Flux Compactifications and Intersecting Sources

by

Danielle Smith

A Dissertation

Presented to the Graduate Committee

of Lehigh University

in Candidacy for the Degree of

Doctor of Philosophy

in

Physics

Lehigh University

May 2024

This dissertation is accepted and approved in partial fulfillment of the requirements for the Ph.D. in Physics.

Date Approved

Timm Wrase, Committee Chair

Sera Cremonini

Bitan Roy

Jerome Licini

Huai-Dong Cao

Contents

List of Figures

Abstract

When discussing the expansion of the universe, we often use a scalar potential as a model. To build this type of model, one can use flux compactifications in type IIB string theory. In the introduction of this thesis, we cover this and other relevant concepts. Then, in the next section, we discuss our work in [5], where we examine a nongeometric model and discuss the local minima of its associated scalar potential. Finally, in the last section, we look at our work in [6], where we study the intersection between two higher dimensional objects in string theory, known as D-branes or O-planes. The intersection of these objects in string theory are of interest when building models of particle physics and for flux compactifications.

1 Introduction

The work in this thesis will be based off of our work done in [5] and [6]. To motivate this, let's first talk about some basic concepts.

1.1 Scalar fields

A scalar field is a function which ascribes some scalar value to every point in spacetime. A common example of this is a temperature map, like those used in weather forecasts on news channels, as shown in Figure 1. Given this

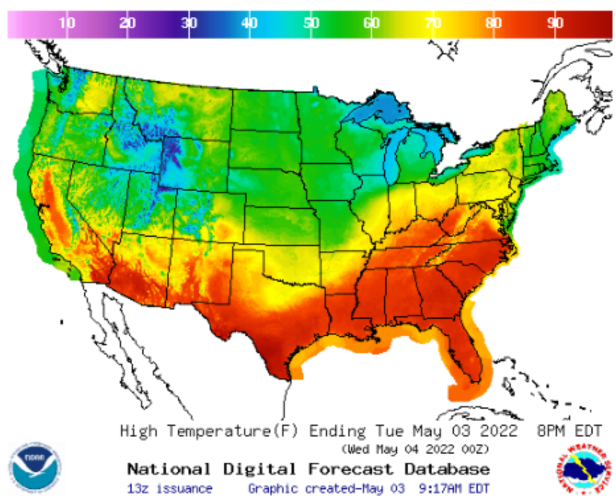

Figure 1: This is an example of a temperature map of the United States of America, provided by the National Weather Service [1].

definition, we can use a scalar field to describe some aspect of spacetime. In particular, consider a scalar field ϕ with the following action [7]:

$$S = \int d^4x\sqrt{-g}\left(-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right) \tag{1.1}$$

where $V(\phi)$ is the potential of the scalar field, g is the spacetime metric, and $\mu = 0, 1, 2, 3$.

Then, by varying the action with respect to the metric, the energy density of ϕ is found to be

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\frac{(\nabla\phi)^2}{a^2} + V(\phi) \tag{1.2}$$

and the pressure of ϕ is found to be

$$P_\phi = \frac{1}{2}\dot{\phi}^2 - \frac{1}{6}\frac{(\nabla\phi)^2}{a^2} - V(\phi). \tag{1.3}$$

For a slowly-varying scalar field, the derivatives in these equations would be small, leading to the simplification:

$$\rho_\phi \approx V(\phi) \approx -P_\phi. \tag{1.4}$$

If we compare this to the relationship between energy density and pressure for the cosmological constant:

$$\rho_\Lambda = \frac{\Lambda}{8\pi G} = -P_\Lambda, \tag{1.5}$$

then we see that these two equations are similar. This leads to the conclusion that the potential of a slowly-varying scalar field could play the role of the cosmological constant.

1.2 Inflation

Now that we've motivated the idea of a slowly-varying scalar field, let's give more details to the idea of slow-roll inflation. This is the general type of scalar potential discussed in section 2. This can be described by a scalar field coupled to general relativity with the action [2]:

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{2}M_P^2 R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right) \tag{1.6}$$

where M_P is the reduced Planck mass and $\mu = 0, 1, 2, 3$.

A graph of a potential is shown in Figure 2.

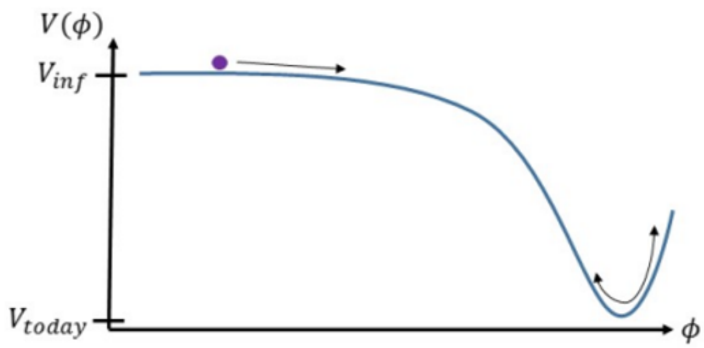

Figure 2: This is the general shape of the potential expected for slow-roll inflation [2].

This leads to slow roll inflation if the following conditions are met:

$$\epsilon_V \equiv \frac{M_P^2}{2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2 \ll 1 \tag{1.7}$$

and

$$|\eta_V| \equiv M_P^2\left|\frac{V''(\phi)}{V(\phi)}\right| \ll 1. \tag{1.8}$$

Because space gets stretched exponentially during inflation, the spatial variations die off quickly, meaning $\partial_\mu\phi\partial^\mu\phi \approx \partial_t\phi\partial^t\phi \ll V(\phi)$ [2]. So the action reduces to

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{2}M_P^2 R - V(\phi)\right). \tag{1.9}$$

Since the potential depends on the scalar field, the potential is approximately constant because ϕ is changing slowly. This means that the scalar field acts like general relativity with the non-vanishing cosmological constant $\Lambda M_P^2 = V(\phi)$ [2].

1.3 Compactifications

Another important concept for this research is that of compactification. Supersymmetric string theory requires 10 dimensions, but we only see 4 dimensions in our universe (3 spatial dimensions plus time). Calabi-Yau (CY) manifolds are generally used to explain this, so let's look at a toy example. Consider a parallelogram folded to become the shape of a torus, as seen in Figure 3.

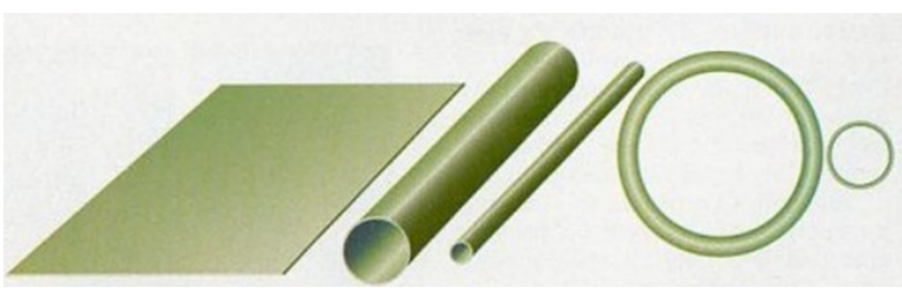

Figure 3: This is a diagram showing how a parallelogram can be folded into the shape of a torus [2].

If we consider the parallelogram, we can see that it has three real parameters. Let's identify the length of the bottom by R_1, the height by R_2, and the angle betweeen the two sides by θ. The overall volume of the torus is controlled by $R_1 R_2$, and the overall shape of the torus is determined by both $\frac{R_1}{R_2}$ and θ. If a theory of general relativity is compactified on this torus, then these parameters are part of the internal metric and give rise to 4-dimensional scalar fields [2].

The above is also true for more complicated string compactifications on 6 real dimensional CY-manifolds. A limit of a CY-manifold can be constructed by taking three identical copies of the torus described above. This gives rise to three real scalar fields [2].

If we restrict ourselves to type IIB string theory, we can take the low energy limit, which gives a 10D $\mathcal{N} = 2$ supergravity. In this theory, there are two 10D scalars (the dilaton which sets the string coupling and the axion C_0), two real 2-forms (B_{MN} and C_{MN}), and a 4-form (C_{MNOP}). The dilaton and the axion can be combined into a complex scalar, called the axio-dilaton [2]:

$$S = C_0 + \frac{i}{g_S} = C_0 + ie^{-\phi} \tag{1.10}$$

where g_S is the string coupling.

Although B_{MN} and C_{MN} can combine to give complex 4D scalars if M and N extend along the internal directions, we will restrict ourselves to compactifications with no appropriate 2-cycles along which these indices can extend. C_{MNOP} in this compactification gives rise to a real scalar field which combines with the volume modulus $(R_1R_2)^2$ to give a complex modulus T, called the Kähler modulus. In more generic string compactifications of this type on CY-manifolds, there can be multiple Kähler moduli, T^k, whose imaginary parts control the volumes of the internal cycles. But because we are making the three tori identical, all of the T fields are equal. The two real scalar fields $\frac{R_1}{R_2}$ and θ from above combine to give a complex modulus U, called the complex structure modulus. Again, there can in principle be multiple of these (U^k), but making the three tori equal identifies all of the U fields with each other. This model gives rise to a 4D $\mathcal{N} = 1$ supergravity and is called the STU-model [2].

Some important restrictions we have to make to trust our theory are that Im(S) $\gg 1$ and Im(T) $\gg 1$. This ensures that the string coupling is small (so we do not have large corrections) and that the volume of the internal space is large [2]. For our specific model, there are extra dimensions threaded with electric and magnetic field lines and potentially wrapped by D-branes and O-planes, as shown in Figure 4.

1.4 The Swampland

Effective field theories (EFTs) are very useful in describing physical phenomena up to a certain energy scale, denoted as Λ. Modifications to the EFT are required above this scale, but it accurately describes the physics below this scale according to both theory and experiments [4].

When given some UV (ultraviolet) complete theory, meaning a theory that is well-defined at arbitrarily high energies, it is possible to integrate out the UV degrees of freedom. This gives the low energy EFT. So for a d-dimensional

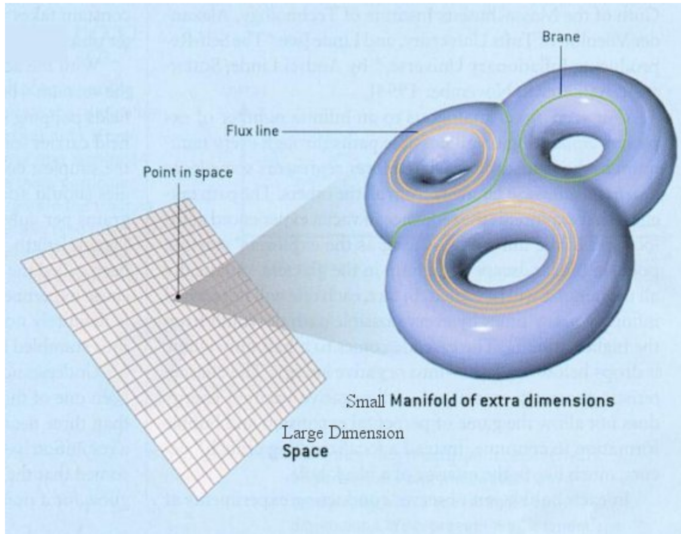

Figure 4: This is a sketch of a string compactification [3].

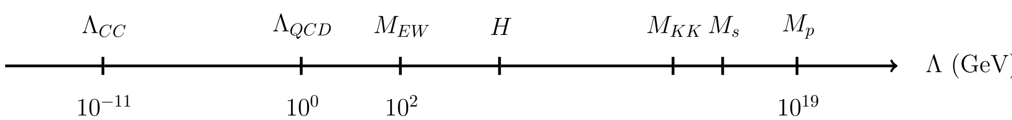

Figure 5: This figure shows typical reference scales in high energy physics, including the cosmological constant, quantum chromodynamics, electroweak symmetry breaking, and the Planck scale, M_p. H indicates a possible estimation for the Hubble scale during inflation, M_{KK} indicates the Kaluza-Klein compactification scale, and M_s indicated the string scale [4].

theory, the effective Lagrangian can be separated into the renormalizable part and a tower of non-renormalizable operators [4]

$$L_{eff} = L_{ren} + \sum_{n=d}^{\infty} \frac{\mathcal{O}_n}{M_p^{n-d}} \tag{1.11}$$

where M_p is the Planck mass.

The aim of the Swampland program is to explore whether some quantum EFT weakly coupled to Einstein gravity can always be UV completed to a consistent theory of quantum gravity. In other words, can the process always be reversed? No, as it turns out, we cannot UV complete any EFT in a way that is consistent with quantum gravity. Although the string landscape is large, not everything is covered. The question then becomes: what conditions does an EFT need to satisfy in order for this to be possible? The Swampland is defined as "those apparently consistent (anomaly free) quantum EFTs that cannot be embedded in a UV consistent theory of quantum gravity" [4]. Essentially, if we say that the EFTs that are consistent with quantum gravity are in "the landscape," then what is the boundary that separates the Swampland from the landscape [4]? To help visualize this question, Figure 6 has been included below.

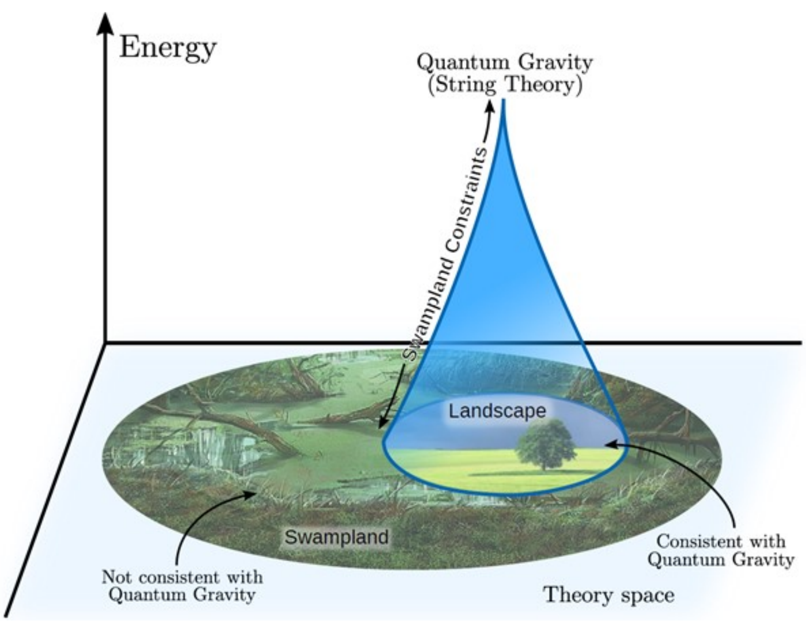

Figure 6: This figure shows the Swampland and the Landscape of EFTs. The cone is formed because the constraints on the Swampland become stronger at high energies [4].

1.5 Sources in String Theory

String theory has not just string but extended objects that we briefly review here.

1.5.1 D-branes

Writing the open string mode expansion:

$$
\begin{aligned}
X^\mu(z,\bar{z}) &= X^\mu(z) + X^\mu(\bar{z}) \\
X^\mu(z) &= \frac{x^\mu}{2} + \frac{x'^\mu}{2} - i\alpha' p^\mu \ln(z) + i\left(\frac{\alpha'}{2}\right)^{1/2} \sum_{n\neq 0} \frac{1}{n}\alpha_n^\mu z^{-n} \\
X^\mu(\bar{z}) &= \frac{x^\mu}{2} - \frac{x'^\mu}{2} - i\alpha' p^\mu \ln(\bar{z}) + i\left(\frac{\alpha'}{2}\right)^{1/2} \sum_{n\neq 0} \frac{1}{n}\alpha_n^\mu \bar{z}^{-n}
\end{aligned} \tag{1.12}
$$

where x'^μ is an arbitrary number [8]. This number cancels out when doing the usual open string coordinate. If we place X'^{25} on a circle of radius R, then the T-dual coordinate is

$$
\begin{aligned}
X'^{25}(z,\bar{z}) &= X^{25}(z) - X^{25}(\bar{z}) \\
&= x'^{25} - i\alpha' p^{25} \ln\left(\frac{z}{\bar{z}}\right) + i(2\alpha')^{1/2} \sum_{n\neq 0} \frac{1}{n}\alpha_n^{25} e^{-in\tau}\sin(n\sigma) \\
&= x'^{25} + 2\alpha' p^{25}\sigma + i(2\alpha')^{1/2} \sum_{n\neq 0} \frac{1}{n}\alpha_n^{25} e^{-in\tau}\sin(n\sigma) \\
&= x'^{25} + 2\alpha' \frac{n}{R}\sigma + i(2\alpha')^{1/2} \sum_{n\neq 0} \frac{1}{n}\alpha_n^{25} e^{-in\tau}\sin(n\sigma)
\end{aligned} \tag{1.13}
$$

The momentum usually comes from the zero mode term in the mode expansion, and we can see that there is no τ dependence in this term. This means that there is no momentum. Because the oscillator terms equal zero at the endpoints $\sigma = 0, \pi$, we see that the endpoints cannot move in the X'^{25} direction. We can see that we have the Dirichlet boundary condition, $\partial_t X = i\partial_\tau X = 0$, instead of the typical Neumann boundary condition, $\partial_n X = \partial_\sigma X = 0$. For clarity, ∂_n refers to the normal partial derivative, and ∂_t refers to the tangential partial derivative. More precisely, the ends are fixed in place [8]:

$$X'^{25}(\pi) - X'^{25}(0) = \frac{2\pi\alpha' n}{R} = 2\pi n R' \tag{1.14}$$

As we can see, the values of the coordinate X'^{25} at each end are the same up to an integer multiple of the periodicity of the dual dimension. This is consistent with the idea that the definition of the normal and tangential derivatives get exchanged under T-duality [8]:

$$\begin{aligned} \partial_n X^{25}(z,\bar{z}) &= \frac{\partial X^{25}(z)}{\partial z} + \frac{\partial X^{25}(\bar{z})}{\partial \bar{z}} = \partial_t X'^{25}(z,\bar{z}), \\ \partial_t X^{25}(z,\bar{z}) &= \frac{\partial X^{25}(z)}{\partial z} - \frac{\partial X^{25}(\bar{z})}{\partial \bar{z}} = \partial_n X'^{25}(z,\bar{z}). \end{aligned} \tag{1.15}$$

It is important to realize that this all pertains to just X^{25}, the direction we T-dualised. The ends can still move in the remaining 24 spatial directions, which constitutes a hyperplane called a ‘D-brane’. Since there are 24 spatial dimensions, this is more specifically called a D24-brane [8].

1.5.2 O-planes

The effect of T-duality can also be understood as a one-sided parity transformation. In the case of closed strings, the original coordinate is $X^m(z,\bar{z}) =$

$X^m(z) + X^m(\bar{z})$, and the dual coordinate is $X'^m(z, \bar{z}) = X^m(z) - X^m(\bar{z})$. A world sheet parity reversal is the action of exchanging $X^\mu(z)$ and $X^\mu(\bar{z})$. For the dual coordinate, this gives [8]:

$$X'^m(z, \bar{z}) \leftrightarrow -X'^m(\bar{z}, z). \tag{1.16}$$

Notice that this is the product of world-sheet and spacetime parity operation. For unoriented strings, they are invariant under the world sheet parity operation, while the dual coordinate, they are invariant under the product of the world-sheet parity operation and a spacetime parity operation. This generalisation of the usual unoriented theory is called an 'orientifold'. This term mixes the term 'orbifold' and orientation reversal. Unlike a D-brane, an O-plane is not a dynamical object [8].

1.5.3 Smearing Sources

The process of smearing a source is usually done by removing the delta function for the source and replacing it with a constant in the equations of motion. This delta function localizes the brane, so by removing it, we are spreading the source into all dimensions. This means that we are no longer constraining the source to extend along only particular dimensions, leaving no transverse directions.

1.6 Supersymmetry

Many people have been interested in symmetries that extend Poincaré symmetry since the 1960s as a part of certain mathematical proofs by Coleman and Mandula [9]. However, Golfand and Likhtman found the first supersymmetric (SUSY) model in four dimensions in 1970 [10]. In this theory, they extended the Poincaré algebra to a "graded-Lie algebra," meaning algebra with anticomutators and spinor generators. This allowed for the possibility of a symmetry between bosons and fermions. Then, Haag et. al. extended the Coleman-Mandula theorem to allow for these algebras. They showed that the only possible extension of the Poincaré algebra is SUSY and found the most general form of the SUSY algebra [9].

The generators of SUSY include the usual Poincaré generators for translations, rotations, and boosts as well as complex, anticommuting spinors Q and their conjugates $Q^\dagger$ [9]:

$$\{Q_\alpha, Q_\beta\} = \{Q^\dagger_{\dot{\alpha}}, Q^\dagger_{\dot{\beta}}\} = 0. \tag{1.17}$$

The nontrivial extension of Poincaré symmetry arises because a translation generator, the momentum operator P_μ, is given from the anticommutator of the spinors and their conjugates [9]:

$$\{Q_\alpha, Q^\dagger_{\dot{\alpha}}\} = 2\sigma^\mu_{\alpha\dot{\alpha}} P_\mu, \tag{1.18}$$

where

$$\sigma^{\mu}_{\alpha\dot{\alpha}} = \left(\mathbb{1}, \sigma^{i}\right), \bar{\sigma}^{\mu}_{\alpha\dot{\alpha}} = \left(\mathbb{1}, -\sigma^{i}\right). \tag{1.19}$$

Note that $\mathbb{1}$ denotes the 2×2 identity matrix and the σ^i are the usual Pauli matrices:

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{1.20}$$

2 Type IIB flux compactifications with $h^{1,1} = 0$

The following work in this section is from a paper published in JHEP, which is also available on arXiv. I completed this work in collaboration with Jacob Bardzell, Eduardo Gonzalo, Muthusamy Rajaguru, and Timm Wrase [5].

2.1 Introduction

Recent years have seen a flurry of activities related to the swampland program [11, 12, 13]. Many new conjectures were proposed and fascinating interconnections between different conjectures became apparent. However, given that it is extremely hard to prove any swampland conjecture (see for example [14]), one might wonder whether all of the conjectures are truly imposed by quantum gravity or whether some arose from our somewhat limited understanding of string theory. Given that many conjectures are motivated and tested against what we observe in controlled string theory setups, there is an apparent need to broaden our tool kits and to get trustworthy results from larger and larger classes of string theory setups.

While this is a noble goal we are faced with the immediate problem that enlarging the landscape of trustworthy string theory constructions is extremely difficult. New approaches are rare (see for example [15]) and in most instances, like for example in the absence of supersymmetry, one can only study a fairly limited set of string theories. While these might already hold surprises (see for example [16, 17, 18, 19, 20, 21]), we are clearly very far from even remotely understanding the landscape of non-supersymmetric string theories.

Several conjectures involving moduli spaces and scalar potentials have been proposed recently in [22, 23, 24, 25]. Sometimes these are influenced by what does and does not work in explicit string theory compactifications. The most studied of such string compactifications are based on geometric compactifications of 10d supergravity theories that arise as low energy limits of string theory. One may be tempted to conjecture that a property that we observe in such a corner of moduli space is indeed a fundamental consistency requirement of quantum gravity. However, the presence of a heuristic argument, for example based on black hole physics, is usually considered a more important hint that the criteria applies more generally. In the absence of such arguments one can only try to enlarge the landscape of four-dimensional theories that one can obtain from string theory, to check existing conjectures against a larger part of the string landscape. This is what we are doing here, by focusing on a corner of the landscape which has not been explored much, namely flux compactifications with orientifolds and purely non-geometric SCFT descriptions for the internal dimensions. In particular, we focus on type IIB and two different Landau-Ginzburg (LG) orientifold models, with F_3 and H_3 fluxes turned on. The tools necessary to determine the low energy effective action for these models were spelled out in [26]. See [27, 28] for additional details regarding the geometrical description of the mirror of the rigid CY that we consider.

In this work we revisit and expand the results of [26, 29, 30]. We find new AdS, Minkowski and dS solutions and discuss them in the context of the swampland program. In the first part of the paper we focus our attention on supersymmetric solutions. At weak coupling and large complex structure we find several infinite families of AdS solutions. In some cases the solutions are mirror dual to the well known type IIA AdS flux vacua found by DeWolfe, Giryavets, Kachru and Taylor (DGKT) [31] (see also [32, 33, 34, 35, 36] for earlier work). We find perfect agreement with the AdS distance conjecture except in one family of solutions which is dual to DGKT. It was argued in [37] that the presence of a $\mathbb{Z}_k$ discrete symmetry forces a modification of the conjecture. We argue that such a symmetry is indeed present in our setup so we find an agreement with the refined version of the conjecture, just like for DGKT.

Surprisingly our setup, which is essentially dual to a generalization of the DGKT model in type IIA, allows also for infinite families of AdS solutions with an ever growing number of D3 branes. Such solutions do naively give rise to AdS_4 spacetimes with arbitrarily large gauge group rank. We are not aware of any such solutions in the literature and they certainly deserve further study and scrutiny.

We also find fully stabilized four-dimensional Minkowski families of solutions, which are to our knowledge the only full-fledged string theory constructions of $\mathcal{N} = 1$ Minkowski vacua without flat directions. Such Minkowski vacua were previously discovered in [26, 29] and their validity beyond the perturbative regime was shown to be guaranteed by a powerful non-renormalization theorem in [26]. We extend the previous analysis to show that these vacua do not only survive despite string loop corrections but we also prove that, although string loop corrections can change the masses, they cannot lead to any flat directions.

We also study non-supersymmetric solutions. In the parametrically weak coupling and large complex structure regime we find a family of non-supersymmetric AdS vacua as in the dual DGKT setting. We find that the masses lie above the BF bound, so the vacuum is perturbatively stable. According to the conjecture in [38] these should be unstable but we leave it for the future to study potential decay. In the not-so-weak coupling regime we find a metastable dS vacuum which requires the presence of D3 branes in order to satisfy the tadpole cancellation condition. These vacua are, however, not protected by the non-renormalization theorem. In particular, the Kähler potential is expected to receive quantum corrections that are not under control and therefore these dS vacua are not trustworthy.

The structure of the section is as follows. In the next section 2.2 we review how to obtain the low energy 4d $\mathcal{N} = 1$ theories and revisit the no-go theorems protecting the superpotential. Then, in section 2.3 we discuss fully stabilized, supersymmetric Minkowski vacua and contrast their existence with related swampland conjectures. In section 2.4 we find several new families of supersymmetric and non-supersymmetric AdS vacua and we discuss their connection to the AdS distance conjecture. Lastly, we study dS vacua in section 2.5 before concluding in section 2.6. We include several useful formulas regarding Type IIA flux compactifications on Calabi-Yau manifolds in appendix A.

2.2 Review of the setup

In type IIA flux compactifications on Calabi-Yau manifolds with smeared O6-plane sources and NSNS and RR fluxes it is possible to stabilize all moduli at tree-level if $h^{2,1} = 0$, i.e. if we are dealing with a rigid Calabi-Yau manifold [31]. In a mirror symmetric type IIB compactification, using the SYZ conjecture [39], one would then expect to be able to stabilize all moduli on 'spaces' with $h^{1,1} = 0$. The RR fluxes F_p with $p = 0, 2, 4, 6$ on the IIA side all transform into RR F_3 flux. The IIA H_3 flux could in principle transform to a mixture of NSNS H_3, geometric and non-geometric fluxes in IIB.[1] However, on the type IIA side, for $h^{2,1} = 0$, we have a space with only one 3-cycle (and its dual). Turning on the H_3 flux in type IIA so that it does not thread the T^3 fibration of the SYZ setup, we expect that after the three T-dualities, we end up in type IIB with a setup that only involves H_3 flux and neither geometric nor non-geometric fluxes. Intuitively, this might also be expected from the work of Giddings, Kachru and Polchinski (GKP) [41] that showed that in type IIB it is possible to stabilize the axio-dilaton and all complex structure moduli using only F_3 and H_3 fluxes. This means of course that we can stabilize all moduli in the absence of Kähler moduli, i.e. for $h^{1,1} = 0$.

This idea of studying how all moduli are stabilized at tree-level in type

[1]See [40] for a recent detailed discussion of the T-duality between type IIA and type IIB flux compactifications.

IIB flux compactifications with F_3 and H_3 fluxes on 'spaces' with $h^{1,1} = 0$ was first fleshed out in [26, 29]. There the authors studied orbifolds of certain Landau-Ginzburg models and searched successfully for completely stabilized, supersymmetric 4d $\mathcal{N} = 1$ Minkowski and AdS vacua. Such Minkowski vacua are absent in geometric type IIA flux compactifications [42, 43] and require non-geometric fluxes, which are not well-controlled due to potential α' corrections. However, as mentioned above, under mirror symmetry the H_3 flux can become geometric and non-geometric fluxes. So, even if we only turn on the H_3 flux on the type IIB side, we actually probe a genuinely larger part of the string landscape than DGKT. Furthermore, due to powerful no-go theorems that we will review in the next subsection, these settings are very well-controlled.

Landau-Ginzburg orbifold models provide a way of analytically continuing Calabi-Yau compactifications to small volume and can even be used to describe the mirror dual of compactification on a rigid Calabi-Yau manifold [44]. A Landau-Ginzburg theory is determined by the superpotential $W(\Phi_i)$, which is a quasi-homogeneous analytic function of the (worldsheet) chiral superfields Φ_i. In this paper, following [26], we will consider two models. Firstly, we consider the 1^9 model with a superpotential given by

$$W_{ws} = \sum_{i=1}^{9} \Phi_i^3 \,, \tag{2.1}$$

and secondly we will consider the 2^6 model with a superpotential given by

$$W_{ws} = \sum_{i=1}^{6} \Phi_i^4 . \tag{2.2}$$

In the 1^9 model one can orbifold by the $\mathbb{Z}_3$ symmetry $\Phi_i \to \omega\Phi_i$ where $\omega = e^{\frac{2\pi \mathrm{i}}{3}}$, while in the 2^6 model we use the $\mathbb{Z}_4$ symmetry with $\omega = e^{\frac{\pi \mathrm{i}}{2}}$. For the 1^9 orientifold, σ_1 in [26], one combines worldsheet parity with $(\Phi_1, \Phi_2, \Phi_3, ..., \Phi_9) \to -(\Phi_2, \Phi_1, \Phi_3, ..., \Phi_9)$. The orientifold for the 2^6 model is the σ_0 orientifold in [26] that acts on the fields as $(\Phi_1, \Phi_2, \Phi_3, ..., \Phi_6) \to e^{2\pi \mathrm{i}/8}(\Phi_1, \Phi_2, \Phi_3, ..., \Phi_6)$. In both of the cases one ends up with O3-planes whose charge can be cancelled by turning on F_3 and H_3 fluxes and/or by adding D3 branes.

Before turning on the fluxes, it is easy to check which are the corresponding Calabi-Yau (CY) manifolds. We need to compute the dimensions of the ring of superprimary chiral operators $R = \frac{C[\Phi]}{\partial_j W(\Phi)]}$. The (c, c) ring correspond to $(2, 1)$ harmonic forms while the chiral-antichiral ring (c, a) corresponds to $(1, 1)$ forms. For the 1^9 model it is easy to check that there are $h_{2,1} = 63$ monomials $\Phi_i\Phi_j\Phi_k$ which are invariant under the $\mathbb{Z}_3$ and the orientifold action. One also obtains $h_{1,1} = 0$ [26], that is, there are no corresponding Kähler moduli in the would be CY manifold. It corresponds to the mirror of $\frac{T^6}{\mathbb{Z}_3 \times \mathbb{Z}_3}$. Thus we see that in the absence of fluxes the model is dual to a DGKT construction, i.e. to a compactification of type IIA on a rigid CY manifold.[2] Similarly, for the 2^6 orientifold one obtains $h^{1,1} = 0$ and $h_{2,1} = 90$ and it corresponds to the mirror of $\frac{T^6}{\mathbb{Z}_4 \times \mathbb{Z}_4}$.

In this work, following [29, 30], we will restrict ourselves to what would be the bulk moduli in the mirror dual toroidal orbifold. We will furthermore set the three bulk complex structure moduli equal and study a rather simple 4d $\mathcal{N} = 1$ SU model. This allows us to find many analytic families of solutions

[2]The actual model that was explicitly worked out by DGKT is a slightly different $\frac{T^6}{\mathbb{Z}_3 \times \mathbb{Z}_3}$ that differs from this model in the twisted sector [29].

and thereby truly study the parameter space of this model in great detail.[3] It is expected that all of our findings carry over to the full model. In the simplest, somewhat restricted setup where our model is dual to the DGKT model, this follows from the explicit check of the blow-up modes in the DGKT paper [31]. When talking about fully stabilized Minkowski vacua, then we can refer to the paper [26] where Minkowski vacua were found even when including all moduli. In particular, our new proof below that the masses cannot become zero even when including all corrections applies equally well to our SU model and the full model studied in [26]. However, although we do not expect surprises, it would of course be interesting to extend our analysis to a generic setup with arbitrary many moduli.

The careful reader might worry that stabilizing blow-up modes requires turning on many additional fluxes that then contribute to the tadpole which then might become much larger than the fixed negative charge induced by the O3 planes in our models. This expectation would be in line with the recently proposed tadpole conjecture [45, 46, 47, 48, 49]. However, it does not apply here for two reasons: Firstly, in the case where our models are dual to the DGKT model, all blow-up moduli are stabilized in the dual model by using F_4 fluxes that do not appear in any non-trivial tadpole condition in the type IIA model. This means that the dual F_3 flux quanta, that stabilize blow-up modes, likewise do not appear in the tadpole condition on the type IIB side.

[3] While the previous work [26, 29] studied particular solutions of these models, the more recent paper [30] picked random flux numbers within a finite range and generated large *generic* solution sets that were compared with a variety of swampland conjectures. Here, we actually test several swampland conjectures against new infinite families of analytic solutions.

Secondly, as we explain in the next subsection, the large volume intuition that fluxes contribute with the same sign as D3 branes to the tadpole is not correct in these non-geometric settings. Fluxes are no longer required to be ISD and can even in supersymmetric solutions contribute to the tadpole with the same sign as orientifold planes.

Type IIB string theory compactifications on the above two Landau Ginzburg models, after including the above discussed O3 orientifold projections, give rise to 4d $\mathcal{N} = 1$ theories. The superpotential is generated by H_3 and F_3 fluxes and takes the standard form $W = \int_M (F_3 - SH_3) \wedge \Omega$ [50, 51]. However, given that we are in these setting in a small volume regime, the usual Kähler potential $K = -\ln[-\mathrm{i}(S - \bar{S})] - \ln[\mathrm{i} \int_M \Omega \wedge \overline{\Omega}]$ does receive corrections as discussed in subsection 3.2 of [29]. These corrections can be derived by using mirror symmetry (see appendix A), which leads to the following the Kähler potential $K = -\mathbf{4} \ln[-\mathrm{i}(S - \bar{S})] - \ln[\mathrm{i} \int_M \Omega \wedge \overline{\Omega}]$. In our simple case where we restrict to two moduli, the axio-dilaton $S = C_0 + \mathrm{i}e^{-\phi}$ and a complex structure modulus U, both the 1^9 and the 2^6 model give rise to the following Kähler and superpotential

$$K = -4 \ln[-\mathrm{i}(S - \bar{S})] - 3 \ln[-\mathrm{i}(U - \bar{U})]\,, \tag{2.3}$$

$$W = \left(f^0 - Sh^0\right) U^3 - 3 \left(f^1 - Sh^1\right) U^2 + 3 \left(f_1 - Sh_1\right) U + \left(f_0 - Sh_0\right)\,. \tag{2.4}$$

This restricted model, corresponding to $h^{2,1} = 1$, is dual to a similarly restricted model in type IIA where, for example, one sets the three Kähler moduli in the original DGKT model equal, to get an effective model with $h^{1,1} = 1$ on the type IIA side (an ST model). The four F_3 flux components, labelled by f_0, f_1, f^1, f^0 above correspond on the type IIA side to F_6, F_4, F_2 and F_0 fluxes, while the four H_3 flux components h_0, h_1, h^1, h^0 correspond on the type IIA side to H_3 flux, metric flux and non-geometric Q and R fluxes, respectively (see table 1 in [52]). Thus, this flux compactification on the type IIB side is indeed extending the original DGKT construction [31] in a very important way. Furthermore, as we will explain in the next subsection, there are non-renormalization theorems that allows one to obtain trustworthy results in regimes that have not really been probed much in the existing literature.

As is familiar from any flux compactification with orientifolds, one has to cancel the net charge induced by the fluxes, O-planes and potentially D-branes. In our case this will be the O3 plane charge and the tadpole condition is given by

$$\int_M F_3 \wedge H_3 + N_{D3} = \frac{1}{2} N_{O3} \,. \tag{2.5}$$

This allows us now to clarify, why we discussed above the 1^9 model and the 2^6 model although they both give rise to the same (restricted) Kähler and superpotential in equations (2.3) and (2.5): The above mentioned orientifold projection for the 1^9 model gives rise to $N_{O3} = 24$, while the orientifold projection for the 2^6 model gives rise to $N_{O3} = 80$ [26]. This means that the flux contribution to the tadpole

$$N_{\text{flux}} = \int_M F_3 \wedge H_3 = -h^0 f_0 - 3h^1 f_1 + h_0 f^0 + 3h_1 f^1 \,, \tag{2.6}$$

would have to equal either 12 or 40, if we want to satisfy the tadpole condition in equation (2.5) without the addition of D3 branes.

However, it is also important and interesting in these models to include D3 branes. The reason for this is that the flux contribution to the tadpole N_{flux} has no definite sign (see subsection 3.3 in [29]). This means in particular that fluxes can contribute with the same sign as O3 planes in the tadpole and we will see below that there are even infinite families in which $N_{\text{flux}} \to -\infty$ and at the same time $N_{D3} \to \infty$. One may ask why this is possible, since in the well-known geometric type IIB CY orientifolds with 3-form fluxes, studied in GKP [41], the N_{flux} is always positive. This follows in that case simply from the requirement that the flux $F_3 - SH_3$ has to be imaginary self dual (ISD). The latter in turn follows from the vanishing of the covariant derivatives of the superpotential with respect to the axio-dilaton and the complex structure modulus, i.e. $D_S W = D_U W = 0$. In our setup there are small volume corrections to the Kähler potential in equation (2.3). In particular, the factor of 4 changes $D_S W = 0$ in such a way that one can no longer derive the ISD requirement, as is discussed in more detail in [29].

The above property might be surprising.[4] Therefore we quickly discuss it also in the dual type IIA models. In the case where we only turn on a single H_3 flux quanta our model is dual to a type IIA flux compactification à la DGKT with $h^{2,1} = 0$. There is then a single O6 plane tadpole condition. In this case, for supersymmetric AdS vacua, the flux contribution to the tadpole

[4] At small volume there are a plethora of instances were the large volume understanding of mutually supersymmetric objects changes completely due to stringy corrections. So, it shouldn't necessarily be surprising that fluxes can carry anti-D3 brane charge and still be mutually supersymmetric with D3 branes and O3 planes.

has to have the same sign as D6-branes. If that were not the case, then we could add D6 branes in addition to the O6 planes and thereby completely cancel their negative contribution to the scalar potential. Then there would be no negative term in the scalar potential (and therefore no AdS vacua) since RR fluxes and H_3 flux contribute positive definite terms only. Thus in this case, which is the dual of the DGKT model with $h^{2,1} = 0$, the fluxes induce a charge in the tadpole that has the same sign as D branes and N_{flux} is therefore bounded by zero from below and $N_{O3}/2$ from above. Now when we turn on more general H_3 flux on the type IIB side then this corresponds to type IIA flux compactifications in the presence of geometric and non-geometric fluxes. These fluxes can contribute to the scalar potential with either sign and the O plane term is no longer the only negative term in the scalar potential. Thus, there is no immediate obstruction to over-cancelling the O plane contribution by adding a very large number of D branes. We will see how this works in explicit examples below, when we discuss concrete solutions.

2.2.1 Non-renormalization theorem

In this subsection we first recall the absence of perturbative and non-perturbative corrections to the superpotential [26, 29]. First of all, α' corrections are already taken into account in the LG theory. Thus, one only has to focus on g_s perturbative and non-perturbative corrections. However, it was argued in [26, 29] that the superpotential does not receive any perturbative or non-perturbative corrections at all, which follows for example from the non-renormalization of the BPS tension of a D5-brane domain wall but also passes other non-trivial checks [26]. This means that the superpotential is exact even at strong coupling. Note however, that the Kähler potential can and will receive perturbative and non-perturbative corrections, which is something we will return to in the next paragraph. The cautious reader might worry about the familiar brane instanton corrections to the IIB superpotential. Let us therefore recall that our models have $h^{1,1} = 0$ and thus no Euclidean D3-brane instantons. The absence of D(-1) instantons was argued for in footnote 6 in [26] as follows: Since the D(-1) instantons do not depend on the volume and they are not there in the decompactification limit due to higher supersymmetry, they should also not appear here. This is also consistent with the recent analysis in [53], which trivially covers our setup since we have $h^{1,1} = 0$ and therefore no 4-cycles and no D7 branes or O7 planes. Alternative it was argued for the absence of any brane instanton corrections in [29] using the duality to the type IIA setting of DGKT: There the only 3-cycle in models with $h^{2,1} = 0$ has H_3 flux and therefore there are no brane instantons [54].

When studying Minkowski vacua we will assume that the non-renormalization theorem holds and the superpotential receives no corrections even in those vacua where g_s is of order 1 or larger. The conditions for supersymmetric Minkowski vacua are $\partial_i W = W = 0$ and do not depend on the Kähler potential. Thus, the very existence of Minkowski vacua does not change even if one includes arbitrary corrections because those can only appear in the Kähler potential. Previously, such explicit supersymmetric, fully stabilized Minkowski vacua were constructed in [26, 29, 30]. However, it was stated in [29] that these are necessarily at strong coupling[5] and thus receive large corrections to the Kähler potential. This then leads to the following important question: Are these truly fully stabilized 4d $\mathcal{N} = 1$ Minkowski vacua or can the corrections to K lead to flat directions?

We will prove here that even arbitrary, unknown corrections to K cannot lead to flat directions in these models: We assume that one has been able to a find a fully stabilized SUSY Minkowski vacuum as was the case in [26, 29, 30] (see also section 2.3 below). Then the Hessian matrix of second derivatives of the scalar potential has only positive eigenvalues and is given by[6]

$$H_{i\bar{j}} = \partial_i \partial_{\bar{j}} V = (\partial_i \partial_k W) K^{k\bar{\ell}} (\partial_{\bar{\ell}} \partial_{\bar{j}} \overline{W}), \tag{2.7}$$

or in matrix form

$$H = \mathcal{W}\mathcal{K}\overline{\mathcal{W}}\,. \tag{2.8}$$

Now compute the determinant

$$\det\, H = \det\, \mathcal{W}\, \det\, \mathcal{K}\, \det\, \overline{\mathcal{W}} = |\det\, \mathcal{W}\,|^2 \det\, \mathcal{K}\,. \tag{2.9}$$

Given that *all* eigenvalues of H were positive to begin with we can conclude that $|\det\, \mathcal{W}\,|^2 > 0$.

Now let us take into account arbitrary and unknown corrections to the

[5]We find that they cannot be at parametrically weak coupling but there are certainly examples with $g_s < 1$.

[6]For simplicity we work here with the Hessian. The actual masses squared are the eigenvalues of $H_{i\bar{j}} K^{\bar{j}k}$. However, given that the Kähler metric is positive definite, this does not change our conclusion.

Kähler potential and denote the inverse Kähler metric after including all these corrections $\mathcal{K}_c$. The new Hessian for this corrected Minkowksi vacuum is now given by

$$H_c = \mathcal{W}\mathcal{K}_c\overline{\mathcal{W}}\,. \tag{2.10}$$

We again compute the determinant

$$\det\, H_c = \det\, \mathcal{W}\, \det\, \mathcal{K}_c\, \det\, \overline{\mathcal{W}} = |\det\, \mathcal{W}\,|^2 \det\, \mathcal{K}_c\,. \tag{2.11}$$

Since the superpotential did not receive any corrections we have from above that $|\det\, \mathcal{W}\,|^2 > 0$. Since the Kähler metric controls the kinetic terms of the scalar fields, its eigenvalues have to be positive. This remains true even after including arbitrary corrections and therefore $\det\, \mathcal{K}_c \neq 0$. This, combined with the preservation of $|\det \mathcal{W}|^2$ implies that $\det\, H_c \neq 0$. Thus, *all* the eigenvalues of H_c must be nonzero.

In supersymmetric Minkowski vacua eigenvalues of the Hessian matrix have to be positive for stabilized moduli or zero for flat directions. It was just shown that the eigenvalues of H_c are nonzero, so we can conclude that these Minkowski vacua cannot have flat directions even when including unknown and arbitrary corrections to the Kähler potential.

One can actually prove also the existence of AdS vacua at strong coupling using the non-renormalization of W [26]. While this is not so important since there are infinite families of AdS vacua with parametrically weak coupling, let us nevertheless briefly recall the argument. For supersymmetric AdS vacua, satisfying $D_iW = \partial_i W + (\partial_i K)W = 0$, the $\partial_i K$ term can receive corrections. The authors of [26] expanded the corrected Kähler potential around the minimum which one can choose to be at $\phi^i = 0$, so that $K_c = K + f(\phi^i) + \bar{f}(\bar{\phi}^{\bar{\imath}}) + \phi^i\bar{\phi}^{\bar{\jmath}} g_{i\bar{\jmath}}(\phi^i, \bar{\phi}^{\bar{\imath}})$. At the minimum $\phi^i = 0$ the only correction to $\partial_i K$ arises from $f(\phi^i)$. However, this can be interpreted as a Kähler transformation: $K \to K + f + \bar{f}$, $W \to We^{-f}$, which changes $D_i W \to e^{-f} D_i W$. Therefore, supersymmetric AdS vacua satisfying $D_i W = 0$ cannot disappear even when including arbitrary unknown Kähler corrections. However, for example the mass spectrum is expected to be corrected (within the limits allowed by $\mathcal{N} = 1$ supergravity).

Finally, there is no argument for preventing corrections to non-supersymmetric vacua. So, if one finds them at strong coupling, they could disappear or become unstable when including string loop corrections.

2.3 Fully stabilized $\mathcal{N} = 1$ Minkowski vacua

As mentioned previously, the first fully stabilized 4d $\mathcal{N} = 1$ Minkowski vacua were found in [26]. In the dual type IIA case, such vacua do not exist in geometric compactifications [42, 43], which means that in the type IIB models at least two components of the H_3 flux have to be turned on. It was also show in [29] that these IIB Minkowski vacua are never arising at large complex structure, i.e. on the dual type IIA side they cannot arise at large volume. However, as we reviewed above the Landau-Ginzburg models take all α' corrections into account and therefore do not require us to be at large complex structure. It was furthermore stated [29] that these Minkowski vacua are confined to strong coupling. Given the non-renormalization theorem from the previous section, we can trust Minkowski vacua even at strong coupling. However, we also find that only parametrically weak coupled solutions are forbidden in this setup and $g_s < 1$ is possible with a model dependent lower bound on g_s. In the next subsection we present a new infinite family of fully stabilized supersymmetric Minkowski vacua and in the following subsection we discuss how this family of solutions fits into the swampland program.

2.3.1 Minkowski solutions

In order to find Minkowski vacua we have to solve $W = \partial_S W = \partial_U W = 0$ for the W given in equation (2.5) above. A particular family of solutions with properly integer quantized fluxes arises for

$$f^0 = -4, \quad f^1 = 0, \quad f_1 = 0, \quad f_0 = 4, \quad h^0 = -3 - h_0, \quad h^1 = 1, \quad h_1 = -1\,. \tag{2.12}$$

Here $h_0 \in \mathbb{Z}$ is a free parameter that actually does not appear in the tadpole condition since N_{flux} in equation (2.5) reduces to $N_{\text{flux}} = 12$ independent of h_0. Thus, this is a solution to the 1^9 model which does not require D3 branes since the fluxes cancel the negative O3 plane charge.

The moduli are stabilized at the following values

$$\begin{aligned} \mathrm{Re}(U) &= -\tfrac{1}{2}\,, & \mathrm{Im}(U) &= \frac{\sqrt{3}}{2}\,,\\ \mathrm{Re}(S) &= \tfrac{6+4h_0}{3+h_0(3+h_0)}\,, & \mathrm{Im}(S) &= \frac{2\sqrt{3}}{3+h_0(3+h_0)}\,. \end{aligned} \tag{2.13}$$

While the complex structure modulus is stabilized at a fixed value, the inverse string coupling $\mathrm{Im}(S)$ changes when we vary the free parameter $h_0 \in \mathbb{Z}$. It takes on its maximal value of $\mathrm{Im}(S) = 2\sqrt{3}$ for $h_0 = -1$ and for $h_0 = -2$. For $h_0 \to \pm\infty$ we enter parametrically strong coupled regions with $\mathrm{Im}(S) \propto 1/h_0^2$. We stress again that even in this parametrically strong coupled regime there are no corrections to W due to the above non-renormalization theorem.

The positive masses squared for the two complex scalar fields in the Minkowski vacuum are given by

$$m_-^2 = \frac{(11-4\sqrt{7})(3+h_0(3+h_0))^3}{192\sqrt{3}}, \qquad m_+^2 = \frac{(11+4\sqrt{7})(3+h_0(3+h_0))^3}{192\sqrt{3}}. \tag{2.14}$$

We see that in the limit $h_0 \to \pm\infty$ the masses grow like h_0^6. For the largest inverse string coupling value $\mathrm{Im}(S) = 2\sqrt{3} \approx 3.46$ which is obtained for $h_0 = -1$ and for $h_0 = -2$, the masses squared reduce in both cases to $m_-^2 = \frac{11-4\sqrt{7}}{192\sqrt{3}} \approx 0.00125$ and $m_+^2 = \frac{11+4\sqrt{7}}{192\sqrt{3}} \approx 0.0649$.

2.3.2 Minkowski vacua and the swampland

It is easy to find string compactifications that give rise to 4d Minkowski vacua with $\mathcal{N} \geq 2$, for example, by compactifying type II string theory on a Calabi-Yau manifold or a torus. However, to the best of our knowledge all these Minkowksi vacua with $\mathcal{N} \geq 2$ have flat directions, i.e. massless scalar fields. These flat directions can be protected by the high amount of supersymmetry. However, in 4d $\mathcal{N} = 1$ theories there is no such protection and it is expected that all flat direction would be lifted by corrections which likely leads to runaway directions. To the best of our knowledge, the Minkowski vacua first discovered in [26, 29] are the only fully stabilized $\mathcal{N} = 1$ Minkowski vacua that arise in full-fledged string theory constructions. Given that corrections to the scalar potential are not forbidden by $\mathcal{N} = 1$ supersymmetry, one would have thought that it would not be possible to really argue for the existence of these vacua when including all perturbative and non-perturbative corrections. However, the non-renormalization of the superpotential [26] and our argument above about the mass matrix are implying that these vacua do indeed exist in a strongly coupled corner of string theory.

Given the more recent objections to the existence of dS vacua in string theory [55, 56], the very existence of fully stabilized 4d $\mathcal{N} = 1$ Minkowksi vacua was questioned as well. The reason is that any small, SUSY breaking, positive energy contribution to the scalar potential turns these Minkowksi vacua into metastable dS solutions. Following this logic, the authors of [57] conjectured that strongly stabilized AdS vacua should be forbidden. Here by strongly stabilized one means that the mass of the lightest field satisfies $m_{\text{light}} L_{AdS} \gg 1$, where L_{AdS} is the length scale of the AdS space. This AdS moduli conjecture seems to imply that if we take the limit $L_{AdS} \to \infty$ to go to Minkowksi space, then $m_{\text{light}} \to 0$ in contradiction with the Minkowski vacua discussed here and previously in [26, 29, 30]. Note however, that these Minkowski vacua cannot arise as the limit of any of the infinite families of AdS solutions that we find in these models and that will be discussed in the next subsection. Likewise, there is no obvious small SUSY breaking correction or change to the model that leads to dS vacua. All string loop corrections do not change W and only modify the values of the positive masses squared of the scalar fields in the Minkowski vacuum. Changing some flux quanta to break supersymmetry is a large effect and the same probably applies to any other change given that the complex structure modulus is stabilized at order 1 and we are at strong coupling. However, it would definitely be interesting to study this further.

The existence of these vacua and the absence of corrections is surprising, maybe even more so given the recent paper [58] that finds that generically in quantum gravity any allowed correction should appear. The exception to this rule is stated in the same paper and is formalized in the supersymmetric genericity conjecture [58]. This conjecture says that quantities that are protected in higher supersymmetric theories should only vanish in lower supersymmetric theories, if the lower supersymmetric theory is related to a higher supersymmetric theory. In particular, the authors discuss 4d $\mathcal{N} = 1$ Minkowski vacua with everywhere vanishing superpotential, $W = 0$. They find that the equation $W = 0$ can only survive all corrections if the theory is related to a higher dimensional theory via for example a simple orbifold projection. While our setup with fluxes and a non-zero W generate by those fluxes is not covered by the analysis in [58], our findings seem nevertheless compatible with the supersymmetric genericity conjecture since our setups are simple orbifolds of toroidal models that preserve higher amounts of supersymmetry.

Summarizing, these non-geometric type IIB setups give rise to fully stabilized 4d $\mathcal{N} = 1$ Minkowksi vacua that seem to survive all stringy corrections, which makes them to our knowledge the only full-fledged string theory constructions of this type. These vacua arise only at relatively strong coupling in a barely studied part of the string landscape.

2.4 Infinite families of AdS vacua

In this section we study exemplary families of AdS solutions that arise in these non-geometric type IIB flux compactifications. As discussed above, due to the non-renorma-lization of W even supersymmetric AdS solutions at strong coupling will persist when including all potential corrections. However, for example the masses and the cosmological constant in these solutions might get significantly modified when we are not at weak string coupling. All the different families of AdS solutions that we present below, allow us to go to parametrically weak coupling and thus we have parametric control over them. This enables us to perform trustworthy and detailed studies even when these solutions are not supersymmetric. Given that the exact number of O3 planes in these infinite families plays essentially no role, we will restrict ourselves to the 1^9 model with $N_{O3} = 24$. We will introduce representative examples to illustrate the different behaviors that these infinite families display. Firstly, we present families that are dual to the AdS vacua found in DGKT [31] but we also find other infinite families of AdS vacua that arise in our more general setup. Secondly, we study interesting and very different sets of solutions, where by increasing the number of D3 branes the contribution of the fluxes to the tadpole can become negative and very large. In the $N_{\text{flux}} \to -\infty$ limit the number of D3 branes needs to become infinite $N_{D3} \to \infty$ as well, in order to satisfy the tadpole condition. We discuss how all these solutions fit into the web of swampland conjectures at the end of this section.

2.4.1 Infinite families of AdS vacua without D3 branes

The DGKT dual In [29] two infinite families of SUSY AdS solutions were presented. The first solution is related to the infinite family of SUSY AdS vacua that were found in DGKT [31].[7] To find it one has to necessarily set three H_3 flux quanta to zero, $h^0 = h^1 = h_1 = 0$. The tadpole condition (2.5) then implies

$$h_0 = \frac{12}{f^0}\,, \tag{2.15}$$

which means that due to flux quantization $f^0 \in \{1,2,3,4,6,12\}$. We will not plug in any specific flux values but keep in mind that f^0 and h_0 are bounded due to tadpole cancellation condition but the other fluxes are not.

One can easily solve $D_S W = D_U W = 0$ and find that the axio-dilaton is stabilized at

$$\mathrm{Re}(S) = \frac{f_0 (f^0)^2 + 3 f_1 f^0 f^1 - 2(f^1)^3}{12 f^0}\,, \qquad \mathrm{Im}(S) = 2\sqrt{\frac{5}{3}}\frac{(f_1 f^0 - (f^1)^2)^{\frac{3}{2}}}{9 f^0}\,, \tag{2.16}$$

while the complex structure modulus is stabilized at

$$\mathrm{Re}(U) = \frac{f^1}{f^0}\,, \qquad \mathrm{Im}(U) = \sqrt{\frac{5}{3}}\frac{(f_1 f^0 - (f^1)^2)^{\frac{1}{2}}}{f^0}\,. \tag{2.17}$$

Given that f_1 is unconstrained by the tadpole, we can make it large and even send it to infinity. In that limit the string coupling $1/\mathrm{Im}(S)$ becomes parametrically small and the complex structure modulus becomes parametrically

[7] In the second SUSY AdS solution in subsection 4.3.2 in [29], there seems to be a typo. We find that either $\mathrm{Im}(U)$ or $\mathrm{Im}(S)$ are necessarily negative, so this does not seem to be a physically meaningful solution.

large. This is the mirror dual of the large volume, weak coupling families of AdS vacua that arise in type IIA flux compactifications if one makes the F_4 flux large [31].

The scalar potential at the minimum is

$$V_{AdS} = \frac{-19683\sqrt{\frac{3}{5}}(f^0)^3}{3200(f_1 f^0 - (f^1)^2)^{\frac{9}{2}}} \,. \tag{2.18}$$

The four masses squared in this family can be conveniently expressed in terms of the above value of the scalar potential as

$$m^2 = \left\{ \frac{10}{3}, 6, \frac{70}{3}, \frac{88}{3} \right\} |V_{AdS}| \,. \tag{2.19}$$

Since the AdS radius in 4d is given by $R_{AdS} = \sqrt{3/|V_{AdS}|}$ one finds surprisingly that all the masses squared in AdS units, i.e. all $m^2 R_{AdS}^2$, are integers. This was recently discovered in [59] (see also [60]). Furthermore, the integers are such that the operator scaling dimensions in the dual CFT_3, i.e.

$$\Delta = \frac{1}{2}\left(3 + \sqrt{9 + 4m^2 R_{AdS}^2}\right) = \{5, 6, 10, 11\} \,, \tag{2.20}$$

are integers as well [59, 61, 62]. This fascinating feature of this family of AdS vacua currently awaits an explanation and we check below in our other families whether the same is true or not.

Given that we want to compare our infinite families with the AdS distance conjecture, it is important to determine the mass scale of a tower of states that becomes light in the large flux limit. In the dual DGKT construction [31] the large flux limit corresponds to a large volume limit and the KK scale sets the scale of a tower with a mass scale that goes to zero as the flux quanta go to infinity. Using mirror symmetry, as further discussed in appendix A, we can determine the dual mass scale of a tower that becomes light in this large flux limit[8]

$$m_{\text{tower}}^2 \sim \frac{1}{\text{Im}(U)\text{Im}(S)^2} \sim \frac{(f^0)^3}{(f_1 f^0 - (f^1)^2)^{\frac{7}{2}}} \sim \frac{1}{f_1^{\frac{7}{2}}} \,. \tag{2.21}$$

As we discuss below, the AdS distance conjecture [63], constrains the parameter α that relates the mass scale of the tower to the cosmological constant via $m_{\text{tower}} \sim |\Lambda|^{\alpha}$. In this solution we have $\alpha = 7/18$ since

$$m_{\text{tower}} \sim \frac{1}{f_1^{\frac{7}{4}}} \sim |V_{AdS}|^{\frac{7}{18}} \,. \tag{2.22}$$

SUSY families with $\alpha = 1/2$ Next we discuss another infinite family of AdS vacua that is also parametrically controlled but not dual to the DGKT model since we have two H_3 flux quanta turned on. In particular, we fix the following fluxes

$$f^0 = 0 \,, \qquad f_1 = 0 \,, \qquad h^0 = -3 \,, \qquad h^1 = 0 \,, \qquad h_0 = 0 \,. \tag{2.23}$$

The tadpole condition in equation (2.5) is satisfied if we set $f_0 = 4 - h_1 f^1$ and we are left with two free flux parameters $h_1, f^1 \in \mathbb{Z}$. In this solution the real parts of S and U are equal to zero and the imaginary parts are given by

[8] By mirror symmetry the large volume limit becomes a large complex structure limit in which winding modes should become light and lead to this tower of states.

$$
\begin{aligned}
\mathrm{Im}(U) &= \frac{\sqrt{\frac{9f^1h_1+2(-9+\sqrt{81+24f^1h_1(-4+f^1h_1)})}{f^1}}}{\sqrt{15}}\,, \qquad (2.24)\\
\mathrm{Im}(S) &= \left[\frac{\left(-16f^1h_1+3\left(9+\sqrt{81+24f^1h_1(-4+f^1h_1)}\right)\right)}{2h_1^2}\right]\mathrm{Im}(U)\,.
\end{aligned}
$$

In the limit $f^1 \to \infty$ (and for negative $h_1 < 0$) we find the following scaling of the moduli

$$
\begin{aligned}
\mathrm{Im}(U) &\approx \frac{\sqrt{(9-4\sqrt{6})h_1}}{\sqrt{15}}\,,\\
\mathrm{Im}(S) &\approx \frac{\sqrt{6+8\sqrt{\frac{2}{3}}f^1}}{\sqrt{-h_1}}\,. \qquad (2.25)
\end{aligned}
$$

So, we have parametric control since we can go to parametrically small string coupling. We can in principle also make the complex structure modulus large by an appropriate choice of h_1, however, this is not necessary since the Landau-Ginzburg model already takes all α' corrections into account [26].

In the above limit of very large f^1 the value of the potential at the minimum is given by

$$
V_{AdS} \approx -\frac{27(-h_1)^{\frac{5}{2}}}{32\sqrt{1329+544\sqrt{6}}}\frac{1}{(f^1)^2}\,. \qquad (2.26)
$$

Comparing the mass of the light tower from equation (A.6) with the value of the scalar potential in this limit, we find that $m_{\text{tower}} \sim |V_{AdS}|^{\frac{1}{2}}$, i.e. $\alpha = 1/2$.

In the limit where $f^1 \to \infty$ the masses squared are,

$$\begin{aligned} {m_{1\pm}}^2 &= \frac{2}{9}\left(17+\sqrt{6}\pm\sqrt{127+46\sqrt{6}}\right)|V_{AdS}|\,, \\ {m_{2\pm}}^2 &= \frac{1}{9}\left(25-2\sqrt{6}\pm\sqrt{337+68\sqrt{6}}\right)|V_{AdS}|\,. \end{aligned} \tag{2.27}$$

The smallest of these masses squared, $m_{2-}^2 = \frac{1}{9}(25-2\sqrt{6}-\sqrt{337+68\sqrt{6}})|V_{AdS}|$ $\approx -0.260|V_{AdS}|$, is above the Breitenlohner-Freedman bound $m_{BF}^2 = -\frac{3}{4}|V_{AdS}|$ [64], as required by supersymmetry. Obviously, none of these masses are integers in AdS units and the same is true for the dual conformal scaling dimensions. Since we kept h_1 finite in this example, the complex structure remains finite and therefore the mirror dual type IIA families should have likewise a fixed finite volume, which might (or might not) be related to the absence of integer conformal scaling dimensions.

Non-supersymmetric AdS vacua Lastly, we discuss here a single non-supersymmetric family of AdS vacua. We have not performed an all encompassing search for such solutions but given that they exist in the type IIA models of DGKT and are related to the supersymmetric solutions by simple sign flips of F_4 flux quanta, they have to exist here as well. We found one such family that is related to the supersymmetric AdS solution discussed above in subsection 2.4.1, by setting $f^1 = f_0 = 0$ and flipping the sign of f_1.

Concretely, for $h^0 = h^1 = h_1 = f^1 = f_0 = 0$, and f^0 essentially fixed by the tadpole as in equation (2.15) above, we find a one parameter family of non-SUSY AdS vacua parameterized by f_1. The real parts of the two moduli vanish in this family, Re(S) =Re(U) = 0 and the imaginary parts are given by

$$\mathrm{Im}(U) = \sqrt{\frac{5}{3}}\sqrt{-\frac{f_1}{f^0}}\,, \qquad\qquad \mathrm{Im}(S) = \frac{2}{9}\sqrt{\frac{5}{3}}(-f_1)^{\frac{3}{2}}\sqrt{f^0}\,. \tag{2.28}$$

So, we see that both grow in the limit $f_1 \to -\infty$ and we have parametric control over these non-supersymmetric solutions. The scalar potential is given by

$$V_{AdS} = -\sqrt{\frac{3}{5}}\frac{19683}{3200(f^0)^{\frac{3}{2}}(-f_1)^{\frac{9}{2}}}\,. \tag{2.29}$$

Since the moduli and the cosmological constant scale as for the supersymmetric counter part in subsection 2.4.1 above, one again finds $\alpha = 7/18$.

The four masses squared for these solutions are given by

$$m^2 = \left\{\frac{70}{3}, \frac{40}{3}, 6, -\frac{2}{3}\right\}|V_{AdS}|\,. \tag{2.30}$$

The smallest of these masses squared, $m^2 = -\frac{2}{3}|V_{AdS}|$, is above the Breitenlohner-Freedman bound $m^2_{BF} = -\frac{3}{4}|V_{AdS}|$ [64] and this solutions is stable, although in this case this is not guaranteed by supersymmetry.

We note that the masses squared above again give rise to dual conformal dimensions $\Delta = \{10, 8, 6, 2 \text{ or } 1\}$ that are all integers. This was previously noticed for non-supersymmetric DGKT solutions in [59, 61] and it would be interesting to extend the general analysis of [62] to non-supersymmetric AdS vacua.

2.4.2 AdS vacua with a large number of D3 branes

Given the fact that supersymmetric fluxes in this setup can contribute to the tadpole condition in the same way as O3 planes, we do not necessarily need the latter, however, we will keep them in the models below. We can furthermore ask whether we can find infinite families of supersymmetric vacua where a flux contribution in the tadpole can cancel an arbitrarily large number of D3 branes. This is indeed the case and we will present below two exemplary families where $N_{\text{flux}} \to -\infty$, $N_{D3} \to \infty$ while the tadpole $N_{\text{flux}} + N_{D3} = N_{O3}/2 = 12$ is satisfied. To the best of our knowledge such types of solution have never been discussed in the flux compactification literature before. We will present them below and then discuss potential problems and detailed features of these solutions in more detail below in subsection 2.4.3.

An infinite family with $\alpha = 1/2$ and $\mathbf{N_{D3} \to \infty}$ We will set the following four fluxes to zero $f^1 = f_0 = h^0 = h_1 = 0$. Then we solve the SUSY equations $D_S W = D_U W = 0$. We find supersymmetric AdS solutions with Re(S) =Re(U)=0 and the imaginary parts are stabilized at

$$\begin{aligned} \mathrm{Im}(U) &= \sqrt{\frac{-3f^0h_0 - 9f_1h^1 + \sqrt{9(f^0h_0)^2 + 74f_1f^0h_0h^1 + 81(f_1h^1)^2}}{2f^0h^1}}\,, \qquad (2.31)\\ \mathrm{Im}(S) &= \left(\frac{-3f^0h_0 + 9f_1h^1 + \sqrt{9(f^0h_0)^2 + 74f_1f^0h_0h^1 + 81(f_1h^1)^2}}{8h_0h^1}\right) \mathrm{Im}(U)\,. \end{aligned}$$

The tadpole equation (2.5) in this case reduces to

$$-3h^1f_1 + h_0f^0 + N_{D3} = 12\,. \qquad (2.32)$$

Keeping h_0 and f^0 fixed and choosing a positive h^1, we can send $f_1 \to \infty$. This gives rise to an infinite family of solution that requires an ever growing number of D3 branes to be present, with $N_{D3} \propto f_1$. For simplicity we study the particular example $h^1 = f^0 = 1$. In the $f_1 \to \infty$ limit the moduli are approximately given by

$$\text{Im}(U) \approx \frac{\sqrt{5h_0}}{3}\,, \qquad \text{Im}(S) \approx \frac{3\sqrt{5}f_1}{4\sqrt{h_0}}\,.$$

Thus we are at parametrically weak coupling and we can even make $\text{Im}(U)$ very large by choosing an appropriate fixed but arbitrarily large value for h_0.

In the limit where f_1 goes to infinity we have:

$$V_{AdS} \approx -\frac{2(h_0)^{3/2}}{25\sqrt{5}f_1^2}\,. \tag{2.33}$$

In the large f_1 limit the mass of the light tower (in Planck units) is

$$m^2_{\text{tower}} \sim \frac{1}{\text{Im}(U)\text{Im}(S)^2} \approx \frac{16\sqrt{h_0}}{15\sqrt{5}f_1^2}\,, \tag{2.34}$$

which corresponds to $\alpha = 1/2$. The masses squared in this limit are

$$m_{1\pm}{}^2 \approx \frac{1}{27}(41 \pm 4\sqrt{181})|V_{AdS}|\,, \qquad m_{2\pm}{}^2 \approx \frac{1}{27}(26 \pm \sqrt{181})|V_{AdS}|\,. \tag{2.35}$$

The smallest mass squared, $m^2_{1-} \approx \frac{1}{27}(41 - 4\sqrt{181})|V_{AdS}| \approx -0.475|V_{AdS}|$, is above the Breitenlohner-Freedman bound $m^2_{BF} = -\frac{3}{4}|V_{AdS}|$ [64], as required by supersymmetry.

An infinite family with $\alpha = 3/2$ and $N_{D3} \to \infty$ Lastly, we present an infinite family that gives rise to a different value of α, while still requiring an ever growing number of D3 branes. We choose the following fixed flux values

$$f^1 = 1, \quad f_0 = 1, \quad h^0 = 0, \quad h^1 = -1, \quad h_1 = 0, \quad h_0 = -1, \quad f^0 = 1\,, \tag{2.36}$$

leaving us with f_1 as the free parameter. There exist then supersymmetric AdS vacua in which the moduli take on the following values

$$\begin{aligned} \mathrm{Re}(U) &= 0\,, \qquad & \mathrm{Im}(U) &= \frac{\sqrt{-3-9f_1+\sqrt{9+f_1(74+81f_1)}}}{\sqrt{2}}\,, \\ \mathrm{Re}(S) &= -1\,, \qquad & \mathrm{Im}(S) &= \left(\frac{3-9f_1-\sqrt{9+f_1(74+81f_1)}}{8}\right)\mathrm{Im(U)}\,. \end{aligned} \tag{2.37}$$

The tadpole equation (2.5) in this case reduces to

$$3f_1 + N_{D3} = 13\,. \tag{2.38}$$

In the limit $f_1 \to -\infty$ the above tadpole requires $N_{D3} \sim -3f_1 \to \infty$. The value of the scalar potential in this limit is

$$V_{AdS} \approx -\frac{729}{32768(-f_1)^{\frac{1}{2}}}\,. \tag{2.39}$$

The moduli scale for $f_1 \to -\infty$ like

$$\mathrm{Im}(S) \approx \frac{8\sqrt{-f_1}}{3}\,, \qquad \mathrm{Im}(U) \approx 3\sqrt{-f_1}\,, \tag{2.40}$$

and therefore

$$m^2_{\rm tower} \sim \frac{1}{{\rm Im}(U){\rm Im}(S)^2} \sim \frac{1}{(-f_1)^{\frac{3}{2}}}\,. \tag{2.41}$$

This actually means that $m_{\rm tower} \sim |V_{AdS}|^{\frac{3}{2}}$, i.e. $\alpha = 3/2$ in this case.

In the limit where $f_1 \to -\infty$ the masses become

$$m^2 \approx \left\{6, \frac{10}{3}, \frac{22}{7}, -\frac{8}{27}\right\} |V_{AdS}|\,. \tag{2.42}$$

The masses squared are above the Breitenlohner-Freedman bound $m^2_{BF} = -\frac{3}{4}|V_{AdS}|$ [64], as required by supersymmetry. Interestingly the first two masses squared give again rise to dual conformal scaling dimensions that are integers, while the later two give rise to fractional scaling dimensions: $\Delta = \{6, 5, 11/3, 8/3\}$.

2.4.3 AdS vacua and the swampland

Many explicit and widely studied constructions of AdS vacua in string theory exhibit the following two features: First, there are usually some light fields whose masses are comparable (or smaller) than the AdS scale $M_{AdS} = 1/R_{AdS} = \sqrt{|V_{AdS}|/3}$ and this was conjectured to be true in all string compactifications in [57]. Second, the most widely studied AdS vacua in string theory are of Freund-Rubin type [65, 66] or exhibit similar features, by which we mean that the size of the internal space R_{KK} is not parametrically smaller than R_{AdS}. This property was recently studied for example in [67, 68, 69, 70, 71, 61, 72, 73] and has led to the AdS distance conjecture [63] that states that for infinite families of AdS vacua with $V_{AdS} \to 0$, there exist a tower of massive states with masses that satisfy $m_{\text{tower}} \sim |V_{AdS}|^{\alpha}$ for some positive α of order one. The strong version of this conjecture says that for supersymmetric AdS vacua $\alpha = 1/2$. This conjecture has been refined in [74, 37]. Lastly, it was conjectured that no stable AdS vacua exist at all [38] and all these conjectures have been used to derive important implications for the standard model of particle physics [75, 76, 77, 78, 79, 80].

Against the backdrop of the above results, let us start by examining our infinite families of AdS vacua. First, let us note that in all the above families of solution the masses of the light fields S and U are always of the same order as $\sqrt{|V_{AdS}|}$. This means that they are all consistent with the AdS/moduli conjecture proposed in [57].

Let us now look at the $N_{\text{flux}} = 12$ solutions, which do not require the presence of D3 branes and that were discussed above in subsection 2.4.1. The supersymmetric AdS solutions with $\alpha = 7/18$ violate the strong version of the AdS distance conjecture. A refined version of the conjecture was proposed in [37] where a 4d discrete $\mathbb{Z}_k$ 3-form gauge symmetry was identified in the DGKT model and the following refined conjecture was proposed: $m_{\text{tower}} \sim \sqrt{k|V_{AdS}|}$. Given that our family of solutions is mirror dual to the DGKT AdS vacua we have a discrete $\mathbb{Z}_{f_1}$ symmetry and our solutions indeed satisfy $m_{\text{tower}} \sim \sqrt{f_1|V_{AdS}|}$.[9]

The next family of supersymmetric AdS vacua that we discuss above sat-

[9] One could in principle work this out explicitly following [37]: A 3-form gauge field with $U(1)$ gauge group arises from $F_7 = dC_6$ wrapping an internal 3-cycle. This 3-form gauge field couples to the F_3 flux component f_1 and the complex structure axion $\text{Re}(U)$, which leads to the breaking of the symmetry to $\mathbb{Z}_{f_1}$. However, given the non-geometric nature of our compactifications things are more involved and it is easiest to simply rely on mirror symmetry.

isfies the strong version of the AdS distance conjecture since it has $\alpha = 1/2$. This absence of scale separation was also discovered in related IIA models in [81].

This leaves us with a non-supersymmetric family of AdS solutions that is also dual to DGKT and that has $\alpha = 7/18$. This is again consistent with the refined AdS distance conjecture due to the presence of a discrete symmetry that is unaffected by a simple sign flip of a flux quanta. Since these solutions are non-supersymmetric they are predicted to decay perturbatively or non-perturbatively [38]. Given that we find that the masses squared of S and U are above the Breitenlohner-Freedman bound [64], it is not clear whether there is a perturbative instability. Studying all possible non-perturbative decay channels or trying to identify one explicit non-perturbative decay channel is a daunting task, so we restrict ourselves here to referring to a related study of non-supersymmetric AdS vacua in the dual DGKT model [82].

Finally, let us discuss the most interesting families of supersymmetric 4d $\mathcal{N} = 1$ AdS vacua, namely the new families that allow for the inclusion of an arbitrarily large number of D3 branes and that are discussed in subsection 2.4.2. While the first one has $\alpha = 1/2$ and is therefore consistent with the strong version of the AdS distance conjecture, the second one has $\alpha = 3/2$, which means that the light tower is becoming light much more quickly. These solution can be made consistent with the strong version of the AdS distance conjecture by demanding $\alpha \geq 1/2$, as is already discussed in the original paper [63]. Nevertheless, given that these vacua with $\alpha = 3/2$ are different from all the other solutions which had $\alpha = 1/2$ or smaller, they are interesting and deserve further study.

Since the later two families of supersymmetric AdS vacua have an ever increasing number of D3 branes one should worry about what that means exactly. In geometric compactifications we would expect an ever growing number of light open string modes associated with these N_{D3} branes. Concretely, for N_{D3} branes at separate locations the number of light open string degrees of freedom should grow like N_{D3}. If there is a an actual potential being generated for the D3 brane position moduli, then it seems likely that they all settle into the minimum.[10] We can of course also always choose to place all the N_{D3} on top of each other and since they are mutually BPS there should be no force between them. This would then lead to a number of light degrees of freedom that grows even faster like N_{D3}^2. Due to the species bound [83, 84, 85, 86], this leads to a UV cutoff that goes like $\Lambda_{UV} \sim M_{pl}/\sqrt{N_{D3}^2} = M_{pl}/N_{D3}$. In our first family of AdS vacua one finds that $\Lambda_{UV} \sim 1/f_1 \sim m_{\text{tower}}$. So, the UV cutoff from the species bound scales in the same way as the infinite tower of light modes. In the second example with $\alpha = 3/2$ one finds that $\Lambda_{UV} \sim 1/f_1 \sim m_{\text{tower}}^{\frac{4}{3}}$. This means that the species bound is even lower than the tower of light states that comes down rather quickly in this case anyways. Note that the previous discussion is based on the geometric intuition that might well carry over to these non-geometric setups. However, the actual open string spectrum for D3 branes in these model was not worked out in the previous literature. We leave it as an interesting task for the future to check the light open string degrees

[10] At least in a geometric compactification the moduli space is compact so there are no runaway directions and for a non-trivial potential there has to exist a global minimum. Any potential that is generated for the D3 brane position moduli should be small in our limit of parametrically weak string coupling, so these position moduli should be light.

of freedoms in these models.

Slightly disconnected from the different AdS conjectures discussed above, we lastly would like to point out the most interesting and most surprising feature of these AdS solutions with $N_{D3} \to \infty$: The fluctuations along the AdS_4 directions of the open string modes on these branes should give rise to gauge groups with arbitrarily large rank. For example, if we place all N_{D3} branes on top of each other one would naively expect an $SU(N_{D3}/2)$ gauge group.[11] String universality in higher dimensions with higher amount of supersymmetry leads to fairly low ranks for the gauge group, which seems in stark contrast with the solutions above. This is a by now very active area of research following the initial work of [87, 88, 89, 90, 91]. However, there is no argument in the literature that forbids 4d $\mathcal{N} = 1$ (not scale separated) AdS solutions with an $SU(N)$ gauge group for arbitrarily large N. Furthermore, there exist families of AdS_7 solutions with arbitrarily large gauge group rank (see for example [92, 93, 94, 95, 96] for early work on this). So, it seems reasonable that related AdS_4 solutions do exist as well in the barely explored part of the string landscape that we study here. As discussed in section IV of [63], since the AdS_7 solutions are not scale separated one should think of the gauge group as living on a defect in the higher dimensional $\mathrm{AdS}_7 \times S^4/\mathbb{Z}_k$ theory. For this AdS_7 case one can increase the gauge group rank by making k large and this does not lead to a decompactification. However, in our setup when we increase the rank of the gauge group we send the cosmological

[11] The tadpole condition in equation (2.5) counts D3 branes in the covering space, hence there can be at most $\lfloor N_{D3}/2 \rfloor$ freely moving D3 branes in the quotient space. If N_{D3} is odd then one D3 brane would necessarily be stuck on top of an O3 plane.

constant to zero $V_{AdS} \to 0$. The internal space is also not geometric. So, although our solutions are not scale separated and there is a tower of light string modes, it is not necessarily natural to think of the D3 branes as defects in a higher dimensional geometric space. We again add as a word of caution that the open string spectrum for these D3 branes has not been worked out and therefore it could hypothetically not contain any massless open strings or no gauge fields at all. It would be very interesting to check this explicitly and we hope to do this in the future.

2.5 de Sitter vacua

Lastly, we would like to comment on the existence of dS vacua in this setup. Given that the Kähler potential can receive string loop corrections, one finds that non-supersymmetric solutions can cease to exist, if corrections are large. Thus, unless they are at weak coupling one should not trust non-supersymmetric solutions. All dS solutions in the models discussed here will have a string coupling that is not that much smaller than 1 and it is therefore not clear whether they can be trusted. Nevertheless, we discuss them for the following two reasons: Firstly, they were recently studied in [30] and in the dual type IIA picture in [97] and we would like to comment on and extend these previous results. Secondly, dS vacua are notoriously difficult to construct in purely classical scalar potentials [98] and only very few explicit solutions without tachyons exist in the literature [99, 100, 101, 102]. Therefore, it is interesting to check whether they also exist in our simple models or not.

Unstable dS solutions, i.e. solutions with a tachyonic direction and the correct tadpole for the 1^9 model, $N_{\text{flux}} = 12$, were found in [30]. Interestingly, the authors of that paper performed a scan over flux values that do not satisfy the tadpole condition and they found that stable dS vacua exist for a large $N_{\text{flux}} \sim \mathcal{O}(100)$. They also noticed that the ratio of stable dS vacua to all randomly generated vacua grows with N_{flux} (see figure 9 in [30]). The smallest N_{flux} value that was giving rise to a stable dS solution in figure 9 in [30] is larger than 66 and the smallest, explicitly listed, stable dS solution in table 5 of that paper has $N_{\text{flux}} = 74$. While this is substantially larger than the allowed $N_{\text{flux}} = 12$ in the 1^9 model, it is not that much larger than the allowed $N_{\text{flux}} = 40$ in the 2^6 model.

2.5.1 Explicit dS solutions

An explicit tachyonic dS extremum with $N_{\text{flux}} = 12$ was previously found in [30]. The corresponding fluxes are

$$\begin{aligned} &f^0 = 4, \quad f^1 = 8, \quad f_1 = 7, \quad f_0 = -17, \\ &h^0 = 1, \quad h^1 = 1, \quad h_1 = 1\,, \quad h_0 = -2\,. \end{aligned} \tag{2.43}$$

Given that $N_{\text{flux}} = 12$ this is a solution to the 1^9 model which does not require D3 branes since the fluxes cancel the negative O3 plane charge. The moduli are stabilized at the following values

$$\begin{aligned} \mathrm{Re}(U) &\approx 0.544\,, \qquad & \mathrm{Im}(U) &\approx 1.11\,, \\ \mathrm{Re}(S) &\approx 7.72\,, \qquad & \mathrm{Im}(S) &\approx 5.19\,. \end{aligned} \tag{2.44}$$

The value of the scalar potential is given by $V_{dS} \approx 1.72 \times 10^{-4}$. The masses squared for the four real scalar fields in the unstable dS extremum are given by

$$m_1^2 \approx 0.0226\,, \qquad m_2^2 \approx 0.0157\,, \qquad m_3^2 \approx 0.00143\,, \qquad m_4^2 \approx -0.00119\,. \tag{2.45}$$

So, there are unstable dS solutions like the one above and, as mentioned previously, there are also metastable dS vacua, if one ignores the tadpole and lets N_{flux} become fairly large. Therefore, one should ask what the lowest possible value for N_{flux} is that still gives rise to metastable dS solutions. We have not been able to answer this question in full generality. However, we noticed that unstable and metastable dS solutions still exist when we set four fluxes to zero: $f^1 = f_0 = h^0 = h_1 = 0$. We then studied the full parameter space spanned by the remaining four fluxes, while ignoring the tadpole. This led us to discover infinite families of solutions that transition from AdS to unstable dS and then to metastable dS, if we vary the fluxes. Within these family we identified the smallest possible N_{flux} that has integer quantized fluxes and gives rise to metastable dS solutions. We find that the only possible value below $N_{O3}/2 = 40$ for the 2^6 model is $N_{\text{flux}} = 30$.[12] For this value there are four different metastable dS solutions. Three have $\text{Im}(S) < 1$ and are therefore expected to receive substantial string loop corrections. The fourth one with

[12]The next larger values of N_{flux} that give rise to metastable dS solutions in our restricted model with only four non-zero fluxes are $N_{\text{flux}} = \{59, 60, 61\}$. This is too large to be compatible with the tadpole cancellation condition.

the fluxes

$$
\begin{aligned}
&f^0 = 33, \quad f^1 = 0, \quad f_1 = -1, \quad f_0 = 0, \\
&h^0 = 0, \quad h^1 = -1, \quad h_1 = 0\,, \quad h_0 = 1\,,
\end{aligned}
\tag{2.46}
$$

has a metastable dS vacuum at

$$
\begin{aligned}
\mathrm{Re}(U) &= 0\,, \qquad & \mathrm{Im}(U) &\approx 0.299\,, \\
\mathrm{Re}(S) &= 0\,, \qquad & \mathrm{Im}(S) &\approx 1.32\,.
\end{aligned}
\tag{2.47}
$$

The value of the scalar potential is given by $V_{dS} \approx 0.00524$. The masses squared for the four real scalar fields in the dS minimum are given by

$$
m_1^2 \approx 3.31\,, \qquad m_2^2 \approx 1.29\,, \qquad m_3^2 \approx .302\,, \qquad m_4^2 \approx 0.0999\,. \tag{2.48}
$$

Given that $N_{\text{flux}} = 30$ this is a solution to the 2^6 model which does require $N_{D3} = 10$ D3 branes. Thus, there should be additional light open string moduli associated with those D3 branes.

It would be interesting to extend our full analysis beyond the restriction $f^1 = f_0 = h^0 = h_1 = 0$ and check whether there exist metastable dS solutions in these models that are at smaller string coupling and/or that do not require D3 branes in order to satisfy the tadpole. Due to the mirror symmetry that relates our above models to models with H_3 flux and non-geometric Q flux there should be also a connection to the metastable dS solution found in 2009 in [103]. Note however, that the latter also required geometric and/or non-geometric fluxes in the type IIB duality frame since $h^{1,1} \neq 0$ and thus they are less controlled then the models we discussed in this paper due to the risk of large α' corrections.

2.5.2 dS extrema and the swampland

The very existence of dS vacua in string theory was first questioned in [55, 56] and a variety of refined dS swampland conjectures were proposed in 2018 in for example [104, 105, 106, 22, 107]. All of these conjectures forbid metastable dS solutions. However, given that our metastable dS solution above is expected to receive substantial string loop corrections, it does not invalidate these conjectures. The previously discovered unstable dS extremum of [30] has $e^{\phi} \approx .5$ and does not require D3 branes. It is thus in much better shape, however, given that it is unstable with large $|\eta| \approx 7$ it is not really incompatible with any of the refined dS swampland conjectures.

It would be interesting to study this simplified model or related more complicated setups to see whether one can find metastable dS vacua at weak coupling and without D3 branes. While there is no obstruction to this, it was recently shown in the context of type IIA flux compactifications that dS solutions cannot exist in a parametrically controlled region [108, 109]. While these papers mostly focused on geometric type IIA flux compactifications they also discuss more exotic ingredients like non-geometric fluxes which makes them applicable to all the type IIA flux compactifications that are the mirror dual of our type IIB setup. Thus, they actually apply also to our non-geometric type IIB models. This means there should be no parametrically controlled dS solutions, i.e. no solutions with a free flux parameter that we can send to infinity to get $\mathrm{Im}(S) \to \infty$. However, there is no obvious reason why well-controlled dS solutions with $\mathrm{Im}(S) \gg 1$ cannot exist in the setup discussed in this paper.

2.6 Conclusion

In this section we have studied type IIB flux compactifications based on Landau-Ginzburg orientifolds. We have focused on models that are non-geometric in the sense that $h^{1,1} = 0$, i.e., there are no Kähler moduli. This barely studied class of models was originally introduced in [26, 29] and allows for full moduli stabilization. We have revisited these models and discovered a variety of interesting new families of solutions. We have contrasted these solutions with several swampland conjectures (see [30] for recent related work).

Concretely, we have explored the four dimensional landscape of two models which are mirror duals to type IIA string theory on rigid Calabi-Yau orientifolds, i.e., Calabi-Yau manifolds with $h^{2,1} = 0$. After including H_3 and F_3 fluxes our models are dual to type IIA flux compactifications with both metric and non-geometric fluxes, so our analysis goes beyond (and includes) setups such as DGKT [31]. However, while non-geometric fluxes are not under control in type IIA supergravity models, we have only regular (and well understood) H_3 and F_3 fluxes in the mirror dual Landau-Ginzburg models in IIB. Furthermore, there exists a very powerful non-renormalization theorem that protects the superpotential from receiving any corrections at all [26]. For simplicity we have focused here on an isotropic two moduli (SU) model, which is not the most general setup, but it is already enough to provide us with new interesting results that we now sum up.

In this work we have provided additional arguments which point to the existence of fully stabilized 4d $\mathcal{N} = 1$ Minkowski vacua. While these were originally discovered in [26, 29], we managed to find infinite families of Minkowski vacua and we have argued that they are in principle compatible with existing swampland conjectures like [57, 58]. Furthermore, we have proven that although the masses do receive corrections, they can never become zero and there cannot arise any flat directions even when including all unknown corrections.

We have also found several new infinite families of AdS vacua, which are not connected to the aforementioned family of Minkowski vacua. By taking some particular flux combinations to infinity (often simply one of the fluxes) these AdS solutions approach Minkowski space. However, in every example we have argued using mirror symmetry that there is a tower of states becoming light with a certain power $\alpha \geq 1/2$ of the cosmological constant, i.e. $m_{\text{tower}} \sim |V_{AdS}|^{\alpha}$. Thus, our results in this regard are consistent with the AdS distance conjecture [63]. However, since our models are essentially a generalization of the DGKT models in type IIA, we also identified supersymmetric and non-supersymmetric infinite families of AdS vacua in a subset of our model, which have $\alpha = 7/18$ like the original examples in DGKT [31]. For similar reasons as discussed in [37] we find agreement of these families with the refined version of the AdS distance conjecture due to a large discrete 3-form gauge symmetry. For the nonsupersymmetric infinite family of AdS vacua, our moduli S and U acquire masses squared above the BF bound. These vacua arise in a regime of parametric control but should be unstable according to the conjecture in [38]. It would be interesting to analyze possible decay modes for our family of non-supersymmetric AdS solutions.

As explained in [29], due to the non-geometric nature of our models, the Kähler potential acquires an unfamiliar factor of 4 whose main effect is to allow supersymmetric fluxes that are not imaginary self-dual. This actually allows the H_3 and F_3 fluxes to contribute to the D3/O3 tadpole condition with either sign. Interestingly, this enables us to construct new infinite families of supersymmetric AdS_4 vacua with an unbounded number of spacetime filling D3 branes. This is possible because the flux contribution to the tadpole can have the same sign as that of O3 orientifold planes and we can make it arbitrarily large. This arbitrarily large flux then requires an arbitrarily large number of D3 branes. Given that all these solutions are consistent with the AdS distance conjecture there is an infinite tower of massive states becoming light when we increase the flux and the number of D3 branes at the same time. Furthermore, it is expected that there are large numbers of massless open string modes that are associated with those D3 branes, leading to an ever decreasing species bound $\Lambda_{UV} \sim M_{pl}/N_{D3}$. Nevertheless, it seems naively possible to get a very large rank for the gauge group in these 4d $\mathcal{N} = 1$ AdS vacua. It would be very interesting to study this further and see whether these solutions are indeed trustworthy or suffer from some inconsistencies.

Finally, we have been able to find a metastable de Sitter vacuum that requires some number of D3 branes to be present. However, this vacuum does not arise at weak coupling and there is no argument preventing perturbative and non-perturbative corrections from destroying it.

Given the current large amount of activity in the swampland program, it is very important to keep exploring all different areas of the string landscape, in particular, areas that are truly stringy in the sense that they do not have a geometric supergravity description. In this section we have revisited and extended previous studies of type IIB flux compactifications in the absence of Kähler moduli, i.e. for $h^{1,1} = 0$. We found several intriguing results which could be natural in this rather unexplored corner of the string landscape and that deserve further study in the future.

3 On the absence of supergravity solutions for localized, intersecting sources

The following work in this section is from a paper available for preprint on arXiv. I completed this work in collaboration with Jacob Bardzell, Kevin Federico, and Timm Wrase [6].

3.1 Introduction

For more than two decades string theory compactifications with intersecting D-branes and O-planes have played an important role in string phenomenology. On the one hand, intersecting D-brane models are used to obtain particle physics models that can resemble the supersymmetric standard model and extension thereof, see for example the review article [110]. On the other hand, orientifold planes are needed in flux compactifications to partially break supersymmmetry and to provide a source of negative energy in the scalar potential, see for example [111, 112] for early review articles. For flux compactifications on toroidal orbifolds the orientifold planes generically intersect in the internal space. So, both settings lead to supergravity equations of motion that have localized sources that intersect in a non-trivial way.

For such intersecting sources one then has to solve the equations of motion for the electromagnetic field strengths that are being sourced. This is rather simple since the equations are linear and the field strengths for each individual source can simply be added up. However, Einstein's equations are non-linear and extremely hard to solve. This has led to the often-employed simplification of a so-called smearing of the sources over their transverse directions. Mathematically speaking one replaces the delta function sources with constants, which dramatically simplifies the equations of motion. If one does that, one would then have to try to understand how close such a smeared solution is to the actual localized solution one started with, which is not an easy question to answer [113, 114, 115].

One can of course try to solve the equations of motions for intersecting objects without smearing or by only partially smearing the sources. For example, one could smear only over the mutual transverse directions of all sources, or one smears the sources only over directions that are transverse to one and parallel to another, etc. This leads to a plethora of possibilities that are discussed for example in the review article [116] (see also [117] for an earlier review article). The upshot of this endeavor is that fully localized solutions are known essentially only for parallel sources and in all other cases one has to do at least some partial smearing in order to solve the equations of motion. One exception is the case of two intersecting NS5-branes extending along $x^0, x^1, x^2, x^3, x^4, x^5$ and $x^0, x^1, x^6, x^7, x^8, x^9$ respectively (without any mutually transverse directions), see [118] for a discussion of this solution.

Within the swampland program [119] in string theory many flux compactifications have recently been revisited and scrutinized. In particular, flux compactifications of massive type IIA give rise to infinite families of weakly coupled 4d $\mathcal{N} = 1$ AdS vacua [31, 36]. The viability of these solution was questioned for example by the AdS swampland conjecture [63]. One criticism pertaining to these type IIA flux compactifications is that they are using smeared orientifolds planes, i.e., the full 10d supergravity equations of motion have not been explicitly solved [120]. Two papers recently revisited this problem [121, 122] and found approximate solution with localized sources (see also [69, 70, 62, 72, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133] for closely related recent work). These approximate solutions in [121, 122] arose from an expansion in the large F_4-flux quanta and they capture the leading order backreaction of the localized orientifold planes. However, at this order the actual effects of the intersection of the O-planes is not taken into account. It would therefore be extremely important to extend these approximate solutions to higher order. However, given the importance of intersecting sources in many parts of string theory, a broader approach is also certainly warranted.

In this section we study the equations of motion for two localized Dp-branes or Op-planes in flat space. We take them to intersect perpendicular with four Neumann/Dirichlet directions and $(p-2)$ common directions (often denoted

D$p \perp$ D$p\,(p-2)$). This means the setup preserves 8 supercharges, which allows us study the SUSY transformations of the fermions. Demanding that these vanish, as required for a supersymmetric solution, we find that a fully localized solution cannot exist for a generic diagonal metric Ansatz, even when allowing for fully generic fluxes. While this might come as a surprise, similar results were previously obtained. For example, it was shown in [134] that no solution can exist for localized, intersecting D3/D5-branes.

The outline of the section is as follows: In section 3.2 we review the supergravity solution for a single source. Then we discuss two perpendicularly intersecting objects in section 3.3 and show that the corresponding equations of motion have no solution. In section 3.4 we discuss our findings and provide an outlook on important open questions.

3.2 Review of a single source

In this section we will solve the equations of motion of type II supergravity coupled to a stack of Dp-branes or an Op-plane in 10d flat space. Such a solution is textbook material [135] but we review it here to set up our notation and to remind the reader of some features that will be important in the next section.

3.2.1 Type II supergravity

We are using the notation and conventions of [113] but we will change to string frame. The trace reversed Einstein equations are given by

$$\begin{aligned} R_{ab} &= -2\nabla_a \partial_b \phi + \frac{1}{4} g_{ab} \left(2g^{cd}\partial_c\phi\partial_d\phi - \nabla^2\phi \right) + \frac{1}{2}|H|^2_{ab} - \frac{1}{8} g_{ab}|H|^2 \\ &+ \sum_{n\leq 5} \mathrm{e}^{2\phi} \left(\tfrac{1}{2(1+\delta_{n5})}|F_n|^2_{ab} - \tfrac{n-1}{16(1+\delta_{n5})} g_{ab}|F_n|^2 \right) + \frac{1}{2}\mathrm{e}^{\phi} \left(T^{loc}_{ab} - \frac{1}{8} g_{ab} T^{loc} \right) . \end{aligned} \tag{3.1}$$

The sum over n includes all even/odd numbers from 0 to 5 for IIA/IIB. The δ_{n5} is the Kronecker delta, and squares of q-forms are defined via $|A|^2_{\alpha\beta} = \frac{1}{(q-1)!} A_{\alpha a_2 \ldots a_q} A_\beta{}^{a_2\ldots a_q}$, $|A|^2 = \frac{1}{q!} A_{a_1\ldots a_q} A^{a_1 \ldots a_q}$. We restrict to parallel (stacks) of Dp-branes or Op-planes so that the local stress tensor is given by

$$T^{loc}_{\mu\nu} = \mu_p \, g_{\mu\nu} \, \delta(p) \, . \tag{3.2}$$

Here μ_p is negative for Dp-branes and positive for an Op-plane.[13] $\delta(p)$ denotes a delta function that localizes us on the $p+1$ dimensional world volume of the source. For multiple parallel Dp-branes or Op-planes $\delta(p)$ should be understood as a sum of δ-functions. μ, ν are denoting the directions along the worldvolume of the source and $g_{\mu\nu}$ is the pullback of the spacetime metric g_{ab} to the worldvolume of the source.

[13] While we do not need the exact values, the charge and tension of a stack of N_p Dp-branes is $-N_p \tilde{\mu}_p = -N_p (2\pi\sqrt{\alpha'})^{-p}/\sqrt{\alpha'}$. The charge and tension of an Op-plane is $-2^{p-5}\tilde{\mu}_p$ in the quotient space. The quantity appearing in our equations is $\mu_p = -N_p 2\kappa^2_{10}\tilde{\mu}_p = -N_p(2\pi\sqrt{\alpha'})^{7-p}$ for a stack of Dp-branes and $\mu_p = 2^{p-5}(2\pi\sqrt{\alpha'})^{7-p}$ for an Op-plane.

The equation of motion for the dilaton is given by

$$\nabla^2\phi = 2g^{ab}\partial_a\phi\partial_b\phi - \frac{1}{2}|H|^2 + \sum_{n<5} \tfrac{5-n}{4}\mathrm{e}^{2\phi}|F_n|^2 - \tfrac{p-3}{4}\mathrm{e}^{\phi}\mu_p\delta(p)\,. \qquad (3.3)$$

Plugging the above into equation (3.1), we find that it simplifies to

$$\begin{aligned} R_{ab} \;&=\; -2\nabla_a\partial_b\phi + \frac{1}{2}|H|^2_{ab} + \sum_{n\leq 5}\mathrm{e}^{2\phi}\left(\tfrac{1}{2(1+\delta_{n5})}|F_n|^2_{ab} - \tfrac{1}{4(1+\delta_{n5})}g_{ab}|F_n|^2\right) \\ &\quad + \frac{1}{2}\mathrm{e}^{\phi}\left(T^{loc}_{ab} - \frac{1}{2}g_{ab}\mu_p\delta(p)\right)\,. \end{aligned} \qquad (3.4)$$

In the absence of NS5-branes, the Bianchi identities for the field strengths are

$$\begin{aligned} dH \;&=\; 0\,, \\ dF_n \;&=\; H\wedge F_{n-2} - \mu_{8-n}\delta_{n+1}(8-n)\,, \end{aligned} \qquad (3.5)$$

where $\delta_{n+1}(8-n)$ is a shorthand notation for the delta function $\delta(8-n)$ multiplied by a normalized $(n+1)$ volume form transverse to the source.

The equations of motion for the gauge fields in the absence of NSNS sources are given by

$$\begin{aligned} \mathrm{d}\left(\mathrm{e}^{-2\phi}\star H\right) \;&=\; -\frac{1}{2}\sum_{n\leq 10}\star F_n\wedge F_{n-2}\,, \\ d\left(\star F_n\right) \;&=\; -H\wedge\star F_{n+2} - (-1)^{\frac{n(n-1)}{2}}\mu_{n-2}\delta_{11-n}(n-2)\,. \end{aligned} \qquad (3.6)$$

The equations of motion for the RR fields can be obtained from the Bianchi identities in equation (3.5) by using that $F_n = (-1)^{\frac{(n-1)(n-2)}{2}}\star F_{10-n}$.

For supersymmetric solutions one has to require that the SUSY transformations of the fermions vanish. This provides a simpler set of first order equations that often completely fixes the system and thereby automatically solves the Einstein and dilaton equations. We use the conventions of [136, 111] so that the transformations of the gravitino and gaugino are given by

$$
\begin{aligned}
\delta_\epsilon \psi_a &= \left(\partial_a + \frac{1}{4}\underline{\omega_a} + \frac{1}{4}\underline{H_a}\mathcal{P}\right)\epsilon + \frac{1}{8}e^{\phi}\sum_n \frac{1}{1+\delta_{n5}}\,\underline{F_n}\Gamma_a\mathcal{P}_n\epsilon\,, \\
\delta_\epsilon \lambda &= \left(\underline{\partial\phi} + \frac{1}{4}\underline{H}\mathcal{P}\right)\epsilon + \frac{1}{4}e^{\phi}\sum_n (-1)^n(5-n)\underline{F_n}\mathcal{P}_n\epsilon\,.
\end{aligned}
\tag{3.7}
$$

The sum over n includes all even/odd numbers from 0 to 5 for IIA/IIB. As above $a = 0, 1, \ldots, 9$ is a curved space index and we denote the corresponding tangent space indices as $A, B = 0, 1, \ldots, 9$. The underlined quantities are given by

$$
\begin{aligned}
&\underline{\omega_a} = {\omega_a}^{AB}\Gamma_{AB}\,, \qquad \underline{H_a} = \tfrac{1}{2}H_{abc}\Gamma^{bc}\,, \qquad \underline{H} = \tfrac{1}{3!}H_{abc}\Gamma^{abc}\,, \\
&\underline{F_n} = \tfrac{1}{n!}F_{a_1\ldots a_n}\Gamma^{a_1\ldots a_n}\,, \qquad \underline{\partial\phi} = \partial_a\phi\Gamma^a\,,
\end{aligned}
\tag{3.8}
$$

where $\Gamma^{a_1 a_2 \ldots a_n} = \Gamma^{[a_1}\Gamma^{a_2}\ldots\Gamma^{a_n]}$, and we also define $\Gamma_{10} = \Gamma_{012\ldots9}$. Furthermore, we have that

$$
\begin{aligned}
&\mathcal{P} = \Gamma_{10} \quad \text{in IIA}, \qquad \mathcal{P} = -\sigma_3 \quad \text{in IIB}, \\
&\mathcal{P}_n = (\Gamma_{10})^{\frac{n}{2}} \quad \text{in IIA}, \qquad \mathcal{P}_n = \begin{cases} \sigma_1 \text{ for } \frac{n+1}{2} \text{ even}, \\ \mathrm{i}\sigma_2 \text{ for } \frac{n+1}{2} \text{ odd}, \end{cases} \quad \text{in IIB}\,.
\end{aligned}
\tag{3.9}
$$

The spinor ϵ in type IIA has 32 real components, which could be split into two 16 component Majorana-Weyl spinors with opposite chiralities: $\Gamma_{10}\epsilon_1 = \epsilon_1$,

$\Gamma_{10}\epsilon_2 = -\epsilon_2$. For IIB $\epsilon = (\epsilon_1, \epsilon_2)^T$ is a doublet of two 16 component Majorana-Weyl spinors with positive chirality so that $\Gamma_{10}\epsilon_i = \epsilon_i$. The Pauli matrices σ_i above act on this doublet.

In the presence of Dp-branes along the first $p+1$ directions or when doing the corresponding orientifold projection we break half of the supersymmetry via the following projection (involving the flat space Γ-matrices)

$$\epsilon_2 = \Gamma_{01\ldots p}\epsilon_1 \,. \tag{3.10}$$

3.2.2 A single p-dimensional source

We consider first a single Op-plane or a stack of Dp-branes. These localized objects are magnetic sources for F_{8-p} due to their Chern-Simons coupling to C_{p+1}. So, the only sourced RR-field is $F_{8-p} = \star F_{p+2}$. We can set all other RR-fields and the NSNS-flux H equal to zero.

We can choose our coordinates in such a way that the Op-plane or the stack of Dp-branes extend along x^μ with $\mu = 0, 1, \ldots, p$ and are located at the origin in the transverse directions $x^i = 0$, for $i = p+1, p+2, \ldots, 9$. This then preserves an $SO(p,1) \times SO(9-p)$ symmetry group, where the first $SO(p,1)$ factor is enhanced to the full Poincaré group. The most general metric Ansatz that is compatible with these symmetries is

$$g = e^{2A_1(r)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{2A_2(r)}\delta_{ij}dx^i dx^j \,. \tag{3.11}$$

Here $e^{2A_1(r)}$ and $e^{2A_2(r)}$ can only depend on $r = \sqrt{(x^{p+1})^2 + \ldots + (x^9)^2}$, the overall transverse distance from the localized source.

The solution to the equations above in subsection 3.2.1 can be found in the textbook [135, (10.38)] and we write it as

$$e^{-4A_1(r)} = e^{4A_2(r)} = 1 - \frac{\tilde{\mu}_p}{r^{7-p}}\,, \tag{3.12}$$

$$e^{\phi(r)} = e^{\phi_0+(p-3)A_1(r)}\,, \tag{3.13}$$

$$C_{p+1}(r) = \left(1 - e^{4A_1(r)}\right) e^{-\phi_0}\; dx^0 \wedge dx^1 \wedge \ldots \wedge dx^p\,. \tag{3.14}$$

Here e^{ϕ_0} is the asymptotic value of the dilaton infinitely far away from the source. We fixed the metric to be asymptotically Minkowski and we chose $C_{p+1}(r)$ to asymptotically vanish. We also defined $\tilde{\mu}_p = \frac{(-1)^{p+1}e^{\phi_0}\Gamma(\frac{9-p}{2})}{2(7-p)\pi^{\frac{9-p}{2}}}\mu_p$.

Note that due to the minus sign in equation (3.12) and the fact that $\tilde{\mu}_p$ is positive for an Op-plane, there is actually a singularity at a finite distance $r = \tilde{\mu}_p^{\frac{1}{7-p}}$ from the Op-plane. This singularity is at a distance that is of the order of the string length, $l_s = 2\pi\sqrt{\alpha'}$. At this point stringy corrections modify the equations of motion and remove this singularity.

Since we will need this later, we derive here explicitly the solution to the non-trivial Bianchi identity (cf. equation (3.5)). We rewrite it using the transverse metric determinant $g_{9-p} = e^{2(9-p)A_2}$ as follows

$$\begin{aligned} dF_{8-p} &= -\mu_p\, \delta_{9-p}(p) \\ &= -\mu_p\, \delta_{9-p}(p) \star_{9-p} 1 \\ &= -\mu_p\, \frac{1}{\sqrt{g_{9-p}}}\, \delta(x^{p+1})\delta(x^{p+2})...\delta(x^9)\, \sqrt{g_{9-p}}\; dx^{p+1} \wedge dx^{p+2} \wedge ... \wedge dx^9 \\ &= -\mu_p\, \delta(x^{p+1})\delta(x^{p+2})...\delta(x^9)\, dx^{p+1} \wedge dx^{p+2} \wedge ... \wedge dx^9 \end{aligned}$$

$$= -\mu_p \, \tilde{\delta}(\vec{r}) \, \tilde{\star}_{9-p} 1 \,. \tag{3.15}$$

The tilde indicates that we are working with the flat space metric so there is no warp factor dependence anymore. The solution is given by

$$F_{8-p} = \tilde{\star}_{9-p} \, d \left(\frac{\tilde{\mu}_p}{r^{7-p}} \right) , \tag{3.16}$$

since

$$\begin{aligned} dF_{8-p} &= \mathrm{d}\tilde{\star}_{9-p} d \left(\frac{\tilde{\mu}_p}{r^{7-p}} \right) \\ &= (-1)^p (\tilde{\star}_{9-p} 1) \tilde{\nabla}^2 \left(\frac{\tilde{\mu}_p}{r^{7-p}} \right) \\ &= (-1)^{p+1} (\tilde{\star}_{9-p} 1) \tilde{\mu}_p \frac{2(7-p)\pi^{\frac{9-p}{2}}}{\Gamma(\frac{9-p}{2})} \tilde{\delta}(\vec{r}) \\ &= (\tilde{\star}_{9-p} 1) \, \mu_p \, \tilde{\delta}(\vec{r}) \,. \end{aligned} \tag{3.17}$$

Summarizing, we see that it is possible to solve the supergravity equations exactly for a single source. Similarly, one can solve the equations of motion for parallel sources that are located not necessarily at $\vec{r} = 0$ but at different positions $\vec{r}_0^{(\alpha)}$, $\alpha = 1, 2, \ldots$. In this case we can simply add up the individual solutions for each source and the solution is given by

$$\begin{aligned} e^{-4A_1(\vec{r})} = e^{4A_2(\vec{r})} &= 1 - \sum_\alpha \frac{\tilde{\mu}_p^{(\alpha)}}{\left| \vec{r} - \vec{r}_0^{(\alpha)} \right|^{7-p}} \,, \\ e^{\phi(\vec{r})} &= e^{\phi_0 + (p-3) A_1(\vec{r})} \,, \\ C_{p+1}(\vec{r}) &= \left(1 - e^{4A_1(\vec{r})} \right) e^{-\phi_0} \; dx^0 \wedge dx^1 \wedge \ldots \wedge dx^p \,. \end{aligned} \tag{3.18}$$

Note that the Bianchi identities in equation (3.5) are linear and we can always simply add up the field strengths for any arbitrarily complicated configuration of sources. However, it is highly unusual, and special to this case of parallel sources, that the non-linear general relativity equation in (3.1) is also solved if we simply add up solutions.

3.3 Two perpendicularly intersecting sources

In this section we want to solve the equations of motion for two perpendicularly intersecting p-dimensional sources in flat space. These could be either two Op-planes or two stacks of Dp-branes or one of each. We restrict to $1 \leq p \leq 6$ so that we can have four directions that are along one of the objects and transverse to the other and there is at least one common transverse direction. The configuration that preserves eight supercharges in flat space is shown below.

Directions	0	...	$p-2$	$p-1$	p	$p+1$	$p+2$	$p+3$	...	9
First source	$\times$	$\times$	$\times$	$\times$	$\times$	-	-	-	-	-
Second source	$\times$	$\times$	$\times$	-	-	$\times$	$\times$	-	-	-

The above intersecting sources respect an $SO(p-2,1) \times SO(2) \times SO(2) \times SO(7-p)$ symmetry.[14] The first $SO(p-2,1)$ group is actually enhanced to the full Poincaré group. This symmetry group together with the specific source configuration shown above allows the metric (warp factors) to only depend on $\rho_1 = \sqrt{(x^{p+1})^2 + (x^{p+2})^2}$, $\rho_2 = \sqrt{(x^{p-1})^2 + (x^p)^2}$ and $\rho_T =$

[14]For the special case of $p = 1$ there are no directions common to both sources and therefore no $SO(p-2,1)$ factor. However, this does not affect our reasoning.

$\sqrt{(x^{p+3})^2 + ... + (x^9)^2}$. We make the following diagonal metric Ansatz

$$ds^2 = e^{2A_1(\rho_1,\rho_2,\rho_T)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{2A_2(\rho_1,\rho_2,\rho_T)}\left((dx^{p-1})^2 + (dx^p)^2\right) \quad (3.19)$$
$$+ e^{2A_3(\rho_1,\rho_2,\rho_T)}\left((dx^{p+1})^2 + (dx^{p+2})^2\right) + e^{2A_4(\rho_1,\rho_2,\rho_T)}\left((dx^{p+3})^2 + ... + (dx^9)^2\right),$$

with $\mu, \nu = 0, 1, \ldots, p-2$. Poincaré invariance ensures that the first part of the metric is generic and there cannot be any off-diagonal terms like for example $g_{\mu\rho_1}dx^\mu d\rho_1$ since there are no invariant constant vectors of $SO(p-2,1)$. Non-constant vectors like $\eta_{\mu\nu}x^\mu dx^\nu$ are forbidden by translational invariance. However, in general there could be terms involving $d\rho_1 d\rho_T$, etc. and also terms involving the corresponding angles $d\theta_1$ and $d\theta_2$ when going to polar coordinates, $(x^{p+1}, x^{p+2}) \to (\rho_1, \theta_1)$ and $(x^{p-1}, x^p) \to (\rho_2, \theta_2)$. Here we are restricting to a diagonal metric to make the problem tractable. Since the source setup is invariant under the exchanges $x^{p-1} \leftrightarrow x^p$ and $x^{p+1} \leftrightarrow x^{p+2}$ we can impose the same symmetry on the metric Ansatz, making equation (3.19) the most general diagonal metric Ansatz compatible with the source configuration.

We choose to work with Cartesian coordinates that have the following property that will be important below

$$\partial_{x^{p-1}} e^{2A_n(\rho_1,\rho_2,\rho_T)} = \frac{x^{p-1}}{\rho_2}\partial_{\rho_2} e^{2A_n(\rho_1,\rho_2,\rho_T)},$$
$$\partial_{x^p} e^{2A_n(\rho_1,\rho_2,\rho_T)} = \frac{x^p}{\rho_2}\partial_{\rho_2} e^{2A_n(\rho_1,\rho_2,\rho_T)},$$
$$\partial_{x^{p+1}} e^{2A_n(\rho_1,\rho_2,\rho_T)} = \frac{x^{p+1}}{\rho_1}\partial_{\rho_1} e^{2A_n(\rho_1,\rho_2,\rho_T)},$$
$$\partial_{x^{p+2}} e^{2A_n(\rho_1,\rho_2,\rho_T)} = \frac{x^{p+2}}{\rho_1}\partial_{\rho_1} e^{2A_n(\rho_1,\rho_2,\rho_T)}. \quad (3.20)$$

For the dilaton the most general Ansatz is $\phi = \phi(\rho_1, \rho_2, \rho_T)$. We also define the transverse coordinates for the two O-planes

$$r_1 = \sqrt{\rho_1^2 + \rho_T^2} = \sqrt{(x^{p+1})^2 + (x^{p+2})^2 + (x^{p+3})^2 + ... + (x^9)^2}\,,$$
$$r_2 = \sqrt{\rho_2^2 + \rho_T^2} = \sqrt{(x^{p-1})^2 + (x^{p})^2 + (x^{p+3})^2 + ... + (x^9)^2}\,. \tag{3.21}$$

Using the metric Ansatz as given in equation (3.19), we seek the solution for the above source configuration. We first solve the linear Bianchi identity (cf. equation (3.5))

$$dF_{8-p} = -\mu_p^{(1)}\delta_{9-p}^{(1)}(p_1) - \mu_p^{(2)}\delta_{9-p}^{(2)}(p_2)\,. \tag{3.22}$$

We solve the above equation by writing $F_{8-p} = F_{8-p}^{(1)} + F_{8-p}^{(2)} + F_{8-p}^{(c)}$, where $dF_{8-p}^{(c)} = 0$ is closed[15] and

$$dF_{8-p}^{(1)} = -\mu_p^{(1)}\delta_{9-p}^{(1)}(p_1) \qquad \text{and} \qquad dF_{8-p}^{(2)} = -\mu_p^{(2)}\delta_{9-p}^{(2)}(p_2)\,. \tag{3.23}$$

So, this manifests the linearity of the electromagnetic equations and allows us to simply add up the two fields strengths for the two sources, i.e., we can add up the results for two single sources in flat space. The solution for the first source is (cf. equation (3.16))

$$F_{8-p}^{(1)} = \tilde{\star}_{9-p}^{(1)} d\left(\frac{\tilde{\mu}_p^{(1)}}{r_1^{7-p}}\right). \tag{3.24}$$

[15]We are indebted to Daniel Junghans for pointing out this additional closed piece in F_{8-p}.

From equation (3.24) we can read off the non-zero components of $F^{(1)}_{8-p}$

$$
\begin{aligned}
F^{(1)}_{8-p} =& \tilde{\star}^{(1)}_{9-p} d\left(\frac{\tilde{\mu}^{(1)}_p}{r_1^{7-p}}\right) \\
=& -\frac{\tilde{\mu}^{(1)}_p(7-p)}{r_1^{8-p}} \tilde{\star}^{(1)}_{9-p} dr_1 \\
=& -\frac{\tilde{\mu}^{(1)}_p(7-p)}{r_1^{9-p}} \tilde{\star}^{(1)}_{9-p} \left(x^{p+1}dx^{p+1} + x^{p+2}dx^{p+2} + ... + x^9 dx^9\right) \\
=& \frac{\tilde{\mu}^{(1)}_p(7-p)}{r_1^{9-p}} \Bigg(x^{p+1}dx^{p+2} \wedge dx^{p+3} \wedge ... \wedge dx^9 \\
& \qquad - x^{p+2}dx^{p+1} \wedge dx^{p+3} \wedge ... \wedge dx^9 \\
& \qquad + ... \\
& \qquad + (-1)^p \left(x^9 dx^{p+1} \wedge dx^{p+2} \wedge ... \wedge dx^8\right) \Bigg) . \qquad (3.25)
\end{aligned}
$$

Explicitly we find the following component that we will use below

$$
\left(F^{(1)}_{8-p}\right)_{(p+1)(p+3)(p+4)...9} = -\frac{\tilde{\mu}^{(1)}_p(7-p)}{r_1^{9-p}}\, x^{p+2} . \qquad (3.26)
$$

$F^{(2)}_2$ can be obtained by exchanging x^{p+1}, x^{p+2} with x^{p-1}, x^p in equation (3.25). In particular, it has the component

$$
\left(F^{(2)}_{8-p}\right)_{(p-1)(p+3)(p+4)...9} = -\frac{\tilde{\mu}^{(2)}_p(7-p)}{r_2^{9-p}}\, x^{p} . \qquad (3.27)
$$

Note that the above $F_{8-p} = F^{(1)}_{8-p} + F^{(2)}_{8-p} + F^{(c)}_{8-p}$ is the most generic and exact solution to the Bianchi identity in equation (3.5). It is independent of our particular metric Ansatz since the warp factors do not appear.

3.3.1 The Einstein and dilaton equations

Now we can look at Einstein's equations from equation (3.1) that reduce to

$$\begin{aligned} R_{ab} = & -2\nabla_a \partial_b \phi + \frac{1}{4} g_{ab}(2g^{cd}\partial_c\phi\partial_d\phi - \nabla^2\phi) \\ & e^{2\phi}\left(\frac{1}{2(1+\delta_{(8-p))5})}|F_{8-p}|^2_{ab} - \frac{7-p}{16(1+\delta_{(8-p)5})} g_{ab}|F_{8-p}|^2\right) \\ & + \frac{1}{2}e^{\phi}(T^{loc}_{ab} - \frac{1}{8}g_{ab}T^{loc}) \,. \end{aligned} \tag{3.28}$$

Calculating the Ricci scalar for the above metric Ansatz in equation (3.19) we find for $a = p-1$, $b = p+1$ (essentially from equation (3.20) but also via an explicit computation) that

$$R_{(p-1)(p+1)} = x^{p-1}x^{p+1} f_R(\rho_1, \rho_2, \rho_T)\,, \tag{3.29}$$

where $f_R(\rho_1, \rho_2, \rho_3)$ is a specific function that one can calculate from the above metric Ansatz in equation (3.19). The important point is that the entire $R_{(p-1)(p+1)}$ component of the Ricci tensor is proportional to derivatives with respect to x^{p-1} and x^{p+1}. This then leads (cf. equation (3.20)) to the above prefactor $x^{p-1}x^{p+1}$ in front of $f_R(\rho_1, \rho_2, \rho_3)$.

Likewise we find that the dilaton Ansatz $\phi = \phi(\rho_1, \rho_2, \rho_T)$ leads to

$$-2\nabla_{p-1}\partial_{p+1}\phi = x^{p-1}x^{p+1} f_\phi(\rho_1, \rho_2, \rho_T)\,. \tag{3.30}$$

Let us assume first that the F_{8-p}-flux is simply the superposition of the fluxes from the two single sources as might be expected due to the linearity of the corresponding equation (3.22). That means we are setting the closed piece $F_{8-p}^{(c)}$ to zero in the F_{8-p}-flux. This also means that neither the other RR-fluxes nor the H-flux are sourced.

All non-diagonal entries of the metric in equation (3.19) vanish and the source terms vanish away from the sources as well. Therefore, the off-diagonal entry of the Einstein equation (3.28) for $(ab) = (p-1\,p+1)$ is given by

$$
\begin{aligned}
R_{p-1\,p+1} &= -2\nabla_{p-1}\partial_{p+1}\phi + \frac{1}{2}e^{2\phi}|F_{8-p}|^2_{p-1\,p+1} \qquad (3.31)\\
x^{p-1}x^{p+1}f_R(\rho_1,\rho_2,\rho_T) &= x^{p-1}x^{p+1}f_\phi(\rho_1,\rho_2,\rho_T)\\
&\quad + e^{2\phi}\frac{1}{2(p-1)!}\\
&F_{8-p,p-1\,a_1\dots a_{7-p}}g^{a_1b_1}\dots g^{a_{7-p}b_{7-p}}F_{8-p,p+1\,b_1\dots b_{7-p}}\\
&= x^{p-1}x^{p+1}f_\phi(\rho_1,\rho_2,\rho_T)\\
&\quad + e^{2\phi}\frac{1}{2}F_{8-p,p-1\,p+3\dots 9}g^{p+3p+3}\dots g^{99}F_{8-p,p+1\,p+3\dots 9}\\
&= x^{p-1}x^{p+1}f_\phi(\rho_1,\rho_2,\rho_T)\\
&\quad + e^{2\phi}\frac{1}{2}\tilde{\mu}_p^{(2)}(7-p)\frac{x^p}{{r_2}^{9-p}}e^{-2(7-p)A_4}\tilde{\mu}_p^{(1)}(7-p)\frac{x^{p+2}}{{r_1}^{9-p}}.
\end{aligned}
$$

We rewrite this as

$$
x^{p-1}x^{p+1}(f_R - f_\phi) = x^p x^{p+2}\left(\frac{e^{2\phi-2(7-p)A_4}}{2}\frac{\tilde{\mu}_p^{(1)}(7-p)}{{r_1}^{9-p}}\frac{\tilde{\mu}_p^{(2)}(7-p)}{{r_2}^{9-p}}\right). \qquad (3.32)
$$

The above equation has to be true for all $x^{p-1}, x^p, x^{p+1}, x^{p+2}$. In particular, the left-hand-side is odd under the sign flips $x^{p-1} \to -x^{p-1}$ or $x^{p+1} \to -x^{p+1}$ and even under the sign flips $x^p \to -x^p$ or $x^{p+2} \to -x^{p+2}$. Since ϕ, A_4, r_1 and

r_2 are all functions of (ρ_1, ρ_2, ρ_T), we find that the symmetry properties of the right-hand-side are exactly opposite. This means the left- and right-hand-side have to vanish independently. Since the dilaton and the component e^{2A_4} of the diagonal metric cannot vanish everywhere we conclude that the vanishing of the right-hand-side implies that

$$\tilde{\mu}_p^{(1)} \tilde{\mu}_p^{(2)} = 0 \,. \tag{3.33}$$

The above equation implies that one of the two sources is absent. Or, if we insist that both of the intersecting sources are present, we have shown that there is no solution to the supergravity equations of motion for our two intersecting localized sources with our generic diagonal metric Ansatz. To make this proof fully general, we have to allow for the closed piece $F_{8-p}^{(c)}$ as solution to the Bianchi identity (3.22). Then this closed form piece can source the H-flux and other RR-fluxes via the equations of motion for the fluxes given above in (3.6). Thus, in order to give a full proof we have to actually allow for all possible RR-fluxes and the most generic H-flux compatible with our $SO(p-2,1) \times SO(2) \times SO(2) \times SO(7-p)$ symmetry group. This makes the Einstein and dilaton equations too complicated to analyze directly. Therefore, in the next subsection we study the spinor equations and show that there is indeed no supersymmetric localized solution to the supergravity equations of motion.

3.3.2 Spinor equations for the most generic fluxes

Let us discuss the most generic forms that are invariant under the assumed symmetry group $SO(p-2,1)\times SO(2)\times SO(2)\times SO(7-p)$ for the backreacted solution. Since the first factor $SO(p-2,1)$ is enhanced to the full Poincaré group, the only invariant forms are the always present 0-form and its Hodge dual which is the volume form that is proportional to $dx^0\wedge dx^1\wedge\ldots\wedge dx^{p-2}$. The other three spaces all have an $SO(n)$ symmetry so we can discuss them together: In addition to the 0-form and the dual volume form, there are two more forms. There is one 1-form which is d acting on the radial coordinates, $d\rho_1$, $d\rho_2$, $d\rho_T$ in our case, and then there is the dual $(n-1)$-form. For an $SO(2)$ symmetry this would be another 1-form, which we denote $d\theta_1$ and $d\theta_2$, where (ρ_i,θ_i) are simply polar coordinates. For the $SO(7-p)$-symmetry we would go to spherical coordinates $(\rho_T,\theta_T^{(1)},\theta_T^{(2)},\ldots,\theta_T^{(6-p)})$ and an invariant $(6-p)$-form is given by $\sin(\theta_T^{(1)})^{5-p}\sin(\theta_T^{(2)})^{4-p}\ldots\sin(\theta_T^{(4-p)})^2\sin(\theta_T^{(5-p)})\,d\theta_T^{(1)}\wedge\theta_T^{(2)}\wedge\ldots\wedge\theta_T^{(6-p)}$. Lastly, we note that all functions like the metric, the warp factors, the dilaton or the prefactors that appear in front of the forms when spelling out the fluxes, can only depend on ρ_1,ρ_2,ρ_T due to the preserved symmetry.

Let us give a concrete example to clarify the above discussion. We choose $p=6$ and want to find a localized solution that describes two O6-planes (or D6-branes) that extend along the directions $(x^0,x^1,x^2,x^3,x^4,x^5,x^6)$ and $(x^0,x^1,x^2,x^3,x^4,x^7,x^8)$, respectively. We take them to be localized at the origin in their transverse spaces. We assume that the metric is given by equation (3.19) above for $p=6$. We take all warp factors and the dilaton to be functions of the three variables

$$\rho_1 = \sqrt{(x^5)^2 + (x^6)^2}\,, \qquad \rho_2 = \sqrt{(x^7)^2 + (x^8)^2}\,, \qquad \rho_T = x^9\,. \tag{3.34}$$

We then make the most generic flux Ansatz that is compatible with the symmetry group $SO(4,1) \times SO(2) \times SO(2)$

$$\begin{aligned}
F_2 &= F_2^{(1)} + F_2^{(2)} + F_2^{(c)}\,,\\
F_2^{(1)} &= \tilde{\star}_3^{(1)} d\left(\frac{\tilde{\mu}_6^{(1)}}{\sqrt{\rho_1^2+\rho_T^2}}\right)\,,\\
F_2^{(2)} &= \tilde{\star}_3^{(2)} d\left(\frac{\tilde{\mu}_6^{(2)}}{\sqrt{\rho_2^2+\rho_T^2}}\right)\,,\\
F_2^{(c)} &= \sum_{i=1}^{10} f_2^{(i)}(\rho_1,\rho_2,\rho_T)\,Y_i^2\,,\\
F_4 &= \sum_{i=1}^{5} f_4^{(i)}(\rho_1,\rho_2,\rho_T)\,Y_i^4\,,\\
H &= \sum_{i=1}^{10} h^{(i)}(\rho_1,\rho_2,\rho_T)\,Y_i^3\,.
\end{aligned} \tag{3.35}$$

Here the $f_2^{(i)}$, $f_4^{(i)}$, $h^{(i)}$ are unknown functions and the Y_i^2, Y_i^3, Y_i^4 denote the invariant and closed forms that form a basis of invariant forms. Since the $f_2^{(i)}(\rho_1,\rho_2,\rho_T)$ are generic functions and the Y_i^2 include for example $d\theta_1 \wedge d\theta_2$, this Ansatz does not yet satisfy $dF_2^{(c)} = 0$. We furthermore allow for a constant and non-zero F_0. The two 16 component spinors that are present in 10d flat space are constrained due to the presence of the O6-planes (or D6-branes) and have to satisfy

$$\epsilon_2 = \Gamma_{0123456}\epsilon_1\,,$$

$$\epsilon_2 = \Gamma_{0123478}\epsilon_1 \,. \tag{3.36}$$

This breaks one quarter of the supersymmetry and leaves us with 8 real independent spinor components. The fully backreacted solution should preserve these eight supercharges. We therefore assume that these eight spinors are independent (and also functions of (ρ_1, ρ_2, ρ_T)).

We now demand that there is a supersymmetric solution and therefore demand that the spinor transformations in equation (3.7) satisfy $\delta_\epsilon \psi_a = \delta_\epsilon \lambda = 0$. This leads directly to $F_0 = F_4 = H = 0$, while F_2 has to be of the following form

$$\begin{aligned}
F_2 &= \tilde{\star}_3^{(1)} d\left(\frac{\tilde{\mu}_6^{(1)}}{\sqrt{\rho_1^2+\rho_T^2}}\right) + \tilde{\star}_3^{(2)} d\left(\frac{\tilde{\mu}_6^{(2)}}{\sqrt{\rho_2^2+\rho_T^2}}\right) \\
&\quad + f_2^{(3)}(\rho_1,\rho_2,\rho_T) d\theta_1 \wedge d\rho_1 + f_2^{(8)}(\rho_1,\rho_2,\rho_T) d\theta_1 \wedge d\rho_T \\
&\quad + f_2^{(7)}(\rho_1,\rho_2,\rho_T) d\theta_2 \wedge d\rho_2 + f_2^{(9)}(\rho_1,\rho_2,\rho_T) d\theta_2 \wedge d\rho_T \\
&= \left(f_2^{(3)}(\rho_1,\rho_2,\rho_T) + \frac{\mu_{6,1}\rho_1\rho_T}{(\rho_1^2+\rho_T^2)^{\frac{3}{2}}}\right) d\theta_1 \wedge d\rho_1 \\
&\quad + \left(f_2^{(8)}(\rho_1,\rho_2,\rho_T) - \frac{\mu_{6,1}\rho_1^2}{(\rho_2^2+\rho_T^2)^{\frac{3}{2}}}\right) d\theta_1 \wedge d\rho_T \\
&\quad + \left(f_2^{(7)}(\rho_1,\rho_2,\rho_T) + \frac{\mu_{6,2}\rho_2\rho_T}{(\rho_1^2+\rho_T^2)^{\frac{3}{2}}}\right) d\theta_2 \wedge d\rho_2 \\
&\quad + \left(f_2^{(9)}(\rho_1,\rho_2,\rho_T) - \frac{\mu_{6,2}\rho_2^2}{(\rho_2^2+\rho_T^2)^{\frac{3}{2}}}\right) d\theta_2 \wedge d\rho_T \,.
\end{aligned} \tag{3.37}$$

Recall that we have made a fully generic Ansatz for the closed piece in F_2 and we have not yet imposed that it is actually closed.

Let us briefly discuss the above solution in equation (3.37). We see that without imposing the Bianchi identities and equations of motions for the fluxes we can only have a very limited number of flux components $f_2^{(i)}(\rho_1, \rho_2, \rho_T)$ in addition to the source terms. These extra flux components actually combine with the source terms which makes perfect sense. For example, we know that there are solutions for a single source and we can for example use the $f_2^{(i)}(\rho_1, \rho_2, \rho_T)$ to remove one of the sources and then we actually reproduce the result for a single source discussed above in subsection 3.2.2. Here however, we are interested in solutions that describe two intersecting sources and we therefore do not want to cancel any source terms. We therefore proceed to study the remaining equations of motion.

We want that the source terms containing $\mu_{6,1}$ and $\mu_{6,2}$ give rise to the delta function sources and that the rest is closed (see the discussion around equation (3.22) above). Thus, we have to demand that $dF_2 = 0$ away from the source and therefore we find that

$$\begin{aligned} \partial_{\rho_2} f_2^{(3)}(\rho_1, \rho_2, \rho_T) &= 0\,, \\ \partial_{\rho_1} f_2^{(7)}(\rho_1, \rho_2, \rho_T) &= 0\,, \\ \partial_{\rho_2} f_2^{(8)}(\rho_1, \rho_2, \rho_T) &= 0\,, \\ \partial_{\rho_1} f_2^{(9)}(\rho_1, \rho_2, \rho_T) &= 0\,. \end{aligned} \tag{3.38}$$

Additionally, the spinor equations in (3.7) did not only set most of the flux components to zero but they also fixed the first derivatives of the warp factors via the spin connection term in $\delta_\epsilon \psi_a = 0$ and the first derivatives of the dilaton via $\delta_\epsilon \lambda = 0$. Concretely, they fix $\partial_{\rho_1} e^{A_2(\rho_1,\rho_2,\rho_T)}$ and $\partial_{\rho_2} e^{A_2(\rho_1,\rho_2,\rho_T)}$ to be two different functions of the warp factors, the dilaton, the $f_2^{(i)}(\rho_1,\rho_2,\rho_T)$ and the source terms

$$\begin{aligned}
\partial_{\rho_1} e^{A_2(\rho_1,\rho_2,\rho_T)} &= F_1(e^{A_i}, e^{\phi}, f_2^{(i)}, \rho_1, \rho_2, \rho_T)\,, \\
\partial_{\rho_2} e^{A_2(\rho_1,\rho_2,\rho_T)} &= F_2(e^{A_i}, e^{\phi}, f_2^{(i)}, \rho_1, \rho_2, \rho_T)\,. \qquad (3.39)
\end{aligned}$$

Now we can impose the conditions above in equation (3.38) and the following consistency condition

$$\begin{aligned}
0 &= \partial_{\rho_2}\partial_{\rho_1} e^{A_2(\rho_1,\rho_2,\rho_T)} - \partial_{\rho_1}\partial_{\rho_2} e^{A_2(\rho_1,\rho_2,\rho_T)} \qquad (3.40) \\
&= \partial_{\rho_2} F_1(e^{A_i}, e^{\phi}, f_2^{(i)}, \rho_1, \rho_2, \rho_T) - \partial_{\rho_1} F_2(e^{A_i}, e^{\phi}, f_2^{(i)}, \rho_1, \rho_2, \rho_T) \\
&= \frac{e^{A_2 - 2A_4 + 2\phi}}{2\rho_1\rho_2} \left(f_2^{(8)}(\rho_1,\rho_T) - \frac{\mu_{6,1}\rho_1^2}{(\rho_1^2+\rho_T^2)^{\frac{3}{2}}} \right) \left(f_2^{(9)}(\rho_2,\rho_T) - \frac{\mu_{6,2}\rho_2^2}{(\rho_2^2+\rho_T^2)^{\frac{3}{2}}} \right).
\end{aligned}$$

Since the prefactor in the above equation cannot vanish everywhere, we see that

$$\left(f_2^{(8)}(\rho_1,\rho_T) - \frac{\mu_{6,1}\rho_1^2}{(\rho_1^2+\rho_T^2)^{\frac{3}{2}}} \right) \left(f_2^{(9)}(\rho_2,\rho_T) - \frac{\mu_{6,2}\rho_2^2}{(\rho_2^2+\rho_T^2)^{\frac{3}{2}}} \right) = 0\,. \qquad (3.41)$$

This shows that there is no fully localized solution with our generic diagonal metric Ansatz. The above equation requires us to at least partially remove (or smear) one of the sources.

Let us pursue the above further by setting without loss of generality

$$f_2^{(9)}(\rho_2,\rho_T) = \frac{\mu_{6,2}\rho_2^2}{(\rho_2^2+\rho_T^2)^{\frac{3}{2}}}\,. \qquad (3.42)$$

This cancels the last term in F_2 above in equation (3.37) and the closure $dF_2 = 0$ then imposes the additional constraint that

$$f_2^{(7)}(\rho_1, \rho_2, \rho_T) = \frac{\mu_{6,2}\rho_2^2}{(\rho_2^2 + \rho_T^2)^{\frac{3}{2}}} + f(\rho_2)\,, \tag{3.43}$$

where $f(\rho_2)$ is an undetermined function. With that F_2 becomes

$$\begin{aligned} F_2 \;=\; & \left(f_2^{(3)}(\rho_1, \rho_T) + \frac{\mu_{6,1}\rho_1\rho_T}{(\rho_1^2 + \rho_T^2)^{\frac{3}{2}}} \right) d\theta_1 \wedge d\rho_1 \\ & + \left(f_2^{(8)}(\rho_1, \rho_T) - \frac{\mu_{6,1}\rho_1^2}{(\rho_2^2 + \rho_T^2)^{\frac{3}{2}}} \right) d\theta_1 \wedge d\rho_T \\ & + f(\rho_2) d\theta_2 \wedge d\rho_T\,. \end{aligned} \tag{3.44}$$

So, we have effectively removed the second source completely. Actually the equations of motion for F_2 fix $f(\rho_2) = c\rho_2$ and using that in the solution to the spinor equations, we find that all derivatives of the warp factors and the dilaton with respect to ρ_2 vanish: $\partial_{\rho_2} e^{A_i} = \partial_{\rho_2} e^{\phi} = 0$. This is indicative of a smeared source and we indeed see from

$$dF_2 \supset d(f(\rho_2)d\theta_2 \wedge d\rho_T) = c\, d\rho_2 \wedge d\theta_2 \wedge d\rho_T\,, \tag{3.45}$$

that we can have at best a smeared second source in which the delta function source (see equation (3.22)) is replaced with the constant c. Thus, in addition to proving the absence of a solution with two fully localized sources, our equations pass consistency checks and do not forbid solutions with partially smeared sources.

We have repeated the above analysis for two intersecting sources with $p = 1, 2, 3, 4, 5$ and explicitly reproduced the same absence of localized solutions. This might have been expected from T-duality invariance of type II string theory, however, there is an important subtlety: If we have an Op-plane (or a Dp-brane) in flat space, then we can T-dualize along any of its worldvolume directions. The reason is that the dilaton, metric and everything else does not depend on these coordinates. This leads to an O$(p-1)$-plane (or a D$(p-1)$-brane) that is actually smeared over the direction we T-dualized. Similarly, we cannot T-dualize along a transverse direction since these are not isometries. We would have to first smear the source along this transverse direction and then we can T-dualize to get a $(p+1)$-dimensional source. So, strictly speaking we cannot use T-duality invariance in the strict sense and therefore we checked the equations for each $p = 1, 2, 3, 4, 5, 6$ explicitly.

3.4 Discussion and Outlook

The above surprising result, that shows the absence of localized supergravity solutions for intersecting objects in flat space, raises many important questions: Does the result also hold for a generic non-diagonal metric? Can such setups be described explicitly in the full string theory? Does our result carry over to compactifications? In this subsection we will briefly discuss these questions. However, we will not be able to answer them and leave many avenues for further research.

First it seems clear that intersecting sources can arise in string theory and corresponding solutions will exist. Our two intersecting Op-planes can arise from a single orientifold projection combined with a $\mathbb{Z}_2$ orbifold of flat space. For example, we can do an orientifold involution consisting of the worldsheet parity operator Ω_p and a spatial involution that flips the signs of x^7, x^8, x^9. This leads to a single O6-plane localized at $x^7 = x^8 = x^9 = 0$. Doing a $\mathbb{Z}_2$ orbifold that flips the signs of x^5, x^6, x^7, x^8 then introduces a second O6-plane localized at $x^5 = x^6 = x^9 = 0$. In principle one should be able to study the full string theory on such an orientifolded orbifold of flat space. Supergravity as a low energy approximation of the full string theory might simply not allow for a solution because we neglect higher derivative corrections, string loop corrections and/or did not include the full spectrum of the string states.

For the case of intersecting stacks of D-branes in particular we neglected all the open strings on the D-branes. These open strings give rise to gauge theories and one can study the dynamics of these gauge theories. It is possible that the gauge dynamics leads to a (partial) smearing of the D-branes and partially smeared solutions do certainly exist, see for example [137, 138, 139, 140]. Some of these papers also discuss the near core (near horizon) limit of these brane setups and manage to find localized solutions in this limit. For the particular case of two intersecting D6-branes or O6-planes one can also try to lift things to M-theory and try to find a solution in 11d supergravity. Such a lift of two intersecting D6-branes was discussed in [141].

Let us mention that it is known that multiplying together the two harmonic functions (warp-factors) for the two sources cannot solve the localized equations of motion but rather requires smearing (see for example [137, eqns. (1)-(2)] and references therein). We reproduce the same result with a generic diagonal metric Ansatz. A loophole to our findings is exploited by the only (to us) known fully localized supergravity solution of two intersecting branes [118]. The two intersecting NS5 branes in this setup have no mutually transverse direction since they extend along 012345 and 016789. In our equations we crucially use the fact that there are $7-p>0$ transverse directions.[16] It would be interesting to study further brane setups without mutually transverse directions.

We crucially assumed here that there is an unbroken $SO(p-2,1)\times SO(2)\times SO(2)\times SO(7-p)$ symmetry group. This seems to be justified for static objects or in the probe limit but it is possible that the dynamics of the D-branes (or the dynamics of O-planes at strong coupling) could break this symmetry group. It would therefore be interesting to see whether one can relax the requirement of this large unbroken symmetry group. Here let us note that [134] discusses the D3/D5-brane intersection, where the D5 brane extends along $x^0, x^1, x^2, x^3, x^4, x^5$ and the D3-brane along x^0, x^1, x^2, x^6. In this case the authors only assume the presence of an $SO(2,1)$ Poincaré group and an $SO(3)$-symmetry group in the mutually transverse x^7, x^8, x^9 directions. While solving the equations of motion they discover the necessary presence of an extra $SO(3)$-symmetry acting on the x^3, x^4, x^5 directions, before they find that no localized solution exists. Thus, it is conceivable that our result might still hold even if we were to give up the $SO(2)\times SO(2)$ symmetry and/or allow for a non-diagonal metric along the corresponding directions. It would

[16] Recall that here we restrict ourselves to $1 \le p \le 6$. It thus might be possible to write down fully localized solutions for $p=7$ but such setups are better described in F-theory [142].

be interesting to check this explicitly in particular given that the orbifold blow ups discussed recently in [132] would break this $SO(2) \times SO(2)$ symmetry. In our setup one could glue in a $\mathbb{P}^1$ to remove the orbifold singularity and this would correspond to giving a non-zero vev to the Kähler modulus that controls its size. However, it is unclear to us that this would happen dynamically and what could fix the scale of a non-zero vev for the Kähler modulus.

A different approach to solving the equations of motion for intersecting sources was pursued in [143, 144]. In those papers it is required that the sources preserve a certain amount of supersymmetry and then the constraints from the equations of motion and the vanishing of the supersymmetry transformations is being studied. The sources are then derived from the equations of motion rather than being specified from the outset. This seems to allow for localized solutions that asymptotically become flat space, but not for our specific set of two perpendicularly intersecting sources.[17]

It is a far stretch to go from our setup of two intersecting sources in flat space to a full compactification of 10d supergravity like the massive type IIA flux compactifications discussed in [31, 36]. However, we note here that the two papers [121, 122] only worked to first order in the sources, i.e., in our language to the first order in the $\tilde{\mu}_p$. This means that contradictions like equations (3.33) or (3.41) above that are quadratic would not be visible when working at linear order. It would be therefore of great importance to extend the work of [121, 122] to higher order. Already in the simplest case of a toroidal compactification we note that the preserved symmetry group gets dramatically

[17]We are indebted to Iosif Bena for bringing these references to our attention and discussing their consistency with our results.

reduced and it is conceivable that then localized solutions exist. We plan to study this as well as a generic off-diagonal metric Ansatz in the future.

A Details of the dual type IIA models

A.1 Kähler potential

As mentioned in the text the way in which one derives the formula for the Kähler potential is by using mirror symmetry. The usual formula for the Kähler potential in Type IIA flux compactifications on Calabi-Yau manifolds is [35]

$$K = -\log\left[\frac{4}{3}\int J\wedge J\wedge J\right] - 2\log\left[2\int \mathrm{Re}\,(C\Omega_3)\wedge\star\mathrm{Re}\,(C\Omega_3)\right]\,, \qquad \text{(A.1)}$$

where J is the Kähler form and Ω the holomorphic 3-form. The volume is given by $\mathrm{vol}_6 = \frac{1}{6}\int J\wedge J\wedge J$. The so-called 4d dilaton is defined via $e^D = e^\phi/\sqrt{\mathrm{vol}_6}$ and $C\overline{C}\int\Omega\wedge\overline{\Omega} = e^{-2D}$. The supergravity fields are introduced by expanding the complexified Kähler form and the complexified holomorphic 3-form [35]

$$J_c = B_2 + \mathrm{i}J = \sum_{a=1}^{h^{(1,1)}} T^a\omega_a\,, \qquad \text{(A.2)}$$

$$\Omega_c = C_3 + 2\mathrm{i}\mathrm{Re}\,(C\Omega_3) = S\alpha_0 + \sum_{k=1}^{h^{(2,1)}} U^k\alpha_k\,. \qquad \text{(A.3)}$$

When $h^{(2,1)} = 0$ there are no complex structure moduli. We can always write the volume in terms of the triple intersection number $\kappa_{abc} = \int\omega_a\wedge\omega_b\wedge\omega_c$ of the

Calabi-Yau manifold, which leads (up to a constant) to the Kähler potential

$$K = -\log\left[\frac{\mathrm{i}}{6}\kappa_{abc}\left(T^a - \overline{T}^a\right)\left(T^b - \overline{T}^b\right)\left(T^c - \overline{T}^c\right)\right] - 4\log\left[-\frac{\mathrm{i}}{2\sqrt{2}}\left(S - \overline{S}\right)\right]. \tag{A.4}$$

Mirror symmetry simply exchanges the $h^{1,1}$ Kähler moduli T^a with the $h^{2,1}$ complex structure moduli U^k. Since we have no complex structure moduli the mirror dual Kähler potential is the one given above in equation (2.3), if one restricts to the torus bulk moduli and sets them all equal [29]. The superpotential can be derived in the same way but was also argued for directly in type IIB in [26].

A.2 KK towers

In this section, following the original work [31], we quickly review how to derive the KK scale in type IIA flux compactifications. Using mirror symmetry we can then derive the mass scale for a light tower in the non-geometric type IIB flux compactifications discussed in this paper. As on the type IIA side, this is not proven to be always the lightest tower but no other lighter tower is expected to arise in the type IIA side, so presumably the same is true on the type IIB side. Also, our infinite families of AdS vacua are all consistent with the refined AdS distance conjecture [63, 37], which means that this is likely the relevant tower of massive states.

The KK scale in type IIA flux compactifications is controlled by the internal volumes of 2-cycles, $\mathrm{Im}(T^a)$. In the isotropic limit where we set the three bulk 2-cycles of the torus equal we will simply use $\mathrm{Im}(T)$ to describe this volume.

So, we know that m^2_{KK} scales like $1/\mathrm{Im}(T)$. Compactifying from 10d to 4d and then going to 4d Einstein frame introduces an extra factor and the correct KK scale is given by

$$m^2_{KK} \sim \frac{1}{\mathrm{vol}_6 e^{-2\phi}\mathrm{Im}(T)} = \frac{1}{(\mathrm{Im}(S))^2\mathrm{Im}(T)} \,. \tag{A.5}$$

Again using mirror symmetry, we find a dual massive tower with masses that scale like

$$m^2_{\mathrm{tower}} \sim \frac{1}{\left(\mathrm{Im}S\right)^2 \mathrm{Im}(U)} \,. \tag{A.6}$$

References

[1] National Weather Service, *National forecast maps*, https://www.weather.gov/forecastmaps.

[2] T. Wrase, dS vacua and Inflation in String Theory.

[3] String Theory Group - Institute for Theoretical Physics - TU Wien, *Flux compactifications and generalised geometries*, http://hep.itp.tuwien.ac.at/string/research_flux.html.

[4] M. van Beest, J. Calderón-Infante, D. Mirfendereski and I. Valenzuela, *Lectures on the swampland program in string compactifications*, *Physics Reports* **989** (2022) 1–50.

[5] J. Bardzell, E. Gonzalo, M. Rajaguru, D. Smith and T. Wrase, *Type IIB flux compactifications with $h^{1,1} = 0$*, *JHEP* **06** (2022) 166 [2203.15818].

[6] J. Bardzell, K. Federico, D. Smith and T. Wrase, *On the absence of supergravity solutions for localized, intersecting sources*, 2403.09873.

[7] T. Wrase, ASTR/PHY 395 – Cosmology.

[8] C.V. Johnson, *D-Branes*, Cambridge Monographs on Mathematical Physics, Cambridge University Press (2002).

[9] J. Terning, *Modern Supersymmetry: Dynamics and Duality*, International Series of Monographs on Physics, Oxford University Press (2006).

[10] M.A. Shifman, *Introduction to the Yuri Golfand memorial volume: Many Faces of Superworld*, 9, 1999 [hep-th/9909016].

[11] E. Palti, *The Swampland: Introduction and Review*, *Fortsch. Phys.* **67** (2019) 1900037 [1903.06239].

[12] M. van Beest, J. Calderón-Infante, D. Mirfendereski and I. Valenzuela, *Lectures on the Swampland Program in String Compactifications*, *Phys. Rept.* **989** (2022) 1 [2102.01111].

[13] M. Graña and A. Herráez, *The Swampland Conjectures: A Bridge from Quantum Gravity to Particle Physics*, *Universe* **7** (2021) 273 [2107.00087].

[14] D. Harlow and H. Ooguri, *Symmetries in quantum field theory and quantum gravity*, *Commun. Math. Phys.* **383** (2021) 1669 [1810.05338].

[15] R. Blumenhagen, M. Brinkmann, A. Makridou, L. Schlechter and M. Traube, *dS Spaces and Brane Worlds in Exotic String Theories*, *JHEP* **06** (2020) 077 [2002.11746].

[16] K.R. Dienes, *Solving the hierarchy problem without supersymmetry or extra dimensions: An Alternative approach*, *Nucl. Phys. B* **611** (2001) 146 [hep-ph/0104274].

[17] I. Basile, J. Mourad and A. Sagnotti, *On Classical Stability with Broken Supersymmetry*, *JHEP* **01** (2019) 174 [1811.11448].

[18] N. Cribiori, S. Parameswaran, F. Tonioni and T. Wrase, *Misaligned Supersymmetry and Open Strings*, *JHEP* **04** (2021) 099 [2012.04677].

[19] I. Basile, *Supersymmetry breaking, brane dynamics and Swampland conjectures*, *JHEP* **10** (2021) 080 [2106.04574].

[20] S. Abel and K.R. Dienes, *Calculating the Higgs mass in string theory*, *Phys. Rev. D* **104** (2021) 126032 [2106.04622].

[21] N. Cribiori, S. Parameswaran, F. Tonioni and T. Wrase, *Modular invariance, misalignment and finiteness in non-supersymmetric strings*, *JHEP* **01** (2022) 127 [2110.11973].

[22] H. Ooguri, E. Palti, G. Shiu and C. Vafa, *Distance and de Sitter Conjectures on the Swampland*, *Phys. Lett. B* **788** (2019) 180 [1810.05506].

[23] E. Palti, *The Weak Gravity Conjecture and Scalar Fields*, *JHEP* **08** (2017) 034 [1705.04328].

[24] E. Gonzalo and L.E. Ibáñez, *A Strong Scalar Weak Gravity Conjecture and Some Implications*, *JHEP* **08** (2019) 118 [1903.08878].

[25] E. Gonzalo and L.E. Ibáñez, *Pair Production and Gravity as the Weakest Force*, *JHEP* **12** (2020) 039 [2005.07720].

[26] K. Becker, M. Becker, C. Vafa and J. Walcher, *Moduli Stabilization in Non-Geometric Backgrounds*, *Nucl. Phys. B* **770** (2007) 1 [hep-th/0611001].

[27] P. Candelas, E. Derrick and L. Parkes, *Generalized Calabi-Yau manifolds and the mirror of a rigid manifold*, *Nucl. Phys. B* **407** (1993) 115 [hep-th/9304045].

[28] S. Sethi, *Supermanifolds, rigid manifolds and mirror symmetry*, *Nucl. Phys. B* **430** (1994) 31 [hep-th/9404186].

[29] K. Becker, M. Becker and J. Walcher, *Runaway in the Landscape*, *Phys. Rev. D* **76** (2007) 106002 [0706.0514].

[30] K. Ishiguro and H. Otsuka, *Sharpening the boundaries between flux landscape and swampland by tadpole charge*, *JHEP* **12** (2021) 017 [2104.15030].

[31] O. DeWolfe, A. Giryavets, S. Kachru and W. Taylor, *Type IIA moduli stabilization*, *JHEP* **07** (2005) 066 [hep-th/0505160].

[32] K. Behrndt and M. Cvetic, *General N=1 supersymmetric fluxes in massive type IIA string theory*, *Nucl. Phys. B* **708** (2005) 45 [hep-th/0407263].

[33] J.-P. Derendinger, C. Kounnas, P.M. Petropoulos and F. Zwirner, *Superpotentials in IIA compactifications with general fluxes*, *Nucl. Phys. B* **715** (2005) 211 [hep-th/0411276].

[34] D. Lust and D. Tsimpis, *Supersymmetric AdS(4) compactifications of IIA supergravity*, *JHEP* **02** (2005) 027 [hep-th/0412250].

[35] T.W. Grimm and J. Louis, *The Effective action of type IIA Calabi-Yau orientifolds*, *Nucl. Phys. B* **718** (2005) 153 [hep-th/0412277].

[36] P.G. Camara, A. Font and L.E. Ibanez, *Fluxes, moduli fixing and MSSM-like vacua in a simple IIA orientifold*, *JHEP* **09** (2005) 013 [hep-th/0506066].

[37] G. Buratti, J. Calderon, A. Mininno and A.M. Uranga, *Discrete Symmetries, Weak Coupling Conjecture and Scale Separation in AdS Vacua*, *JHEP* **06** (2020) 083 [2003.09740].

[38] H. Ooguri and C. Vafa, *Non-supersymmetric AdS and the Swampland*, *Adv. Theor. Math. Phys.* **21** (2017) 1787 [1610.01533].

[39] A. Strominger, S.-T. Yau and E. Zaslow, *Mirror symmetry is T duality*, *Nucl. Phys. B* **479** (1996) 243 [hep-th/9606040].

[40] P. Shukla, *Dictionary for the type II nongeometric flux compactifications*, *Phys. Rev. D* **103** (2021) 086009 [1909.07391].

[41] S.B. Giddings, S. Kachru and J. Polchinski, *Hierarchies from fluxes in string compactifications*, *Phys. Rev. D* **66** (2002) 106006 [hep-th/0105097].

[42] A. Micu, E. Palti and G. Tasinato, *Towards Minkowski Vacua in Type II String Compactifications*, *JHEP* **03** (2007) 104 [hep-th/0701173].

[43] M. Ihl, D. Robbins and T. Wrase, *Toroidal orientifolds in IIA with general NS-NS fluxes*, *JHEP* **08** (2007) 043 [0705.3410].

[44] K. Hori and C. Vafa, *Mirror symmetry*, hep-th/0002222.

[45] I. Bena, J. Blåbäck, M. Graña and S. Lüst, *The tadpole problem*, *JHEP* **11** (2021) 223 [2010.10519].

[46] I. Bena, J. Blåbäck, M. Graña and S. Lüst, *Algorithmically Solving the Tadpole Problem*, *Adv. Appl. Clifford Algebras* **32** (2022) 7 [2103.03250].

[47] E. Plauschinn, *The tadpole conjecture at large complex-structure*, *JHEP* **02** (2022) 206 [2109.00029].

[48] S. Lüst, *Large complex structure flux vacua of IIB and the Tadpole Conjecture*, 2109.05033.

[49] I. Bena, C. Brodie and M. Graña, *D7 moduli stabilization: the tadpole menace*, *JHEP* **01** (2022) 138 [2112.00013].

[50] K. Dasgupta, G. Rajesh and S. Sethi, *M theory, orientifolds and G - flux*, *JHEP* **08** (1999) 023 [hep-th/9908088].

[51] S. Gukov, C. Vafa and E. Witten, *CFT's from Calabi-Yau four folds*, *Nucl. Phys. B* **584** (2000) 69 [hep-th/9906070].

[52] J. Shelton, W. Taylor and B. Wecht, *Nongeometric flux compactifications*, *JHEP* **10** (2005) 085 [hep-th/0508133].

[53] M. Kim, *D-instanton superpotential in string theory*, *JHEP* **03** (2022) 054 [2201.04634].

[54] M. Marino, R. Minasian, G.W. Moore and A. Strominger, *Nonlinear instantons from supersymmetric p-branes*, *JHEP* **01** (2000) 005 [hep-th/9911206].

[55] U.H. Danielsson and T. Van Riet, *What if string theory has no de Sitter vacua?*, *Int. J. Mod. Phys. D* **27** (2018) 1830007 [1804.01120].

[56] G. Obied, H. Ooguri, L. Spodyneiko and C. Vafa, *De Sitter Space and the Swampland*, 1806.08362.

[57] F.F. Gautason, V. Van Hemelryck and T. Van Riet, *The Tension between 10D Supergravity and dS Uplifts*, *Fortsch. Phys.* **67** (2019) 1800091 [1810.08518].

[58] E. Palti, C. Vafa and T. Weigand, *Supersymmetric Protection and the Swampland*, *JHEP* **06** (2020) 168 [2003.10452].

[59] J.P. Conlon, S. Ning and F. Revello, *Exploring the holographic Swampland*, *JHEP* **04** (2022) 117 [2110.06245].

[60] F. Marchesano, D. Prieto, J. Quirant and P. Shukla, *Systematics of Type IIA moduli stabilisation*, *JHEP* **11** (2020) 113 [2007.00672].

[61] F. Apers, M. Montero, T. Van Riet and T. Wrase, *Comments on classical AdS flux vacua with scale separation*, *JHEP* **05** (2022) 167 [2202.00682].

[62] F. Apers, J.P. Conlon, S. Ning and F. Revello, *Integer conformal dimensions for type IIa flux vacua*, *Phys. Rev. D* **105** (2022) 106029 [2202.09330].

[63] D. Lüst, E. Palti and C. Vafa, *AdS and the Swampland*, *Phys. Lett. B* **797** (2019) 134867 [1906.05225].

[64] P. Breitenlohner and D.Z. Freedman, *Stability in Gauged Extended Supergravity*, *Annals Phys.* **144** (1982) 249.

[65] P.G.O. Freund and M.A. Rubin, *Dynamics of Dimensional Reduction*, *Phys. Lett. B* **97** (1980) 233.

[66] J.M. Maldacena, *The Large N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [hep-th/9711200].

[67] F. Farakos, G. Tringas and T. Van Riet, *No-scale and scale-separated flux vacua from IIA on G2 orientifolds*, *Eur. Phys. J. C* **80** (2020) 659 [2005.05246].

[68] G.B. De Luca and A. Tomasiello, *Leaps and bounds towards scale separation*, *JHEP* **12** (2021) 086 [2104.12773].

[69] N. Cribiori, D. Junghans, V. Van Hemelryck, T. Van Riet and T. Wrase, *Scale-separated AdS4 vacua of IIA orientifolds and M-theory*, *Phys. Rev. D* **104** (2021) 126014 [2107.00019].

[70] T.C. Collins, D. Jafferis, C. Vafa, K. Xu and S.-T. Yau, *On Upper Bounds in Dimension Gaps of CFT's*, 2201.03660.

[71] D. Tsimpis, *Relative scale separation in orbifolds of* S^2 *and* S^5, 2201.10916.

[72] M. Emelin, F. Farakos and G. Tringas, *O6-plane backreaction on scale-separated Type IIA* AdS_3 *vacua*, 2202.13431.

[73] N. Cribiori and G. Dall'Agata, *Weak gravity versus scale separation*, 2203.05559.

[74] R. Blumenhagen, M. Brinkmann and A. Makridou, *Quantum Log-Corrections to Swampland Conjectures*, *JHEP* **02** (2020) 064 [1910.10185].

[75] L.E. Ibáñez, V. Martin-Lozano and I. Valenzuela, *Constraining Neutrino Masses, the Cosmological Constant and BSM Physics from the Weak Gravity Conjecture*, *JHEP* **11** (2017) 066 [1706.05392].

[76] Y. Hamada and G. Shiu, *Weak Gravity Conjecture, Multiple Point Principle and the Standard Model Landscape*, *JHEP* **43** (2017) [1707.06326].

[77] E. Gonzalo, A. Herraez and L.E. Ibáñez, *AdS-phobia, the WGC, the Standard Model and Supersymmetry*, *JHEP* **06** (2018) 051 [1803.08455].

[78] E. Gonzalo and L.E. Ibáñez, *The Fundamental Need for a SM Higgs and the Weak Gravity Conjecture*, *Phys. Lett. B* **786** (2018) 272 [1806.09647].

[79] E. Gonzalo, L.E. Ibáñez and I. Valenzuela, *Swampland constraints on neutrino masses*, *JHEP* **02** (2022) 088 [2109.10961].

[80] E. Gonzalo, L.E. Ibáñez and I. Valenzuela, *AdS swampland conjectures and light fermions*, *Phys. Lett. B* **822** (2021) 136691 [2104.06415].

[81] A. Font, A. Herráez and L.E. Ibáñez, *On scale separation in type II AdS flux vacua*, *JHEP* **03** (2020) 013 [1912.03317].

[82] P. Narayan and S.P. Trivedi, *On The Stability Of Non-Supersymmetric AdS Vacua*, *JHEP* **07** (2010) 089 [1002.4498].

[83] G. Veneziano, *Large N bounds on, and compositeness limit of, gauge and gravitational interactions*, *JHEP* **06** (2002) 051 [hep-th/0110129].

[84] N. Arkani-Hamed, S. Dimopoulos and S. Kachru, *Predictive landscapes and new physics at a TeV*, hep-th/0501082.

[85] G. Dvali, *Black Holes and Large N Species Solution to the Hierarchy Problem*, *Fortsch. Phys.* **58** (2010) 528 [0706.2050].

[86] G. Dvali and M. Redi, *Black Hole Bound on the Number of Species and Quantum Gravity at LHC*, *Phys. Rev. D* **77** (2008) 045027 [0710.4344].

[87] A. Adams, O. DeWolfe and W. Taylor, *String universality in ten dimensions*, *Phys. Rev. Lett.* **105** (2010) 071601 [1006.1352].

[88] H.-C. Kim, G. Shiu and C. Vafa, *Branes and the Swampland*, *Phys. Rev. D* **100** (2019) 066006 [1905.08261].

[89] H.-C. Kim, H.-C. Tarazi and C. Vafa, *Four-dimensional $\mathcal{N} = \mathbf{4}$ SYM theory and the swampland*, *Phys. Rev. D* **102** (2020) 026003 [1912.06144].

[90] M. Cvetič, M. Dierigl, L. Lin and H.Y. Zhang, *String Universality and Non-Simply-Connected Gauge Groups in 8d*, *Phys. Rev. Lett.* **125** (2020) 211602 [2008.10605].

[91] M. Montero and C. Vafa, *Cobordism Conjecture, Anomalies, and the String Lamppost Principle*, *JHEP* **01** (2021) 063 [2008.11729].

[92] A. Hanany and A. Zaffaroni, *Chiral symmetry from type IIA branes*, *Nucl. Phys. B* **509** (1998) 145 [hep-th/9706047].

[93] I. Brunner and A. Karch, *Branes at orbifolds versus Hanany Witten in six-dimensions*, *JHEP* **03** (1998) 003 [hep-th/9712143].

[94] A. Hanany and A. Zaffaroni, *Branes and six-dimensional supersymmetric theories*, *Nucl. Phys. B* **529** (1998) 180 [hep-th/9712145].

[95] S. Ferrara, A. Kehagias, H. Partouche and A. Zaffaroni, *Membranes and five-branes with lower supersymmetry and their AdS supergravity duals*, *Phys. Lett. B* **431** (1998) 42 [hep-th/9803109].

[96] C.-h. Ahn, K. Oh and R. Tatar, *Orbifolds of AdS(7) x S**4 and six-dimensional (0,1) SCFT*, *Phys. Lett. B* **442** (1998) 109 [hep-th/9804093].

[97] P. Shukla, *Rigid nongeometric orientifolds and the swampland*, *Phys. Rev. D* **103** (2021) 086010 [1909.10993].

[98] D. Andriot, *Open problems on classical de Sitter solutions*, *Fortsch. Phys.* **67** (2019) 1900026 [1902.10093].

[99] R. Kallosh and T. Wrase, *dS Supergravity from 10d*, *Fortsch. Phys.* **67** (2019) 1800071 [1808.09427].

[100] J. Blåbäck, U. Danielsson and G. Dibitetto, *A new light on the darkest corner of the landscape*, 1810.11365.

[101] D. Andriot, *Tachyonic de Sitter Solutions of 10d Type II Supergravities*, *Fortsch. Phys.* **69** (2021) 2100063 [2101.06251].

[102] P. Shukla, *On T-dual pairs of stable AdS/dS vacua ?*, 2202.12840.

[103] B. de Carlos, A. Guarino and J.M. Moreno, *Complete classification of Minkowski vacua in generalised flux models*, *JHEP* **02** (2010) 076 [0911.2876].

[104] D. Andriot, *On the de Sitter swampland criterion*, *Phys. Lett. B* **785** (2018) 570 [1806.10999].

[105] S.K. Garg and C. Krishnan, *Bounds on Slow Roll and the de Sitter Swampland*, *JHEP* **11** (2019) 075 [1807.05193].

[106] A. Hebecker and T. Wrase, *The Asymptotic dS Swampland Conjecture - a Simplified Derivation and a Potential Loophole*, *Fortsch. Phys.* **67** (2019) 1800097 [1810.08182].

[107] D. Andriot and C. Roupec, *Further refining the de Sitter swampland conjecture*, *Fortsch. Phys.* **67** (2019) 1800105 [1811.08889].

[108] D. Junghans, *Weakly Coupled de Sitter Vacua with Fluxes and the Swampland*, *JHEP* **03** (2019) 150 [1811.06990].

[109] A. Banlaki, A. Chowdhury, C. Roupec and T. Wrase, *Scaling limits of dS vacua and the swampland*, *JHEP* **03** (2019) 065 [1811.07880].

[110] R. Blumenhagen, M. Cvetic, P. Langacker and G. Shiu, *Toward realistic intersecting D-brane models*, *Ann. Rev. Nucl. Part. Sci.* **55** (2005) 71 [hep-th/0502005].

[111] M. Grana, *Flux compactifications in string theory: A Comprehensive review*, *Phys. Rept.* **423** (2006) 91 [hep-th/0509003].

[112] R. Blumenhagen, B. Kors, D. Lust and S. Stieberger, *Four-dimensional String Compactifications with D-Branes, Orientifolds and Fluxes*, *Phys. Rept.* **445** (2007) 1 [hep-th/0610327].

[113] J. Blaback, U.H. Danielsson, D. Junghans, T. Van Riet, T. Wrase and M. Zagermann, *Smeared versus localised sources in flux compactifications*, *JHEP* **12** (2010) 043 [1009.1877].

[114] J. Blaback, U.H. Danielsson, D. Junghans, T. Van Riet, T. Wrase and M. Zagermann, *The problematic backreaction of SUSY-breaking branes*, *JHEP* **08** (2011) 105 [1105.4879].

[115] J. Blaback, U.H. Danielsson, D. Junghans, T. Van Riet, T. Wrase and M. Zagermann, *(Anti-)Brane backreaction beyond perturbation theory*, *JHEP* **02** (2012) 025 [1111.2605].

[116] D.J. Smith, *Intersecting brane solutions in string and M theory*, *Class. Quant. Grav.* **20** (2003) R233 [hep-th/0210157].

[117] J. Gauntlett, *Intersecting branes*, in *APCTP Winter School on Dualities of Gauge and String Theories*, pp. 146–193, 5, 1997, DOI [hep-th/9705011].

[118] J.P. Gauntlett, D.A. Kastor and J.H. Traschen, *Overlapping branes in M theory*, *Nucl. Phys. B* **478** (1996) 544 [hep-th/9604179].

[119] C. Vafa, *The String landscape and the swampland*, hep-th/0509212.

[120] B.S. Acharya, F. Benini and R. Valandro, *Fixing moduli in exact type IIA flux vacua*, *JHEP* **02** (2007) 018 [hep-th/0607223].

[121] D. Junghans, *O-Plane Backreaction and Scale Separation in Type IIA Flux Vacua*, *Fortsch. Phys.* **68** (2020) 2000040 [2003.06274].

[122] F. Marchesano, E. Palti, J. Quirant and A. Tomasiello, *On supersymmetric AdS_4 orientifold vacua*, *JHEP* **08** (2020) 087 [2003.13578].

[123] V. Van Hemelryck, *Scale-Separated AdS3 Vacua from G2-Orientifolds Using Bispinors*, *Fortsch. Phys.* **70** (2022) 2200128 [2207.14311].

[124] F. Apers, *Aspects of AdS flux vacua with integer conformal dimensions*, *JHEP* **05** (2023) 040 [2211.04187].

[125] G. Shiu, F. Tonioni, V. Van Hemelryck and T. Van Riet, *AdS scale separation and the distance conjecture*, *JHEP* **05** (2023) 077 [2212.06169].

[126] N. Cribiori and C. Montella, *Quantum gravity constraints on scale separation and de Sitter in five dimensions*, *JHEP* **05** (2023) 178 [2303.04162].

[127] F. Farakos, M. Morittu and G. Tringas, *On/off scale separation*, *JHEP* **10** (2023) 067 [2304.14372].

[128] D. Junghans, *de Sitter-eating O-planes in supercritical string theory*, *JHEP* **12** (2023) 196 [2308.00026].

[129] R. Carrasco, T. Coudarchet, F. Marchesano and D. Prieto, *New families of scale separated vacua*, *JHEP* **11** (2023) 094 [2309.00043].

[130] G. Tringas, *Anisotropic scale-separated* AdS_4 *flux vacua*, 2309.16542.

[131] D. Andriot and G. Tringas, *Extensions of a scale-separated* AdS_4 *solution and their mass spectrum*, *JHEP* **01** (2024) 008 [2310.06115].

[132] D. Junghans, *A note on O6 intersections in AdS flux vacua*, *JHEP* **02** (2024) 126 [2310.17695].

[133] F. Farakos and M. Morittu, *Scale-separated* $AdS_3 \times S^1$ *vacua from IIA orientifolds*, *Eur. Phys. J. C* **84** (2024) 98 [2311.08991].

[134] L. Patino and D.J. Smith, *On the non-existence of totally localised intersections of D3 / D5 branes in type IIB SUGRA*, *JHEP* **04** (2005) 003 [hep-th/0410207].

[135] C.V. Johnson, *D-branes*, Cambridge Monographs on Mathematical Physics, Cambridge University Press (2005), 10.1017/CBO9780511606540.

[136] E. Bergshoeff, R. Kallosh, T. Ortin, D. Roest and A. Van Proeyen, *New formulations of D = 10 supersymmetry and D8 - O8 domain walls*, *Class. Quant. Grav.* **18** (2001) 3359 [hep-th/0103233].

[137] D. Youm, *Partially localized intersecting BPS branes*, *Nucl. Phys. B* **556** (1999) 222 [hep-th/9902208].

[138] D. Marolf and A.W. Peet, *Brane baldness versus superselection sectors*, *Phys. Rev. D* **60** (1999) 105007 [hep-th/9903213].

[139] A. Gomberoff, D. Kastor, D. Marolf and J.H. Traschen, *Fully localized brane intersections - the plot thickens*, *Phys. Rev. D* **61** (2000) 024012 [hep-th/9905094].

[140] S. Arapoglu, N.S. Deger and A. Kaya, *A Note on supergravity solutions for partially localized intersecting branes*, *Phys. Lett. B* **578** (2004) 203 [hep-th/0306040].

[141] A.M. Uranga, *Localized instabilities at conifolds*, hep-th/0204079.

[142] C. Vafa, *Evidence for F theory*, *Nucl. Phys. B* **469** (1996) 403 [hep-th/9602022].

[143] O. Lunin, *Strings ending on branes from supergravity*, *JHEP* **09** (2007) 093 [0706.3396].

[144] O. Lunin, *Brane webs and 1/4-BPS geometries*, *JHEP* **09** (2008) 028 [0802.0735].

Vita

Danielle Smith was born in Tampa, FL and raised in various location throughout the eastern half of the United States. She began her undergraduate studies at University of Dayton, located in Dayton, OH. There, she participated in a research program with Southern Ohio Council of Higher Education at the Air Force Research Laboratory on Wright-Patterson Air Force Base conducting experiments concerning semiconductor characterization. In May 2019, she received her Bachelor's degree in Physics. Starting August 2019, she enrolled in the Ph.D. program in Physics at Lehigh University. During her time in this program, she studied string phenomenology with Professor Timm Wrase. She received her Master's degree in May 2021, and obtained her Ph.D. in May 2024.