TOPICS IN SYPERSYMMETRY

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Abstract

We briefly review the most popular supersymmetric extensions of the "standard" model, insisting on the arbitrariness left in the phenomenological effective Langrangian. We also discuss the possibility of building completely finite theories based on N=2 supersymmetry.

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1. Motivation for Supersymmetry.

The motivation for supersymmetry remains fairly theoretical. The initial image of a new symmetry relating known fermions to known bosons does not withstand the analysis.

While supersymmetry establishes some connections between gauge bosons and scalars, this connection is relatively loose and rather accidental in the N=1 theories. Indeed, while the masses of scalars and gauge bosons are related for unbroken SUSY[1)], the choice of the group representations is left arbitrary (e.g.: in the "standard" model, supersymmetry does not explain why scalars should lie in doublets rather than triplets). Such relations become, however, much more compelling in N>2 supersymmetric theories.

The most economical solution would identify the scalar partners $\tilde{e}$, $\tilde{\nu}$ of the e and ν with the scalars* responsible for breaking SU(2) x U(1). This, however, would violate lepton number conservation. Also, another "Higgs" doublet H_1 is necessary in SUSY models to give mass to the charge 2/3 quarks, and the cancellation of anomalies then requires a similar particle H_2 with opposite hypercharge - nothing is saved in terms of particle content! (see below for more discussion of $\langle\tilde{\nu}\rangle$).

<u>The motivation which may be considered closest to the preoccupations of a phenomenologist is provided by an attempt at solving the "hierarchy problem."</u> This problem arises very generally from the assumption that the presently known interactions may be unified into a single gauge group at some scale. An estimate of the scale at which this would happen is then provided by the study of the renormalization group equations, as established from the currently known particle spectrum (of from some assumed spectrum if SUSY is considered). The unification of the electromagnetic and strong coupling then suggest a very high mass scale (> 10^{15} GeV).

The simultaneous presence of such a large mass scale and of fundamental

*Also known as BEGH^{2}K bosons or "Higgs" bosons.

light scalars leads to some difficulties in perturbation theory, where quadratic divergences appear which destroy the initially assumed potential. Formally there is nothing here which cannot be blamed on the perturbation expansion or cured by an appropriate order by order renormalization.

However, the very fact that the low energy theory is so sensitive to minute fluctuations of the couplings associated with the high energy structure is disturbing in itself, and may prove a real problem when gravity is eventually coupled to the model. This is usually referred to as the "hierarchy" problem. Supersymmetry can cure this difficulty: adding up fermionic and bosonic contributions kills the troubling quadratic divergencies. If the effective breaking of SUSY sppears at some scale μ, one expects then the corrections to the Higgs masses squared to be of $O(\alpha\mu^2)$. Asking that such corrections be no larger than the typical v.e.v.'s then gives some estimate of the expected scale, $\mu \sim 300\text{GeV}/\sqrt{\alpha}$ -in other words, and depending upon which amount of tuning is judged acceptable, $\mu \sim$ several TeV's.

There are other, more theoretical justifications for the extension of the present models to SUSY, such as the unification of gravity in the framework of supersymmetry, or the desire to build a perturbatively finite model. These do not demand in any way that SUSY be broken at low energy, and the scale of the breaking is anybody's guess.

In practice, we must thus accept that extending the "standard model" to supersymmetry involves the association of at least one unobserved SUSY-partner to each known particle. The separation in mass between SUSY partners may be expected to be of the same order as the effective SUSY-breaking scale which is only limited in practice by our desire to avoid the "hierarchy" problem. The following abbreviations will be used.

Particle	SUSY Partner	
	Scalars	Spinors
e_L	S. election $\tilde{e}_L$	
e_R	$\tilde{e}_R$	
ν_L	S. neutrino $\tilde{\nu}_L$	
w^μ		$\tilde{w}$ wino
B^μ		$\tilde{B}$ bino
H		$\tilde{h}$ higgino
g^μ		$\tilde{g}$ gluino

Although the couplings of those SUSY partners are similar to the corresponding vertices in the standard model, they can escape detection, due to their mass and the fact that they need to be pair produced [this latter feature, while usually implemented in models suffers some exceptions, see below].

In the following sections we will

- quickly review the existing patterns of SUSY breaking, and some of their phenomenological implications.
- present the framework for finite, N=2, softly broken SUSY models.

2. Softly Broken SUSY.

Since the anticommutator of the SUSY charges is related to the 4-momentum the supersymmetric vacuum if it exists, has automatically the lowest possible energy:

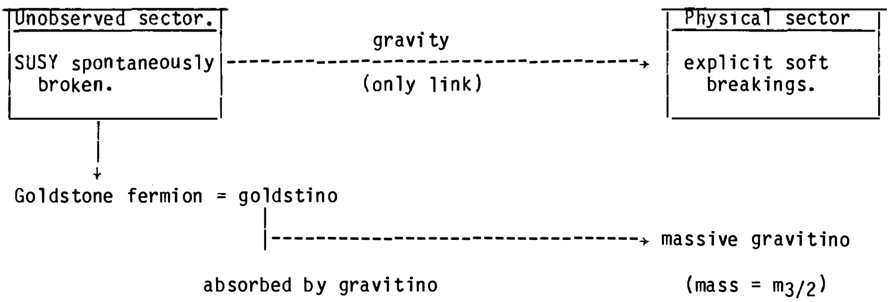
$$\{Q_\alpha, \overline{Q}_{\dot\beta}\} = 2\sigma^\mu_{\alpha\dot\beta} P^\mu \tag{1}$$

In order to break SUSY spontaneously one is led to look for situations where no possible SUSY vacuum exists. Such models, although somewhat difficult to build are possible[2]. They require either the introduction of extra scalar multiplets or of an extra U(1) gauge group. The phenomenological consequences of earlier versions of these models have been reviewed, e.g., in ref.3; some significant progress has been made recently in models using the extra U(1)[4]. The main problem of that extra gauge symmetry consisted in the presence of anomalies, which can be removed at the cost of parity doubling; it is interesting to note that this operation can lead quite naturally to N=2 theories.

An obviously easier solution consists in the explicit breaking of supersymmetry. Of course, introducing explicit breakings by hand cannot be a fundamental solution to the problem. A suitable set of breaking parameters which do not reintroduce quadratic divergences, could, however, constitute a technical solution to the "hierarchy" problem. Such terms, usually referred to as "soft breaking" terms have been enumerated in ref.5 -they involve mass terms for the scalar partners of quarks and leptons, or even for the fermionic partners of gauge bosons (gauginos).

This "technical" solution happily received some comfort from the consideration of local SUSY models (including gravity). These models are considered at the lowest order in the gravitational interaction, and provide an effective Lagrangian for low-energy supersymmetry[6].

Of course, even in the presence of gravity, some SUSY-breaking mechanism is still needed, and can be implemented by the use of a "hidden sector," where one of the usual[2] spontaneous symmetry breaking schemes is used. Since that "hidden" sector is only coupled to ordinary matter via gravity, the news of SUSY breaking is transferred to the visible sector in a perfectly universal way (irrespective of colour, flavour ...)



As a consequence of this universal character of the relations between the hidden and the observed sector, the effective SUSY breaking parameters depend only upon the spin of the particles.

As a typical example, we have:

$$\mathcal{L}_{\text{broken}} = \mathcal{L}_{\text{global SUSY}} - m^2_{3/2} A^+_i A_i - Bm^2_{3/2} \Sigma m_{ij} A_i A_j - Am^2_{3/2} \Sigma g_{ijk} A_i A_j A_k \qquad (2)$$

where A_i are the scalar components of the various superfields (e.g., higgs scalar, scalar neutrino, scalar quarks), while m_{ij} and g_{ijk} are respectively the ordinary mass terms and Yukawa couplings. A and B are in principle calculable constants, but model dependent. This scheme, with the justification arising from Supergravity itself seems both simple and predictive, in view of the few parameters involved.

At first sight, it could be applied as such, using the bilinear coupling to induce gauge symmetry breaking without putting any mass scale by hand, $m_{3/2}$ being then the only dimensional parameter. It is not difficult however to check that such a scheme, which would indeed break the gauge symmetry is unacceptable phenomenologically , as it would lead, e.g., to the non-conservation of electric charge[7]. Other less restrictive models are however possible[3].

Alternative mechanisms have been suggested, which assume that the coefficient of the trilinear term is small enough that it does not play an important role in the symmetry breaking process.

While it would be impossible to give a negative mass to all the scalars without making the potential unbounded from below, it it conceivable that radiative corrections push one of these masses down, thereby allowing the familiar gauge symmetry breaking mechanism to take place. The leading logarithm corrections to the n point vertices associated to (2) can be evaluated by a renormalization group -improved calculation based on the relevant 1-loop diagrams.

When it comes to writing the corresponding renormalized Langrangian, one is obviously free to choose the most convenient renormalization point -the one which minimizes further radiative corrections. Since we want to use a grand unified theory, it is natural to use the "grand unification scale" as a substraction point, and to impose the values of the soft breaking terms at that scale. This defines the theory once and for all.

This choice of renormalization constants guaranties that, e.g., the 3-point function associated to $\tilde{e}_R H \tilde{e}_L$ has value $m_{3/2}.A.g_e$ when evaluated at a momentum transfer $-p^2 = \mu^2_{GUT}$. This, however, does not tell us directly what the value of that function is for low energy scattering! This value can be computed by summing the perturbation series, according to the renormalization procedure presented above. Alternatively, one may find it convenient to rewrite the Lagrangian in terms of another substraction point, using the renormalization group equations, so as to minimize the value of further radiative corrections evaluated at low energy. The same is true for the coefficient of, say, H^+H in (2). While its value is fixed to $+m^2_{3/2}$ when the theory is renormalized at the GUT scale, this does not imply that the vacuum is stable. One possible test for the stability of the vacuum is the presence of Tachyons: the low-energy behavior of the 2 point function associated to H^+H must, therefore, be calculated by summing the corresponding diagrams at low energy. In other terms, the renormalized Lagrangian does not tell us the whole story without calculation, and what matters is in fact the effective potential. While the two substraction points lead to strictly equivalent theories, according to a re-parameterization associated to the renormalization group equations, one minimizes the radiative corrections at high energies and is, therefore, useful in establishing symmetrical boundary conditions, while the other, which minimizes the radiative corrections at low energy, is closer to the effective potential, and therefore indicative of the (in)stability of the trivial vacuum.

The renormalization group equations for the various parameters appearing in (2) are at present well-known. We list the most relevant ones, following the notations of ref.9b) (G is the Higgs doublet coupled to u_R, H is coupled to d_R)

$$4\pi^2 \frac{dM^2_G}{d\ell n\Lambda} = 6[M_G^2 \, Tr\lambda_u\lambda_u^\dagger + Tr\lambda_u M_Q^2\lambda_u^\dagger + Tr\lambda_u^\dagger M_u^2\lambda_u + Tr\eta_u^\dagger\eta_u] - 8 \sum_{\alpha=1,2} C_\alpha(G)^2\mu_\alpha^2 g_\alpha \tag{3.a}$$

$$4\pi^2 \frac{dM^2_Q}{d\ell n\Lambda} = 2[\tfrac{1}{2}\{\lambda_u^\dagger\lambda_u + \lambda_D^\dagger\lambda_D, M_Q^2\} + M_G^2 \, Tr\lambda_u\lambda_u^\dagger + M_H^2 \, Tr\lambda_D\lambda_D^\dagger + \lambda_u^\dagger M_u^2\lambda_u + \lambda_D^\dagger M_D^2\lambda_D + \eta_u^\dagger\eta_u + \eta_D^\dagger\eta_D] - 8 \sum_{\alpha=1,2,3} C_\alpha(Q)\mu_\alpha^2 \, g_\alpha^2 \tag{3.b}$$

$$4\pi^2 \frac{dM^2_L}{d\ell n\Lambda} = 2[\tfrac{1}{2}\{\lambda_L^\dagger\lambda_L, M_L^2\} + M_H^2 Tr\lambda_L\lambda_L^\dagger + \lambda_L^\dagger M_L^2\lambda_L + \eta_L^\dagger\eta_L - 8 \sum_{\alpha=1,2} C_\alpha(L)\mu_\alpha^2 g_\alpha^2 \tag{3.c}$$

$$4\pi^2 \frac{d\eta_u}{d\ell n\Lambda} = \eta_u[5\lambda_u^\dagger\lambda_u + 3Tr\lambda_u\lambda_u^\dagger + \lambda_D^\dagger\lambda_D - 2C_\alpha^w g_\alpha^2] + 2\lambda_u[2\lambda_u^\dagger\eta_u + 3Tr\eta_u\lambda_u^\dagger + \lambda_D^\dagger\eta_D + 2C_\alpha^u\mu_\alpha g_\alpha^2] \tag{3.d}$$

$$4\pi^2 \frac{d\lambda_u}{d\ell n\Lambda} = \lambda_u[3(\lambda_u^\dagger\lambda_u + Tr\lambda_u\lambda_u^\dagger) + \lambda_D^\dagger\lambda_D - 2C_\alpha^u g_\alpha^L] \tag{3.e}$$

where Q , L represent quarks and lepton doublets, λ_u, λ_D are the Yukawa couplings which provide the quark masses and mixing angles, η_u is the coefficient of the trilinear coupling $\tilde{u}_R\ G\ \tilde{q}_L$, and obeys $\eta_u = Am_{3/2}\lambda_u$ at the G.U. scale. C_α (X) are the Casimir coefficients for the representation X of the gauge subgroup SU(α) (the charges squared for α=1); G is the "Higgs" field coupled to the up quarks. [M_Q^2, λ_u, ... are matrices in generation space]. $C_\alpha^2 = C_\alpha(Q) + C_\alpha(U) + C_\alpha(G)$; $C_\alpha^d = C_\alpha(Q) + C_\alpha(D) + C_\alpha(H)$.
It is easy to check from eq.(3) that the mass of the scalar fields is "pushed" down by the Yukawa couplings when Λ decreases. On the other hand, gaugino masses μ seem to increase M_G^2. As a result we expect that the particle with the largest Yukawa couplings and the smallest gauge couplings will be the first to develop a negative "mass" term. This points immediately to the Higgs field coupled to the top quark, whose Yukawa couplings are further enhanced by a color factor which the top s-quark does not enjoy. Furthermore, the top squark is protected by its gauge interaction if the gaugino masses turn out to be large.

This far the model seems to remain quite predictive, since eq.(3) only depends on the physical Yukawa couplings and on the parameters A,B, $m_{3/2}$ appearing in (2). (no gaugino mass is present in (2).) Several models have been suggested along this line, usually requesting a fairly heavy top quark.

It should be remarked, however, that the hypothesis of vanishing gaugino masses is not justified. It is indeed simple to check that (2) generates such masses at the one-loop level; furthermore, they become logarithmically divergent at the 2-loop level,[7] which imposes some renormalization. It is therefore fair to say that the gaugino masses μ_α appearing in (3) should be treated as arbitrary parameters; the number of those parameters being only reduced by the requirement of grand unification. (see ref.10)

As a consequence of this, the various models become almost unconstrained, with predicted gluino and top masses varying between 0 and 200 GeV (see e.g., ref.11). The reason why the solution of eq.(3) is so sensitive to gaugino masses (more specifically gluino masses) is somewhat indirect: while μ_3 does not appear in (3.a), it enters in (3.6) where it pushes up the s-quark masses, which in turn enter (3.a) to push down M_G^2.

Lepton Number Violation

We will have more to extract from equations (3), specially when we will deal with quark mixings. One intriguing possibility deserves to be examined; namely, the question of lepton number conservation.

As is well-known the scalar partners of the lepton doublets have the same quantum numbers under SU(3) x SU(2) x SU(1) or SU(5) as the "higgs" bosons. For this reason, some discrete symmetry is used to avoid explicit non-conservation of lepton number at the Lagrangian level (one can e.g. require invariance under a transformation where all lepton fields change sign, while "higgses" stay unchanged). Such a symmetry is usually inplemented as part of the susy R. symmetry [it is interesting to note that such precautions are unnecessary in other gauge groups, like SO(10) where Higgses and leptons occur in different representations].

Even if the bare Lagrangian conserves lepton number, the possibility still exists that spontaneous symmetry breakdown violates it. This can indeed be the case in the present approach: there is a zero direction of the quadratic term of the potential corresponding to $\langle \tilde{L}^o\rangle^2 = \langle G^o\rangle^2 - \langle H^o\rangle^2$; the issue of spontaneous lepton number violation then depends on the evolution of the scalar masses, according to eq.(3). The situation has been studied in detail in ref.12, which showed that a necessary condition would be $m_t > 5m_{\tilde{\ell}}$.

In view of the present limits on $m_{\tilde{\ell}}$ this seems unlikely; it should nevertheless, be kept in mind that most limits are still conditional, and that, on the other hand, the presence of a 4th generation could satisfy these bounds. It is interesting to mention some of the particular consequences of such lepton number violations.

First, if the violation is indeed spontaneous, a Goldstone boson - the Majoron - is expected to appear[13]. This causes some trouble with the stability of red giant stars, but can be avoided at the cost of an explicit breaking of lepton numbers, which is easily realized by introducing right-handed neutrinos and Majorana masses (the phenomenological consequences of such breaking can be minimized by the use of large Majorana masses and small couplings between left and right handed neutrinos). More interestingly, $<\tilde{\nu}_\tau>$ would mix wino's and leptons, leading to neutrino masses, departures from lepton universality and, last but not least, production of odd numbers of supersymmetric partners[14].

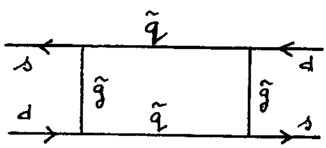
Looking for Sypersymmetry.

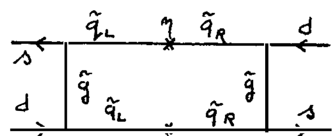
With the exception of an (unlikely) violation of lepton number, the yet unseen spectrum of the above models consists at least, in heavy scalar leptons and quarks (left and right handed partners slightly mixed), heavy gluinos, two Dirac Fermions made out of the 4 Weyl spinors ($\tilde{w}^+, \tilde{w}^-, \tilde{h}^+, \tilde{h}^-$), and 4 Majorana spinors which are linear combinations of ($\tilde{W}^o, \tilde{b}^o, \tilde{h}^o, \tilde{h}^o$). We will use ($\tilde{\gamma}_1 \ldots \tilde{\gamma}_4$) to label those neutral mass eigenstates. In any case, <u>there is no fundamental reason why any of these particles should be light</u>; their masses are related to the effective scale of SUSY breaking, which can be several TeV's. This is essential to keep in mind when experimental data are examined, as should be remembered that only correlated limits can be given (see an example below).

Not only can the masses be large, but the actual eigenstates depend strongly on the type of model chosen. For a discussion of the gaugino masses and mixing, see e.g., ref.15. In two extreme cases, we may form Dirac spinors out of ($\tilde{w}^- \tilde{w}^+$) and ($\tilde{h}^- \tilde{h}^+$) on one hand, or ($\tilde{w}^-, \tilde{h}^+$) and ($\tilde{h}^-, \tilde{w}^+$) on the other hand. The first case provides for a vectorlike theory (no forward-backward asymmetry in the production), with the first fermion more strongly coupled to W and Z than the second, while the other case provides for 2 particles with similar couplings to W, Z and a forward-backward asymmetry in e^+e^-production somewhat smaller than a standard lepton pair. These particles will decay into standard leptons or quarks, plus some neutral "photino" ($\tilde{\gamma}_1, \ldots \tilde{\gamma}_4$). Since the neutral particle is massive, less energy will be available for the outgoing leptons (hadrons) than in a typical lepton sequential decay, which may hamper their detection.

Flavour Changing Transitions and CP Violation

Since susy particles are assumed to be produced in pairs, the simplest process where to look for them is where 0 pairs are produced. It is readily apparent from (2) and (3) that the scalar quarks will not be mass degenerate; therefore, they can mediate flavor changing transitions.[16]. Contributions to the $K^0\bar{K}^0$ mass differences and CP violation parameters arise, e.g., from the graphs



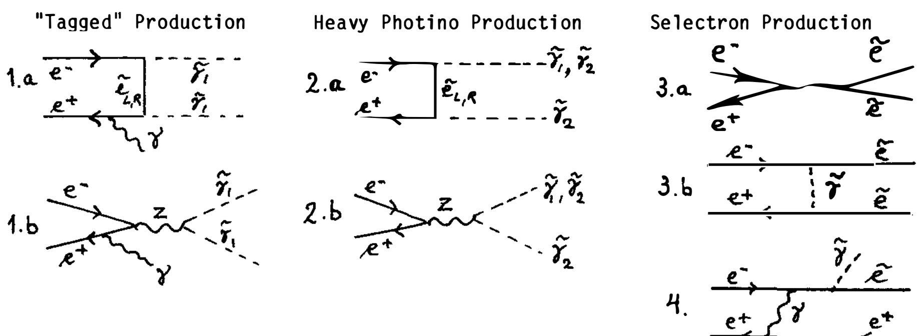


The scalar quarks and the quarks themselves cannot in general be diagonalized simultaneously, and more mixing parameters should thus be introduced. Ref.9 has shown that only mild constraints on the squark masses can be extracted from the $K\bar{K}$ mass difference.

As far as the CP violation parameters are concerned, only simplified cases have been considered so far, where the squark mixing was identified to the usual Kobayashi-Mashkawa matrix, and the extra phases associated with SUSY[18] have been neglected. It is noteworthy that already in this approximation[19][20], the squarks/gluino contribution can flip the sign of ε'/ε with respect to the standard model; small or negative values can be fitted. The low experimental value quoted for ε'/ε (these proceedings) could be the only experimental hint in favor of supersymmetry. However, the uncertainities associated with the prediction of ε'/ε in the standard model do not allow such a conclusion; also, several other schemes could account for the value of ε'/ε (e.g., L.R. models).

SUSY Pair Production

This has certainly been the most investigated topic in SUSY searches. For a general review, see e.g. ref.21. The associated production of gluinos and/or squarks will be dealt with in great detail by M. Barnett (these proceedings), in order to illustrate the difficulty to give model-independent limits I will focus on the simplest possible system of SUSY particles, and deal with scalar electrons $\tilde{e}_L$, $\tilde{e}_R$ and neutralinos $\tilde{\gamma}_1 \ldots \tilde{\gamma}_4$ (all neutral SUSY fermions are denoted $\tilde{\gamma}$ below; the index refers to the mass, $\tilde{\gamma}_1$ being the lighest; while $\tilde{\gamma}_1$ is often assumed to be "the" photino, there is no compulsory reason for this). Several processes have been suggested to observe those particles[22]. The following graphs summarize them:

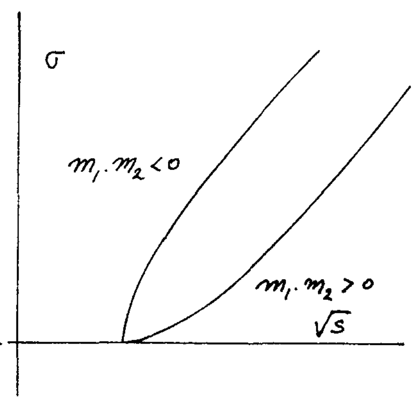


Which of these processes is most favorable for observation depends strongly on the model. In the case of photino production, $\tilde{e}$ exchange (1, a, 2, a) is important; if $\tilde{\gamma}_1$ is a higgino these graphs are negligible. On the other hand 1b vanishes for a pure photino but not for a higgino. The processes described in 1 are further suppressed by the electromagnetic coupling;

therefore, the processes in (2) (where $\tilde{\gamma}_1 \to \tilde{\gamma}_2$ + quarks, or leptons and $\tilde{\gamma}_1$ escapes, which gives a "one sided event") can be competitive if $m_{\tilde{\gamma}1} + m_{\tilde{\gamma}2} < \sqrt{s}$ and the mixing is not negligible, or if $2m_{\tilde{\gamma}2} < \sqrt{s}$. The relative interest of (3) and (4) depends upon the ratio $m_{\tilde{\gamma}}/m_{\tilde{e}}$, but also on the nature of the involved photino, since a higgino would be very lightly coupled.

Experimental bounds will be presented by several groups at this meeting (see e.g., talks by Bohn, Hollebeek, Prepost). While the experimental results are usually formulated in terms of massless photinos, degenerate scalar electrons, and assume no mixing, this should be considered a convenient way of presenting the data rather than a real discussion of the excluded region in parameter space. Such an enterprise, as we have tried to show, would involve dealing with at least a 4 or 5 parameter space and seems somewhat premature at the present stage.

More subtle differences may arise; e.g., in the "one-sided" process (2), the threshold for $\tilde{\gamma}_2\,\tilde{\gamma}_2$ is always P-wave while, if $\tilde{\gamma}_1\,\tilde{\gamma}_2$ are produced, the nature of the threshold behavior depends upon the relative sign of their Majorana masses. Intermediary situations are possible if CP is violated[23]. These peculiarities of Majorana particles, while a potential challenge for experimentalists, would be very interesting to observe.



As a conclusion for this section, possible SUSY signatures are many, but no one can be pointed at as the crucial test; due to the high value of the permitted SUSY effective breaking scale, negative searches even conducted at the next generation of accelerators would not completely exclude the existence of SUSY partners. But a positive evidence for SUSY may appear every day.

3. Finite Models†

The infinities related to the perturbative expansion of gauge theories are adequately dealt with by the renormalization procedure. As we have seen when dealing with the "hierarchy" problem this, however, mixes the various scales of the model and often results in the introduction of more phenomenological parameters.

It has been shown recently[24)] that a large class of finite theories could be built. They rest on the N = 2 extension of supersymmetry [25)] (hypersymmetry), and are fairly restrictive in terms of the particle contents and couplings. The basic structure of such a theory contains the following physical fields:

	vectors	spinors	scalars	group representation
gauge multiplet	V^μ	λ_1	M	adjoint
		λ_2		
scalar multiplet		ψ_1^i	A_1^i	r
(i)		ψ_2^i	A_2^i	$\bar{r}$

With respect to N = 1 SUSY, the number of fermions for each multiplet is doubled; for each scalar multiplet, ψ_1^i and ψ_2^i are left handed fermions transforming respectively under the representation r and $\bar{r}$ of the gauge group $\mathcal{G}$. The "gauge scalar" M transforms according to the adjoint representation of $\mathcal{G}$. The Lagrangian of such theories is severely constrained. Let us first mention the finiteness condition which states the vanishing of the 1-loop β function:

$$\sum_i n_{R_i} C_2(R_i) = n_{adj} C_2(adj) \qquad (4)$$

†This work was done in collaboration with Y.-P. Yao

where n and C_2 respectively stand for the dimension and Casimir coefficient of the representation. As we will see below, this condition strongly limits the number of possible matter representations for a given group (SU(5) is excluded).

The only Yukawa couplings permitted are determined by the gauge interaction[26] and read

$$-ig\sqrt{2}(\overline{\lambda}_2\overline{\lambda}_1 M + \overline{\psi}_1^i\overline{\lambda}_1\psi_1^i + \overline{A}_2^i\lambda_2\psi_1^i + \psi_2^i M\psi_1^i - A_2^m\overline{\lambda}_1\overline{\psi}_2^m) + h.c. \tag{5}$$

(notice that λ_1 acts diagonally on the indices 1,2, while M and λ_2 mix fermions carrying the index 1 with their "mirror partner" carrying index 2.) An explicit mass term for the matter fields is also allowed for each multiplet i:

$$+ m_i\psi_2^i\psi_1^i \tag{6}$$

This far we have only dealt with unbroken N = 2 SUSY. As was the case with N = 1, we may now ask what are this time the "soft breaking terms," which, while breaking SUSY, preserve finiteness.

This question has been dealt with for various groups[27),28)]. Are these soft breaking terms sufficiently general to allow a realistic breaking pattern for the gauge group? Can they also help us get rid of the unobserved mirror symmetry implied by N = 2?

We will consider here the special case where the soft breakings are diagonal in the matter representations (full expressions can be found in ref.28). In addition to N = 1 terms, we may introduce:

$$\delta m_M^2\, MM^+ + \delta m_{1\ell}^2\, |A^1|^2 + \delta m_{2\ell}^2 (A^2|^2 \tag{7.a}$$

$$m_M^{12}\, M^2 + h.c. \tag{7.b}$$

$$\delta m'^2_{\substack{ij\\mn}}\, A_m^i U A_n^j \tag{7.c}$$

$$\frac{i}{2}\,\delta P_{\substack{ij\\mn}}\, A^{+i}_m M A_n^j - \frac{1}{2}\, m_{ij}\lambda^i\lambda^j + h.c. \tag{7.d}$$

where A^iUA^j is only permitted for real or pseudoreal representations, and U is the matrix which projects out the singlet out of A A (e.g., $\varepsilon_{k\ell}$ in the case of SU(2)). The Majorana-like terms (7.b) and (7.c) appear totally unrestricted by perturbative finiteness, while the terms (4.a) and (4.d) have to obey:

$$\delta m_M^2 - \frac{1}{n}\left[(\delta m_1^2)_\ell^{\ \ell} + (\delta m_2^2)_\ell^{\ \ell}\right] = m_{ij}\bar{m}^{ij} \qquad (8.a)$$

$$\left(\delta m_1^2 + \delta m_2^2\right)_\ell^{\ \ell'} = \frac{1}{n}\left(\delta m_1^2 + \delta m_2^2\right)_k^{\ k}\delta_{\ell\ell'} \qquad (8.b)$$

$$\delta P_{\substack{ij\\mn}} = -\,2\sqrt{2}\; m_{ij}\, g\, \delta_{mn} \qquad (8.c)$$

where n is the number of matter representations. As appears readily from (7) and (8), the mirror symmetry which relates A_1 and A_2 in $\mathcal{L}$ can be easily broken by choosing $\delta m_1^2 \neq \delta m_2^2$, since only the sum of those quantities is fixed for each matter representation by (8.a).

This breaking of the $1 \to 2$ symmetry only applies this far to the bosonic sector, and the real concern we have is about the fermionic sector. Before dealing with this we should attract attention to the fact that, while conditions (8) guarantee the perturbative finiteness of the theory, they say nothing of the stability of the vacuum. Introducing negative mass terms, as is customary in spontaneously broken gauge theories may prove dangerous. The danger is quite general in view of the existence of numerous flat directions in the quadratic part of the potential, leading to unboundedness from below. While a general study seems extremely difficult, a few interesting no-go theorems can be found in ref.28. There is, however, a breaking mechanism which is safe with respect to those flat directions. Indeed, the quartic potential has no flat direction where <MA> is vanishing, which is also the condition for a (negative) contribution to arise from the trilinear terms. It is easy to check (27) that such a breaking is indeed both safe and possible.

Which Group?

A general review of the possible grand unification groups can be found in ref.29. An interesting mechanism for the breaking of mirror symmetry has been suggested recently[30)] but unfortunately not in the framework of a grand unified theory-and such a theory is essential to ensure finiteness, since small groups and a fortiori U(1) factors cannot satisfy eq.(4), which has to be true for each factor group.

This is a biased review of groups suitable for the construction of a grand unified finite model. The bias comes from the fact that we give special importance to the breaking scheme in which the trilinear coupling plays a central role, as exemplified in the previous section. In general, both the adjoint and at least one matter representation will then develop v.e.v.'s $\langle M\rangle$ and $\langle A_1\ell\rangle$ respectively.

As a consequence of the presence of the only allowed Yakawa coupling, $g\psi_2{}^\ell M\psi_1\ell$, "Dirac" mass terms will be induced, linking $\psi_1{}^{\ell\alpha}$ and $\psi_2{}^{\ell\alpha}$, where α is an index in group space. Since under the unbroken little group g_1 $\langle M\rangle$ necessarily transforms as a singlet while ψ_1 and ψ_2 transform under reducible conjugate representations R_1 and $\overline{R}_1$ we would get massive Dirac fermions interacting in a vector-like way with the gauge boson representing g_1. (This picture would be modified for the "generation" directly linked to $\langle A_1{}^\ell\rangle$, since we have the further entry $\psi 2^\ell\lambda_2\langle A_1{}^\ell\rangle + \psi_1\lambda\langle A_1{}^{+\ell}\rangle$, but would still obtain for most of the fermions involved).

As a typical example, let us imagine a toy model based on SU(5) [a realistic model is impossible, since all observed particles cannot be included]. The usual breaking along the adjoint (24) leaves $SU(3)^x SU(2)^x U(1)$ invariant; however, it joins the $10 + \overline{10}$, $5 + \overline{5}$ into Dirac fermions; in the 5 representation we have:

$$\psi_2\psi_1 \langle M\rangle \sim (d\ d\ d\ \bar{e}\ \bar{\nu}) \begin{vmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3 \end{vmatrix} \begin{vmatrix} \bar{d} \\ \bar{d} \\ \bar{d} \\ e \\ \nu \end{vmatrix}$$

This may prove a major drawback in the way to construct models. There are, of course, several ways around. We will list them briefly, and concentrate on the direction which seems most promising.

-If the group G is large enough that the physically interesting particles are not affected by <M>, one may avoid the above trouble. However, the difficulty will pop up again at the level of the unbroken subgroup g_1. The breakdown of that group will then have to proceed via the matter representation alone. Such breaking schemes are usually not very promising, however, because the defining representation above breaks SU(5) into SU(4), SU(3), SU(2)... assuming that enough independent sets of scalars are available.

Notice, however, that for each matter representation, one subset of the particles appearing in (9) can be made massless by introducing suitable mass for the matter multiplet, resulting here in a cancellation betwen (6) and (9) for either the "quarks" or "leptons". This means that in such a scheme, physical leptons and quarks cannot be found in the same multiplet (as a very long shot, this can be seen as an argument for an extended proton lifetime).

-By adjusting m_1^2, m_2^2, δm^2 in eq.(1.4) one can tune the ratio <M>/<A>; if <M> is made neglibible, one finds directly the above situation, where the matter representations above are to be held responsible for the whole breaking pattern. We must keep in mind, however, that only a very limited set of matter representations is available due to the finiteness conditions.

-A more expedient way would consist in giving ab initio a large (Majorana) mass to the fermions corresponding to, say ψ_2^{ℓ}. The presence of <M> would then only induce a slight perturbation in the diagonalization of the mass matrix of ($\psi_1{}^\ell$, $\psi_2{}^\ell$, λ_1, λ_2), and leave a state close to ψ_1 essentially massless. Such a Majorana mass $\mu\ \psi_2{}^\ell\ \psi_2{}^\ell$ is usually forbidden, for it would break the gauge symmetry. The necessary condition to allow it is obviously that $R \times R \supset \mathbf{1}$, namely that the representation R be real or pseudoreal. Such a mass term is always allowed if included in an N = 1 soft term $\phi_1{}^\ell U \phi^\ell{}_m$, where ϕ_1 stands for the superfield (A_1, ψ_1, F_1). This solution seems to be leading from bad to worse! Instead of having a "reality" problem associated to the presence of $R + \overline{R}$ for any representation R in use, we further demand that R itself be (pseudo) real!

The advantage, however, is that such a doubling allows us to completely eliminate $\overline{R}$ from the observable spectrum, and liberates us from the unique but unwanted Yukawa coupling $\psi_1 M \psi_2$. A corollary of this is that the light fermion masses will have to be generated beyond the tree level. The loop diagrams involved may prove considerably less transparent to evaluate; on the other hand, they constitute very "soft" effective mass terms for the fermions, which may be an interesting property.

With the above motivation in mind, we now turn to a list of the groups suitable for grand unification, paying special attention to the real or pseudoreal representations.

For each group, the tables below list the representations which are permitted by the finiteness condition (1.2), their real/complex character, their indices $\left(\sim \frac{n_R \cdot C_2(R)}{n_{adj}}\right)$. We have examined successively the groups* SU(N), SO(2N), SO(2N=1) and the exceptional groups E7, E8.

While SO(9) comes close to the correct particle content, using the 16 representation, it is known not to have the correct charge assignments (the right handed leptons transform like doublets under the $SU(2)_{weak}$ group). We therefore do not consider it here.

		Real Pseudo-Complex	Index	Max Number Allowed
SO(9)	16	R	4	3 *
(B4)	9	R	2	
	36	adj	14	
SO(11)	32	P	8	2
(B5)	11	R	2	
	55	adj	18	
SO(13	64	P	16	1
(B6)	13	R	2	
	78	adj	22	
SO(15	128	R	32	NO
(B7)	15	R	2	
	105	adj	26	

		Real Pseudo-Complex	Index	Max Number Allowed
SO(10)	16	c	4	4
(D5)	10	R	2	
	45	adj	16	
SO(12)	32	P	8	2
(D6	12	R	2	
	66	adj	20	
SO(14)	64	c	16	1
(D7)	14	R	2	
	91	adj	24	
SO(18)	128	R	32	forbidden
(D8)	16	R	2	
	120	adj	28	

E7	56	R	12	3
	133	adj	36	
E8	248		60	(N=4)

*wrong particle content

SU(5)	5	C	1	
	10	C	3	
	24	adj	10	
SU(6)	20	R	4	3 *
(A5)	35	adj	12	
SU(8)	70	R	20	NO.
(A7)	63	adj	16	

If we insist on having 3 equivalent "generations" included within the matter fields we see that none of the SU or SO groups can satisfy the finiteness conditions, while retaining an acceptable particle content. (We exclude a priori the real N representations of SO(N), in view of the familiar problems associated with charge-2 exchanges and have not considered here the symplectic groups.) The only groups accepting a triplicate matter generation structure with real or pseudoreal representation are E_7 and E_8. For E_7, taking 3 times the spinorial representation 56 exactly satisfies the finiteness condition: this group, therefore, appears as a very strong candidate. E_8 is a special example; since one cannot distinguish between the gauge and matter fermions, its more natural framework is N=4 supersymmetric theory.

The constraint to have equivalent triplication of the matter generations as a unique solution is obviously very attractive. We may nonetheless think of relaxing it, and take into account the gauge fermions as was done for the

*Same additional solutions can be found in the case of pseudoreal representations for a different realization of N = 2 SUSY[31].

case of E_8. The smallest group then turns out to ge given by SO(11) [as the $\underline{16}$ representation of SO(10) is not real]. (Of course, larger groups than SO(11) may also satisfy our criteria, but we should note that the index of the spinoidal representation grows geormetrically, while that of the adjoint only linearly. Therefore, groups larger than SO(16) must be excluded from our analysis.) Focusing on SO(11), we may satisfy the vanishing beta function criteria (1.2) by including 2(32) + 1(11) as matter representations. When decomposed under SU(5), this gives:

$$32 = \overline{5} + 10 + 1 + 1 + \overline{10} + 5$$

$$11 = \overline{5} + 5 + 1$$

$$55 = 24 + 1 + 10 + \overline{10} + 5 + 5$$

and we, therefore, obtain the required $3(\overline{5}+10)$ fermionic content. We have checked that a satisfactory breaking patterm down to SU(5) was indeed possible; in view of the many parameters still present, study of the further breaking steps proves difficult.

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