $\Delta F = 2$ operators and the scale of new physics*, *JHEP* **03** (2008) 049 [[arXiv:0707.0636](#)] [[INSPIRE](#)].
- [78] C. Bobeth et al., *$B_{s,d} \rightarrow l^+l^-$ in the Standard Model with Reduced Theoretical Uncertainty*, *Phys. Rev. Lett.* **112** (2014) 101801 [[arXiv:1311.0903](#)] [[INSPIRE](#)].
- [79] M. Beneke, C. Bobeth and R. Szafron, *Power-enhanced leading-logarithmic QED corrections to $B_q \rightarrow \mu^+\mu^-$* , *JHEP* **10** (2019) 232 [Erratum *ibid.* **11** (2022) 099] [[arXiv:1908.07011](#)] [[INSPIRE](#)].
- [80] S. Neshatpour, T. Hurth, F. Mahmoudi and D. Martinez Santos, *Neutral Current B-Decay Anomalies*, *Springer Proc. Phys.* **292** (2023) 11 [[arXiv:2210.07221](#)] [[INSPIRE](#)].
- [81] W. Altmannshofer and P. Stangl, *New physics in rare B decays after Moriond 2021*, *Eur. Phys. J. C* **81** (2021) 952 [[arXiv:2103.13370](#)] [[INSPIRE](#)].
- [82] LHCb collaboration, *Measurement of lepton universality parameters in $B^+ \rightarrow K^+\ell^+\ell^-$ and $B^0 \rightarrow K^{*0}\ell^+\ell^-$ decays*, *Phys. Rev. D* **108** (2023) 032002 [[arXiv:2212.09153](#)] [[INSPIRE](#)].
- [83] LHCb collaboration, *Test of lepton universality in $b \rightarrow s\ell^+\ell^-$ decays*, *Phys. Rev. Lett.* **131** (2023) 051803 [[arXiv:2212.09152](#)] [[INSPIRE](#)].
- [84] M. Algueró et al., *To (b)e or not to (b)e: no electrons at LHCb*, *Eur. Phys. J. C* **83** (2023) 648 [[arXiv:2304.07330](#)] [[INSPIRE](#)].
- [85] B. Capdevila, *Status of the global $b \rightarrow s\ell^+\ell^-$ fits*, *PoS FPCP2023* (2023) 010 [[INSPIRE](#)].
- [86] S. Descotes-Genon, J. Matias, M. Ramon and J. Virto, *Implications from clean observables for the binned analysis of $B \rightarrow K^*\mu^+\mu^-$ at large recoil*, *JHEP* **01** (2013) 048 [[arXiv:1207.2753](#)] [[INSPIRE](#)].
- [87] HPQCD collaborations, *$B \rightarrow K$ and $D \rightarrow K$ form factors from fully relativistic lattice QCD*, *Phys. Rev. D* **107** (2023) 014510 [[arXiv:2207.12468](#)] [[INSPIRE](#)].
- [88] A. Greljo, J. Salko, A. Smolkovič and P. Stangl, *Rare b decays meet high-mass Drell-Yan*, *JHEP* **05** (2023) 087 [[arXiv:2212.10497](#)] [[INSPIRE](#)].
- [89] M. Ciuchini et al., *Constraints on lepton universality violation from rare B decays*, *Phys. Rev. D* **107** (2023) 055036 [[arXiv:2212.10516](#)] [[INSPIRE](#)].
- [90] T. Hurth, F. Mahmoudi and S. Neshatpour, *B anomalies in the post $R_{K^{(*)}}$ era*, *Phys. Rev. D* **108** (2023) 115037 [[arXiv:2310.05585](#)] [[INSPIRE](#)].
- [91] A. Khodjamirian, T. Mannel, A.A. Pivovarov and Y.-M. Wang, *Charm-loop effect in $B \rightarrow K^{(*)}\ell^+\ell^-$ and $B \rightarrow K^*\gamma$* , *JHEP* **09** (2010) 089 [[arXiv:1006.4945](#)] [[INSPIRE](#)].
- [92] N. Gubernari, D. van Dyk and J. Virto, *Non-local matrix elements in $B_{(s)} \rightarrow \{K^{(*)}, \phi\}\ell^+\ell^-$* , *JHEP* **02** (2021) 088 [[arXiv:2011.09813](#)] [[INSPIRE](#)].
- [93] A.J. Buras, *Weak Hamiltonian, CP violation and rare decays*, in the proceedings of the Les Houches Summer School in Theoretical Physics, Session 68: Probing the Standard Model of Particle Interactions, Les Houches, France, July 28 – September 05 (1997) [[hep-ph/9806471](#)] [[INSPIRE](#)].
- [94] A.J. Buras, D. Buttazzo, J. Girrbach-Noe and R. Knegjens, *$K^+ \rightarrow \pi^+\nu\bar{\nu}$ and $K_L \rightarrow \pi^0\nu\bar{\nu}$ in the Standard Model: status and perspectives*, *JHEP* **11** (2015) 033 [[arXiv:1503.02693](#)] [[INSPIRE](#)].

- [95] NA62 collaboration, *Measurement of the very rare $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ decay*, *JHEP* **06** (2021) 093 [[arXiv:2103.15389](#)] [[INSPIRE](#)].
- [96] G. Anzivino et al., *Workshop summary: Kaons@CERN 2023*, *Eur. Phys. J. C* **84** (2024) 377 [[arXiv:2311.02923](#)] [[INSPIRE](#)].
- [97] C. Ahdida et al., *Post-LS3 Experimental Options in ECN3*, [arXiv:2310.17726](#) [[INSPIRE](#)].
- [98] K. Aoki et al., *Extension of the J-PARC Hadron Experimental Facility: Third White Paper*, [arXiv:2110.04462](#) [[INSPIRE](#)].
- [99] BELLE-II collaboration, *The Belle II Physics Book*, *PTEP* **2019** (2019) 123C01 [Erratum *ibid.* **2020** (2020) 029201] [[arXiv:1808.10567](#)] [[INSPIRE](#)].
- [100] S. Glazov, *Belle II physics highlights*, in the proceedings of the *European Physical Society Conference on High Energy Physics (EPS-HEP)*, Universität Hamburg, Germany, August 20–25 (2023) [<https://indico.desy.de/event/34916/contributions/149769/attachments/84417/111854/Belle.%20II%20highlights.pdf>].
- [101] W. Altmannshofer et al., *Light New Physics in $B \rightarrow K^{(*)} \nu \bar{\nu}$?*, *Phys. Rev. D* **109** (2024) 075008 [[arXiv:2311.14629](#)] [[INSPIRE](#)].
- [102] HFLAV collaboration, *Averages of b -hadron, c -hadron, and τ -lepton properties as of 2021*, *Phys. Rev. D* **107** (2023) 052008 [[arXiv:2206.07501](#)] [[INSPIRE](#)].
- [103] G. D’Ambrosio, G.F. Giudice, G. Isidori and A. Strumia, *Minimal flavor violation: An effective field theory approach*, *Nucl. Phys. B* **645** (2002) 155 [[hep-ph/0207036](#)] [[INSPIRE](#)].
- [104] R.K. Ellis et al., *Physics Briefing Book: Input for the European Strategy for Particle Physics Update 2020*, [arXiv:1910.11775](#) [[INSPIRE](#)].
- [105] J.M. Lizana, J. Matias and B.A. Stefanek, *Explaining the $B_{d,s} \rightarrow K^{(*)} \bar{K}^{(*)}$ non-leptonic puzzle and charged-current B -anomalies via scalar leptoquarks*, *JHEP* **09** (2023) 114 [[arXiv:2306.09178](#)] [[INSPIRE](#)].
- [106] MU3E collaboration, *The Mu3e Experiment*, in the proceedings of the *Snowmass 2021*, Seattle, U.S.A., July 17–26 (2022) [[arXiv:2204.00001](#)] [[INSPIRE](#)].
- [107] LHCb collaboration, *Physics case for an LHCb Upgrade II — Opportunities in flavour physics, and beyond, in the HL-LHC era*, [arXiv:1808.08865](#) [[INSPIRE](#)].
- [108] COMET collaboration, *Conceptual design report for experimental search for lepton flavor violating $\mu^- - e^-$ conversion at sensitivity of $10^{(-16)}$ with a slow-extracted bunched proton beam (COMET)*, KEK-2009-10 (2009) [[INSPIRE](#)].
- [109] MU2E collaboration, *Mu2e Technical Design Report*, [arXiv:1501.05241](#) [[DOI:10.2172/1172555](#)] [[INSPIRE](#)].
- [110] A. Cerri et al., *Report from Working Group 4: Opportunities in Flavour Physics at the HL-LHC and HE-LHC*, *CERN Yellow Rep. Monogr.* **7** (2019) 867 [[arXiv:1812.07638](#)] [[INSPIRE](#)].
- [111] FCC collaboration, *FCC-ee: The Lepton Collider: Future Circular Collider Conceptual Design Report Volume 2*, *Eur. Phys. J. ST* **228** (2019) 261 [[INSPIRE](#)].
- [112] FCC collaboration, *FCC Physics Opportunities: Future Circular Collider Conceptual Design Report Volume 1*, *Eur. Phys. J. C* **79** (2019) 474 [[INSPIRE](#)].
- [113] CEPC STUDY GROUP collaboration, *CEPC Conceptual Design Report: Volume 2 — Physics & Detector*, [arXiv:1811.10545](#) [[INSPIRE](#)].

- [114] J. de Blas et al., *Global SMEFT Fits at Future Colliders*, in the proceedings of the *Snowmass 2021*, Seattle, U.S.A., July 17–26 (2022) [[arXiv:2206.08326](#)] [[INSPIRE](#)].
- [115] M. Dam, *Tau-lepton Physics at the FCC-ee circular e^+e^- Collider*, *SciPost Phys. Proc.* **1** (2019) 041 [[arXiv:1811.09408](#)] [[INSPIRE](#)].
- [116] J. Davighi and J. Tooby-Smith, *Electroweak flavour unification*, *JHEP* **09** (2022) 193 [[arXiv:2201.07245](#)] [[INSPIRE](#)].
- [117] J. Davighi, G. Isidori and M. Pesut, *Electroweak-flavour and quark-lepton unification: a family non-universal path*, *JHEP* **04** (2023) 030 [[arXiv:2212.06163](#)] [[INSPIRE](#)].
- [118] B. Grzadkowski, M. Iskrzynski, M. Misiak and J. Rosiek, *Dimension-Six Terms in the Standard Model Lagrangian*, *JHEP* **10** (2010) 085 [[arXiv:1008.4884](#)] [[INSPIRE](#)].
- [119] J. Fuentes-Martín et al., *A proof of concept for matchete: an automated tool for matching effective theories*, *Eur. Phys. J. C* **83** (2023) 662 [[arXiv:2212.04510](#)] [[INSPIRE](#)].
- [120] CMS collaboration, *Inclusive nonresonant multilepton probes of new phenomena at $\sqrt{s} = 13$ TeV*, *Phys. Rev. D* **105** (2022) 112007 [[arXiv:2202.08676](#)] [[INSPIRE](#)].
- [121] A. Crivellin, F. Kirk, C.A. Manzari and M. Montull, *Global Electroweak Fit and Vector-Like Leptons in Light of the Cabibbo Angle Anomaly*, *JHEP* **12** (2020) 166 [[arXiv:2008.01113](#)] [[INSPIRE](#)].
- [122] HFLAV collaboration, *Averages of b -hadron, c -hadron, and τ -lepton properties as of 2018*, *Eur. Phys. J. C* **81** (2021) 226 [[arXiv:1909.12524](#)] [[INSPIRE](#)].
- [123] A. Crivellin, *Explaining the Cabibbo Angle Anomaly*, [arXiv:2207.02507](#) [[INSPIRE](#)].
- [124] PARTICLE DATA GROUP collaboration, *Review of Particle Physics*, *PTEP* **2020** (2020) 083C01 [[INSPIRE](#)].
- [125] G. Buchalla, A.J. Buras and M.E. Lautenbacher, *Weak decays beyond leading logarithms*, *Rev. Mod. Phys.* **68** (1996) 1125 [[hep-ph/9512380](#)] [[INSPIRE](#)].
- [126] M. Bona et al., *Global fits of the Unitarity Triangle within the Standard Model and beyond. Updates from the UTfit collaboration*, in the proceedings of the *European Physical Society Conference on High Energy Physics (EPS-HEP)*, Universität Hamburg, Germany, August 20–25 (2023) [https://indico.desy.de/event/34916/contributions/147139/attachments/84166/111441/EPS2023_UTfit.pdf].
- [127] J. Fuentes-Martín, P. Ruiz-Femenia, A. Vicente and J. Virto, *DsixTools 2.0: The Effective Field Theory Toolkit*, *Eur. Phys. J. C* **81** (2021) 167 [[arXiv:2010.16341](#)] [[INSPIRE](#)].
- [128] A. Crivellin, S. Najjari and J. Rosiek, *Lepton Flavor Violation in the Standard Model with general Dimension-Six Operators*, *JHEP* **04** (2014) 167 [[arXiv:1312.0634](#)] [[INSPIRE](#)].
- [129] SINDRUM collaboration, *Search for the Decay $\mu^+ \rightarrow e^+e^+e^-$* , *Nucl. Phys. B* **299** (1988) 1 [[INSPIRE](#)].
- [130] Y. Kuno and Y. Okada, *Muon decay and physics beyond the standard model*, *Rev. Mod. Phys.* **73** (2001) 151 [[hep-ph/9909265](#)] [[INSPIRE](#)].
- [131] M. Ardu, S. Davidson and S. Lavignac, *Constraining new physics models from $\mu \rightarrow e$ observables in bottom-up EFT*, *Eur. Phys. J. C* **84** (2024) 458 [[arXiv:2401.06214](#)] [[INSPIRE](#)].
- [132] SINDRUM II collaboration, *A search for muon to electron conversion in muonic gold*, *Eur. Phys. J. C* **47** (2006) 337 [[INSPIRE](#)].