TWO-NUCLEON POTENTIAL WITH FULL RECOIL AND THE DETERMINATION OF THE TYPE OF COUPLING BETWEEN THE PION AND THE NUCLEON

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We have calculated the non-static one- and two-pion-exchange potentials (referred to hereafter as OPEP and TPEP respectively) without making use of the expansion in the inverse mass of the nucleon, as a step to establish the interaction between two nucleons when they are not too close to each other ($x = \mu r \gtrsim 0.7$, μ^{-1} is the pion Compton wave length).

First we derived the potentials in momentum space and found that they can be written in the form of local potentials in x-space very accurately for two nucleon scatterings below three or four hundred MeV.

Main results of our investigation [1] are the following: the concept of the local potential in the x-space is a very good approximation for the treatment of the nucleon-nucleon scattering up to 300 or 400 MeV.

Thus, the usual analyses based on the use of the potential is justified. For momentum transfer small compared to 3μ, or for distances large compared to $(3\mu)^{-1}$, the non-static OPEP is determined unambiguously for the case of the ps-coupling, and is given by the second order perturbation calculation with the observed coupling constant [2].

It seems physically plausible that the same situation would also hold for the case of the pv-coupling, if pions have some sort of a structure (as in the composite model) [2].

Nonstatic OPEP with full recoil may be expressed in the following form for low energy ($\lesssim 300$ MeV) phenomena

$$V^{1\pi}(x,q) = \mu\frac{f^2}{4\pi}(\tau^{(1)}\cdot\tau^{(2)})\cdot\Big\{V_\sigma(x)\sigma^{(1)}\cdot\sigma^{(2)} + V_\tau(x)S_{12} + V_q(x)\frac{\sigma^{(1)}\cdot q\,\sigma^{(2)}\cdot q}{M^2} + V_{\sigma q}(x)\sigma^{(1)}\cdot\sigma^{(2)}\frac{q^2}{M^2} + V_{LL}(x)[\sigma^{(1)}\cdot\sigma^{(2)}L^2 - \sigma^{(1)}\cdot L\sigma^{(2)}\cdot L]\Big\}$$

where

$$q\psi(r) = -i\nabla\psi(r) \tag{1}$$

and

$$V_\sigma(x) = \frac{1}{3}\frac{e^{-x}}{x} \tag{2a}$$

$$V_T(x) = \frac{1}{3}\left(1+\frac{3}{x}+\frac{3}{x^2}\right)\frac{e^{-x}}{x} \tag{2b}$$

$$V_q(x) = \pm\frac{1}{2}\left\{1\mp\left(\frac{1}{x}+\frac{1}{x^2}\right)\right\}\frac{e^{-x}}{x} \tag{2c}$$

$$V_{\sigma q}(x) = \mp\frac{1}{2}\left(1+\frac{1}{x}\right)\frac{e^{-x}}{x^2} \tag{2d}$$

$$V_{LL}(x) = \pm\frac{1}{2}\left(\frac{\mu}{M}\right)^2\left(1+\frac{3}{x}+\frac{3}{x^2}\right)\frac{e^{-x}}{x^3} \tag{2e}$$

the upper (lower) sign corresponds to the case of the ps(pv)-coupling. Okubo and Marshak [3] calculated previously the corresponding expressions up to the order of g^2M^{-2} using the M^{-1} expansion in the case of the ps-coupling.

Non-static parts of the OPEP (Eqs. (2c)-(2e)) have different signs for different types of the coupling between the pion and the nucleon. This is the first definite difference derived nearly unambiguously from the theories with ps- and pv-couplings.

In the ^1S state, V_T does not contribute and V_q and $V_{\sigma q}$ are negligible compared with the kinetic energy term if x is not very small. Hamada[4] has shown that

the potential similar to $V_{LL}(x)$ (Eq.(2*e*)) is manifestly required to fit both the $^1S_0$ and $^1D_2$ phase shifts up to 310 MeV.

He has found that the phenomenological potentials of the following form should be added to fit the above-mentioned data :

$$V_{LL}^H(a) = 0.0054\mu L(L+1)\frac{e^{-x}}{x^3} \qquad (3a)$$

$$V_{LL}^H(b) = 0.000155\mu L(L+1)\frac{e^{-x}}{x}\left(1+8\frac{e^{-x}}{x}+6\frac{e^{-2x}}{x^2}\right) \qquad (3b)$$

Eq.(3*a*) has similar form and magnitude with Eq.(2*e*), which is, in the ^1S state, given by

$$\mp 0.0018\mu L(L+1)\frac{e^{-x}}{x}\left(1+\frac{3}{x}+\frac{3}{x^2}\right) \qquad (4)$$

Apparently the pv-coupling case gives the right sign, while the ps-coupling case gives the wrong sign and disagrees with the angular distribution data at 210 and 310 MeV data. (see Fig. 1) [5].

Detailed examination, which takes into consideration the possible ambiguities in the V_{LL}-term of TPEP, shows that the strength of the ps-coupling does not exceed ½ of that of the pv-coupling. More precise determination of the ratio of two couplings will become possible if more thorough experimental data become available at lower energies ($\sim$100 MeV) [5].

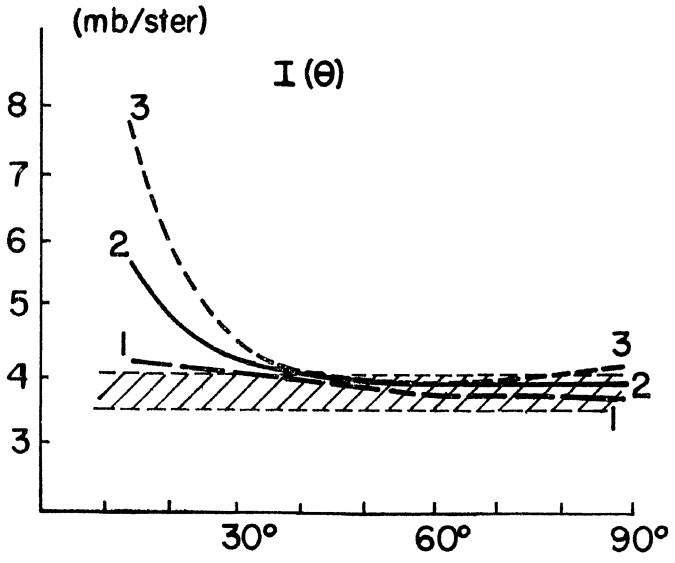

Fig. 1 Differential $p-p$ scattering cross section at 310 MeV. The shaded region shows the experimental cross section. — — indicates solution 1 : central plus Eq. (3 *b*), which nearly corresponds to the pv-coupling. —— indicates solution 2 : central force only. - - - - - indicates solution 3 : central minus Eq. (3 *b*), which nearly corresponds to the ps-coupling.

We have also evaluated the two-pion-exchange potential, up to the fourth order of the coupling constant, without using the expansion with respect to M^{-1} [6]. It is shown that the previous results making use of the M^{-1}-expansion are modified seriously by taking the recoil fully into account. Also in this case, the results may be expressed by local potentials in x-space very accurately for energies below three or four hundred MeV. (Eq. (5)) One may fit the experimental phase shifts adopting the nonstatic OPEP and TPEP derived by us in the outer region, $x<0.7$.

$$V^{2\pi}(x) = V_0(x)+V_\sigma(x)\sigma^{(1)}\cdot\sigma^{(2)}+V_T(x)S_{12}+$$
$$+V_{LS}(x)L\cdot S+V_q(x)\sigma^{(1)}\cdot q\sigma^{(2)}\cdot q+$$
$$+V_{LL}(x)\sigma^{(1)}\cdot L\sigma^{(2)}\cdot L+V_{\sigma L}(x)\sigma^{(1)}\cdot\sigma^{(2)}L^2$$

where

$$V_i(x) = 1\cdot V_i^1(x)+(\tau^{(1)}\cdot\tau^{(2)})V_i^\tau(x) \qquad (5)$$
$$i = 0,\sigma,T,LS,\ldots$$

Momenta of virtual pions were cut off at 6μ.

$$\begin{cases} V_0^\tau = -\mu\dfrac{0.65\times 3^2}{4\pi}\dfrac{e^{-3x}}{x} \\ V_0^1 = -\frac{1}{2}V_0^\tau \end{cases}$$

$$\begin{cases} V_\sigma^1 = \mu\dfrac{0.02\times 3.5^4}{3\pi}\dfrac{e^{-3.5x}}{x} \\ V_\sigma^\tau = -0.15V_\sigma^1 \end{cases}$$

$$\begin{cases} V_T^1 = -\mu\dfrac{0.01\times 3.5^5}{3\pi}\left(1+\dfrac{3}{3.5x}+\dfrac{3}{(3.5x)^2}\right)\dfrac{e^{-3.5x}}{3.5x} \\ V_T^\tau = -0.15V_T^1 \end{cases}$$

The above formulae hold for both ps- and pv-couplings. Furthermore we have

$$\begin{cases} V_{LS}^{1} = -\mu \dfrac{0.018\times 4^{5}}{4\pi}\left(1+\dfrac{1}{4\pi}\right)\dfrac{e^{-4x}}{(4x)^{2}} \\ V_{LS}^{\tau} = -0.56 V_{LS}^{1} \text{ for ps coupling,} \end{cases}$$

$$\begin{cases} V_{LS}^{1} = -\mu \dfrac{0.016\times 3^{2}}{(2\pi)^{2}}\left\{\dfrac{3}{x^{2}}K_{0}(3x)+\dfrac{4}{x^{3}}K_{1}(3x)\right\} \\ V_{LS}^{\tau} = -\mu \dfrac{0.041\times 4^{5}}{4\pi}\left(1+\dfrac{1}{4x}\right)\dfrac{e^{-4x}}{4x} \text{ for pv coupling,} \end{cases}$$

$$\begin{cases} V_{LL}^{1} = -\mu \dfrac{0.06\times 10^{-2}\times 3^{7}}{4\pi}\left(1+\dfrac{3}{3x}+\dfrac{3}{(3x)^{2}}\right)\dfrac{e^{-3x}}{(3x)^{3}} \\ V_{LL}^{\tau} = 0.4 V_{LL}^{1} \text{ for pv coupling,} \end{cases}$$

$$\begin{cases} V_{LL}^{1} = \mu \dfrac{0.01\times 10^{-2}\times (2.5)^{7}}{4\pi}\left(1+\dfrac{3}{2.5x}+\dfrac{3}{(2.5x)^{2}}\right)\dfrac{e^{-2.5x}}{(2.5x)^{3}} \\ V_{LL}^{\tau} = -0.7 V_{LL}^{1} \text{ for ps coupling,} \end{cases}$$

$$\begin{cases} V_{\sigma L}^{1} = \mu \dfrac{4.8\times 10^{-4}\times 3^{7}}{4\pi}\left(1+\dfrac{3}{3x}+\dfrac{3}{(3x)^{2}}\right)\dfrac{e^{-3x}}{(3x)^{3}} \\ V_{\sigma L}^{\tau} = -0.14 V_{\sigma L}^{1} \text{ for ps coupling,} \end{cases}$$

$$\begin{cases} V_{\sigma L}^{1} = -\mu \dfrac{5.0\times 10^{-4}\times 3^{7}}{4\pi}\left(1+\dfrac{3}{3x}+\dfrac{3}{(3x)^{2}}\right)\dfrac{e^{-3x}}{(3x)^{3}} \\ V_{\sigma L}^{\tau} = -0.14 V_{\sigma L}^{1} \text{ for pv coupling} \end{cases}$$

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DISCUSSION

LEVY : I realize that you have calculated the potential without using expansion in $1/M$ and I realize that this is possible in momentum space. You are led then to a non-local potential. But you have translated this into a local type of potential, and I would like to understand what prescription you have used. Is this something which is more like Charap and Fubini or is it something different?

MACHIDA : I think it is different. We have first performed all calculations in momentum space and in that space the potential is a function of x and momentum p and, in general, also of other quantities like angular momentum or energy. We have investigated numerically the detailed properties of the one-pion exchange contribution and the two pion exchange potential in the momentum space. We found that the dependence on energy and the momentum can be expressed by a very simple combination of momentum and angular momentum and energy. We found that this was a very good approximation.

LEVY : If I understand it you put all the non-locality into succeeding powers of the angular momentum.

MACHIDA : No, we have not used any expansion.

LEVY: I am not talking about expansion. I am saying all the non-local aspects appear in derivatives and then these apply to local functions. I will look into the details because I do not see how this can be possible without a $1/M$ expansion.

MACHIDA : We have found this result in a very detailed calculation. I think one cannot see the results before such a very detailed calculation.

BREIT : I should like to ask how it comes about that in Gupta's calculation there are only central potential effects in V_4?

MACHIDA : Gupta has used the S matrix method, that is, he first calculated the S matrix. He calculated the elements of the potential in the Born approximation.

If you make a Born approximation, the terms other than the central part vanish.

BREIT: For some applications such as the one I mentioned, however, it is very convenient to have the result directly in such a form that one can use a first order calculation. So that, as I understand it, there is really no disagreement between you and Gupta.

MACHIDA: I think, to calculate the non-static potential, we cannot use the S matrix method, because the potential has a matrix element with different energy. However, if we use the S matrix method, one just obtains the diagonal elements. Thus, you obtain only the central part and you would miss the non-central part.

BREIT: For other applications I can see that this is of very great interest. But for applications for nucleon-nucleon scattering it would seem that it should be sufficient to have the diagonal elements.

MACHIDA: However, the point is that if you do not use a potential you can be content with the diagonal elements, but if you use potentials you have to calculate the non-diagonal elements, with respect to energy. And in this case, you cannot use the S matrix method to obtain the non-static potential.

BREIT: I agree with that completely, but nevertheless, for applications like nucleon-nucleon scattering it would seem that the other method is already sufficient.

POLARIZATION IN PROTON-PROTON AND PROTON-BERYLLIUM SCATTERING AT 1.7 GeV

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In this paper I report on results of double scattering experiments to investigate proton-beryllium and proton-proton polarization at 1.7 GeV. The experiments were carried out at the Saclay proton synchrotron by P. Bareyre, J. F. Detoeuf, L. W. Smith, R. D. Tripp and myself.

(A) EXPERIMENTAL

The left-right asymmetry of the proton differential cross section in the second scattering has been measured under the various conditions listed in Table I.

In experiments 1, 2 and 3, protons were first scattered through 4° 10′ from an internal beryllium target. The scattered beam was momentum analysed by the synchrotron magnetic field and focused by quadrupoles onto the second target.

In experiments 4 and 5, protons were first scattered through 18° 17′ from a CH_2 or C target located in one of the straight sections of the proton-synchrotron. The scattered beam was momentum analysed by an external magnet and focused by quadrupoles. In the beam from the CH_2 target 70% of the flux impinging on the second target originates from elastic hydrogen scattering and the remainder from quasi-elastic carbon scattering.

In experiments 1 and 2 the protons are detected after the second scattering (Be scattering or small angle hydrogen scattering) by a counter telescope made of two plastic scintillators and an ethylene filled Čerenkov counter utilized to discriminate against inelastically scattered protons.

In experiments 3, 4, and 5 (large angle p-p scattering) both of the outgoing protons were detected in coincidence by plastic scintillators at conjugate angles.