

# Study of the $d(d, p)^3\text{H}$ and $d(d, n)^3\text{He}$ processes at low energies

M. Viviani<sup>1</sup>, L. Girlanda<sup>2,3</sup>, A. Kievsky<sup>1</sup>, D. Logoteta<sup>4</sup>, and L.E. Marcucci<sup>1,4</sup>

<sup>1</sup> INFN-Pisa, Largo B. Pontecorvo 3, I-56127, Pisa, Italy

<sup>2</sup> Department of Mathematics and Physics, Un. of Salento, Via Arnesano, I-73100 Lecce, Italy

<sup>3</sup> INFN-Lecce, Via Arnesano, I-73100 Lecce, Italy

<sup>4</sup> Department of Physics "E. Fermi", Un. of Pisa, Largo B. Pontecorvo 3, I-56127, Pisa, Italy

E-mail: [michele.viviani@pi.infn.it](mailto:michele.viviani@pi.infn.it)

**Abstract.** The processes  $d(d, p)^3\text{H}$  and  $d(d, n)^3\text{He}$  at energies of interest for energy production and for big-bang nucleosynthesis are studied using the hyperspherical harmonic method. The interactions include modern two- and three-nucleon interactions, derived in chiral effective field theory. We report results for the astrophysical S-factor and the quintet suppression factor.

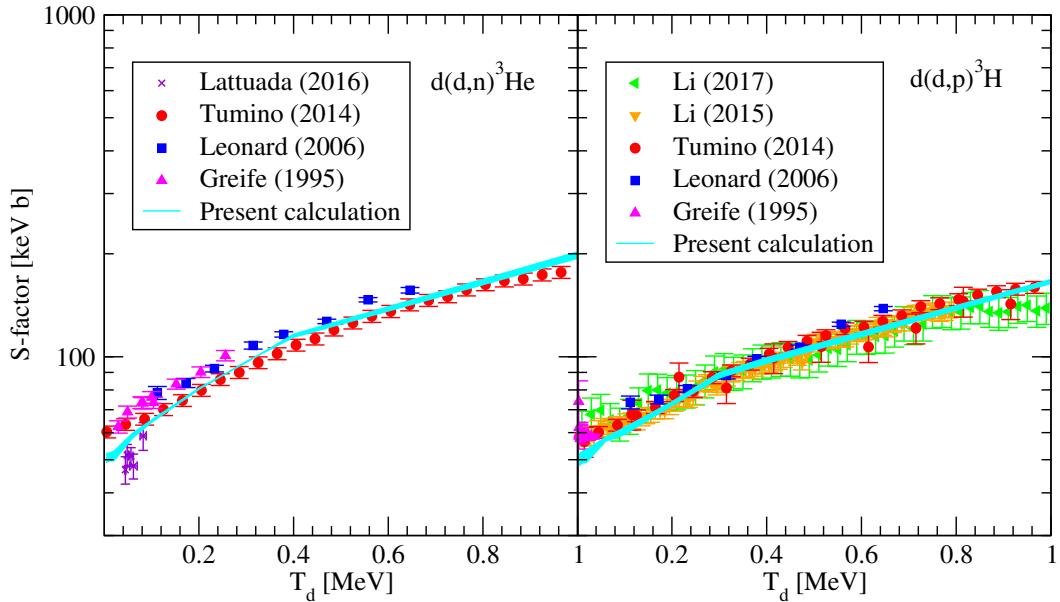
## 1. Introduction

The fusion reactions  $d(d, p)^3\text{H}$  and  $d(d, n)^3\text{He}$  play a crucial role in our understanding of Big-Bang nucleosynthesis (BBN) and hold significance for new fusion reactor designs. Presently, the uncertainties in predicting the deuteron abundance [D/H] in BBN models are largely attributed to the lack of precise knowledge concerning the astrophysical S-factor  $S(E)$  for these processes [1, 2]. Furthermore, there has been speculation about the possibility of reducing the rates of  $d(d, p)^3\text{H}$  and  $d(d, n)^3\text{He}$  reactions by preparing initial deuterons with parallel spins, also known as the "quintet" spin state [3, 4, 5, 6]. This interest arises from the possible construction of "neutron lean reactors" with a  $d + ^3\text{He}$  plasma, which could produce energy via the  $d + ^3\text{He} \rightarrow p + ^4\text{He}$  reaction while aiming to minimize the number of neutrons produced by the  $d + d \rightarrow n + ^3\text{He}$  reaction. The suppression of the  $\vec{d}(\vec{d}, n)^3\text{He}$  (and  $\vec{d}(\vec{d}, p)^3\text{H}$ ) rate is expected when the capture occurs in an S-wave, as this process requires a spin-flip to produce either a  $^3\text{H}$  or  $^3\text{He}$  nucleus, which is generally suppressed. However, this argument does not account for the presence of the deuteron D-state or the possibility of capture in P- and D-waves, which have been found to give significant contributions, especially at low energy [4]. The importance of P- and D-waves can be understood by considering the large extension of deuteron wave functions, which remain sizable even at interparticle distances of 6 fm. Consequently, the two entrance particles interact at relatively large impact parameters.

Measurements of the total cross section (or astrophysical S-factor), unpolarized differential cross section, as well as some vector and tensor analyzing observables, have been conducted at deuteron beam energies  $T_d < 100$  keV (see, for example, Ref. [7]). Theoretical calculations, such as those obtained from the solution of the Faddeev-Yakubovsky (FY) equations [8, 9] and using the Correlated Gaussian method [10, 11], have been reported in the literature. Other



Content from this work may be used under the terms of the [Creative Commons Attribution 3.0 licence](#). Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.



**Figure 1.** (color online) The astrophysical S-factor for the processes  $d(d, n)^3\text{He}$  (left panel) and  $d(d, p)^3\text{H}$  (right panel) calculated with the N3LO500/N2LO500 and N3LO600/N2LO600 interactions. The width of the bands reflects the spread of theoretical results using  $\Lambda = 500$  or  $600$  MeV cutoff values. See the main text for more details. The experimental values are from Refs. [30, 29, 32, 33, 34, 35].

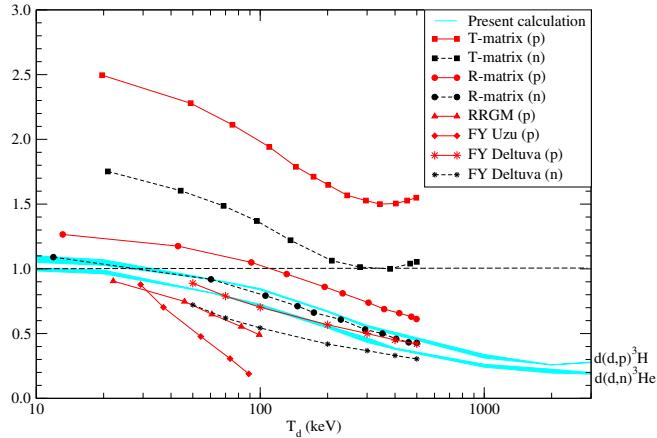
calculations can be found in Refs. [12, 13, 14, 15]. Additionally, accurate calculations of the  $^3\text{H}(d, n)^4\text{He}$  fusion have been achieved using the No-Core Shell Model method [16].

In this paper, we investigate the mentioned processes utilizing the hyperspherical harmonics (HH) expansion method [19, 20, 7]. For our study, we consider potentials based on chiral nucleon-nucleon (NN) interactions, specifically derived at next-to-next-to-next-to-leading order (N3LO) by Entem and Machleidt [21, 22]. These potentials are characterized by a cutoff  $\Lambda$  of 500 and 600 MeV. Additionally, we incorporate a chiral three-nucleon (3N) interaction, obtained at next-to-next-to-leading order (N2LO) according to Refs. [23, 24].

The N2LO 3N potential includes two free parameters typically referred to as  $c_D$  and  $c_E$ . We adjust these parameters to reproduce the experimental values of the binding energies for  $A = 3$  nuclei and the Gamow-Teller matrix element (GTME) related to the tritium  $\beta$  decay [25, 26, 27, 28]. We label the resulting interactions as N3LO500/N2LO500 and N3LO600/N2LO600, respectively. The two values for the cutoff parameter can be taken as representatives of the breakdown scale of the chiral expansion, i.e. to physics scales unresolved by the effective theory.

## 2. Results

We report the calculated S-factors in Fig. 1, where they are compared with recent experimental data [29, 32, 35]. The calculations have been performed using the N3LO500/N2LO500 and N3LO600/N2LO600 interactions and the results are shown as bands, their width reflecting the spread of theoretical results using  $\Lambda = 500$  or  $600$  MeV cutoff values. As it can be seen from the figure, the calculations correctly reproduce the energy dependence of the data. The astrophysical S-factor for  $d(d, n)^3\text{He}$  results to be larger than that of  $d(d, p)^3\text{H}$  for  $T_d > 0.2$  MeV. The calculations are well in agreement with the data of Ref. [32], whereas the data of Ref. [29] are slightly underpredicted, especially at low energy.



**Figure 2.** (color online) The QSF for the processes  $d(d, n)^3\text{He}$  and  $d(d, p)^3\text{H}$  shown as bands, in analogy of Fig. 1. We also report the results obtained with other theoretical approaches: T-matrix [41]; R-matrix [29]; RRGM [12, 13]; FY Uzu [15]; FY Deltuva [8]. The red solid [black dashed] lines connecting the red [black] symbols denote the QSF calculated in the literature for the  $d(d, p)^3\text{H}$  [ $d(d, n)^3\text{He}$ ] reaction.

Next we consider the quintet suppression factor (QSF). Defined  $\sigma_{11}$  ( $\sigma$ ) as the total cross section for both deuterons polarized along the beam direction (the total unpolarized cross section), then  $\text{QSF} = \sigma_{11}/\sigma$ . We report the calculated QSF in Fig. 2, together with other theoretical estimates obtained using various methods [41, 12, 13, 15, 29, 8]. As it can be seen, our calculations agree fairly well with the results of the FY calculation of Ref. [8] and with those obtained from the R-matrix analysis reported in Ref. [29]. Therefore, the trend with energy appears to be well consolidated: the QSF is close to unity at small energies and then slowly decreases. At  $T_d = 1$  MeV, it reaches a sort of plateau. These findings are at variance, however, with what found by other analyses [41, 12, 13, 15]. Clearly an accurate measurement of the QSF would be welcome, in order to clarify this issue.

### 3. Conclusions

In this study, we have investigated the  $d(d, p)^3\text{H}$  and  $d(d, n)^3\text{He}$  processes at energies relevant for Big-Bang nucleosynthesis (BBN) and energy production in fusion reactors. The results of our calculations are presented as bands, the width of which serves as a preliminary estimate of the theoretical uncertainty associated with our limited understanding of nuclear dynamics. In our calculations such bands reflect the variation between theoretical results obtained using the two cutoff parameter values,  $\Lambda = 500$  and  $600$  MeV, for the nuclear interaction. Taking into account the width of the bands, we observe a good agreement between the theoretical results and the data. Specifically, the  $d(d, p)^3\text{H}$  [ $d(d, n)^3\text{He}$ ] astrophysical S-factor at zero energy is estimated to be  $S(0) = 50.8 \pm 1.9$  keV b [51.0  $\pm$  1.4 keV b]. Our ongoing analysis aims to explore the implications of these values for cosmological models. Concerning the QSF, we are currently investigating the potential impact of our study on the design of new reactors.

In future work, we plan to conduct a more precise assessment of the theoretical uncertainties, particularly by employing the new  $\chi$ EFT interactions derived up to next-to-next-to-next-to-next-to-leading order [42] and utilizing the procedures from Refs. [38, 39, 40]. Additionally, we intend to investigate changes in fusion rates induced by the presence of strong high-frequency electromagnetic fields. There have been suggestions that Coulomb barrier penetrability could significantly increase in certain configurations when exposed to such fields [43, 44, 45].

*Acknowledgements* The calculations were made possible by grants of computing time from

the Italian National Supercomputing Center CINECA and from the National Energy Research Supercomputer Center (NERSC). We also gratefully acknowledge the support of the INFN-Pisa computing center. D.L. acknowledges the support of ACTA Srl, and in particular of its CEO Dr. Eng. Davide Mazzini. This article is supported by the Ministry of University and Research (MUR) as part of the PON 2014-2020 “Research and Innovation” resources – Green Action - DM MUR 1062/2021 of title “Study of nuclear reactions of interest for the “green” energy production from nuclear fusion”.

## References

- [1] T. H. Yeh, K. A. Olive, and B. D. Field, *JCAP* **03**, 046 (2021)
- [2] O. Pisanti, G. Mangano, G. Miele, and P. Mazzella, *JCAP* **04**, 020 (2021)
- [3] R. M. Kulsrud, H. P. Furth, E. J. Valeo, and M. Goldhaber, *Phys. Rev. Lett.* **49**, 1248 (1982)
- [4] H. Paetz gen. Schieck, *Eur. Phys. J. A* **44**, 321 (2010)
- [5] G. Ciullo, *Springer Proc. in Phys.* **187**, 1 (2016)
- [6] H. Paetz gen. Schieck, *Springer Proc. in Phys.* **187**, 15 (2016)
- [7] M. Viviani, L. Girlanda, A. Kievsky, D. Logoteta, and L. E. Marcucci, *Phys. Rev. Lett.* **130**, 122501 (2023)
- [8] A. Deltuva and A. C. Fonseca, *Phys. Rev. C* **81**, 054002 (2010)
- [9] A. Deltuva and A. C. Fonseca, *Phys. Rev. C* **95**, 024003 (2017)
- [10] K. Arai, S. Aoyama, Y. Suzuki, P. Descouvemont, and D. Baye, *Phys. Rev. Lett.* **107**, 132502 (2011)
- [11] S. Aoyama, K. Arai, Y. Suzuki, P. Descouvemont, and D. Baye, *Few-Body Syst.* **52**, 97 (2012)
- [12] H.M. Hofmann and D. Fick, *Phys. Rev. Lett.* **52**, 2038 (1984)
- [13] D. Fick and H.M. Hofmann, *Phys. Rev. Lett.* **55**, 1650 (1985)
- [14] J. S. Zhang, K. F. Liu, and G. W. Shuy, *Phys. Rev. Lett.* **57**, 1410 (1986)
- [15] E. Uzu [arXiv:nucl-th/0210026](https://arxiv.org/abs/nucl-th/0210026)
- [16] G. Hupin, S. Quaglioni, and P. Navrátil, *Nat. Commun.* **10**, 351 (2019)
- [17] K. Grigoryev *et al.* *J. Phys.: Conf. Ser.* **295** 012168 (2011)
- [18] A. Solovev *et al.*, *JINST* **15**, C08003 (2020)
- [19] L.E. Marcucci, J. Dohet-Eraly, L. Girlanda, A. Gnech, A. Kievsky, and M. Viviani, *Frontiers in Physics* **8**, 69 (2020)
- [20] M. Viviani, L. Girlanda, A. Kievsky, and L. E. Marcucci, *Phys. Rev. C* **102**, 034007 (2020)
- [21] D.R. Entem and R. Machleidt, *Phys. Rev. C* **68**, 041001(R) (2003)
- [22] R. Machleidt and D.R. Entem, *Phys. Rep.* **503**, 1 (2011)
- [23] E. Epelbaum *et al.*, *Phys. Rev. C* **66**, 064001 (2002)
- [24] P. Navrátil, *Few-Body Syst.* **41**, 117 (2007)
- [25] A. Gardestig and D.R. Phillips, *Phys. Rev. Lett.* **96**, 232301 (2006)
- [26] D. Gazit, S. Quaglioni, and P. Navrátil, *Phys. Rev. Lett.* **103**, 102502 (2009)
- [27] L. E. Marcucci, F. Sammarruca, M. Viviani, and R. Machleidt, *Phys. Rev. C* **99**, 034003 (2019)
- [28] L. E. Marcucci, A. Kievsky, S. Rosati, R. Schiavilla, and M. Viviani, **121**, 049901(E) (2018)
- [29] D.S. Leonard, H.J. Karwowski, C.R. Brune, B.M. Fisher, and E.J. Ludwig, *Phys. Rev. C* **73**, 045801 (2006)
- [30] U. Greife *et al.*, *Z. Phys. A* **351**, 107 (1995)
- [31] T.-S. Wang *et al.*, *Chinese Phys. Lett.*, **24**, 3103 (2007)
- [32] A. Tumino *et al.*, *The Astrophys. J.* **785**, 96 (2014)
- [33] C. Li *et al.*, *Phys. Rev. C* **92**, 025805 (2015)
- [34] C. Li *et al.*, *Phys. Rev. C* **95**, 035804 (2017)
- [35] D. Lattuada *et al.*, *Phys. Rev. C* **93**, 045808 (2016)
- [36] K. A. Fletcher *et al.*, *Phys. Rev. C* **49**, 2305 (1994)
- [37] R. J. Furnstahl, D. R. Phillips and S. Wesolowski, *J. Phys. G: Nucl. Part. Phys.* **42**, 034028 (2015)
- [38] R. J. Furnstahl, N. Klco, D. R. Phillips, and S. Wesolowski, *Phys. Rev. C* **92**, 024005 (2015)
- [39] S. Wesolowski, N. Klco, R. J. Furnstahl, D. R. Phillips, and A. Thapaliya, *J. Phys. G: Nucl. Part. Phys.* **43**, 074001 (2016)
- [40] S. Wesolowski *et al.*, *Phys. Rev. C* **104**, 064001 (2021)
- [41] S. Lemaitre and H. Paetz gen. Schieck, *Few-Body Syst.* **9**, 155 (1990); *Ann. Phys.* **2**, 503 (1993)
- [42] D.R. Entem, R. Machleidt, and Y. Nosyk, *Phys. Rev. C* **96**, 024004 (2017)
- [43] F. Queisser and R. Schützhold, *Phys. Rev. C* **100**, 041601(R) (2019)
- [44] W. J. Lv, H. Duan, and J. Liu, *Phys. Rev. C* **100**, 064610 (2019)
- [45] W. J. Lv, B. Wu, H. Duan, S. W. Liu, and J. Liu, *Eur. Phys. J. A* **58**, 54 (2022)