A method for reconstructing air shower parameters (E_0, X_{max}) from optical measurements based on the universality of showers

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Abstract: Recent experiments like Auger and Telescope Array measure optical images of giant showers while they propagate down in the atmosphere. We show that the universal behaviour of the showers allows one to predict practically exactly the number of photons emitted towards the telescope from any element of the shower track, once the geometry, primary energy E_0 and the depth of the shower maximum X_{max} is known. Thus, E_0 and X_{max} fitting best the measurements can be found (of course, atmospheric conditions must also be known).

Universality in question consist in the following shower characteristics established by detailed computer simulations of EAS: a) energy spectrum of electrons on some level in a shower depends only on the age parameter of this level; b) angular distribution of electrons of a given energy on some level depends only on this energy; c) lateral distribution of electrons (r in Molière units) depends on the age of the level only. In addition, we use the well established shape $\sim N_e(X)$ as a gamma function (known as the Gaisser-Hillas function). Having the above distributions it is possible to predict both the fluorescence and Cherenkov (scattered and direct) instantaneous images for any assumed $N_e\,(X(t))$. A curve fitting best the measurements provides the primary energy E_0 and X_{max} of a shower, that is an information on the primary mass and/or on an interaction model.

Keywords: extensive air showers, Cherenkov light, fluorescence light, experimental method

1 Introduction

One of the methods for detecting extensive air showers produced by ultra-high energy cosmic rays in the atmosphere is measuring their optical images by large telescopes (as in the Fly's Eye, HiRes, the Pierre Auger Observatory and the Telescope Array). The main contribution to the light flux emitted by a shower is the fluorescence of the excited nitrogen molecules. The number of fluorescence photons (emitted isotropically) is proportional to the energy deposited in the atmosphere by the shower particles, so that by integrating it over the shower track one would get practically the energy of the primary particle E_0.

However, the fluorescence light is not the only component of the light flux emitted by shower particles. About $\frac{1}{3}$ of shower electrons (both signs) emit Cherenkov radiation which, although collimated with particle directions, can contribute to the total registered light mainly by being scattered to the sides. Here, we describe how the Cherenkov light (direct and scattered) can be taken into account in evaluating the primary energy of a shower.

Our method is based on universal characteristics of large showers: the shape of the energy spectrum of electrons at a given level of its development depends only on the age parameter of the shower at this level. Also the lateral distribution of electrons (expressed in the Molière radius) and their angular distribution depend on the shower age only.

Thus, assuming a depth of a shower maximum X_{max} and a shower curve $N_e(X)$ (number of particles as a function of depth in the atmosphere) for a given energy E_0 we can predict the fluorescence and Cherenkov fluxes arriving at a detector (assuming that the atmosphere properties are known). X_{max} and E_0 which fit the shower data best are the reconstructed shower characteristics.

This method should work independently of the amount of the Cherenkov light contained in the total light, thus also for showers with a relatively small angles between the line of sight and their directions.

2 The method

Our work has been stimulated by the participation in the Pierre Auger Observatory [1], so we shall assume that the optical detector is an imaging one, i.e. consisting of a mirror and a camera enabling one to measure the angular distribution of arriving photons. The telescope integration time is relatively short (100 ns in Auger) as compared to the total time while the shower is in the field of view.

We assume that we know the shower geometry and its optical image (angular distribution of photons arriving at the telescope) as a function of time as the shower travels through the atmosphere. We also have to know the atmospheric optical conditions at that time.

Thanks to the universality of showers, we can predict the number of fluorescence and Cherenkov photons (and their angular distribution at the detector) for a shower with a given E_0, X_{max} and geometry (distance to the core, zenith and azimuth angles).

2.1 The fluorescence image

Let us assume that a shower is at a slant depth X. The number of the fluorescence photons, Δn_{fl}, produced by the shower along a path length ΔX depends on the energy deposited ΔE by all electrons along this element. Since the main process are the energy losses on ionization ΔE depends on the energy spectrum of electrons at this level. Basing on the shower simulations with CORSIKA [2] it was shown [3, 4] that the shape of the electron energy spectrum depends only on the shower age s at the level in question. It practically does not depend on the primary particle mass or the primary energy. Moreover, such large showers as detected by Auger ($E_0 > 10^{18}$eV) fluctuate very little so that the shape of the spectrum stays the same in any shower for the same age s. Thus, to calculate Δn_{fl} one needs to know the total number of shower electrons on level X, $N_e(X)$, and X_{max} (to determine $s(X)$).

To find the angular distribution of photons Δn_{fl} at the telescope diaphragm (the instantaneous shower image) one needs to know the lateral distribution of electrons at level X (strictly speaking, it is that distribution of electrons lying on the cross section of the shower by a plane perpendicular to the shower-detector plane, the former being a bisector of the angle δ between the shower direction and that from the shower towards the detector [5, 6]).

It was shown [7] that the lateral distribution of electrons, if expressed in units r/r_M, where r_M is the Molière radius referring to the level in question, depends on the shower age only. Thus, the lateral distribution of the energy deposited, as function of r/r_M, depends also only on s. It has been correspondingly parametrized [8] and can be used for finding the fluorescence image of the shower.

The number of photons reaching the detector depends, of course, on the distance of point X to the detector and on the atmospheric conditions (like pressure and aerosol content).

2.2 The Cherenkov (Ch) image

There are two components of the Ch light. One is the Ch light produced by shower particles *above* level X and scattered at X along ΔX towards the detector (by the Rayleigh and Mie processes). The second one called the direct Ch light are the Ch photons *produced at* ΔX at directions towards the detector. The latter is usually very small unless the angle δ is smaller than $\sim 30^\circ$. The former, however, plays a dominant role in determining the optical image of a shower below its maximum [9].

2.2.1 The scattered Ch image

A calculation of the Ch image of the first component (the scattered Ch light) is similar to that of the fluorescence image. But instead of the lateral distribution (LDF) of electrons we need to know the LDF of the Ch photons (LDCh) arriving at level X, and instead of the isotropic fluorescence production we have an anisotropic scattering. This problem has been treated by us [9], although in a somewhat approximate way:

$LDCh(X)$ is the integral of the Ch light produced at all depths $X' < X$. The contribution from a level X', $LDCh(X;X')$, depends on the angular distribution of electrons above the Ch threshold at X', on the $LDF(X')$ and, of course, on the distance between level X' and X. All these depend on s and height in the atmosphere (strictly speaking, on the air density), since the Ch threshold energy and the distances $X' - X$ depend on height. Our approximation consisted in: a) assuming that the angular and lateral distributions of Ch emitting electrons are independent of each other; b) adopting the shape of $LDCh(X;X')$ following from the dominant of the two, the second one being a correction.

Now, we propose a better treatment. We have shown in [6] that the angular distribution of electrons *of a given energy* depends on this energy only. It does not depend even on the shower age. (It is not difficult to understand this – an electron looses quickly its energy, so that its scattering angle does not depend on its history, when it had much larger energy and much smaller scattering angle, which does not influence the final angle as they add in quadrature). Thus we have any reason to assume that the angular distribution of electrons *of a given energy* is independent also of the distance from the shower axis, so that *both distributions, angular and lateral, of electrons with a fixed energy should be independent*.

The LDF (in units of r/r_M) for a fixed E have been shown to depend on shower age s and E only [7]. Taking this into account one can calculate $LDCh(X;X',E)$, separately for each bin of the electron energy E. Now, as the treatment seems to be quite correct, it is worth to drop the approximation "b" mentioned above and calculate exactly $LDCh(X;X',E)$, that is by folding one distribution with another. Integrating over E with the electron energy spectrum $f(E;s(X'))$ as weight and then over $X' < X$ one obtains the $LDCh(X)$.

We do not think that our new approach will change much the LDCh obtained in [9] by the approximate way because, as it was shown there, it is mainly the *angular* distributions of electrons at higher levels that determine $LDCh(X)$.

Nevertheless, seeing a way of calculating it more accurately, without involving time and effort consuming Monte-Carlo shower simulations, we think it is worth doing it.

It has to be stressed that the atmospheric optical conditions are usually changing (as at the Auger site) so that doing a reconstruction of a shower parameters one has to use the actual data about the mean free path lengths for the Rayleigh and Mie scatterings and their height dependences. Thus, the $LDCh(X)$, if parametrised, would have to depend on these conditions.

The question of the LDCh dependence on the shower zenith angle is discussed in [9].

2.2.2 The direct Ch image

The number of Ch photons produced by the shower track element ΔX arriving directly (without being scattered) at the detector together with the fluorescence light produced in this element depends on the angular distribution of the Ch electrons at this level X. From what has already been said it follows that this distribution depends on s and height of this level only. Their *angular* distribution at the diaphragm depends, as in the case of the fluorescence image, on the lateral distribution of the *emitting* electrons.

Thus, to obtain the direct Ch image in an approximate way, one may assume that the angular and lateral distributions of the Ch emitting electrons on a given level are independent of each other, which, as discussed above, is not true if electrons have various energies.

However, a more accurate way is to find the image from electrons with a *fixed* energy and then integrate it over the energy distribution on the level in question.

This may seem not necessary in case of the Auger experiment as for most cases the contribution from the direct Ch light is practically very small. The elevation angles α in Auger are not large: $0 < \alpha < 30°$ what, together with the limit on the zenith angle $\theta < 60°$ of the well reconstructed showers, results in rather large viewing angles δ.

However, the situation is changed for the HEAT extension [10] to Auger, where the elevation angles reach $60°$.

We have derived the distribution of the viewing angle δ for a given elevation angle α, assuming the zenith angle distribution

$$F(\theta) = \frac{2}{1-\cos^2\theta_0}\sin\theta\cos\theta \tag{1}$$

where $\theta_0 = 60°$ is the maximum zenith angle.

The result is:

a) for $\alpha \leq 30°$

$$f(x) = \frac{8}{3\pi}\left\{\sin\alpha\cdot x\left[\frac{\pi}{2}+\arcsin\left(\frac{\sin\alpha\cdot x-\frac{1}{2}}{\cos\alpha\sqrt{1-x^2}}\right)\right] + \sqrt{\cos^2\alpha-\frac{1}{4}+\sin\alpha\cdot x-x^2}\right\} \tag{2}$$

b) for $\alpha \geq 30°$

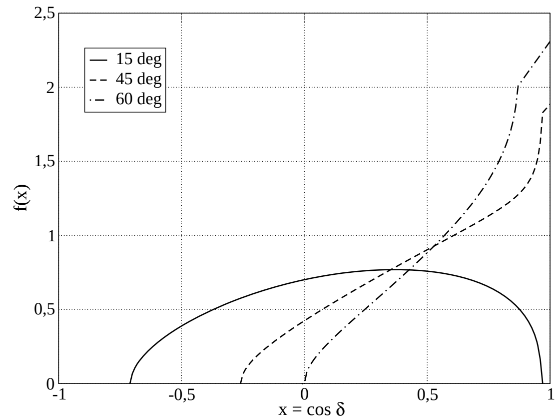

Figure 1: $f(\cos\delta)$ distribution. Numbers referring to curves denote elevation angle α.

$$f(x) = \frac{8}{3}\sin\alpha\cdot x \quad \text{for} \quad 0° \leq \delta \leq \alpha - 30° \tag{3}$$

$$f(x) = \text{ as in case a)} \quad \text{for} \quad \alpha - 30° \leq \delta \leq 150° - \alpha \tag{4}$$

where $x = \cos\delta$.

Figure 1 shows $f(\cos\delta)$ for $\alpha = 15°$ – elevation of the centre of an Auger telescope, $45°$ – centre of HEAT and $60°$ – HEAT upper limit.

A dramatic difference in the δ distributions for Auger and the HEAT telescopes can be seen. For $\delta < 30°$ the corresponding fractions are $\sim 3.8\%$, 21% and 29%. Of course, this is only a rough illustration of the difference as the actual distribution of the number of the camera angular pixels looking at a given δ at a shower in the reconstructed showers will cut out some of the small δ due to selection criteria.

Nevertheless, Figure 1 shows that the effect of direct Ch light plays a bigger role in showers registered by HEAT and needs to be treated in an accurate way.

2.3 Total number of electrons $N_e(X)$

So far we have discussed only the shapes of the electron distributions. To predict the actual number of photons at the detector, arriving from a depth X one needs to know $N_e(X)$. It is well known that this dependence can be described by a gamma function of $(X - X_1)/\Lambda$, $N_{max} = N_e(X_{max})$ and X_{max}, proposed by Gaisser and Hillas [11], X_1 and Λ being some free parameters.

Thus, our final procedure is to find such values of the four parameters describing $N_e(X)$ which predict best the number of photons detected by all camera angular pixels at any time.

In principle, this method should work independently of the amount of the Cherenkov component, therefore, as an alternative to the current Auger reconstruction method [12], it can be useful for reconstruction of HEAT showers, where this component is larger than in those registered by the s-tandard Auger fluorescence detectors.

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