Neutrino masses and mixing in an inverse seesaw (2,3) model augmented with S_4 modular flavor symmetry

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Abstract. In our work, we constructed an inverse seesaw(2,3) model using the modular invariance approach. The predictability of the model is enhanced and the number of flavon fields reduced by using this modular invariance approach. Here, we have used the S_4 modular group to assist us design the model. Within the present framework, the neutrino phenomenology can be studied with the help of the non-trivial transformation of Yukawa couplings. The right-handed neutrino mass can be experimentally verified by reducing it to the TeV range via the application of the inverse seesaw mechanism. In this work, we build the neutrino mass matrix and explain about the neutrino mixing phenomena. We show that the obtained CP violating phase and mixing angles are compatible with the observed 3σ ranges of existing neutrino oscillation data.

1 Introduction

The conclusive evidence of the existence of massive neutrinos implies an emerging area of particle physics beyond the Standard Model (SM). Precision measurements of neutrino data, which includes solar and atmospheric neutrino mass splittings (Δm^2_{ij}) and reactor angles (θ_{ij}) of neutrino flavor mixings, are being carried out by the neutrino experiments. However, there are still certain parameters that need to be determined experimentally, such as the neutrino mass ordering, octant degeneracy of the atmospheric mixing (θ_{23}), CP violating Dirac and Majorana phases δ, α, and β. It is important to look into possibilities beyond the standard model (BSM) in order to generate non-zero neutrino masses. The existence of sterile neutrinos, termed as SM gauge singlets, is a key feature of BSM scenarios. These are considered as right-handed neutrinos connected via Yukawa interactions to the active neutrinos under the standard model. The mass of right-handed neutrinos is thought to be of the order of 10^{15} GeV in the conventional seesaw framework, which is beyond the reach of current neutrino oscillation experiments to explain the eV scale light neutrinos. Nevertheless, there are several low-scale mechanisms that can be verified experimentally, such as the inverse seesaw[3, 4], linear seesaw[5], extended seesaw[6], etc., where the heavy neutrino mass is taken in the TeV scale range.

However, one of the challenges in SM is understanding the flavor structure of fermions. Hence, symmetry can be crucial for describing the flavor structure. In the literature[7–11], some non-abelian discrete flavor symmetry groups S_4, A_4, A_5, etc. are found to be effective for constructing models. According to these approaches, the discrete flavor symmetry cannot

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be spontaneously broken without additional flavon fields[12], and the model becomes less predictable when higher dimensional operators are implemented. A modular invariance technique has been developed in the literature[13] to prevent such challenges, which has several appealing characteristics. This method enhances the predictability of the model by eliminating the need for flavon fields other than the modulus[14]. This encourages us to explore the present model.

In this study, we use modular S_4 symmetry to construct an inverse seesaw model (2,3). We generate the desired neutrino mass matrix in our scenario using the proposed symmetry, and we conduct a numerical analysis for various parameters to fit in with the existing neutrino oscillation data.

The structure of the paper is as follows. The formalism and the neutrino mass matrix from the inverse seesaw process with modular S_4 symmetry are described in Section 2. Section 3 presents the numerical study of correlations between the neutrino sector observables and the input model parameters. Finally, we conclude our results in Section 4.

2 Model Building

In this section, we present our model framework for the inverse seesaw mechanism that introduces modular S_4 symmetry. The fact that S_4 modular symmetry requires fewer flavons than the typical S_4 group emphasizes its significance. The group transformation associated with Yukawa couplings is non-trivial in this case. In this model, we introduce left-handed sterile fermions transformed as $\{1, 1'\}$ under modular S_4 symmetry, and right-handed $SU(2)$ singlet fermions N_R, which are transformed as doublet under the same symmetry. Additionally, considering modular S_4 symmetry, two Higgs doublets H_u, H_d have charge 1.

Table 1 provides the particle content and group charges under $S_4 \times k_I$ for the model. Here, the number of modular weights is indicated by k_I.

Table 1: Particle content of the model and their charges

Fields	L	l_R	N_R	S_i	H_u	H_d	ϕ
$SU(2)_L$	2	1	1	1	2	2	1
S_4	3	$1, 1'$	2	3	1	1	1
k_I	-2	-2	-2	-2	0	0	0

Table 2: Modular weight attributed to the Yukawa couplings

Yukawa coupling	S_4	k_I
Y	3	2

The Lagrangian for the leptonic sector of the model is provided by the above charge assignments and symmetries.

$$\mathcal{L}_{lepton} = \mathcal{L}_L + \mathcal{L}_D + \mathcal{L}_{NS} + \mathcal{L}_S \tag{1}$$

where $\mathcal{L}_L$ is the mass term for charged leptons, $\mathcal{L}_D$ is the Dirac mass term which connects left-handed (ν_L) and right-handed (N_R) components of neutrinos, $\mathcal{L}_{NS}$ reprsents the mixing term for right-handed neutrinos (N_R) and sterile fermions (S_i) and $\mathcal{L}_S$ is the majorana mass term for gauge singlet sterile fermions (S_i).

2.1 Charged Lepton Mass Matrix

Under S_4 symmetry, the left-handed lepton doublets $(l_{L_e}, l_{L_\mu}, l_{L_\tau})$ in this model are transformed into singlet $(1, 1^{''}, 1^{'})$. Further, under S_4, the right-handed lepton l_R is transformed into a triplet. They have been allocated a modular weight of -2. Similarly, under this symmetry, the yukawa couplings $Y = (Y_1, Y_2, Y_3)$ transform as triplet $(3, 3^{'})$, with $(2, 4)$ as the modular weight.

The Lagrangian for the charged leptons is given by

$$\mathcal{L}_L = \alpha(LE_1^c)_{3'}(H_d)_1 Y_{3'} + \beta(LE_2^c)_3(H_d)_1 Y_3^{(4)} + \gamma(LE_3^c)_{3'}(H_d)_1 Y_{3'}^{(4)} \tag{2}$$

where α, β, γ are the free parameters. The charged lepton mass matrix can be written as

$$M_L = v_d \begin{pmatrix} \alpha Y_3 & -2\beta Y_2 Y_3 & 2\gamma Y_1 Y_3 \\ \alpha Y_5 & \beta(\sqrt{3} Y_1 Y_4 + Y_2 Y_5) & \gamma(\sqrt{3} Y_2 Y_4 + Y_1 Y_5) \\ \alpha Y_2 & \beta(\sqrt{3} Y_1 Y_5 + Y_2 Y_4) & \gamma(\sqrt{3} Y_2 Y_5 - Y_1 Y_4) \end{pmatrix} \tag{3}$$

where v_d is the vev for down type Higgs doublet.

2.2 Dirac Mass Matrix for the model

Here, the right-handed $SU(2)$ singlet fermions N_R transformed as doublet under S_4 group. The modular weight assigned to it is -2.

Hence, the Lagrangian for Dirac mass term for the model can be written as

$$\mathcal{L}_D = (LN_R)_3(H_u)_1 Y_3^{(4)} + (LN_R)_{3'}(H_u)_1 Y_{3'}^{(4)} \tag{4}$$

Now, the Dirac mass matrix for the neutrinos is given by

$$M_D = v_u \begin{pmatrix} -2\alpha_D a & -2\beta_D b \\ -\alpha_D(\frac{\sqrt{3}}{2}c + \frac{1}{2}d) + \beta_D(\frac{3}{2}d - \frac{\sqrt{3}}{2}e) & \alpha_D(\frac{3}{2}f + \frac{\sqrt{3}}{2}c) + \beta_D(\frac{3}{2}f + \frac{\sqrt{3}}{2}c) \\ -\alpha_D(\frac{\sqrt{3}}{2}f + \frac{1}{2}c) + \beta_D(\frac{3}{2}c - \frac{\sqrt{3}}{2}f) & \alpha_D(\frac{3}{2}e + \frac{\sqrt{3}}{2}d) + \beta_D(\frac{3}{2}e + \frac{\sqrt{3}}{2}d) \end{pmatrix} \tag{5}$$

where v_u is the vev for the up type Higgs doublet and α_D, β_D are the free parameters. Here, we denote $a = Y_2 Y_3$, $b = Y_1 Y_3$, $c = Y_2 Y_4$, $d = Y_2 Y_5$, $e = Y_1 Y_4$ and $f = Y_1 Y_5$.

2.3 Mass term for the mixing between the right-handed neutrinos and gauge singlet neutral fermions

Here, the additional sterile fermion S_i transforms as triplet under S_4 symmetry and the modular weight assigned to it is -2.

Thus, the Lagrangian for this mixing can be written as

$$\mathcal{L}_{NS} = \alpha_{NS}(Y_3^{(4)} N_R)_3(S_1)_3\phi + \beta_{NS}(Y_{3'}^{(4)} N_R)_3(S_2)_{3'}\phi \tag{6}$$

The resultant mass matrix for this mixing is found to be

$$M_{NS} = v_{NS} \begin{pmatrix} -2\alpha_{NS} a & -\frac{\alpha_{NS}}{2}(\sqrt{3}e + d) + \frac{\sqrt{3}}{2}\beta_{NS}(\sqrt{3}d - e) & -\frac{\alpha_{NS}}{2}(\sqrt{3}f + c) + \frac{\sqrt{3}}{2}\beta_{NS}(\sqrt{3}c - f) \\ -2\beta_{NS} b & \frac{\sqrt{3}}{2}\alpha_{NS}(\sqrt{3}f + c) + \frac{\beta_{NS}}{2}(\sqrt{3}c - f) & \frac{\sqrt{3}}{2}\alpha_{NS}(\sqrt{3}e + d) + \frac{\beta_{NS}}{2}(\sqrt{3}d - e) \end{pmatrix} \tag{7}$$

where α_{NS}, β_{NS} are the free parameters and v_{NS} is the vev for scalar field ϕ.

2.4 Majorana mass term

The Lagrangian for Majorana mass term can be written as

$$\mathcal{L}_S = \mu_0 S S Y_3^{(4)} \tag{8}$$

So, the Majorana mass matrix becomes

$$M_S = \mu_0 \begin{pmatrix} 0 & -(\sqrt{3}f + c) & \sqrt{3}e + d \\ -(\sqrt{3}f + c) & 2a & 0 \\ \sqrt{3}e + d & 0 & -2a \end{pmatrix} \tag{9}$$

2.5 Inverse Seesaw Mechanism for light neutrino masses

The complete neutrino mass matrix for the present model is given by

$$M_\nu = \begin{pmatrix} 0 & M_D & 0 \\ M_D^T & 0 & M_{NS} \\ 0 & M_{NS}^T & M_S \end{pmatrix} \tag{10}$$

where M_D is at electroweak scale, M_{NS} at TeV scale and M_S at keV scale to get SM neutrinos at sub-eV scale[16]. Now, the inverse seesaw formula for light neutrinos is given by

$$M_\nu = M_D M_{NS}^{-1} M_S (M_{NS}^{-1})^T M_D^T \tag{11}$$

3 Numerical Analysis

In this section, we will carry out some calculations for parameters satisfying neutrino oscillation data [15]. Table 3 gives the 3σ ranges for current neutrino oscillation.

Table 3: Current neutrino oscillation parameters from global fits[15]

Oscillation parameters	Normal Ordering		Inverted Ordering	
	Best fit$\pm 1\sigma$	3σ range	Best fit$\pm 1\sigma$	3σ range
$\Delta m_{21}^2/10^{-5}eV^2$	$7.42^{+0.21}_{-0.20}$	$6.82 - 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 - 8.04$
$\lvert\Delta m_{3l}^2\rvert/10^{-3}eV^2$	$2.510^{+0.027}_{-0.027}$	$2.430 - 2.593$	$-2.490^{+0.026}_{-0.028}$	$-2.574 - -2.410$
θ_{12}°	$33.45^{+0.77}_{-0.75}$	$31.27 - 35.87$	$33.45^{+0.78}_{-0.75}$	$31.27 - 35.87$
θ_{23}°	$42.1^{+1.1}_{-0.9}$	$39.7 - 50.9$	$49.0^{+0.9}_{-1.9}$	$39.8 - 51.6$
θ_{13}°	$8.62^{+0.12}_{-0.12}$	$8.25 - 8.98$	$8.61^{+0.14}_{-0.12}$	$8.24 - 9.02$
δ_{CP}°	230^{+36}_{-25}	$144 - 350$	278^{+22}_{-30}	$194 - 345$

Here, the neutrino mass matrix of Eq. 11 can be numerically diagonalized by a unitary matrix U, from which the standard relations can be used to obtain the neutrino mixing angles:

$$\sin^2\theta_{13} = |U_{13}|^2, \; \sin^2\theta_{12} = \frac{|U_{12}^2|}{1 - |U_{13}^2|}, \; \sin^2\theta_{23} = \frac{|U_{23}^2|}{1 - |U_{13}^2|} \tag{12}$$

Apart from this, the Jarlskog invariant which controls the size of CP violation in the quark and leptonic sector is given by the equation

$$\begin{aligned} J_{CP} &= \text{Im}[U_{e1} U_{\mu 2} U^*_{e2} U^*_{\mu 1}] \\ &= \sin\theta_{23} \cos\theta_{23} \sin\theta_{12} \cos\theta_{12} \sin\theta_{13} \cos^2\theta_{13} \sin\delta_{CP} \end{aligned} \tag{13}$$

Now, to satisfy the current neutrino oscillation data, we can choose the following ranges of the free parameters for the model:

$$\text{Re}[\tau] \in [0.5, 2.5],\ \text{Im}[\tau] \in [0.8, 2],\ \{\alpha_D, \beta_D\} \in [0.1, 10],$$
$$\{\alpha_{NS}, \beta_{NS}\} \in [0.1, 1],\ v_{NS} \in [1, 10]\ \text{TeV},\ \mu_0 \in [10^2, 10^3]\ \text{eV}.$$

Here, we have taken only the case for normal ordering. Figure 1 show the variation of modular Yukawa couplings with allowed region of Re(τ) and Im(τ) for NO.

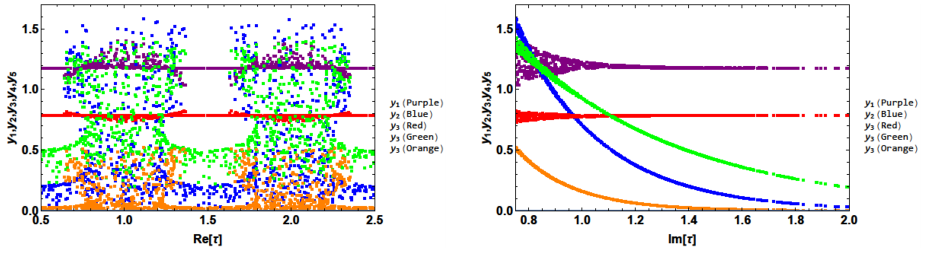

Figure 1: Correlation of modular Yukawa couplings with real and imaginary parts of modulus of τ for NO respectively.

Also, the mixing angles for the model can be obtained from the relations given in Eq. 12. Figures 2, 3, 4 show the variation of the sum of neutrino masses (Σm_ν) with the mixing angles $\sin\theta_{13}$, $\sin\theta_{12}$ and $\sin\theta_{23}$ for both normal ordering.

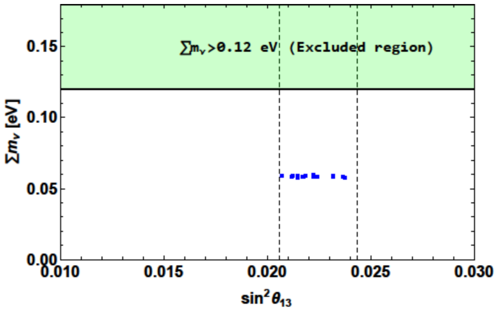

Figure 2: Correlation between sum of neutrino masses (Σm_ν) and $\sin^2\theta_{13}$ for NO.

It is evident from the figures that a wide number of parameter space lie within the allowed region for sum of neutrino masses i.e., ($\Sigma m_\nu \leq 0.12$ eV) and the mixing angles. The lower bound for sum of neutrino masses is found to be around 0.06 eV.

Also, we have calculated Majorana phases for the model. The figure 5 implies that α_{21} takes the value in the range ($0^\circ - 40^\circ$) and the ranges for α_{31} is found to be ($0^\circ - 360^\circ$) for NO.

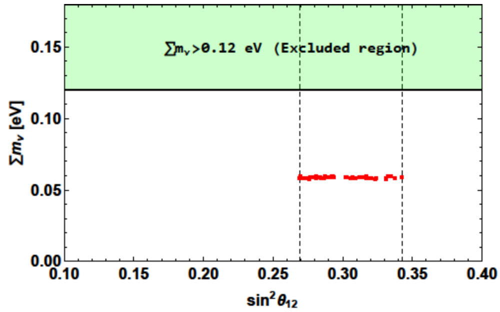

Figure 3: Correlation between sum of neutrino masses (Σm_ν) and $\sin^2\theta_{12}$ for NO.

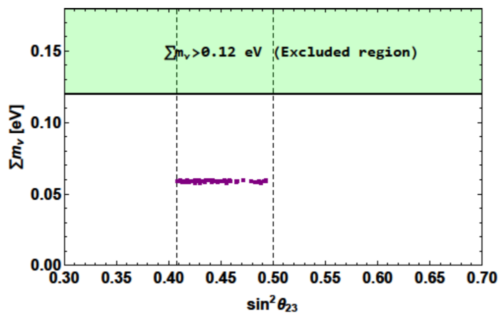

Figure 4: Correlation between sum of neutrino masses (Σm_ν) and $\sin^2\theta_{23}$ for NO.

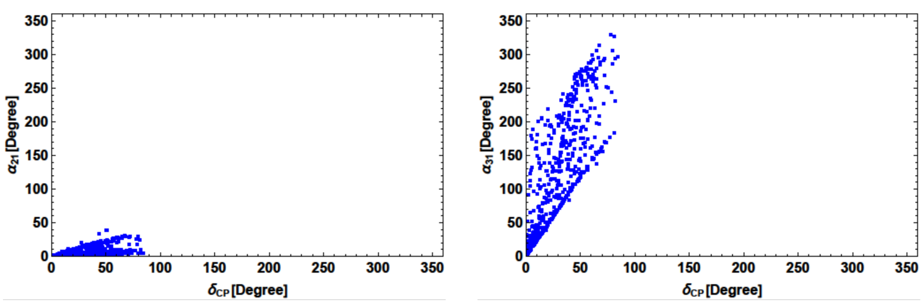

Figure 5: Correlation between Majorana phases and Dirac CP phases for NO.

4 Conclusion

Here, we have studied an inverse seesaw model with modular S_4 symmetry. This symmetry restricts the use of multiple flavon fields. The neutrino mass matrix for the model is characterized by the modulus τ and the free parameters of the model.

After that, we have carried out some numerical analysis in search of various parameter sets to fit into the 3σ ranges of neutrino oscillation data for normal and inverted ordering. Here, the Dirac CP phase is restricted in the region $0°$ to $90°$ and the lower bound for sum of the neutrino masses is around 0.06 eV. Furthermore, we have found the Majorana phases for the model. The ranges for α_{21} and α_{31} are found to be $(0° - 40°)$ and $(0° - 360°)$ respectively.

References

[1] Acciarri, R., M. A. Acero, M. Adamowski, C. Adams, P. Adamson, S. Adhikari, Z. Ahmad et al. "Long-baseline neutrino facility (LBNF) and deep underground neutrino experiment (DUNE) conceptual design report, volume 4 the DUNE detectors at LBNF." arXiv preprint arXiv:1601.02984 (2016).

[2] "Constraint on the matter–antimatter symmetry-violating phase in neutrino oscillations." Nature 580, no. 7803 (2020): 339-344.

[3] González-Garciá, M. Concepción, and José WF Valle. "Fast decaying neutrinos and observable flavour violation in a new class of majoron models." Physics Letters B 216, no. 3-4 (1989): 360-366.

[4] Hernández, AE Cárcamo, Juan Marchant González, and Ulises Jesus Saldaña-Salazar. "Viable low-scale model with universal and inverse seesaw mechanisms." Physical Review D 100, no. 3 (2019): 035024.

[5] Malinský, M. J. C. Romao, and J. W. F. Valle. "Supersymmetric SO (10) seesaw mechanism with low $B - L$ scale." Physical review letters 95, no. 16 (2005): 161801.

[6] Mohapatra, R. N., S. Antusch, K. S. Babu, Gabriela Barenboim, Mu-Chun Chen, A. De Gouvêa, P. De Holanda et al. "Theory of neutrinos: a white paper." Reports on Progress in Physics 70, no. 11 (2007): 1757.

[7] Brahmachari, Biswajoy, Sandhya Choubey, and Manimala Mitra. "A 4 flavor symmetry and neutrino phenomenology." Physical Review D 77, no. 7 (2008): 073008.

[8] Mukherjee, Ananya, and Mrinal Kumar Das. "Neutrino phenomenology and scalar Dark Matter with A4 flavor symmetry in Inverse and type II seesaw." Nuclear Physics B 913 (2016): 643-663.

[9] Di Iura, Andrea, M. L. López-Ibáñez, and Davide Meloni. "Neutrino masses and lepton mixing from A5 and CP." Nuclear Physics B 949 (2019): 114794.

[10] Ma, Ernest. "Neutrino mass matrix from S4 symmetry." Physics Letters B 632, no. 2-3 (2006): 352-356.

[11] Chakraborty, Mainak, R. Krishnan, and Ambar Ghosal. "Predictive S4 flavon model with TM1 mixing and baryogenesis through leptogenesis." Journal of High Energy Physics 2020, no. 9 (2020): 1-48.

[12] Pakvasa, Sandip, and Hirotaka Sugawara. "Discrete symmetry and Cabibbo angle." Physics Letters B 73, no. 1 (1978): 61-64.

[13] Feruglio, Ferruccio. "Are neutrino masses modular forms?." In From My Vast Repertoire… Guido Altarelli's Legacy, pp. 227-266. 2019.

[14] Xing, Zhi-zhong. "Flavor structures of charged fermions and massive neutrinos." Physics Reports 854 (2020): 1-147.

[15] Gonzalez-Garcia, Maria Concepcion, Michele Maltoni, and Thomas Schwetz. "NuFIT: three-flavour global analyses of neutrino oscillation experiments." Universe 7, no. 12 (2021): 459.

[16] Deppisch, F., and J. W. F. Valle. "Enhanced lepton flavor violation in the supersymmetric inverse seesaw model." Physical Review D 72, no. 3 (2005): 036001.