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A clockwork solution to the doublet-triplet splitting problem

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ABSTRACT

Maybe the biggest puzzle in grand unified theories (GUTs) is the apparent large splitting of the doublet and triplet Higgs masses. We suggest a novel mechanism to solve this puzzle, which relies on the clockwork mechanism to generate large hierarchies from order-one numbers. The tension between gauge coupling unification and proton lifetime from minimal $SU(5)$ GUTs is also removed in this scenario, and the theory remains perturbative until the Planck scale. We comment on a possible extra dimensional implementation of the idea.

1. Introduction

The quantum numbers of the Standard Model (SM) fermions strongly suggest the existence of a unified gauge group such as $SU(5)$ or $SO(10)$. Moreover, the minimal supersymmetric (SUSY) extension of the SM, the MSSM, the gauge couplings unify with a good accuracy at a scale of the order of $\sim 10^{16}$ GeV [1–6]. In the simplest $SU(5)$ scenario, the Higgs doublets unify with triplets in fundamental representations of the gauge group. However, while the doublet Higgs bosons need to remain massless all the way down to the weak scale, their triplet partners need to be heavy in order to achieve coupling unification and suppress proton decay. As we will review in the next section, such a splitting of doublets and triplets is highly unnatural in the minimal models, i.e., it requires an accurate cancellation of seemingly unrelated parameters. Several extensions have been proposed to solve this doublet-triplet splitting problem (DTSP). The elegant and minimal sliding singlet mechanism [7] turns out to fail under closer inspection, even though more complicated versions in $SU(6)$ extensions can work [8–10]. Other solutions to the DTSP that have been suggested include the missing partner mechanism [11–13], pseudo Nambu-Goldstone bosons [14–16], and extra dimensions [17–19]. For $SO(10)$, there is the possibility of the Dimopoulos-Wilczek mechanism [20].

In the $SU(5)$ context, the DTSP is actually a triad of three interconnected problems which need to be dealt with together:

1. The natural separation of doublet and triplet masses. Here, with natural we mean the absence of large cancellations between a priori unrelated free parameters.[1]
2. Sufficient suppression of triplet-Higgs mediated dimension-five operators that trigger proton decay (PD).
3. Precision gauge coupling unification (GCU): any additional gauge representation at the GUT scale will contribute threshold corrections that change unification of gauge couplings.

One could add to this a fourth problem, the so called $\mu/B\mu$ problem, that is, the question why the SUSY breaking soft mass between the two Higgs doublets $B\mu$ is of the same order as the supersymmetric doublet mass μ^2. A simple solution to the latter is found in the Giudice Masiero (GM) mechanism [21].

The idea we would like to propose to solve the above mentioned problems relies on the so-called clockwork mechanism [22] which allows for generation of hierarchical couplings and scales from order-one numbers. The basic idea is to add copies of fields and employ spurious symmetries in order to enforce a special kind of mass matrix that only couples "nearest-neighbours" similar to a one-dimensional lattice Hamiltonian. Some of the light modes then exponentially localize at certain points in this lattice (or theory space), creating suppressed couplings with other low-energy modes. The clockwork mechanism was originally proposed in (rel)axion model building [23–25] and has since been used in many different contexts [26–63] in order to create hierarchies in a natural way. In the present implementation, due to a very mild accidental cancellation, the two physical Higgs doublets localize at opposite ends of the lattice, creating a small μ

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[1] Since in SUSY theories the required operators appear in the superpotential, even in the presence of such tuning the theory is technically natural.

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term, while this localization does not take place for the remaining Higgs doublets and the triplets, leaving them to decouple at the GUT scale. We will also see that our mechanism allows to parametrically separate the effective triplet scales relevant for GCU and PD.

2. Review of the problem(s)

Here we briefly review the DTSP as it manifests itself in the minimal $SU(5)$ GUT model. The Higgs dependent terms in the superpotential are

$$W = \bar{H}(m_1 + m_{24} Y) H + \mathcal{Y}_{ij} \bar{H} A_i \bar{F}_j + \frac{1}{2} \mathcal{Y}'_{ij} H A_i A_j \tag{1}$$

The chiral superfields $\bar{H}$ and $\bar{F}_i$ transform in the $\bar{\mathbf{5}}$ representation, H in the $\mathbf{5}$, and A_i in the $\mathbf{10}$. Furthermore, Y denotes hypercharge,

$$Y \equiv \begin{pmatrix} -\frac{1}{3} & & & & \\ & -\frac{1}{3} & & & \\ & & -\frac{1}{3} & & \\ & & & \frac{1}{2} & \\ & & & & \frac{1}{2} \end{pmatrix} \tag{2}$$

and the coupling proportional to Y in Eq. (1) results from a trilinear $\bar{H}\Sigma H$ coupling of the Higgs fields with the $SU(5)$ breaking adjoint superfield Σ. The DTSP arises from the fact that we need $\mu_D \equiv m_1 + \frac{1}{2} m_{24}$ to be of the weak scale (μ_D is of course nothing but the μ parameter), while the corresponding parameter for the triplet, $\mu_T \equiv m_1 - \frac{1}{3} m_{24}$ has to be of the order of the GUT scale. This is only possible if m_1 and m_{24} are both of the order of the GUT scale, thus implying a delicate cancellation of the two contributions in m_D at the level of one part in 10^{12}.

The reason why μ_T needs to be of the order of the GUT scale is twofold, which under closer inspection reveals another problem. Firstly, let us consider gauge coupling unification (GCU). The weak scale gauge couplings $\alpha_i(m_Z)$ are a function of the unified high scale coupling $\alpha_5(\Lambda)$, and the masses of all the fields, at one loop they read [64]

$$\begin{aligned} \frac{1}{\alpha_3(m_Z)} &= \frac{1}{\alpha_5(\Lambda)} + \frac{1}{2\pi} \left[-4 \log \frac{m_{\text{susy}}}{m_Z} - 3 \log \frac{\Lambda}{m_Z} \right. \\ &\quad \left. -4 \log \frac{\Lambda}{m_V} + 3 \log \frac{\Lambda}{m_\Sigma} + \log \frac{\Lambda}{\mu_T} \right] \end{aligned} \tag{3}$$

$$\begin{aligned} \frac{1}{\alpha_2(m_Z)} &= \frac{1}{\alpha_5(\Lambda)} + \frac{1}{2\pi} \left[-\frac{25}{6} \log \frac{m_{\text{susy}}}{m_Z} + \log \frac{\Lambda}{m_Z} \right. \\ &\quad \left. -6 \log \frac{\Lambda}{m_V} + 2 \log \frac{\Lambda}{m_\Sigma} \right] \end{aligned} \tag{4}$$

$$\begin{aligned} \frac{1}{\alpha_1(m_Z)} &= \frac{1}{\alpha_5(\Lambda)} + \frac{1}{2\pi} \left[-\frac{5}{2} \log \frac{m_{\text{susy}}}{m_Z} + \frac{33}{5} \log \frac{\Lambda}{m_Z} \right. \\ &\quad \left. -10 \log \frac{\Lambda}{m_V} + \frac{2}{5} \log \frac{\Lambda}{\mu_T} \right] \end{aligned} \tag{5}$$

where m_V and m_Σ are the X/Y boson and adjoint scalar masses, and Λ is any UV scale higher than the masses. For simplicity, we have considered a common sparticle mass m_{susy}, our considerations will not depend on this assumption. It is convenient to consider α_3^{-1} as well as the combinations $-2\alpha_3^{-1} - 3\alpha_2^{-1} + 5\alpha_1^{-1}$ and $-2\alpha_3^{-1} + 3\alpha_2^{-1} - \alpha_1^{-1}$. The former difference only depends on the combination $(m_\Sigma m_V^2)^{\frac{1}{3}}$ but is independent of μ_T while the latter only depends on μ_T:

$$\begin{aligned} &(-2\alpha_3^{-1} + 3\alpha_2^{-1} - \alpha_1^{-1})(m_Z) \\ &\quad = \frac{1}{2\pi} \left(\frac{12}{5} \log \frac{\mu_T}{m_Z} - 2 \log \frac{m_{SUSY}}{m_Z} \right) \end{aligned} \tag{6}$$

This relation completely determines the triplet mass. For instance Ref. [65], including more realistic SUSY thresholds as well as some two loop corrections, constrains μ_T to lie in the narrow corridor

$$3.5\ 10^{14}\ \text{GeV} < \mu_T < 3.6\ 10^{15}\ \text{GeV} \tag{7}$$

at the 90% confidence level.

It is well known that this value is in tension with the lifetime of the proton. In particular, integrating out the triplet Higgs gives rise to dimension-five superpotential

$$W_{d=5} = \frac{1}{\mu_T} \mathcal{Y}_{ij} \mathcal{Y}'_{kl} (Q_i L_j + \bar{U}_i \bar{D}_j)(Q_k Q_l + \bar{U}_k \bar{E}_l) \tag{8}$$

The $QQQL$ and $\bar{U}\bar{U}\bar{D}\bar{E}$ operators violate Baryon number leading to PD via loops, which induce a bound on μ_T [65]

$$\mu_T > 7.6\ 10^{16}\ \text{GeV} \tag{9}$$

that is evidently in conflict with Eq. (7). These bounds apply to generic sfermion masses and mixings. It has been pointed out that decoupling the first two sfermion generations and choosing a peculiar pattern of sfermion mixings with the third generation, one can tune the proton decay constraints away [66]. Another option would be to push the scale of all sfermion masses up. However, this requires giving up on naturalness more than presently required experimentally. For the present paper, we are assuming a generic sfermion spectrum with the benchmark bounds Eq. (7) and (9), and explore how our DTS mechanism can ease the tension between GCU and PD.

3. Model

The basic idea behind our proposal is to clone the Higgs sector to include N Higgs fields of $\mathbf{5}$ and $\bar{\mathbf{5}}$ each, with a clockwork-type mass matrix. The superpotential is taken to be

$$W = \bar{H}^T (\mathcal{M}_1 + \mathcal{M}_{24} Y) H + \mathcal{Y}_{ij} \bar{H}_1 A_i \bar{F}_j + \frac{1}{2} \mathcal{Y}'_{ij} H_N A_i A_j. \tag{10}$$

The mass matrices $\mathcal{M}_1$ and $\mathcal{M}_{24}$ are taken of the following structure

$$\mathcal{M}_1 = \alpha_1 M - \beta_1 K \qquad \mathcal{M}_{24} = \alpha_{24} M - \beta_{24} K \tag{11}$$

where $\alpha_{1,24}$ and $\beta_{1,24}$ are dimensionless constants, and the $N \times N$ matrices M and K are two spurions given by

$$(M)_{ij} = m_i \delta_{ij} \qquad (K)_{ij} = k_i \delta_{i,j+1} \tag{12}$$

that is, the mass matrix has all m's in the N diagonal entries and k's in the $N-1$ lower sub-diagonal ones. In the absence of the superpotential, $W = 0$, the theory has a large $G = U(N)' \times U(N)$ chiral symmetry acting on $\bar{H}$ and H respectively (we take H transforming in $(1, N)$ and $\bar{H}$ transforming in $(\bar{N}, 1)$). The spurions M and K then transform in $(N, \bar{N})$. The stabilizer groups are the Abelian subgroups $H_M = U(1)^N$ and $H_K = U(1)^{N+1}$ generated by

$$\mathfrak{h}_M = \{\mathsf{Q}'_i + \mathsf{Q}_i\} \qquad \mathfrak{h}_K = \{\mathsf{Q}'_1, \mathsf{Q}_N, \mathsf{Q}'_i + \mathsf{Q}_{i-1}, i > 1\} \tag{13}$$

The intersection of the two is the vectorlike $U(1)$ generated by $\sum_i \mathsf{Q}_i + \mathsf{Q}'_i$. The Yukawa couplings $\mathcal{Y}$ and $\mathcal{Y}'$ transform in the $(N, 1)$ and $(1, \bar{N})$ respectively and leave the groups $H_{\mathcal{Y}} = U(N-1)' \times U(N)$ and $H_{\mathcal{Y}'} = U'(N) \times U(N-1)$ unbroken, which breaks the remaining $U(1)$. We will assume these are the only spurions breaking G, and explore the phenomenology of this ansatz.

Let us then define the doublet and triplet mass matrices ($X = D, T$)

$$\mathcal{M}_X \equiv M_X - K_X = \begin{pmatrix} m_{X,1} & & & & \\ -k_{X,1} & m_{X,2} & & & \\ & -k_{X,2} & m_{X,3} & & \\ & & \ddots & \ddots & \\ & & & -k_{X,N-1} & m_{X,N} \end{pmatrix} \tag{14}$$

where

$$m_{D,i} \equiv \left(\alpha_1 + \frac{1}{2}\alpha_{24}\right) m_i\,, \qquad k_{D,i} \equiv \left(\beta_1 + \frac{1}{2}\beta_{24}\right) k_i\,, \tag{15}$$

and

$$m_{T,i} \equiv \left(\alpha_1 - \frac{1}{3}\alpha_{24}\right) m_i\,, \qquad k_{T,i} \equiv \left(\beta_1 - \frac{1}{3}\beta_{24}\right) k_i\,. \tag{16}$$

The mass matrices $\mathcal{M}_{D,T}$ are of the clockwork type [22], however with the difference that they have equal number of rows and columns (the spectrum is vector-like), and hence there is no chiral zero mode. What is peculiar in this vector-like clockwork is that for $m_{D,i} \ll k_{D,j}$, there is a single mode with an exponentially suppressed mass

$$\mu_{D,1} \approx \frac{\prod_{i=1}^{N} m_{D,i}}{\prod_{i=1}^{N-1} k_{D_i}} \tag{17}$$

with all the other modes of order $\mu_{D,i+1} \approx k_{D,i}$ $(1 \leq i < N)$. This result can easily be obtained by treating M_D as a small perturbation to K_D and integrating out the vectorlike pairs with masses $k_{D,i}$. Notice that setting any of the $m_{D,i}$ to zero restores a chiral $U(1)$ and thus creates a pair of zero modes, implying that all of the m_D must be nonzero to avoid this case (this is also clear from looking at the determinant of M_D). If, say, m_ℓ vanishes we end up with two traditional chiral clockwork chains, with (for $m_{D,i} \ll k_{D,j}$) one $\bar{H}$ zero mode localized near site 0 and one H zero mode localized near site N. Their wave functions are suppressed at site ℓ, and turning on m_ℓ gives thus an exponentially suppressed Dirac mass to this pair of chiral zero modes. In fact, since there is a large mass gap between the lightest and the other mass eigenvalues, we can use the prescription of Ref. [67] to calculate the corresponding eigenvectors. To this end, define a function LC which when acting on a matrix selects its longest column (in the canonical norm of vectors). For $\mathcal{N} \equiv \mathcal{M}_D^{-1}$, the infinite sequence of vectors

$$\mathrm{LC}(\mathcal{N})\,,\ \mathrm{LC}(\mathcal{N}\mathcal{N}^\dagger)\,,\ \mathrm{LC}(\mathcal{N}\mathcal{N}^\dagger\mathcal{N})\,,\ldots \tag{18}$$

rapidly converges to the (unnormalized) eigenvector of $\mathcal{M}_D^\dagger\mathcal{M}_D$ to eigenvalue $|\mu_{D,1}|^2$ [67]. For the corresponding eigenvector of $\mathcal{M}_D\mathcal{M}_D^\dagger$, one substitutes $\mathcal{N} \to \mathcal{N}^\dagger$. For instance, for $m_{D,i} = m_D$ and $k_{D,i} = k_D$ this gives the leading approximations for the wave functions

$$f_i \approx \left(\frac{m_D}{k_D}\right)^{N-i} \qquad \bar{f}_i \approx \left(\frac{m_D^*}{k_D^*}\right)^{i-1} \tag{19}$$

such that the lightest fields can be written approximately as $h = f^\dagger H \approx H_N$ and $\bar{h} = \bar{f}^\dagger \bar{H} \approx \bar{H}_1$ respectively. We see that the two chiralities participating in the formation of the μ term localize at opposite ends of the theory space. The characteristics of this light mode are reminiscent of the lightest fermion mode in five dimensional compactifications on the interval $[0, L]$ with twisted boundary conditions $(1+\gamma_5)\psi(0) = 0 = (1-\gamma_5)\psi(L)$ [68]. We will comment on this extra-dimensional picture at the end of this letter.

We will now suppose a partial cancellation to happen between the two parameters α_1 and $\frac{1}{2}\alpha_{24}$, such that $m_{D,i} \ll k_{D,i}$ (we will quantify this in a moment), and a light mode given by Eq. (17) exists. Of course, such a cancellation cannot occur simultaneously for the triplets, and such a light mode is absent. In the following, we will take all the $m_i = m$ and $k_i = k$, for unequal parameters we need to replace k^{N-1} and m^N by the products of the individual parameters in all of the expressions below, this will not change our conclusions. The μ term is then simply given as

$$\mu \equiv \mu_{D,1} \approx \frac{(m_D)^N}{(k_D)^{N-1}} \tag{20}$$

Moreover, we can derive very simple exact formulas regarding GCU and dimension-five PD. For GCU, each additional $5+\bar{5}$ with masses μ'_T and μ'_D contribute $\frac{1}{2\pi}\Delta_i$ to the gauge couplings in Eq. (3)–(5) where

$$\Delta_3 = \log\frac{\Lambda}{\mu'_T}\,, \qquad \Delta_2 = \log\frac{\Lambda}{\mu'_D}\,, \tag{21}$$

$$\Delta_1 = \frac{3}{5}\log\frac{\Lambda}{\mu'_D} + \frac{2}{5}\log\frac{\Lambda}{\mu'_T}\,. \tag{22}$$

Then, in Eq. (6) we must replace μ_T with an effective triplet mass

$$\mu_T \to \mu_T^{\mathrm{eff}} \equiv \left|\frac{\mu_{T,1}\cdots\mu_{T,N}}{\mu_{D,2}\cdots\mu_{D,N}}\right| = \frac{|\det\mathcal{M}_T|}{|\det\mathcal{M}_D|}|\mu| \approx \frac{|m_T|^N}{|k_D|^{N-1}}\,. \tag{23}$$

It is this quantity to which the constraint Eq. (7) applies.

On the other hand the coefficient of the dimension-five operator in Eq. (8) is replaced by

$$\frac{1}{\mu_T} \to \frac{1}{\tilde{\mu}_T^{\mathrm{eff}}} \equiv (\mathcal{M}_T^{-1})_{N1} = \frac{(k_T)^{N-1}}{(m_T)^N}\,. \tag{24}$$

Now, the constraint in Eq. (9) applies to $\tilde{\mu}_T^{\mathrm{eff}}$, which is different from μ_T^{eff}. More explicitly, we have

$$\left|\frac{\tilde{\mu}_T^{\mathrm{eff}}}{\mu_T^{\mathrm{eff}}}\right| = \left|\frac{k_D}{k_T}\right|^{N-1}. \tag{25}$$

For instance, consider the case where $\beta_1 = 0$ in Eq. (11), then $k_D/k_T = \frac{3}{2}$ and for small to moderate N, one very efficiently separates the two scales and completely removes the tension of the minimal $SU(5)$ model. As far as this separation is concerned, our mechanism is related to the idea of Ref. [69] which considers a similar mass matrix with $N = 2$.

Next, consider the ratio

$$\left|\frac{\mu_T^{\mathrm{eff}}}{\mu}\right| = \left|\frac{m_T}{m_D}\right|^N \tag{26}$$

For a $|\mu| \sim 1$ TeV, we find that we need $|m_T/m_D| \approx 6.3$ (15.8) for $N = 15$ ($N = 10$). The cancellation between α_1 and α_{24} for this hierarchy is at the level of 25% (10%), in other words mild when compared to the original doublet-triplet splitting. We should point out that if the masses are not site-independent there is more tuning involved, as a priori there is no reason why $k_{D,i} > m_{D,i}$ is more likely then $k_{D,i} < m_{D,i}$. One possible rationale to enforce a uniform choice of parameters is a translational symmetry inherited from a fifth dimension, which we comment on at the end of this letter.

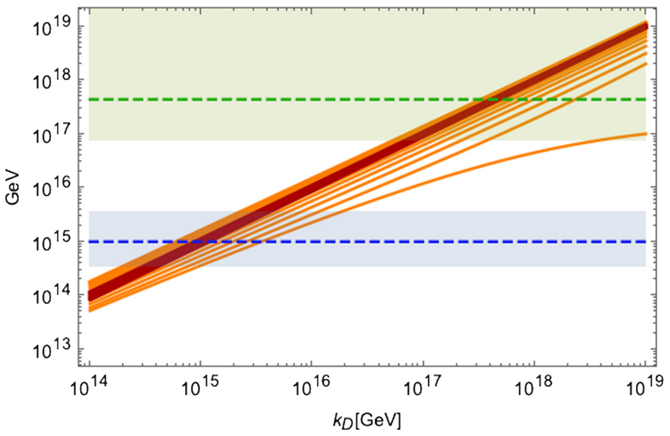

Fig. 1. Spectrum for the heavy doublets (red) and triplets (orange) as a function of the parameter k_D, for $\mu = 1$ TeV and $N = 15$. The blue and green dashed lines are the parameters $\mu_T^{\rm eff}$ (fixed to 10^{15} GeV) and $\tilde{\mu}_T^{\rm eff}$ respectively, and the shaded regions mark their allowed values (from GCU and PD respectively).

Notice that the overall scale of the mass parameters k and m is not fixed by these considerations alone. Setting $\beta_1 = 0$, we are left with three more parameters $\beta_{24}k$, $\alpha_1 m$ and $\alpha_{24}m$, two of which can be eliminated by fixing μ and $\mu_T^{\rm eff}$. We take as the remaining free parameter $k_D = \frac{1}{2}\beta_{24}k$, which is to a good approximation the (nearly degenerate) mass of heavy doublets. We show in Fig. 1 the spectrum as a function of $|k_D|$. For increasing k_D, the GUT breaking vacuum expectation value v_{24}, defined by $\langle\Sigma\rangle = v_{24}Y$ should also go up to avoid a nonperturbative $\bar{H}\Sigma H$ coupling. This means that in this region, the dimension-six proton decay operators are more suppressed, while there is a splitting $m_V \gg m_\Sigma$ as $m_\Sigma m_V^2$ is also tightly constrained by GCU.

Since we are adding a potentially large number of representations, one might wonder if the gauge coupling becomes nonperturbative below the Planck scale. However, this only happens when $N \gtrsim 40$, and such large numbers are not required to solve the DTSP. In this sense there is an advantage over the missing partner mechanism, which employs exotic representations with large Dynkin indices. For instance, the $\mathbf{50} + \overline{\mathbf{50}} + \mathbf{75}$ [11] contribute to the running as much as the equivalent of 60 $\mathbf{5} + \bar{\mathbf{5}}$ representations, leading to the well-known non-perturbativity issues of that model that need to be resolved with more sophisticated model building [13].

Let us also comment on the $\mu/B\mu$ problem. A simple way to implement the GM mechanism is to demand that the doublet mass $\mu_{D,1}$ created from the present mechanism is actually even smaller than the TeV scale and that there is another (dominant) contribution to μ coming from the operator $X^*\bar{H}H$ in the Kähler potential (whereas the $B\mu$ term is coming from $|X|^2\bar{H}H$). These contributions are of course completely negligible for all the other doublets and triplets.

Finally, we would like to make some comments on possible extra-dimensional implementations of our mechanism. The fields H and $\bar{H}$ unify into an $\mathcal{N} = 2$ five-dimensional (5D) hypermultiplet. It is however more convenient to retain an $\mathcal{N} = 1$ superfield language [70] with two chiral fields dependent on a continuous coordinate y, which we will discretize to compare to our CW chain. One way to obtain the localization effects needed is via 5D mass terms. In the simplest case of a flat extra dimension compactified on an interval of length L, and imposing the boundary conditions $H(0) = 0$, and $\bar{H}(L) = 0$, one gets

$$K = \frac{L}{N}\left(\sum_{i=1}^{N} H_i^\dagger H_i + \sum_{i=0}^{N-1} \bar{H}_i^\dagger \bar{H}_i\right), \tag{27}$$

and

$$W = \bar{H}_0\mathcal{O} + \frac{1}{2}H_N\bar{\mathcal{O}} + \sum_{i=0}^{N-1}\bar{H}_i(H_{i+1} - H_i) - \frac{L}{N}m_5\sum_{i=1}^{N-1}\bar{H}_i H_i\,, \tag{28}$$

where $\mathcal{O}$ and $\bar{\mathcal{O}}$ are shorthand for the fermion bilinears the two Higgs fields are coupling to. After canonical normalization and renaming $\bar{H}_i \to \bar{H}_{i+1}$ one obtains precisely our clockwork Lagrangian with[2]

$$k = \frac{N}{L} + m_5\,, \qquad m = \frac{N}{L} \tag{29}$$

For $m < k$, that is, for $m_5 > 0$ we expect a light mode to appear. Indeed, by either solving the 5D equations of motion [68] or by diagonalization of the infinite CW mass matrix[3] one finds

$$\mu \approx 2m_5 e^{-m_5 L}\,, \qquad m_5 > 0\,. \tag{30}$$

The wave functions for H and $\bar{H}$ corresponding to this state indeed localize at opposite boundaries [68]

$$f(y) \sim e^{m_5(y-L)} - e^{-m_5(y+L)} \qquad \bar{f}(y) \sim e^{-m_5 y} - e^{m_5(y-2L)}\,, \tag{31}$$

consistent with Eq. (19). If the sign of m_5 is negative however, no such localization takes place and the lightest mode has a mass $\mu \approx |m_5|$ for $|m_5|L \gg 1$ or $\mu \approx \pi/2L$ for $(|m_5|L \ll 1)$.[4] By making m_5 hypercharge dependent (assuming, for simplicity, a constant profile for the GUT symmetry breaking field) one can easily achieve $m_{5,D} > 0$ and $m_{5,T} < 0$, thus obtaining an exponentially light doublet state with the wave functions for H and $\bar{H}$ localized at two different boundaries, while all the triplet modes remain heavy and delocalized. Moving on to PD, one obtains after integrating out the triplet fields the continuum version of Eq. (24)

$$\frac{1}{\tilde{\mu}_T^{\rm eff}} = L e^{m_{5,T}L}\,, \tag{32}$$

such that PD is exponentially suppressed as $m_{5,T} < 0$. From a 5D point of view this suppression is natural: in order to create the dimension-5 operator Eq. (8) one needs to propagate a massive 5D field over a nonzero distance L. Finally, GCU is a little bit different from the four dimensional model, as in the latter we have simply assumed that only the Higgs fields possess a clockwork chain, while in the 5D realization it is mandatory to propagate also the gauge multiplets as well as the GUT breaking Higgs sector in the bulk, which will also contribute to the running. Moreover, at least the **10** representation should also propagate in the extra dimension, as it necessarily couples to both branes. We will leave a more detailed analysis of this model to future work.

Notice that our 5D setup is different from earlier models. Ref. [17] uses $SU(5)$ braking boundary conditions in order to project out the light triplet modes while retaining the doublets.

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[2] Notice that only k but not m can depend on hypercharge via m_5. As we will see, this is sufficient for suppression of the doublet mass as well as PD. GCU on the other hand cannot be computed without further assumptions on the gauge, matter and GUT Higgs sectors, see below.

[3] Here one has to be a bit careful with the formula Eq. (20) which needs to be corrected when $k/m \approx 1$. To obtain the lightest eigenvalue μ directly from the mass matrix we write $\mu^{-2} \approx \mathrm{Tr}(\mathcal{M}^\dagger\mathcal{M})^{-1} = m^{-2}\sum_{n=1}^{N} n\left(\frac{k}{m}\right)^{2(N-n)} = \int_0^L dy\, y\, e^{2m_5(L-y)} \approx \left(\frac{e^{m_5 L}}{2m_5}\right)^2$, in accordance with the 5D result [68].

[4] For a very general analysis of hypermultiplet spectra in the presence of bulk and brane mass terms see Refs. [71–73].

Ref. [74] can be considered a variant of this idea by putting the Higgs doublets as four-dimensional fields on the GUT breaking boundary without any accompanying triplets. Refs. [18,19] consider a non-compact extra dimension and generate Gaussian profiles for both doublets and triplets that peak at different points in space, due to a nontrivial kink profile of the $SU(5)$ breaking field. In our case, the profiles corresponding to the light doublet are simple exponentials peaking at the two opposite boundaries (see Eq. (31)), while the profiles for the triplets are delocalized over the bulk. Still, as we have shown, an exponential suppression of dimension-5 proton decay operators exists due to the nonlocal nature of these operators.

In summary, we have presented a novel mechanism to solve the DTSP in $SU(5)$ grand unification models, which at the same time removes the tension between the limit on the triplet scales for GCU and PD. The 5D realization of this idea is different from previous ones and constitutes a promising new direction itself, but requires more in-depth model-building beyond the scope of the present article.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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