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INTERFEROMETRY ANALYSIS AND INITIAL CONDITIONS IN A+A COLLISIONS

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The behavior of the interferometry radii in central A+A collisions at different energies and also for different nuclei or impact parameters indicates the initial transverse flows at very early stage of the matter evolution. Development of such flows at pre-thermal partonic stage is considered.

1 Introduction

The first results of the femtoscopy, or HBT analysis at RHIC experiments [1] (as it was first announced by the STAR Collaboration) have revealed unexpected results - the so-called RHIC HBT puzzle [2]. The puzzle implies, firstly, that the absolute values of the interferometry radii/volume in central Au+Au collisions do not change essentially at RHIC as compared to the SPS energies for Pb+Pb collisions despite much higher multiplicities. It was in contrast with, expected at that time, possibility of the proportionality law between the interferometry volumes and multiplicities. At the same time there is an approximate proportionality between interferometry volume and different initial volumes which can be associated, e.g., with number of participants (nucleons of nuclei) in the collision process and, thus, with the multiplicity. Secondly, the ratio of outward to sideward transverse radii is opposite to what was expected in standard hydrodynamic and hadronic cascade pictures. The ratio measured by STAR and PHENIX collaborations at RHIC BNL is close to unity in a wide momentum region. At the first sight these observations are in a contradiction with an existence of quark-gluon plasma and mixed phase as it implies a long time pion radiation which usually results in the large ratio of outward to sideward transversal radii. As a result, now the phenomenological parameterizations, like the blast wave model just ignore the emission from the surface of expanding system despite the fact that it should last at least about the extracted life-time of the fireball: 10-12 fm/c.

These notes represent the possible explanation of the peculiarities of the observed behaviors of the interferometry radii based on an analysis of the temporal evolution of observables [3, 4]. As a result, one can conclude that initial flows in pre-thermal partonic matter, which precede hydrodynamical expansion, should develop in the system. We discuss the possible scenario of the pre-thermal evolution of partonic matter and estimate the collective velocities at this early stage of the processes of ultrarelativistic A+A collisions.

2 Analysis and treatment of experimental data

As it was shown in Ref.[3] the phase-space density of thermal pions *totally averaged* over freeze-out hypersurface σ and over momenta except the longitudinal one (rapidity is fixed, e.g., $y = 0$), $\langle f\rangle$, is an approximate integral of motion.

The conservation of the APSD allows one to study the hadronization stage of the matter evolution based on the possibility to define the APSD of thermal pions at the final stage of the matter evolution through the integral (over momentum) representation of this value through the observed spectra and interferometry volumes [3]. The results for the APSD at mid-rapidity for pions at the AGS, SPS, RHIC demonstrate a plateau at low SPS energies that indicates, apparently, a transformation of an excess of initial energy to non-hadronic forms of matter, a saturation of the APSD at RHIC energies can be treated as an existence of the limiting Hagedorn temperature of hadronic matter, or maximal temperature of deconfinement [4].

Let us use these results for an analysis of the behavior of the pion interferometry volumes V_{int}. If one consider them at small transverse momenta, then they can be represented approximately through the APSD as the following:

$$V_{int} \simeq C\frac{dN/dy}{\langle f\rangle T_{eff}^3} \tag{1}$$

It is easy to see then that at any *fixed* energy $\sqrt{s_{NN}}$ the V_{int} is nearly constant in time since the values dN/dy, APSD $\langle f\rangle$ and effective temperature T_{eff} in r.h.s. of Eq. (1) are approximately conserved for the

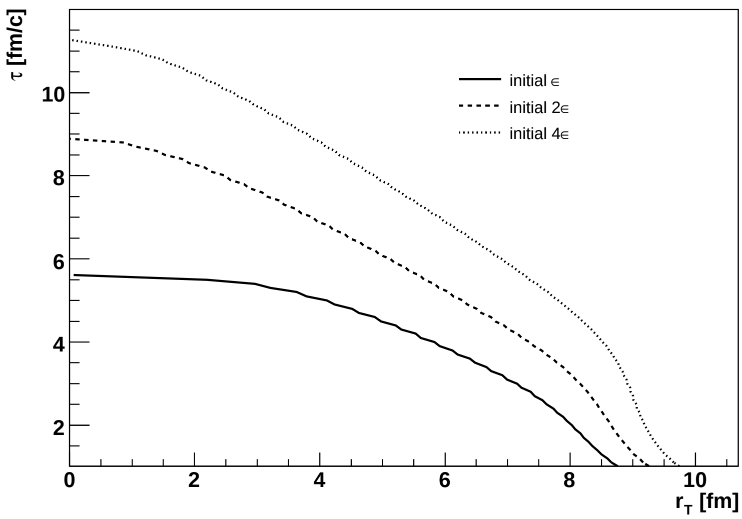

Figure 1. The typical freeze-out hypersurfaces with the *fixed* f.o. energy density presented in $\tau - r$ plane for Bjorken-like azimuthally symmetric hydrodynamic expansion with equation of state $P = \epsilon/3$ and zero initial transverse flow. The curves correspond to the different initial energy densities $\epsilon_i(\tau_i, y, r)$=$\epsilon_0(\tau_i, y, r)$, $2\epsilon_0(\tau_i, y, r)$, $4\epsilon_0(\tau_i, y, r)$ distributed in r-plane according to the Woods-Saxon formula. The initial proper time is $\tau_i = 1$ fm/c.

thermal pions during the chemically frozen hydro-evolution. As the result, the HBT microscope at diverse energies "measures" the radii that are similar to the sizes of colliding nuclei. It explains the experimental observations that at the same collision energy, the V_{int} depends strongly on the sizes of colliding nuclei and on the impact parameters in non-central collisions [5].

The RHIC experiments show clearly that there is no proportionality law between V_{int} and dN^{π}/dy: the later value grows with energy significantly faster than V_{int}. This fact is the main component of the HBT puzzle. According to Eq. (1), a proportionality between V_{int} and the particle numbers dN/dy may be destroyed by a factor $\langle f\rangle T_{eff}^3$. So, if the APSD and V_{int} only slightly grow with energy, mostly an increase of T_{eff}^3 could compensate a growth of dN/dy in Eq. (1). One can see that it is the case: for example, the ratio of cube of effective temperatures of negative pions at $\sqrt{s_{NN}} = 200$ GeV (RHIC) to one at 40 AGeV (CERN SPS) gives approximately 2, while the ratio of correspondent mid-rapidity densities is approximately equal to 3. It can be only in the case of an increase of the pion transverse flows in A+A collisions with energy. If the intensity of flows grows, it leads to a reduction of the corresponding homogeneity lengths which contribute to the interferometry radii. This effect can almost compensate a contribution to observed interferometry volumes of the geometrical system sizes that grow with energy. The question is then: why does the intensity of flow grow? It is clear that an increase of collision energy $\sqrt{s}$ results in a rise of initial energy density ϵ and hence of (maximal) initial pressure p_{max}. At the same time the initial transverse acceleration $a = \mathrm{grad}(p)/\epsilon \propto p_{max}/\epsilon$ does not change. Thus, one can conclude that there could be the two reasons for an increase of transverse pion flows with collision energy. First one is obvious, it is an increase of the time of hydro-evolution that the system needs to reach the same (or less) freeze-out energy density or temperature at higher initial density (see Fig. 1)

However, apparently, relativistic hydrodynamic picture overestimate the increase of the longitudinal interferometry radii, that is associated with life-time of the system, as compare to the experimental data.

The another reason for an increase of the observed transverse flows is the presence of the initial transverse velocity which may develop at the pre-thermal partonic stage and obviously has an influence on the time of evolution and intensity of transverse flow at freeze-out. Moreover, what is essentially important, this factor has the direct connection to the second component of HBT puzzle: the unexpectedly small ratio of outward to sideward interferometry radii. In relativistic hydrodynamics or realistic hydro-inspired parametrization the freeze-out hypersurface should be enclosed, so the protractive surface emission of pions (hadrons) from fairly cold periphery of the expanding system take place. Normally, it should lead to large R_{out} to R_{side} ratio, however, as demonstrated in Ref. [6], it is possible, nevertheless, to describe the data successfully, including R_{out} to R_{side} ratio, if there are positive $r - t$ correlations between the radial r coordinates and times t of surface emission of the particles. The term associated with these correlations gives the negative contribution to R_{out} interferometry radius and so compensates the positive contribution to it from long time surface emission.

The only fit with positive $r - t$ correlations, as it presented in Fig.2, results in good description of the the spectra pions, kaons and protons and pion interferometry data, including R_{side} and R_{out}. All details are presented in Ref.[6].

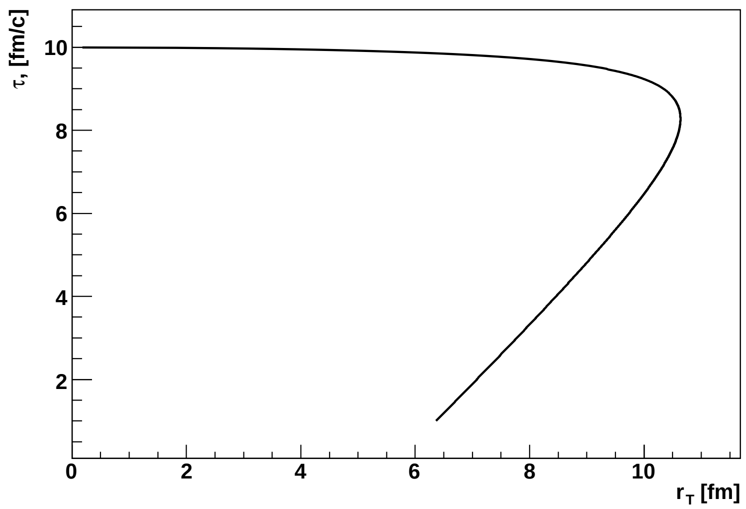

Figure 2. The dynamical realization of the freeze-out with positive $r-\tau$ correlations at constant energy density [7] based on the (3+1)D exact analytical solutions of relativistic hydrodynamics [8] with intensive initial transverse flows

One of the most important observation is that the $r-t$ correlation at the freeze-out hypersurface, according to equations of relativistic hydrodynamics, can be predominantly positive only if the system has at initial moment a developed transverse flow. The typical situation presented in Fig.1 and Fig.2. The former figure corresponds to an absence of the initial transverse flow, the second describe intensive blast-like expansion into vacuum that starts at early stage of the evolution, say, at τ=1 fm/c. In the first case the negative $r-t$ correlations between the surface emission points takes place, and it leads to a positive contribution to R_{out} in addition to big positive contribution associated with protracted surface emission. In the second case the latter positive term is compensated by the positive $r-t$ correlation term. It leads to experimentally observed R_{out} to R_{side} ratio *in presence* of protracted surface emission.

3 Pre-thermal partonic stage : The free-streaming approximation

A problem of formation of the initial transverse velocity at pre-thermal partonic stage leads inevitably to the complex matter of the initial stage in ultrarelativistic A+A collisons and the problem of thermalization. In these notes we will not discuss in details this very complicated topic just keeping in mind quite simple physical picture and apply it phenomenologically.

Let us imagine a box (with size L) that have the ideally reflecting walls and contains the standing (electromagnetic) waves inside. Then collide the two such boxes with the energy that allows to crush them completely. Standing waves then will be destroyed due to a stochastisation that is accompanied by the crashing *processes* [9]. In other words, strong correlations between phases of traveling "backward" and "forward" waves, with discrete momenta, say, $2\pi/L$ and $-2\pi/L$, caused by ideal reflections from the opposite walls, will vanish and instead the random phases $\exp(\alpha_{p_i})$ will appear:

$$\sin\frac{2\pi x}{L}=\frac{1}{2}\left(\exp\frac{i2\pi x_{\rm T}}{L}-\exp\frac{-i2\pi x_{\rm T}}{L}\right)\Rightarrow\sum\rho_{p_i}{\rm e}^{\alpha_{p_i}}{\rm e}^{ip_ix_T}.$$

In the case of very weak field we will see then, say, two incoherent photons traveling, for instance, in transversal plane in opposite directions.

Let us provide an analogy now with high energy nucleus-nucleus collisions by imaging them as the collisions of the two "boxes" (containing many "small boxes" – nucleons). Due to the non-commutativity of the gluon number operator with the operator of Lorenz boost, there is a huge number of coherent partons in the fast moving box – this state probably can be represent within the Color Glass Condensate (CGC) approach [10]. Correspondingly, after collision there will be not just two gluons but the classical color field (because of large occupation number) expanding into vacuum. When occupation number reduces, one can see the picture of the expanding system of incoherent partons. It may call "partonic explosion" when many hidden degrees of freedom, associated with incoherent partons and carried significant transverse momentum, are liberated almost suddenly. An estimate of the thermalization time for this system is a rather complicated problem and we just mention about it later. It seems that partons interact weakly enough and instability mechanism [11] works not so fast, as necessary to reach very small time of momentum symmetrization (thermalization?), less then 1 fm/c, required by hydrodynamics models to describe elliptic flows.

Let us simplify the problem again and consider now the developing of transverse velocity at pre-thermal partonic stage in an approximation of free streaming for this weakly interacting particles.

We start from the simple non-relativistic example. Let us put the initial momentum distribution of particles with mass m to be spherically symmetric Gaussian with the width corresponding to thermal Boltzmann distribution with uniform temperature T_0, no flows: $\mathbf{u}(t=0,\mathbf{r})=0$, and also spherically symmetric Gaussian profile (with radius R_0) for particle density. Let the particles just to free stream. Then according to [3] the collective velocities, which can be defined at any time t according to Eckart:

$$u^i = \int \frac{d^3p}{m^4} p^i f(t,\mathbf{x};p)$$

are

$$\mathbf{u}(t,\mathbf{r}) = \mathbf{r}\frac{tT_0}{mR_0^2+T_0t^2}.$$

As one can see the collective velocities in free streaming system grow with decrease of particle mass, grow with initial parameter T_0 for $m \neq 0$, and are independent of the initial "temperature" at $m=0$. Qualitatively, the same happens for relativistic partonic gas.

Let us consider relativistic partonic picture with the initial momentum distribution at Björken proper time τ=1 fm/c corresponds to "transverse momentum" Fourier components in the color field in the CGC picture found in Ref.[12]. Suppose that after collision the similar transverse spectrum will appear for incoherent partons. As for the longitudinal ones we will use the local 3D isotropic quasi-thermal distribution as it was proposed in Ref.[13] based on the Schwinger mechanism of the partonic production: the partons created by a pulse of the strong chromo-electric field during collision process are distributed (locally) quite isotropically since the limited in time action of the field. Let us use the boost-invariant approximation in mid-rapidity and the Woods-Saxon initial profile for energy density in transverse plane. Then the partonic distribution function at the initial proper time $\tau=\tau_0$ is:

$$f_0 = \frac{1}{\exp\frac{m_T}{T}\cosh\theta - 1}\frac{1}{\exp\frac{1}{\delta}\left(r_T - R\right)+1}, \tag{2}$$

where θ is the difference between particle and fluid rapidities. The main parameters of the distribution is agreed with Refs.[12, 13]: $T = 0.465\Lambda_s$, $\delta = 0.67$ fm, $\Lambda_s = 1.3$ GeV, $\tau_0 = 1$ fm/c, $R = 7.3$ fm, partonic mass is taken to be equal to $m = m_0 = 0.0358\Lambda_s$. The evolution of this function is defined by the equation for free streaming,

$$p^\mu \frac{\partial f}{\partial x^\mu} = 0. \tag{3}$$

The solution of this equation describes the distribution function at any hypersurface τ = const by the use of the following substitution in the arguments of the function f_0 related to the initial proper time τ_0=1 fm/c:

$$\mathbf{r}_\mathrm{T} \to \mathbf{r}_\mathrm{T} - \frac{\mathbf{p}_\mathrm{T}}{m_\mathrm{T}}\left(\tau\cosh\theta - \sqrt{\tau_0^2+\tau^2\sinh^2\theta}\right), \tag{4}$$

$$\theta \to \operatorname{arcsinh}\left(\frac{\tau}{\tau_0}\sinh\theta\right). \tag{5}$$

In what follows we will consider the properties of such a free-streaming expansion of boost-invariant and cylindrically symmetric finite system into vacuum as a first approximation and discuss the possible whole picture of the early pre-thermal stage.

4 Collective velocities and local anisotropy in partonic system

Let us study the free-streaming stage of the evolution, supposing, as it was argued above, that incoherent partonic system arise at the time of order of 1 fm/c as a locally isotopic boost-invariant and weakly interacting gas. Then gas will free stream into vacuum. The process itself will lead to a local anisotropy which we will study in this section. The increase of the anisotropy may be compensated by the process of turbulency (instability) and gradual thermalization associated with Balescu-Lenard term for QCD fields.

The analysis of the local anisotropy of the distribution function can be done in the two ways. The first one deals with a study of distribution function properties, while the second one deals the difference between spatial components of the energy-momentum tensor in local rest frames. In both cases, we are forced to consider the distribution and the energy-momentum tensor in a co-moving reference frame determined by collective velocity. Here we will apply both Eckart and Landau-Lifshitz definitions of the collective velocities $\mathbf{v}(\mathbf{x})$ related to fairly small elements associated with point (x^μ).

The connection between the global and local rest frame moving with 3-velocity $\mathbf{v} = (v^i)$, is Lorentz transformation defined by matrix of the form:

$$(\Lambda^{\mu}{}_{\nu}) = \left(\begin{array}{c|c} \gamma & v^i\gamma \\ \hline v^j\gamma & \delta^{ij} + v^i v^j(\gamma-1)/\mathbf{v}^2 \end{array}\right), \tag{6}$$

where $\gamma = 1/\sqrt{1-v^2}$ is a Lorentz factor; $v \equiv |\mathbf{v}|$.

Making use this matrix, the contravariant vector and tensor transformations read

$$a^{\mu} = \Lambda^{\mu}{}_{\nu} a_*^{\nu}, \qquad a^{\mu\nu} = \Lambda^{\mu}{}_{\lambda} a_*^{\lambda\sigma} \Lambda^{\nu}{}_{\sigma}, \tag{7}$$

where a_*^{μ} and $a_*^{\mu\nu}$ denote these quantities in co-moving reference frame.

Therefore the 3-vector $\mathbf{p}$ is transformed as follows

$$\mathbf{p} = \mathbf{p}_* + \frac{\mathbf{v}}{v^2}\frac{(\mathbf{v}\mathbf{p}_*)(1-\sqrt{1-v^2}) + v^2 E_*}{\sqrt{1-v^2}}, \tag{8}$$

where $E_* = \sqrt{m^2 + \mathbf{p}_*^2}$.

It is possible to examine anisotropy of momentum distribution in different co-moving reference frames associated with different spacial points, where 3-momentum $\mathbf{p}_*$ determines $\mathbf{p}$ in accordance with Eq. (8).

The local anisotropy reveals itself also in structure of the energy-momentum tensor, which in pseudo-Cartesian coordinates reads

$$T^{\mu\nu}(x) = \int p^{\mu} p^{\nu} f(x,p) p_{\mathrm{T}}\, dp_{\mathrm{T}}\, dy\, d\phi, \tag{9}$$

where the Lorentz-invariant integration measure d^3p/E in Cartesian variables is already re-written in Björken variables: $(p^{\mu}) = (m_{\mathrm{T}} \cosh y,\, p_{\mathrm{T}} \cos\phi,\, p_{\mathrm{T}} \sin\phi,\, m_{\mathrm{T}} \sinh y)$.

To find $T^{\mu\nu}$ in central rapidity slice, we numerically calculate energy-momentum tensor (9) at longitudinal coordinate $z = 0$ ($\eta = 0$), when $\tau = t$. Due to the symmetry properties of distribution, one finds $T^{tz} = T^{xz} = T^{yz} = 0$. Let ψ be the angular direction relative to the radial axis x. Note that $T^{xy} = 0$ at $\psi = n\pi/2$, $n = 0, \pm 1, \pm 2, ...$ Fixing $\psi = 0$, the non-vanishing components of the energy-momentum tensor are

$$(T^{\mu\nu}) = \begin{pmatrix} T^{tt} & T^{tx} & 0 & 0 \\ T^{tx} & T^{xx} & 0 & 0 \\ 0 & 0 & T^{yy} & 0 \\ 0 & 0 & 0 & T^{zz} \end{pmatrix}. \tag{10}$$

It is understandable that the direction of collective velocities $\mathbf{v}$ in the global (origin) reference frame at $z = 0$ should coincide with vector $\mathbf{r}_{\mathrm{T}}$ and therefore $\mathbf{v} = (v\cos\psi, v\sin\psi, 0)$.

The tensor $T_*^{\mu\nu}$ in the co-moving reference frame, associated with local velocity $\mathbf{v}$, is defined from (10) by use of the matrix $\Lambda_{\mu}{}^{\nu}$ inverse to (6). (Actually, matrix $\Lambda_{\mu}{}^{\nu}$ is derived from (6) by replacement $v^i \to -v^i$.) In the case of a boost, the components of the energy-momentum tensor in two reference frames are related by

$$T_*^{\mu\nu} = \Lambda_{\lambda}{}^{\mu} T^{\lambda\sigma} \Lambda_{\sigma}{}^{\nu}. \tag{11}$$

4.1 Eckart Frame

Now we are concentrated on the collective velocity computation. In this subsection we deal with 4-velocity defined by Eckart:

$$u_{\mathrm{E}}^{\mu} = \frac{N^{\mu}}{\sqrt{N_{\nu}N^{\nu}}}, \tag{12}$$

where

$$N^{\mu} = \int p^{\mu} f(x,p) p_{\mathrm{T}}\, dp_{\mathrm{T}}\, dy\, d\phi$$

is the particle flux.

The collective 3-velocity is simply given by $\mathbf{v}_{\mathrm{E}} = \mathbf{u}^{\mathrm{E}}/u_0^{\mathrm{E}}$. The dependence of transverse velocity $v_{\mathrm{E}} = \sqrt{v_x^2 + v_y^2}$ on r_{T} at $\eta = 0$ is demonstrated in Fig. 3.

Having got numerically the values of collective velocity, one can re-write the distribution function at the fixed point of space-time in Eckart co-moving reference frame by means of Lorentz transformation (8). At $\tau = \tau_0 = 1$ fm/c, the distribution is isotropic, as must be according to initial conditions. Increasing τ, the distribution becomes more and more anisotropic that is reflected on the collective velocity development.

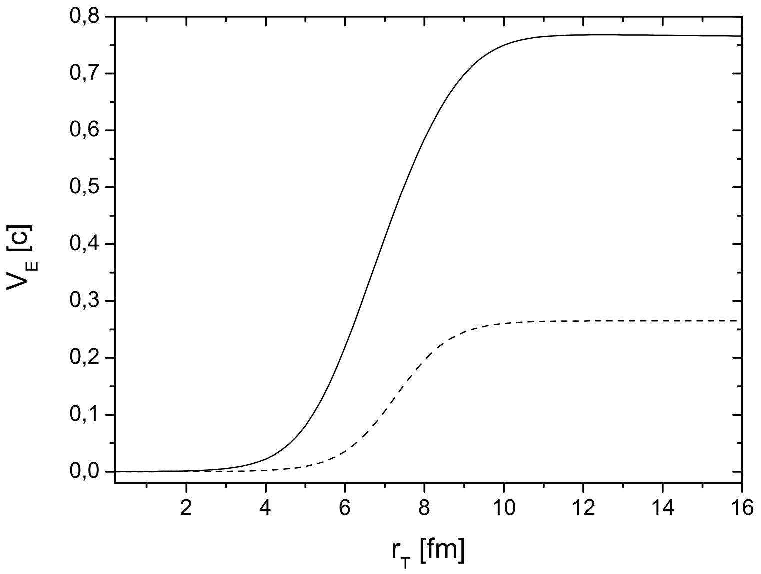

Figure 3. The Eckart collective transverse velocity in weakly interacting partonic system in the approximation of free streaming. The initial state at 1 fm/c is supposed to be quasi-thermal and corresponds to the distribution (2). Dashed curve correspond to τ=1.5 fm/c, solid line – 3 fm/c.

Another possibility to observe the anisotropy in the given systemis to compare the components of energy-momentum tensor in a given co-moving reference frame, which is introduced by means of formula (11). The result of numerical calculations is shown in Fig. 4. Abbreviation "Arb. units" means that the distribution function is not normalized.

We find $T_*^{xx} = T_*^{yy} = T_*^{zz}$ at $\tau = \tau_0 = 1$ fm/c, that also confirms the isotropy at the initial moment. Changing τ, the value of T_*^{zz} turns out essentially less than T_*^{xx} and T_*^{yy}, which also differ.

Remark that, putting $v = v_{\rm E}$ in Eq. (11), it is impossible to cancel T_*^{tx} in whole region of values of $r_{\rm T}$. Further, we will demonstrate that the requirement $T_*^{tx} = 0$ corresponds to definition of Landau-Lifshitz frame.

4.2 Landau-Lifshitz Frame

The Landau-Lifshitz definition of collective velocity can be formulated as

$$u_{\rm L}^{\mu} = \frac{T^{\mu\nu} u_{\nu}^{\rm L}}{u_{\rm L}^{\lambda} T_{\lambda\sigma} u_{\rm L}^{\sigma}}. \tag{13}$$

In general, this expression is equation with respect to $u_{\rm L}^{\mu}$, which should be solved numerically. However, in our case of cylindrical symmetry, when the free streaming is going on along $r_{\rm T}$-axis, the collective velocity can be found explicitly.

Substituting the expression for $T_*^{\mu\nu}$, the components of the collective 4-velocity in co-moving reference frame are

$$(u_{*\rm L}^{\mu}) = (1,0,0,0) = \left(1, \frac{T_*^{tx}}{T_*^{tt}}, 0, 0\right). \tag{14}$$

It meas that $T_*^{tx} = 0$ and then one can get from Eq. (11) the expression for velocity in the global reference frame:

$$v_{\rm L} = \frac{T^{tt} + T^{xx}}{2T^{tx}} \left(1 - \sqrt{1 - \frac{4(T^{tx})^2}{(T^{tt} + T^{xx})^2}}\right). \tag{15}$$

The behavior of $v_{\rm L}$ is shown in Fig. 5. The velocity $v_{\rm L}$ also vanishes at $\tau = \tau_0$. Although $v_{\rm L}$ is close to $v_{\rm E}$ (see Fig. 3) they are not completely coincided since the system is not in locally equilibrated state.

In the case of $v = v_{\rm L}$ (see (11)), the anisotropy of energy-momentum tensor is demonstrated in Fig. 6 and it is qualitatively the same as in the Eckart case presented in Fig. 4.

4.3 Analysis of Weak Anisotropy

As one can see from Figs. 4, 6 the diagonal spatial components of the energy-momentum tensor of partonic system, even if they were equal at the initial formation time [13], are splitting during free-streaming expansion so that $T_*^{yy}(x) > T_*^{xx}(x) > T_*^{zz}(x)$. Thus the components of $T_*^{\mu\nu}(x)$ associated with directions of non-zero

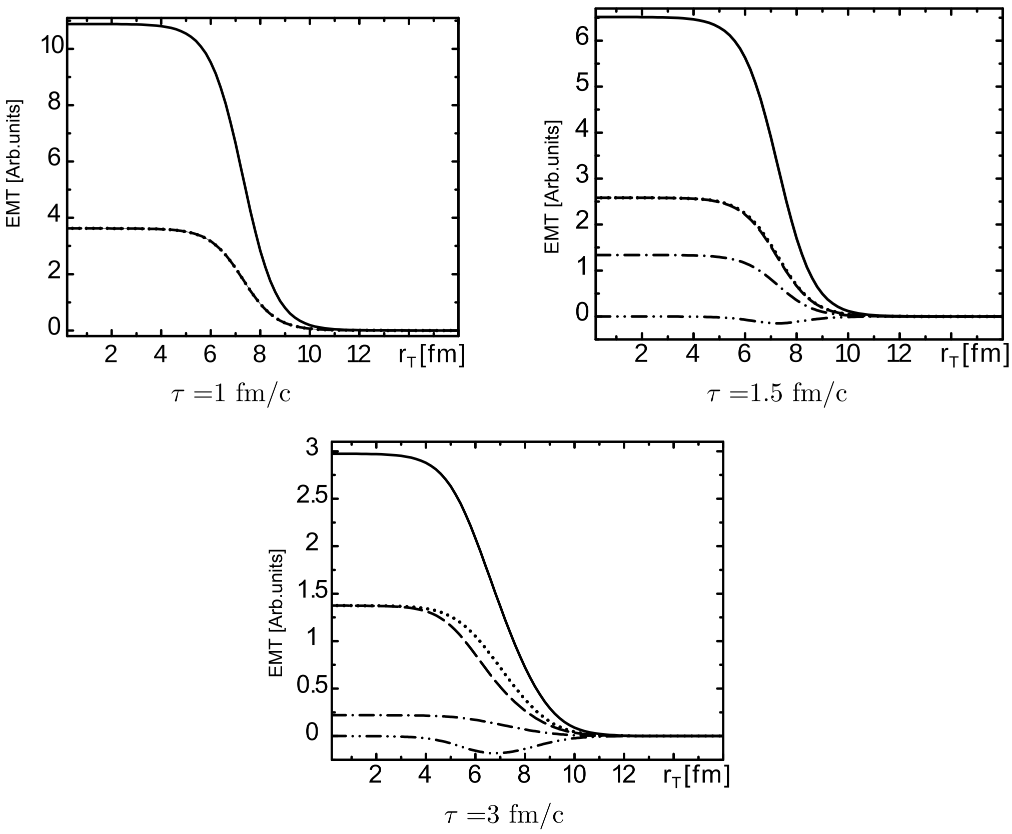

Figure 4. The components of the energy-momentum tensor, T_*^{tt} (solid), T_*^{xx} (dashed), T_*^{yy} (dotted), T_*^{zz} (dot-dashed), T_*^{tx} (dot-dot-dashed), at τ =1, 1.5, 3 fm/c and $\psi = 0$. Eckart co-moving frame.

collective velocities (initial and developed) become suppressed as compare with other ones. Correspondingly, the particle distribution function gradually looses the local momentum isotropy during the expansion. Let us parameterize this anisotropy as depending on $\tau = t$ and $r_{\rm T}$ at fixed $z = 0$.

It is useful to analyze the case of weak anisotropy and relate our result to other models. For this aim we represent distribution function (2), (4), (5) in the form $f = F \cdot W$ where

$$F(a) = \frac{1}{\exp\frac{a}{T} - 1}, \quad W(b) = \frac{1}{\exp\frac{b-R}{\delta} + 1}. \tag{16}$$

Fixing $z = 0$, $\psi = 0$, the arguments of these functions are presented as

$$a^2 = E^2 + \xi p_z^2, \tag{17}$$

$$b^2 = r_{\rm T}^2 - 2\frac{r_{\rm T} p_x}{m^2 + \mathbf{p}_{\rm T}^2}\tau_0(\sqrt{1+\xi}E - a) + \frac{\mathbf{p}_{\rm T}^2}{(m^2 + \mathbf{p}_{\rm T}^2)^2}\tau_0^2(\sqrt{1+\xi}E - a)^2,$$

where we have introduced the parameter of anisotropy $\xi(\tau) = \tau^2/\tau_0^2 - 1$ and $E = \sqrt{m^2 + \mathbf{p}^2}$. Note that the same dependence of ξ on the proper time has already pointed out in Ref.[14] to account for longitudinally boost invariant expansion in partonic system.

Let us write distribution function f in the linear approximation in ξ. The form of such a distribution is

$$f \approx f_{\rm iso} + \frac{\xi(\tau)}{2}\left\{W(r_{\rm T})\frac{dF(E)}{dE}\frac{p_z^2}{E} - F(E)\frac{dW(r_{\rm T})}{dr_{\rm T}}\frac{p^x E\tau_0}{m^2 + \mathbf{p}_{\rm T}^2}\left(1 - \frac{p_z^2}{E^2}\right)\right\}, \tag{18}$$

where $f_{\rm iso} \equiv F(E)W(r_{\rm T})$ is the initial isotropic distribution function in the global reference frame.

We see that the first term in the brackets $\{\}$ corresponds to momentum anisotropy due to initial momentum inhomogeneity, while the second one is related to the initial inhomogeneity in coordinate space.

Since the axial symmetry we can put $\psi = 0$, and the transition to the co-moving frame associated with some point (x, $y = 0$, $z = 0$) is determined in the simple way:

$$p^x = \frac{p_*^x + vE_*}{\sqrt{1-v^2}}, \quad E = \frac{E_* + vp_*^x}{\sqrt{1-v^2}}, \quad p^y = p_*^y, \quad p^z = p_*^z, \tag{19}$$

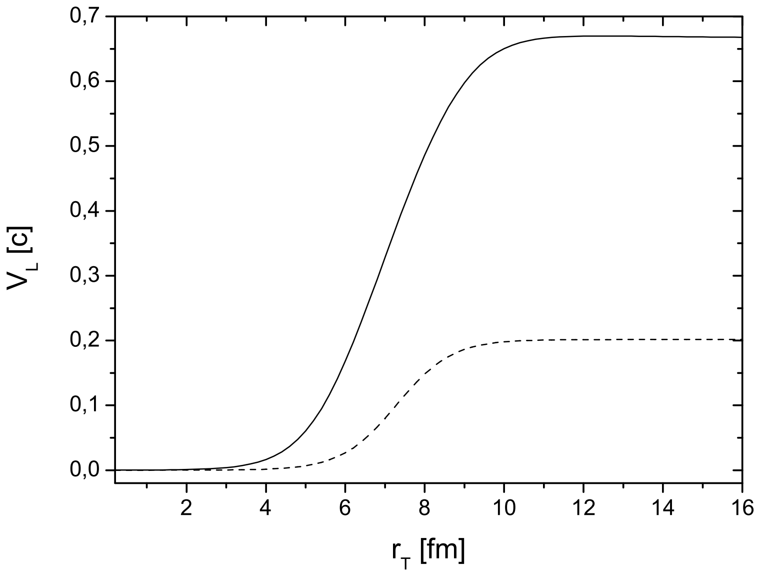

Figure 5. The Landau-Lifshitz collective transverse velocity in weakly interacting partonic system in the approximation of free streaming. The initial state at 1 fm/c is supposed to be quasi-thermal and corresponds to the distribution (2). Dashed curve correspond to τ=1.5 fm/c, solid line – 3 fm/c.

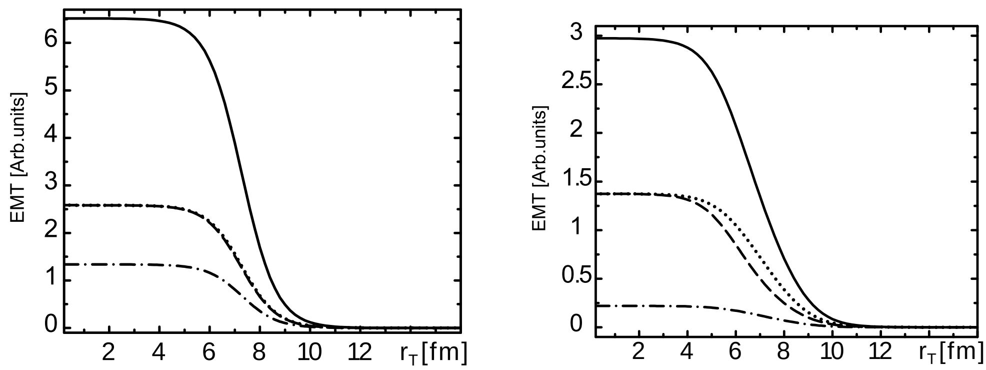

Figure 6. The components of the energy-momentum tensor, T_*^{tt} (solid), T_*^{xx} (dashed), T_*^{yy} (dotted), T_*^{zz} (dot-dashed), at τ =1.5, 3 fm/c (from left to right) and $\psi = 0$. Landau-Lifshitz co-moving frame.

where $E_* = \sqrt{m^2 + \mathbf{p}_*^2}$.

Limiting ourselves by the linear approximation in the parameters of anisotropy, when v is also supposed to be small and discarding the term of order $v\xi$, we find

$$f_* \approx f_{\rm iso}^* - W(r_{\rm T})\frac{dF(E_*)}{dE^*}p_*^x v + \frac{\xi(\tau)}{2}\left\{W(r_{\rm T})\frac{dF(E_*)}{dE_*}\frac{(p_*^z)^2}{E_*}\right.$$
$$\left. -F(E_*)\frac{dW(r_{\rm T})}{dr_{\rm T}}\frac{p_*^x E_* \tau_0}{m^2 + (\mathbf{p}_{\rm T}^*)^2}\left[1 - \left(\frac{p_*^z}{E_*}\right)^2\right]\right\}, \tag{20}$$

where $f_{\rm iso}^* \equiv F(E_*)W(r_{\rm T})$ is the isotropic distribution function in co-moving reference frame.

The radial collective velocity v is actually a dependent parameter. Following the Eckart definition,

$$v_E = \int p^x f(x,p)\frac{d^3p}{E} \bigg/ \int f(x,p)d^3p\,, \tag{21}$$

(where we put $\psi = 0$ again, and then the collective velocity direction coincides with x-axis), we get in linear approximation

$$v_E \approx \xi(\tau)\frac{\lambda_0}{\int f_{\rm iso}d^3p}, \tag{22}$$

where

$$\lambda_n = -\frac{\tau_0}{2}\int \frac{p_x^2 E^n F(E)}{m^2+\mathbf{p}_{\mathrm{T}}^2}\left(1-\frac{p_z^2}{E^2}\right)\frac{dW(r_{\mathrm{T}})}{dr_{\mathrm{T}}}d^3p. \tag{23}$$

Similar computations can be also performed to obtain the form of Landau-Lifshitz collective velocity in the linear approximation in ξ. The result looks like

$$v_{\mathrm{L}} \approx \xi(\tau)\frac{\lambda_1}{T_{tt}^{\mathrm{iso}}+T_{xx}^{\mathrm{iso}}}, \tag{24}$$

where T_{tt}^{iso} and T_{xx}^{iso} are the energy-momentum tensor components found on the basis of the isotropic distribution function $f_{\mathrm{iso}} \equiv F(E)W(r_{\mathrm{T}})$ in the global reference frame.

Now let us compare our results with the parametrization proposed by P. Romatschke with collaborators in Ref.[15]. It was assumed there that the anisotropic distribution function $h(\mathbf{p})$ is independent on space-time coordinates and constructed from an (arbitrary) isotropic distribution function by the rescaling of one direction in momentum space,

$$h(\mathbf{p}) = h_{\mathrm{iso}}(\sqrt{\mathbf{p}^2+\xi(\mathbf{np})^2}), \tag{25}$$

where $\mathbf{n}$ is the direction of anisotropy, $\xi > -1$ is a constant parameter reflecting the strength of anisotropy. We omit here the normalization constant $N(\xi)$ which was used in Ref.[15] as not relevant to our problem since the particle number conservation during the evolution from initially isotropic state is guaranteed by Eq. (3).

Expanding the distribution $h(\mathbf{p})$, in the linear approximation in ξ one can write that

$$h(\mathbf{p}) \approx h_{\mathrm{iso}}(p) + \xi\frac{dh_{\mathrm{iso}}(p)}{dp}\frac{(\mathbf{np})^2}{2p}. \tag{26}$$

It is easy to see that the expression (18) for central slice $z=0, v_z=0$ is reduced to the last formula (26) at $\xi = \tau^2/\tau_0^2 - 1$ and anisotropy vector $\mathbf{n}$ directed along z-axis in the particular case supposed in Ref.[15], namely, spatially homogeneous distribution, $W(r_{\mathrm{T}}) \equiv \mathrm{const}$, and massless particles, $E = |\mathbf{p}_*| \equiv p$.

In our inhomogeneous case, we can present linearized form (20) for distribution function in local rest frame associated with some point (x, $y=0$, $z=0$) as the following

$$f_*(\tau,\mathbf{x},\mathbf{p}_*) \approx f_{\mathrm{iso}}^*(\mathbf{x},|\mathbf{p}_*|) + \xi(\tau)g(\mathbf{x},\mathbf{p}_*). \tag{27}$$

5 Problem of the evolution at pre-thermal stage

As it was demonstrated in [14, 15] the *ansatz* (25), is useful for analytical studies of dispersion law and isotropisation driven by instabilities. The latter can be caused by momentum anisotropy in a system of ultrarelativistic electro- or color- charged particles. The expression (27) also can be utilized for this aim. However, in our case, when initial partonic system is supposed to be formed in pseudo-thermal state due to Schwinger production in the pulse of chromoelectric field, the problem is to estimate whether this state, first, preserve its (local) isotropy due to instability/turbulency mechanism and, then, if it transforms into true thermal state due to the interactions. As we see in previous Section, the anisotropy caused by a free expansion of the finite system into vacuum can be characterized in linear approximation by one parameter ξ, which is a function of proper time τ. One can estimate the possible rate of anisotropy growth at the following

$$R(\tau) \equiv \frac{1}{f^*(\tau)}\frac{df^*(\tau)}{d\tau} \simeq \frac{2\tau}{\tau_0^2}\frac{g^*}{f_{\mathrm{iso}}^*}. \tag{28}$$

Approximate equality is written down for the case $\tau/\tau_0 - 1 \ll 1$.

In order to maintain the initial isotropization of the partonic system during the evolution, it is obvious that the rate $R(\tau)$ should be smaller than $1/\tau_{\mathrm{iso}}$, where τ_{iso} is a characteristic, or relaxation time of isotropization driven by instability. This time τ_{iso} is planning to be estimated in forthcomming work, as well as rate of thermalisation due to color interaction described by Balescu-Lenard term in the kinetic equation.

In previous section we analize the developing of collective transverse flows in the finite non-thermal partonic system in the first free-streaming approximation. The results are presented in Figs. 3, 5. Unlike discussed in a Sec. 3 the specific non-relativistic case, where the initial isotropy in local rest frames of the distribution function is preserved during the further evolution, the ultra-relativistic evolution is not locally equilibrated, the distribution function and energy-momentum tensor become anisotropic in the local rest frames, and thus the development of the transverse velocities is not associated with hydrodynamics of ultrarelativistic gas. Nevertheless, as we demonstrate in Fig. 7, such a development of transverse velocities can be approximated by the hydrodynamic expansion with abnormal hard EoS: $P = 0.45\epsilon$ (“normal” upper limit $P = \epsilon/3$ has ultrarelativistic gas).

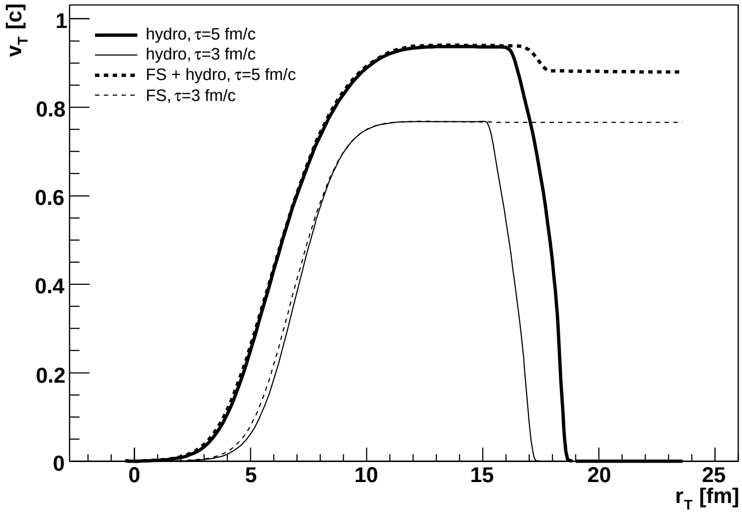

Figure 7. The simulation of the transverse collective velocities (according to Ekkart ≈ Landau-Lifshitz) of the quasi-free and almost massless partons within ideal hydrodynamics with the same initial conditions as for partonic system. The velocity is good approximated by such a hydro-evolution with extra-hard EoS $P = 0.45\epsilon$. Dashed curve corresponds to a weakly interacting partonic system at τ=3 fm/c and 5 fm/c, solid line - to hydro-evolution at corresponding proper times.

Therefore, it might be that a short thermalization time is not necessary for development of the observed radial flows. They can be developed, and even more effectively, at the pre-thermal or pseudo-thermal stage. The natural objection against such a scenario might mean the problem of not radial but the elliptic flows. They need earlier thermalization in order to the initial geometrical asymmetry in transverse plane transforms more effectively into momentum asymmetry. The pre-thermal transverse flows can smear out the asymmetry in momenta coming from the asymmetry in pressure gradients.

The solution of the problem could be an account for the residual – after the exclusion of the non-participants – a transversely directed angular momentum which the system of participants has just after collision due to a shift of the center of masses of colliding nuclei in reaction plane, that is associated with impact parameter [16]. Then, as it is shown in Ref.[17], the corresponding tilt in the major axis of longitudinal expansion gives positive contribution to the asymmetry of the particle momenta in transverse to beam plane, or in v_2 coefficient. The account for an interplay between the initial pre-thermal transverse velocity and the angular momentum which the system of participants obtains in non-central collisions can open the new way in an understanding of the problem of matter evolution in nucleus-nucleus collisions.

6 Conclusions

The approximate conservation of the pion averaged phase-space density (APSD) in A+A collisions during the hadronic stage of the evolution allows one to explain proportionality between interferometry volume and different initial volumes, e.g., in non-central collisions, and also explain the relative independence of the interfometry volumes on energy in central Au+Au and Pb+Pb collisions by an increase of transverse flows with energy. The hydrodynamic picture with initially non-zero transverse flow can help in description of the latter effect.

The another component of the RHIC HBT puzzle - the relatively small ratio of outward to sideward interferometry radii at protracted surface emission also needs in intensive initial transvere flows for its explanation. The reason is that predominantly positive space-time $(r - t)$ correlations for emission points, which reduce the outward radius, can be realized only in hydrodynamic picture with strong enough transverse flow at initial moment.

We demonstrate here that the intensive transverse flows can be developed at the very early pre-thermal partonic stage when many hidden degrees of freedom, associated with incoherent partons, are liberated. It is shown that the development of the transverse velocities at pre-thermal partonic stage can be approximated by the hydrodynamic expansion with abnormal hard equation of state. The interplay of those flows and angular momenta, which the system get in non-central collisions, could lead to new scenario of the matter evolution and help to describe the experimental data in central and non-central A+A collisions.

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