

Electron-capture and its role to explosive neutrino-nucleosynthesis

P.G. Giannaka and T.S. Kosmas

Theoretical Physics Division, University of Ioannina, GR-45110 Ioannina, Greece

E-mail: pgiannak@cc.uoi.gr

Abstract. In explosive nucleosynthesis, among other reactions, prominent position posses the electron capture by nucleons and nuclei. This process plays important role in the core-collapse of massive stars by modifying the quantity Y_e . From a nuclear theory point of view, this process may be studied by using the same nuclear methods (e.g. the quasi-particle random phase approximation, QRPA, employed in the present work) with the one-body charge changing nuclear processes (β -decay modes, charged-current electron-neutrino absorption by nuclei, etc). In this work we concentrate on a detailed study of the orbital e-capture by medium-heavy nuclei (Fe region) which play fundamental role in the evolution of massive stars and in the so called explosive neutrino nucleosynthesis. We study extensively the cross sections and rates of e-capture process by “iron group peaked nuclei”, isotopes that are important for investigating the neutrino nucleosynthesis and other related phenomena.

1. Introduction

The recent studies on presupernova scenarios indicate that e^- -capture on nucleons and nuclei is an important process, mostly in the early stage of the collapse of massive stars. Refinements of the methods used for calculating electron capture on heavy nuclei during the collapse phase shows that the capture on free protons dominates [1, 2], in contrast to the findings of previous works. Captures on nuclei produce neutrinos with rather small energies, making the inclusion of inelastic neutrino-nucleus reactions in supernova simulations more important. Even though capture on free protons is favored compared to that on nuclei, due to the significantly lower Q-value (difference between the nuclear mass of the initial and final state), the number of free protons divided by the total number of nucleons (Y_p) is quite low ($Y_p \sim 10^{-6}$ in the $15M_\odot$ presupernova model) [2, 3].

Obviously, electron capture reduces the ratio Y_e , favoring nuclear composition with more neutron-rich and heavier nuclei, including those with $N > 40$, which dominate the matter composition for densities larger than about $10^{10} g cm^{-3}$. Also recently it was shown that some models for electron capture on nuclei are not realistic, because the Pauli blocking of the GT transitions overestimates the correlations and temperature effects [2].

In this work, we study in detail the orbital electron-capture on medium-heavy nuclei, process represented by reaction

$$(A, Z) + e^- \rightarrow (A, Z - 1)^* + \nu_e \quad (1)$$

Even though the investigation of this process started decades ago, more accurate transition rates are needed due to its significant importance in core-collapse supernova dynamics and

other astrophysical phenomena. To this purpose, in the present work we have chosen to study this process in a set of isotopes which are important from an astrophysical point of view that belong to the “iron group peaked nuclei” [4]. We focus on extensive cross sections calculations following the Donnelly-Walecka multipole decomposition method [7]. For the computation of the many-body nuclear wave functions we utilize the quasi-particle random phase approximation (or QRPA for short) as described below [6, 7, 8].

2. The Donnelly-Walecka multipole decomposition method

In this paper, the method of studying orbital e^- -capture process on nuclei, compared to previous similar works, is improved in the following: 1) In the level where use of compact analytical expressions for the required reduced matrix elements of all basic multipole operators is made. 2) In the level of constructing the nuclear Hamiltonian where the Bonn C-D potential (instead of the Bonn C used up to now) for the realistic two-body nuclear forces is employed [6, 7, 8].

The nuclear calculations start by writing down the weak interaction Hamiltonian \hat{H}_w which in a current-current form describes all the semi-leptonic processes in the presence of nuclei. By denoting the leptonic current as j_μ^{lept} and the hadronic current as \hat{J}_μ , \hat{H}_w is written as:

$$\hat{H}_w = \frac{G \cos \theta_c}{\sqrt{2}} j_\mu^{lept}(\mathbf{x}) \hat{J}_\mu(\mathbf{x}) \quad (2)$$

where $G = 1.1664 \times 10^{-5} \text{ GeV}^{-2}$ and θ_c is the Cabibbo angle ($\cos \theta_c = 0.974$) [7]. The hadronic current, which is of interest from a nuclear theory point of view, consists of the polar-vector and the axial-vector components which can be written by using isovector, axial-vector and pseudoscalar form factors (they depend on the four-momentum transfer q_μ^2) and read:

$$\hat{J}_\mu = g_V \hat{J}_\mu^V - g_A \hat{J}_\mu^A \quad (3)$$

In Eq. (3), g_V (g_A) represent the coupling of the polar (axial) vector interaction (at $q_\mu^2 = 0$, static parameters).

One of the main goals of this effort is to construct an advantageous code for calculating electron-capture cross sections in various currently interesting nuclear isotopes. This code is based on the present formalism and the QRPA that uses realistic Bonn C-D residual interaction [7].

3. Evaluation of the nuclear wave functions within QRPA.

It is well known that the nucleus consists of protons and neutrons which, in a rather rough approximation can be considered as independent particles, attracted by the nuclear center through a central strong force. This attraction can be described by a mean field, as for example the Woods-Saxon potential. In our case, use of a Woods-Saxon potential with Coulomb corrections and a spin orbit part is made.

The two-nucleon correlations, also known as the residual two-body interaction, are necessary to be included in a reliable nuclear Hamiltonian. In our case they are assumed to be described by the Bonn C-D potential. The initially evaluated bare two-body nucleon-nucleon matrix-elements of the latter interaction refer to a specific isotope of mass number A. In the specific isotope (A, Z) studied, a renormalization of the two-body interaction is required. Towards this purpose we use four multiplicative parameters that renormalize the bare residual interaction. The first two $g_{pair}^{p,n}$, known as pairing parameters for protons (p) and neutrons (n) renormalize the monopole interaction, the third g_{pp} tunes the particle-particle channel and the fourth g_{pn} renormalizes the particle-hole interaction.

Table 1. Parameters for the interaction of proton pairs, g_{pair}^p , and neutron pairs, g_{pair}^n . They are fixed in such way that the corresponding experimental gaps, Δ_n^{exp} and Δ_p^{exp} , to be reproduced.

	Abundance(%)	b (h.o.)	g_{pair}^n	g_{pair}^p	S_n (MeV)	S_p (MeV)	Δ_n^{exp} (MeV)	Δ_n^{theor} (MeV)	Δ_p^{exp} (MeV)	Δ_p^{theor} (MeV)
^{96}Ru	5,540	2,158	0.987	0.835	10,694	7,344	1,0824	1,0821	1,4955	1,4954
^{102}Pd	1,020	2,178	0.978	0.958	10,568	7,806	1,3094	1,3085	1,4947	1,4938

^{96}Ru			^{102}Pd		
State	g_{ph}	g_{pp}	State	g_{ph}	g_{pp}
0^+	0.403	0.781	0^+	0.377	0.907
2^+	0.579	1.189	2^+	0.671	1.350
4^+	0.905	0.546	4^+	1.040	0.322
6^+	1.085	1.195	6^+	1.108	0.274

Table 2. Strength parameters for the particle-particle (g_{pp}) and particle-hole (g_{ph}) interaction for various multipolarities.

The ground state of the nucleus in question, is obtained in the context of the Bardeen Cooper Schrieffer (BCS) theory. The solution of the relevant BCS equations gives the probability amplitudes V and U for each single particle level to be occupied or unoccupied, respectively, and the quasi-particle energies.

Subsequently, we construct the excited states of the studied isotope by solving the QRPA equations, which in matrix form are written as

$$\begin{pmatrix} \mathcal{A} & \mathcal{B} \\ -\mathcal{B} & -\mathcal{A} \end{pmatrix} \begin{pmatrix} X^m \\ Y^m \end{pmatrix} = \Omega_{J^\pi}^m \begin{pmatrix} X^m \\ Y^m \end{pmatrix} \quad (4)$$

Their solution is an eigenvalue problem, which gives us the X and Y amplitudes for forward and backward scattering, respectively, as well as the QRPA excitation energies [8]. In the Donelly-Walecka method the solution of the QRPA equations is obtained separately for each multipole set of states $|J^\pi\rangle$.

4. Results and discussion

In Table 1 the values of the BCS parameters are listed through which we construct the wave function for the initial (ground) state. The parameters determined for the final states, of the isotope in question are shown in Table 2.

After the determination of the particle-particle and particle-hole parameters, we proceed by checking the reproducibility of the energy spectrum of the studied isotopes. In Figure 1, as a concrete example we show the energy spectrum of the ^{96}Ru isotope. For comparison, the experimental spectrum is also shown. As can be seen, the agreement is good in the low-lying spectrum.

In the final step we receive the results for cross-sections of the process which, as is known, are proportional to the square of the relevant matrix elements [7, 8]. The required nuclear matrix elements are calculated in the context of the QRPA [7].

The model space chosen in our QRPA calculations includes harmonic oscillator levels as follows: for the ^{96}Ru isotope (14 orbits) $0d_{5/2}, 1s_{1/2}, 0d_{3/2}, 0f_{7/2}, 1p_{1/2}, 1p_{3/2}, 0f_{5/2}, 0g_{9/2}, 1d_{5/2}, 0g_{7/2}, 1d_{3/2}, 2s_{1/2}, 0h_{9/2}, 0h_{11/2}$, and for the ^{102}Pd (11 orbits) $0f_{7/2}, 1p_{1/2}, 1p_{3/2}, 0f_{5/2}, 0g_{9/2}, 1d_{5/2}, 0g_{7/2}, 1d_{3/2}, 2s_{1/2}, 0h_{9/2}, 0h_{11/2}$ [7].

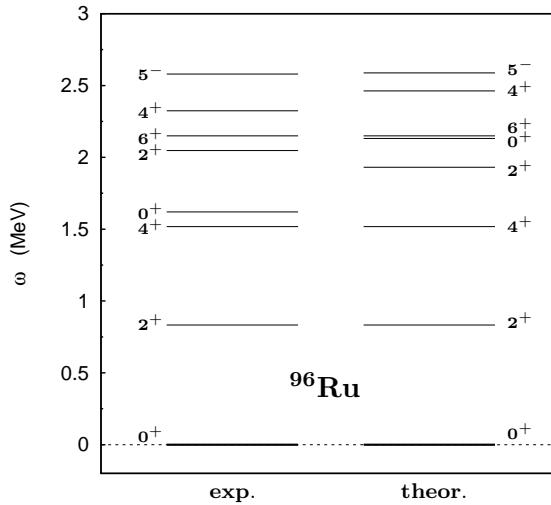


Figure 1. Comparison of the QRPA (theoretical) excitation spectrum (resulting from the solution of the QRPA eigenvalue problem) with the low-lying (up to about 3 MeV) experimental one. The agreement is good.

The reliability of the present calculations, could come out of the comparison of our electron capture cross sections with those evaluated in other similar processes, e.g the corresponding charged-current neutrino nucleus reactions, i.e. the particle conjugate process of the electron capture.

5. Summary and Conclusions

As discussed in the Introduction, during the presupernova and collapse phase, electron captures on nuclei (and in the late stage also on free protons) plays an important role, as the nuclear β -decay during silicon burning does. Electron captures become increasingly possible as the density in the star's center is increased. It is accompanied by an increase of the chemical potential (Fermi energy) of the degenerate electron gas and it reduces the electron-to-baryon ratio Y_e of the matter composition.

In this work, we use an advantageous numerical approach, constructed by our group recently, to calculate all basic multipole transition matrix elements needed for obtained e-capture cross sections. The required nuclear wave functions are obtained within the context of the QRPA using realistic two-body forces. Results for the cross sections are expected to be received soon.

5.1. Acknowledgments

This research has been co-financed by the European Union (European Social Fund ESF) and Greek national funds through the Operational Program Education and Lifelong Learning" of the National Strategic Reference Framework (NSRF) - Research Funding Program: Heracleitus II. Investing in knowledge society through the European Social Fund.

References

- [1] Juodagalvis A , K Langanke et al 2005 *Nucl. Phys. A* **747** 87
- [2] Langanke K, Martinez-Pinedo G, et al 2003 *Phys. Rev. Lett.* **90** 241102.
- [3] Langanke K, Martinez-Pinedo G, et al 2003 *Rev. Mod. Phys.* **75**.
- [4] Frohlich C, Martinez-Pinedo G, et al 2006 *Phys. Rev. Lett.* **96** 142502.
- [5] Kosmas T S and Oset E 1996 *Phys. Rev. C* **53** 1409.
- [6] Donnelly T W and Peccei R D 1979 *Phys. Rep.* **50** 1.
- [7] Chasioti V C and Kosmas T S 2009 *Nucl. Phys. A* **829** 234.
- [8] Tsakstara V and Kosmas T S 2011 *Phys. Rev. C* **83** 054612.