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Noncommuting Wilson Lines from Orbifold Topology and Stringy Construction of Degenerate Orbifolds

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Abstract

We propose a new approach to introduce Wilson lines on orbifold directly after the orbifold twist. A Wilson line corresponds to a representative vector of the conjugacy class of the space group defining the orbifold. Wilson lines corresponding to different conjugacy classes are noncommuting in general. This noncommutativity may reduce the rank of the gauge group. This method makes possible to construct degenerate orbifolds on purely stringy basis without use of the potential argument in the field theoretical approximation.

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The Wilson-line mechanism [1] in the framework of orbifold compactification is a powerful method in constructing four-dimensional chiral superstring models [2-6]. All previous approaches have considered Wilson lines, i.e., constant background gauge fields in the underlying torus, before the orbifold twist, for which the topology of the manifold requires the Wilson lines to be commutative [2-4]. In other words, Wilson lines are introduced as a homomorphism of the translation defining the torus into $E_8\times E_8'$, so that the background gauge field must lie in the Cartan subalgebra.

In this paper we propose another new approach to introduce Wilson lines on orbifold directly, after the orbifold twist. This is done through consideration of a Wilson line corresponding to a representative vector of the conjugacy class of the space group S defining the orbifold. In this case Wilson lines corresponding to different conjugacy classes are not necessary commutative and this noncommutativity may reduce the rank of the unbroken subgroup. Another method to reduce the rank of the gauge group was proposed by [7] by considering non-abelian embedding of the space group in the bosonic formulation. There the background gauge fields were in the Cartan subalgebra so that the Wilson lines were represented by shifts in the $E_8\times E_8'$ lattice. On the other hand, the embedding of the point group was made by Weyl rotations of the $E_8\times E_8'$ lattice. In this paper we work in fermionic representation of the gauge degrees of freedom since it makes possible to treat the abelian and non-abelian embedding in a unified way. Especially we can make purely stringy construction of the degenerate orbifold [8] without recourse to the field theoretical argument such as flat directions of the potential.

Let us consider closed string propagation on space-time with topology $\mathcal{R}^{10-d}\times\mathcal{O}$, where a d-dimensional orbifold $\mathcal{O}$ is constructed by identifying points of d-dimensional euclidean space $\mathcal{R}^d$ under a space group S of rotation and translation v: $\mathcal{O}=\mathcal{R}^d/S$ [2]. A typical element of S takes $z^\alpha\rightarrow(\theta z)^\alpha+v^\alpha$

and will be denoted by (θ, v), where we take the orbifold to be even-dimensional and $\alpha = 1,\ldots,d/2$ in complex notation. Elements of S correspond with the various twist fields or the twisted Hilbert space; in each twisted sector the string field obeys different boundary conditions. There is one sector of the Hilbert space for each conjugacy class of the space group S, since S is non-abelian [2]. In what follows we consider Z_N orbifold with d = 6. Generalization to other orbifold will be straightforward.

For a Z_N orbifold θ is a single rotation of order N in SO(d) and the space group S for this orbifold consists of elements of the form (θ^k, v), where k = 0, 1,...,N-1, θ^k denotes the kth power of θ and v runs over the d-dimensional lattice Γ_α. For k = 0, the translation elements of S belong to conjugacy classes of the form $\{(1, \theta^j v^0)\}$ with v^0 fixed and j = 0,1,...,N-1. These classes describe winding sectors. For k = 1,...,N-1, there are several conjugacy classes in S, $(\theta^k, v_{k,f})$, where $v_{k,f}$ runs over some coset of Γ_d,

$$\begin{aligned} v_{k,f} &= \theta^p v^0_{k,f} + (1 - \theta^k) u \\ p &= 0,1,\ldots,N-1 \\ u &\in \Gamma_d \end{aligned} \tag{1}$$

for some fixed $v^0_{k,f}$, which can be regarded as a representative of a conjugacy class labeled by (k,f). The first index k = 1,...,N-1 denotes one of the N-1 twisted sectors of the Hilbert space. The second index f = 1,...,n(k) labels a conjugacy class within that sector [9]. To each conjugacy class corresponds a representative vector $v^0_{k,f}$ and we associate a Wilson line to this vector.

Embedding of the space group (θ, v) into the internal degrees of freedom is done in both bosonic and fermionic formulation. Let us work in fermionic representation of the gauge degrees of freedom of the heterotic string [10]. We introduce two sets of 16 fermionic coordinates ψ^i and $\tilde{\psi}^i$ (i = 1,...,16) which transform as the vectors of $SO(16)\times SO(16)'$. The background gauge fields A^{ij}_{μ} transform as $(120,1)+(1,120)$. The Wilson line corresponding to a representative vector $v^{o}_{\kappa,f}$ of the conjugacy class (k,f) is given by

$$\Theta_{f,\kappa} = \exp[2\pi i\, \theta_{f,\kappa}] \tag{2}$$

and

$$2\pi\theta_{f,\kappa} = A_{\mu}(v^{o}_{\kappa,f})^{\mu} \tag{3}$$

The coset obtained by setting $v^{o}_{\kappa,f} = 0$ is a sublattice of Γ_d, which is denoted by $(1-\theta)\Gamma_d = \{(1-\theta)u,\ u\in\Gamma_d\}$. For this sublattice the corresponding Wilson line must be identity so that A_{μ} should obey

$$A_{\mu}[(1-\theta)u]^{\mu} = 2\pi N\ , \quad u \in \Gamma_d \tag{4}$$

where N is a matrix with integer eigenvalues.

Embedding of a rotation of order N in $SO(d)$ into the internal degrees of freedom is determined by giving the rotation matrix Ω for the fermionic coordinates ψ^{i} and $\tilde{\psi}^{i}$. By choosing appropriate basis of $SO(16)\times SO(16)'$, we can always diagonalize the Ω such that

$$\begin{aligned}\Omega &= \exp[2\pi i\, \zeta^{\ell} H_{\ell}]\ ,\\ N\zeta^{\ell} &= 0 \mod 1\end{aligned} \tag{5}$$

which obeys $\Omega^{N} = 1$ and H_{ℓ} $(\ell = 1,\dots,16)$ is the Cartan subalgebra of $SO(16)\times SO(16)'$. For d/2 complex z^{α} planes let us denote the unit basis vector e^{α}_{1}, e^{α}_{2} of the Γ_d lattice such that the angle between two vectors is $2\pi/N$. Under the $2\pi/N$ rotation these basis vectors transform as $e^{\alpha}_{1}\to e^{\alpha}_{2}$. So the Wilson lines corresponding to these basis vectors transform as

$$\Omega\,\Theta_{f,1}\,\Omega^{-1} = \Theta_{f,2} \tag{6}$$

where $\Theta_{f,1}$ is defined by (2) with $2\pi\theta_{f,1} = A_\mu(e_1^\alpha)^\mu$, etc., since

$$
\begin{aligned}
(\theta,\ 0)(1,\ e_1)(\theta^{-1},\ 0) &= (\theta,\theta e_1)(\theta^{-1},\ 0) \\
&= (1,\theta e_1) \qquad\qquad (7) \\
&= (1,\ e_2) \quad .
\end{aligned}
$$

For the singly-twisted and singly-antitwisted sectors, $k = \pm 1$, there is a one-to-one correspondence between the conjugacy class $(\theta^k,\ v^o_{k,f})$ and the fixed point f of the rotation θ or θ^{-1} acting on the torus [9]. The conjugacy classes for the higher-twist sectors $(k \neq \pm 1)$ are not usually in one-to-one correspondence with the fixed points of θ^k. Some of the fixed points of θ^k may not be fixed by θ so that physical states are θ-invariant linear combinations of states located at different fixed points of θ^k [9]. The index f labels such a θ-invariant combinations of fixed points. Since the θ-invariant (combination of) fixed points obey $f = \theta^k f$ and $f = \theta f$ mod Γ_d lattice, they are simultaneously fixed for all twisted sectors $k = 1,\ldots,N-1$. Thus the conjugacy classes for the higher-twisted sectors are in one-to-one correspondence with the θ-invariant fixed points.

Now that we have shown that the conjugacy class (k,f) is in one-to-one correspondence with the θ-invariant fixed point labeled by f and also in one-to-one correspondence with the Wilson line $\Theta_{f,k}$ given by (2). Then we can show that the Wilson lines $\Theta_{f,k}$ and $\Theta_{f,h}$ corresponding to the same conjugacy class f obey the following commutation relation:

$$[\Omega^k \Theta_{f,k}\ ,\ \Omega^h \Theta_{f,h}] = 0 \quad . \qquad (8)$$

Proof: $\Omega^k\Theta_{f,k}$ corresponds to $(\theta^k,\ 0)(1,\ v^o_{k,f}) = (\theta^k,\ \theta^k v^o_{k,f})$. Then the commutator in (8) is given by

$$(\theta^k,\ \theta^k v^o_{k,f})(\theta^h,\ \theta^h v^o_{h,f}) - (\ k \leftrightarrow h\ )$$

$$= \quad (\theta^{k+h}, \theta^{k+h} v^{o}_{h,f} + \theta^{k} v^{o}_{k,f}) - (k \leftrightarrow h) . \tag{9}$$

Here we can show that

$$\theta^{k+h} v^{o}_{k,f} + \theta^{k} v^{o}_{k,f} = \theta^{k+h} v^{o}_{k,f} + \theta^{h} v^{o}_{h,f} , \tag{10}$$

since the θ -invariant fixed point f is a common fixed point of both k- and h-twisted sector so that

$$f = (1-\theta^{k})^{-1} \theta^{k} v^{o}_{k,f} = (1-\theta^{h})^{-1} \theta^{h} v^{o}_{h,f} \tag{11}$$

modulo $(1-\theta)\Gamma_{d}$ lattice.

The operator $\omega_{f,k} = \Omega^{k} \Theta_{f,k}$ to the internal degrees of freedom can be thought of an automorphism of space rotation θ^{k} : $\exp[2\pi ik/N]$ for z^{α}, since $[\Omega^{k} \Theta_{f,k}]^{N} = 1$. The commutation relation (8) simply means that Z_{N} is abelian. Note, however, that for different conjugacy classes or fixed points $f \neq g$, we have in general

$$[\Omega^{k} \Theta_{f,k} \, , \, \Omega^{h} \Theta_{g,h}] \neq 0 \tag{12}$$

and this noncommutativity may reduce the rank of the subgroup. Different automorphisms $\omega_{f,k}$ and $\omega_{g,h}$ corresponding to different conjugacy classes or different θ -invariant fixed points may not commute each other. The gauge quantum numbers of various states are determined by the zero modes of generators which commute with the hamiltonian and $\omega_{f,k}$. So the gauge symmetry is broken to the subgroup which commutes with $\omega_{f,k}$ and noncommutative ω's may reduce the rank of the subgroup.

In bosonic representation of the gauge degrees of freedom, an element of S, $z^{\alpha} \rightarrow (\theta^{k} z)^{\alpha} + v^{\alpha}_{k,f}$ is embedded as $X^{I} \rightarrow (\Omega^{k} X)^{I} + V^{I}_{k,f}$, I = 1,...,16. Here Ω is an automorphism of the group lattice, i.e., Weyl rotation and $V^{I}_{k,f}$ is a Wilson line corresponding to the translation $v_{k,f}$ in the conjugacy class (θ^{k}, $v_{k,f}$). In this representation the Wilson lines $V^{I}_{k,f}$ are commutative since they are homomorphism of translations $v_{\kappa,f}$ and they must be chosen to be in the Cartan subalgebra. Note, however, that the basis of the group representation for $V^I_{\kappa,f}$ may be chosen to be different for different conjugacy classes f. If we choose common basis for all conjugacy classes, this reduces to the case considered in [7]. Noncommutativity between an automorphism Ω and translation $V^I_{\kappa,f}$ of the group lattice may reduce the rank of subgroup.

Although we can make parallel argument for bosonic and fermionic formulations, we work in this paper in the fermionic formulation, since it is easier to argue symmetry breaking in a unified way for both abelian and non-abelian embeddings. This also makes possible to construct degenerate orbifolds in a purely stringy way.

In what follows we focus on the Z orbifold for which N = 3. In this case there are only singly-twisted and singly-antitwisted sector and there is a one-to-one correspondence between the conjugacy class and the fixed point of the rotation θ or θ^{-1}. The fixed point on the Z orbifold can be denoted by $(p,q,r)d$ with $d = \sqrt{1/3}\,\exp(i\pi/6)$ and $p,q,r = 0, \pm 1 \bmod 3$. Here p,q,r specify the location of the fixed point in the three complex z^α planes. If we choose the unit basis vector of the Γ_d lattice in the z^α plane as $\tilde{e}^\alpha_1 = e^\alpha_1$ and $\tilde{e}^\alpha_2 = e^\alpha_1 + e^\alpha_2$, these basis vectors transform as $\tilde{e}^\alpha_1 \to \tilde{e}^\alpha_2 - \tilde{e}^\alpha_1$ and $\tilde{e}^\alpha_2 \to -\tilde{e}^\alpha_1$. Then the representative vector of the conjugacy class (θ^κ, f) corresponding to the fixed point $f = (p,q,r)$ can be chosen as $p\tilde{e}^1_\kappa + q\tilde{e}^2_\kappa + r\tilde{e}^3_\kappa$, which is a shift on the Γ_d lattice for the fixed point to be accompanied under a group action [4]. The Wilson line corresponding to this representative vector amounts to

$$\Theta_{f,\kappa} = \exp[2\pi i(p a_\kappa + q b_\kappa + r c_\kappa)] , \qquad (13)$$

where the background-field contribution is written by

$$2\pi a_\kappa = (A^{ij}_\mu T_{ij} + \tilde{A}^{ij}_\mu \tilde{T}_{ij})\, \tilde{e}^{1\mu}_\kappa \qquad (14)$$

and b_K, c_K are given by replacing $\tilde{e}^1_K$ by $\tilde{e}^2_K$ and $\tilde{e}^3_K$, respectively. The generators of $SO(16) \times SO(16)'$ are denoted by T_{ij} and $\tilde{T}_{ij}$. We can introduce at most three independent Wilson lines a_K, b_K and c_K.

Automorphism $\omega_{f,K}$ of the Z_N group action on z^α is represented by the gauge fermions ψ as the following boundary conditions,

$$\psi(\sigma_1+\pi,\sigma_2) = -(-1)^n \Omega^K \Theta_{f,K} \psi(\sigma_1,\sigma_2) \qquad (15)$$

$$\psi(\sigma_1,\sigma_2+\pi) = -(-1)^m \Omega^h \Theta_{f,h} \psi(\sigma_1,\sigma_2) \qquad (16)$$

and matrix notation is understood here. n,m = 0,1 specifies the spin structure. When the background gauge fields are taken in the Cartan subalgebra, all the Wilson-line matrices $\Theta_{f,K}$ and the rotation matrix Ω are commutable and are diagonalized simultaneously:

$$\Omega^K \Theta_{f,K} = \omega_{f,K} = \exp[2\pi i K v_f^\ell H_\ell] \qquad (17)$$

and

$$v_f^\ell = \zeta^\ell + (pa_1 + qb_1 + rc_1)^\ell \qquad (18)$$

where we have used the following relation,

$$a_2 = 2a_1 \ , \quad b_2 = 2b_1 \ , \quad c_2 = 2c_1 \ , \qquad (19)$$

$$3(a_K,\ b_K,\ c_K) = \text{integers} \mod 3 \ , \qquad (20)$$

which are derived by (4) or (6). This is the case of the abelian embedding and we can follow the previous works with the shift vector v_f^ℓ [3,4].

The condition of modular invariance in the presence of Wilson lines is given by the level matching condition at one-loop order [11], which reads

$$N \sum_\ell v_f^\ell = 0 \mod 2 \ , \qquad (21)$$

$$N \{ \sum_\ell (v_f^\ell)^2 - \sum_a (\zeta^a)^2 \} = 0 \mod 2 \ , \qquad (22)$$

where $\xi = \frac{1}{3}(1,1,-2,0)$ is the vector which defines the embedding of the Z_N group action in the spacetime right-moving Neveu-Schwarz-Ramond (NSR) fermions. The string states on the orbifold are invariant under the Z_N group. The group invariant condition is obtained by constructing the projection operator onto the invariant subspace of the string Hilbert space, and is given by

$$(V^{\ell} + k\, v_f^{\ell}/2)v_f^{\ell} + (K^{a} - k\,\xi^{a}/2)\xi^{a} + m_K = 0 \mod 1 \tag{23}$$

where V^{ℓ} is the vector in the $E_8 \times E_8$ root lattice and K^{a} is the vector in the SO(8) vector or spinor lattice [4]. The m_K is the eigenvalue of $\hat{m}_K$, where the twist operator $\hat{g}_K$ for z^{α} is given by $\hat{g}_K = \exp(2\pi i \hat{m}_K)$: $\hat{g}_K z^{\alpha} \hat{g}_K^{-1} = e^{2\pi i/3} z^{\alpha}$.

The gauge bosons are obtained in the untwisted sector by the combination with the right-moving ground states with helicity ± 1 in 8_V of SO(8), for which $K^{a}\xi^{a} = 0$. Then the group invariant condition (23) implies

$$V^{\ell} v_f^{\ell} = 0 \mod 1 \ , \tag{24}$$

and the symmetry corresponding to the root V^{ℓ} obeying (24) for all v_f^{ℓ} remains unbroken. Massless fermions in the untwisted sector are combined with the right-moving ground states with helicity 1/2 in 8_S of SO(8), for which $K^{a}\xi^{a} = 2/3 \mod 1$. The group invariant condition (23) reads

$$V^{\ell} v_f^{\ell} = 1/3 \mod 1 \ , \tag{25}$$

and the states obeying this condition for all v_f^{ℓ} survive as massless fermions. Chiral fermions in the twisted sector must obey the following massless condition:

$$\frac{1}{2}(V^{\ell} + v_f^{\ell})^2 + N_L - \frac{2}{3} = 0 \ , \tag{26}$$

$$\frac{1}{2}(K^{a} - \xi^{a})^2 + N_R - \frac{1}{6} = 0 \ , \tag{27}$$

where N_L and N_R are the occupation numbers for the left- and right-moving oscillators of z^{α}. The group invariant condition (23) is always satisfied by the states obeying (26) and (27). Construction of models based on abelian embedding has been done extensively [3-6].

Now let us consider more general case where the background gauge fields have components other than the Cartan subalgebra. In this case some or all of the automorphisms ω_f's corresponding to different conjugacy classes do not commute each other and are not diagonalized simultaneously. The gauge symmetry will be broken to the lower-rank subgroup which commutes with all ω_f's. In order to quantize string states we need to diagonalize the boundary conditions (15) and (16). Diagonalization is done at each conjugacy class as follows:

$$\omega_{f,k} = \Omega^{k}\Theta_{f,k} = U_f^{-1} \exp[2\pi i k v_f^{\ell} H_{\ell}]\, U_f \tag{28}$$

where U_f belongs to SO(16) $\times$ SO(16) and eigenvalues v_f^{ℓ} must obey $N v_f^{\ell} = 0 \mod 1$ due to Z_N invarience. The string states associated with each conjugacy class are expressed in the different basis which diagonalizes the corresponding boundary conditions. The Z_N invarience of the string Hilbert space imposes now the condition (23) for the eigenvalues v_f^{ℓ} and the additional condition that the string Hilbert space should be invariant under U_f :

$$U_f\, E_V\, U_f^{-1} = E_V\,, \tag{29}$$

$$U_f\, H_{\ell}\, U_f^{-1} = H_{\ell}\,, \tag{30}$$

where E_V is the generator corresponding to the root V of $E_8 \times E_8'$ and H_{ℓ} is the Cartan subalgebra.

Gauge symmetry is determined now by (24) and (30), since the condition (29) is always satisfied for E_V obeying (24) because the transformation generated by E_V leaves v_f invariant and commutes with U_f. This implies that the U_f invariance does not change the non-abelian part of the subgroup determined by (24). The U(1) corresponding to H_{ℓ} which does not obey (30) disappears now and the rank of the subgroup is reduced.

Massless spectra of chiral fermions in the untwisted sector are determined by (25) and (29), (30). Non-singlet multiplets with respect to the unbroken non-abelian subgroup survive under the U_f invariance since the generators of the non-abelian subgroup obey (29). On the other hand, singlets of the non-abelian subgroup may have the U(1) charge which is not invariant under (30) and are discarded. In the twisted sector, massless spectra are determined by the massless condition (26). The number of states in the twisted sector is the same as the case of the abelian embedding.

Here a number of comments are in order.

(1) The discarded zero modes are only singlets of the unbroken non-abelian subgroup, so that this truncation does not change anomaly cancellation with respect to the non-abelian subgroup. When the subgroup contains U(1), some of them might be anomalous, i.e., the trace of the corresponding U(1) generators is non-vanishing [12]. This anomaly is cancelled by the Green-Schwarz mechanism [13].

(2) Since the zero modes which are not invariant under U_f have been discarded, we must check modular invariance of the truncated theory. At one-loop order we can varify modular invariance explicitly by calculating the partition function. It turns out that the transformation U_f to diagonalize the boundary conditions disappears inside the trace of the partition function and we obtain the same partition function as the abelian embedding. Thus the level-matching conditions for the eigenvalue v_f^ℓ given by (21) and (22) are sufficient to keep modular invariance at least at one-loop order. The fact that the partition function does not depend on U_f implies that the (inner) automorphism corresponding to U_f is commutative with the modular transformation. Modular invariance is not affected by the rotation of the basis.

Now we come to an important observation that apart from the U_f-non-singlets the symmetries and particle spectra are completely determined by the eigenvalue v_f^ℓ : Different Wilson lines with the same v_f^ℓ which are connected by U_f give the same symmetry and mass spectra. Since the transformation U_f depends on the continuous parameters, infinitely many Wilson lines are associated with the same eigenvalue v_f^ℓ. This means that the string vacuum is highly degenerate and it corresponds to flat direction of the potential in the field theoretical approximation. This situation has been found by [8] in another approach of non-abelian embedding with the use of the Weyl rotations of the $E_8 \times E_8'$ lattice. In our approach, however, we can construct degenerate orbifolds on purely stringy basis without recourse to the potential by considering possible transformation U_f of the eigen states determined by v_f^ℓ.

Stringy construction of degenerate orbifold is summarized as follows: First we choose the rotation matrix Ω given by (5) to embed the Z_N group action. At this stage the symmetry G_0 is determined such that it is invariant under Ω. Then we introduce Wilson lines by giving eigenvalues v_f^ℓ which obey the condition of modular invariance (21), (22). The symmetry is broken as $G_0 \to G_1 \times U(1)$'s with no rank-reduction. If we rotate the basis by $U_f \subset G_0$, which commutes with G_1, some or all of $U(1)$'s are killed by (30). However, for some particular choice of U_f, (30) happens to lead to no or partial rank-reduction. This corresponds to the multicritical point for which there is an enhanced symmetry. Introduction of Wilson lines corresponds to giving vacuum expectation values (vev's) to scalar fields in such a way that the D- and F-terms vanish. The transformation U_f corresponds to flat directions of the potential. Note that untwisted and twisted flat directions [8] are treated on the same ground by choosing appropreate Wilson lines.

Let us consider simple examples, some of which were given in [8] by looking for flat directions. We can also construct new degenerate orbifolds which were not discovered in the field theoretical argument. We choose Z_3 embedding by giving the rotation matrix Ω with $3\zeta = e_1+e_2+e_5-2e_6+e_7+2e_1'$ in the orthogonal basis. The resulting gauge group is determined by (24) with $v_f = \zeta$ and it turns out to be $SU(9)\times SO(14)'\times U(1)'$. The matter content is $3\times\{(84,1)_0+(1,14)_{-2}+(1,64)_1\}$ in the untwisted sector and $27\times(\bar{9},1)_{4/3}$ in the twisted sector.

Now we introduce Wilson lines. It is possible to choose various Wilson lines and they lead to various degenerate orbifolds.

(i) We introduce one Wilson line, whose eigenvalue is given by $3v_f = 3\zeta \pm (2e_1+e_2+e_3)$ for $f = (\pm1,q,r)$, $q,r = 0,\pm1$ and $v_f = \zeta$ for $f = (0,q,r)$. Then SU(9) breaks down to $SU(3)\times SU(3)\times SU(3)\times [U(1)]^2$, where two U(1)'s correspond to (H_1+H_2) and $(2H_1+H_2+H_3)$. The matter multiplet in the untwisted sector turns out to be $3\times\{(\bar{3},\bar{3},\bar{3}) + 3(1,1,1)\}$ where quantum numbers of three singlets are H_1', (H_1+H_2) and $(2H_1+H_2+H_3)$. Now we rotate the Wilson line by U_f, the subgroup of SU(9), which leaves ζ invariant and commutes with $[SU(3)]^3$. Then two U(1)'s and 3×2 singlets are projected out by (30). The matter content in the twisted sector does not change under U_f and is given by $27\times\{(1,3,1)+(1,1,3)+(3,1,1)\}$. This example corresponds to the model given in [8] with non-vanishing vev's to some components of the 84-dimensional representations in the untwisted sector and we can reproduce their result by purely stringy method.

(ii) As the second example we introduce two Wilson lines whose eigenvalues are given by

$$3\, v_f = 3\zeta + p(2e_1-e_1'+e_2') + q(e_5-e_7+2e_8)\ , \tag{31}$$

for the conjugacy class $f = (p,q,r)$, $p,q,r = 0,\pm1 \bmod 3$. Here the symmetry is broken to $[U(1)]^8\times SO(12)'\times[U(1)']^2$. If we rotate Wilson lines by $U_f = SU(9)$ all U(1)'s from SU(9) are killed by (30) and the remaining gauge group is $SO(12)'\times[U(1)']^2$. In the untwisted sector the matter content turns out to

$$
\begin{aligned}
(84,1) &\longrightarrow 9(1,1) \\
(1,14) &\longrightarrow (1,12) + 2(1,1) \\
(1,64) &\longrightarrow 2(1,32)
\end{aligned}
$$

and in the twisted sector 27×9 singlets. This example corresponds to the model in [8] with vev's to the nine $\bar{9}$'s in the twisted sector and the 14 in the untwisted sector.

In the field theoretical argument it is generally complicated task to look for flat directions of the potential with vanishing D- and F-terms. In stringy construction of degenerate orbifold, the flat direction with vanishing D- and F-terms is automatically chosen by introducing Wilson lines which satisfy the condition of modular invariance. It is easy to construct other models with the choice of Wilson lines. If we take one Wilson line, $3v_{f} = 3\zeta + p(e_1 - e_5 + 2e_2')$, We have the gauge symmetry breaking as $SU(9) \rightarrow SU(7) \times [U(1)]^2$. Rotating the basis by the SU(3) subgroup of SU(9) which leaves SU(7) invariant, two U(1)'s are projected out and the resulting gauge symmetry is $SU(7) \times SO(12)' \times [U(1)']^2$.

To conclude we have proposed a new approach to introduce Wilson lines on orbifold directly after the orbifold twist. A Wilson line corresponds to a representative vector of the conjugacy class so that it is not commutative in general. This noncommutativity may reduce the rank of the gauge group. Our method easily gives purely stringy construction of degenerate orbifold without use of the potential argument. This makes possible to construct wide variety of models including $SU(3) \times SU(2) \times U(1)$, which will be give in detail elsewhere.

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