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SUPERGRAVITY ON ORBIFOLD SPACETIMES :

FROM PHENOMENOLOGY TO $\mathcal{M}$-THEORY

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Sean McReynolds

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The thesis of Sean McReynolds was reviewed and approved* by the following:

Murat Günaydin
Professor of Physics
Thesis Adviser
Chair of Committee

Stephane Coutu
Associate Professor of Physics
Associate Professor of Astronomy and Astrophysics

Richard W. Robinett
Professor of Physics

Luen-Chau Li
Associate Professor of Mathematics

Jayanth Banavar
Professor of Physics
Head of the Department of Physics

*Signatures are on file in the Graduate School.

Abstract

Five-dimensional $\mathcal{N} = 2$ Yang-Mills-Einstein supergravity and its couplings to hyper and tensor multiplets are considered on an orbifold spacetime of the form $\mathcal{M}_4 \times S^1/\Gamma$, where Γ is a discrete group. As is well known in such cases, supersymmetry is broken to $\mathcal{N} = 1$ on the orbifold fixed planes, and chiral $4D$ theories can be obtained from bulk hypermultiplets (or from the coupling of fixed-plane supported fields). Five-dimensional gauge symmetries are broken by boundary conditions for the fields, which are equivalent to some set of Γ-parity assignments in the orbifold theory, allowing for arbitrary rank reduction. Furthermore, Wilson lines looping from one boundary to the other can break bulk gauge groups, or give rise to vacuum expectation values for scalars on the boundaries, which can result in spontaneous breaking of boundary gauge groups. The broken gauge symmetries do not survive as global symmetries of the low energy theories below the compactification scale due to $4D$ minimal couplings to gauge fields. Axionic fields are a generic feature, just as in any compactification of M-theory (or string theory for that matter), and we exhibit the form of this field and its role as the QCD axion, capable of resolving the strong-CP problem. The main motivation for the orbifold theories here is taken to be orbifold-GUTs, wherein a unified gauge group is sought in higher dimensions while allowing the orbifold reduction to handle problems such as rapid proton decay, exotic matter, mass hierarchies, etc. To that end, we discuss the allowable minimal $SU(5)$, $SO(10)$ and E_6 GUT theories with all fields living in five dimensions. It is argued that, within the class of homogeneous quaternionic scalar manifolds characterizing the hypermultiplet couplings in $5D$, supergravity admits a restricted set of theories that yield minimal phenomenological field content. In addition, non-compact gaugings are a novel feature of supergravity theories, and in particular we consider the example of an $SU(5,1)$ YMESGT in which all of the fields of the theory are connected by local (susy and gauge) transformations that are symmetries of the Lagrangian. Such non-compact gaugings allow a novel type of gauge-Higgs unification in higher dimensions. The possibility of boundary-localized fields is considered only via anomaly arguments. In particular, the theories with a $5D$ Chern-Simons form in the Lagrangian will give rise to anomaly inflow classically (which is compensated globally in the $S^1/\mathbb{Z}_2$ case). However, compensation locally requires the quantum theory to have a chiral anomaly on the boundary, which can arise if there is an appropriate bulk fermionic field content, or otherwise by the addition of appropriate boundary-supported fermionic fields with minimal coupling to the gauge fields

propagating there. Some comments are made regarding the phenomenological features of the models, such as the Yukawa couplings and scalar potentials, which depend on the size of the fifth dimension as well as the scalar vacuum. Finally, we sketch the possible M-theoretic origins of these theories, which is left for future work. In particular, the supergravity orbifold theories generally correspond to phases of M-theory distinct from the strongly coupled heterotic string, which has served as the primary phenomenological framework in the past. Throughout, we try to compare and contrast with the phenomenological model building of rigid susy orbifold-GUTs as well as the string/M-theoretic approaches that have dominated the literature for the past decade.

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Notation/Conventions

In this thesis, we will adopt the following conventions:

Symbol/Abbreviation	Meaning
SM	Standard Model
MSSM	Minimal Supersymmetric Standard Model
GUT	Grand Unified Theory
CY	Calabi-Yau
HW	Horava-Witten
RS	Randall-Sundrum
MESGT	Maxwell-Einstein Supergravity Theory
YMESGT	Yang-Mills-Einstein Supergravity Theory
$\mathcal{M}_n$	Generic (smooth) n-dimensional spacetime manifold
$\mathcal{M}_R$	Real Riemannian scalar manifold
$\mathcal{M}_Q$	Quaternionic scalar manifold
$Iso(\mathcal{M})$	Isometry group of the manifold $\mathcal{M}$
G	Global symmetry group of a $5D$ YMESGT
K	$5D$ gauge group
$K_{(\alpha)}$	$4D$ boundary gauge group
$k_{(\alpha)}$	Lie algebra for $K_{(\alpha)}$
$t^{(\hat{a})}$	Algebra of $K_{(\alpha)}$-non-singlets in $K/K_{(\alpha)}$
$t^{(\tilde{a})}$	Algebra of $K_{(\alpha)}$ singlets in $K/K_{(\alpha)}$
C_{IJK}	Rank-3 symmetric G-invariant tensor

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Dedicated To The Memory Of My Mother

Chapter 1

Introduction

The Standard Model of particle physics (SM) that was formulated by the mid-1970s represents a remarkable collision of theory and experiment. It has been tested to great precision since then, and for the most part has held firm. The model can be summarized in a tidy fashion, hiding the great amount of understanding that is contained within it. It is a quantum field theory with spontaneously broken $SU(3)_c \times SU(2)_L \times U(1)_Y$ local symmetry, which manifests itself dynamically as propagating gauge fields (photons, gluons, W, B') coupled to three copies of the following leptons and quarks. Table (1.1) is a list of the first generation of quarks and leptons distinguished by their representations under the SM gauge group.

Field	$SU(3) \times SU(2) \times U(1)$ rep
$(e^-, \nu_e)_L$	$(1, 2, -1)$
e_R^-	$(1, 1, -2)$
$(u, d)_L$	$(3, 2, 1/3)$
u_R	$(3, 1, 4/3)$
d_R	$(3, 1, -2/3)$

Table 1.1: First generation of Standard Model fermions and their representations

There are three independent couplings of $SU(3)_c \times SU(2)_L \times U(1)_Y$, which we can parametrize by g_c, $\tan\theta_W = g_Y/g_L$, $e = g_L g_Y/\sqrt{(g_L^2 + g_Y^2)}$, where g_c is the $SU(3)_c$ strong coupling; θ_W is called the weak mixing (or Weinberg) angle; and e is the electromagnetic coupling of $U(1)_{em}$.

There are a few tagged-on items to this model: (1) there must be a mechanism for the spontaneous breakdown of $(SU(2) \times U(1))_{ew}$ to $U(1)_{em}$, which is popularly taken to be due to a scalar (Higgs) field with an appropriate potential. These scalars must be in the real $\mathbf{2} \oplus \bar{\mathbf{2}}$ of $SU(2)_L$.[1] (2) Neutrinos were found to have a mass, which means that the SM must

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[1] See appendix for notational conventions.

be modified to allow for this. This can naturally arise from the existence of a $(\nu_e)_R$ field with the same quantum numbers as $(\nu_e)_L$ after electroweak (EW) symmetry breaking to the gauge group $SU(3)_c \times U(1)_{em}$.

Even as the model came into shape in the 1970s, the arbitrariness of the ~ 20 independent parameters was questioned. In light of the fact that next-generation colliders such as the Large Hadron Collider (LHC) will involve dramatically higher energies and luminosities than in the past decades, these questions are more relevant than ever. Perhaps the most suggestive features pointing to physics beyond the Standard Model at the time of its formulation were the phenomenological assignments of representations of the SM gauge group to the quarks and leptons (table (1.1)). After the work of Pati and Salam in 1974 [PS74], Georgi and Glashow showed [GG74] that one family of left-handed leptons and quarks could be grouped into the reducible representation $\bar{\mathbf{5}} \oplus \mathbf{10}$ of $SU(5)$ (which contains $SU(3) \times SU(2) \times U(1)$ as a maximal compact subgroup). This was a hint that perhaps the three known gauge interactions were part of a hidden simple gauge group such as the $SU(5)$ they considered; the three independent gauge couplings of the SM would be reduced to one. Electroweak-strong unification scenarios in which the SM gauge group is embedded in a larger simple gauge group (that is unbroken at energies somewhere above $\mathcal{O}(100)GeV$ are now popularly known as Grand Unified Theories (GUTs).

While GUTs reduce the number of free parameters of the Standard Model, they introduce issues of their own that must be addressed if they are to be taken seriously. For example, in the Georgi-Glashow model, the Higgs scalars of the Standard Model must fall into representations of $SU(5)$; the minimal way this can be done is by introducing additional scalars and putting them all into the $\mathbf{5} \oplus \bar{\mathbf{5}}$. The scalars form weak doublets and weak triplets under the SM gauge group. Since only the doublets should be involved in the Standard Model, this introduces the "doublet-triplet" splitting problem: how are the remaining scalars decoupled from the low energy theory (at the electroweak breaking scale M_{ew})? Furthermore, these color triplets, along with the new gauge bosons of the larger $SU(5)$ gauge group, allow for decay of the proton. Searches for such decays have resulted in a lower bound on the proton lifetime of up to 2.3×10^{33} years at 90% confidence level (this is for the decay $p \to \bar{\nu} K^+$) [K05]; this in turn constrains the GUT models one can construct.

Soon after the $SU(5)$ model, GUTs based on the simple compact gauge groups $SO(10)$ and E_6 were proposed. Georgi [G75], and Fritzsch with Minkowski [FM75], showed that the left-handed quarks and leptons of one generation could sit in the irreducible $\mathbf{16}$ of an $SO(10)$ gauge group by introducing a left-handed fermionic SM singlet. This extra field can be identified as a left-handed anti-neutrino, which is necessarily accompanied by a right-
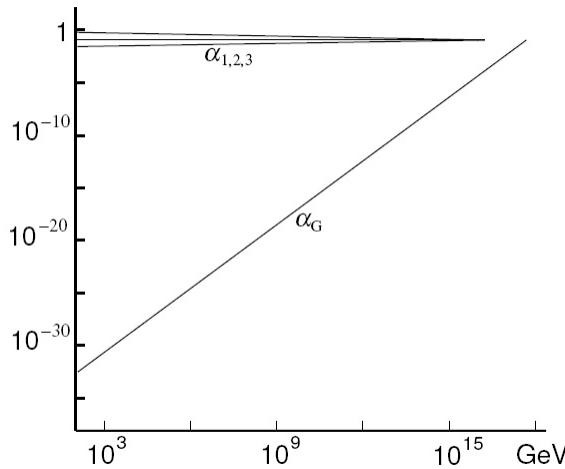
Figure 1.1: Within the framework of the MSSM, the renormalization group flow of the dimensionless $SU(3) \times SU(2) \times U(1)$ gauge couplings α_3, α_2 and α_1, and the dimensionless gravitational coupling $\alpha_G = \kappa E^2$, are plotted vs. energy in GeV. Figure is from [P99].

handed neutrino. Again, after introducing additional scalars, the Higgs scalars can sit in the real **10** of $SO(10)$. Next, Gursey, Ramond and Sikivie[GRS76] proposed a model with E_6 gauge group under which one generation of left-handed fermions, including two new SM-singlets, and the Higgs fields in the **10** of $SO(10)$ form the irreducible **27**. In these two theories, the presence of a right-handed neutrino and left-handed anti-neutrino allows one to form a light neutrino that's charged under the weak interaction, and a more massive (but unobserved) neutrino that is a SM singlet. As a downside to these models, an additional, large Higgs sector must be introduced if the GUT groups are to be broken spontaneously.

It became apparent in the late 1970s that a number of attractive features could be brought to the table if spacetime supersymmetry (susy) was included in a theory of particle physics. In GUTs without supersymmetry, the value of the weak mixing angle (in the form of $\sin^2 \theta_W$) can be predicted [DRW81]; it is suggestively close to, but nevertheless discordant with, the experimentally determined value. The presence of spacetime supersymmetry modifies the assumption of gauge coupling unification leads to prediction of $\sin^2 \theta_W$ such that it is in better agreement with experiment. The running of the dimensionless gauge couplings is shown in figure (1.1). Supersymmetry also offers a partial resolution to an important question that arises in electroweak theory: how is the mass of the Higgs scalar characterized by the electroweak scale M_{ew}, when it should receive radiative corrections on the order of larger scales (such as a GUT scale M_U and ultimately the Planck scale M_P)? This Higgs naturalness

(or gauge hierarchy) issue is partially resolved as follows: as long as supersymmetry is unbroken, the massless Higgs scalars sit in supermultiplets that are protected from receiving radiative mass corrections; once supersymmetry is broken, the remaining (massive) Higgs scalar receives radiative corrections to its mass that are bounded by the supersymmetry breaking scale. For the issue to be completely resolved, however, one must determine the mechanism for low energy supersymmetry breaking that does not involve the naturalness issue in a different guise.

The search for GUTs left out the question of gravitation in part due to the difficulties surrounding this interaction; but also due to the fact that the predicted unification scale in $4D$ field theory GUTs is $10^{16} GeV$, which is several orders of magnitude below the usual "quantum gravity" scale, so that one may assume that quantum gravitational effects are negligible. But a supersymmetric theory can only be coupled to gravitation if the supersymmetry is made local, yielding supergravity [SW78, FvN78, CJSvNFG78, CJSFGvN79, CFGvP82, CFGvP83]. Supergravity was once a promising candidate for a renormalizable or even finite quantum field theory of gravity coupled to other gauge and matter fields. However, it became apparent that any theory whose involvement with gravitation was restricted to simply an Einstein-Hilbert term in the Lagrangian would be non-renormalizable, even with the softening of divergences among particles and superparticles [DKS77, DS99, D99].

The original idea of the unification of seemingly distinct interactions came in the form of Kaluza-Klein unification of classical electromagnetism and gravitation: the two interactions in four dimensions could be realized by general relativity in five spacetime dimensions. Supergravity in $D > 4$ dimensions does not embody this type of unification since there are higher dimensional fields in addition to the spacetime metric. However, the $11D$ supergravity [CJS78] gets closest to embodying this idea: it is a unique theory [D97b] without vector gauge fields consisting of a gravity supermultiplet (graviton, gravitino, and 3-form field). Not all of the vector fields upon compactification come from gravitation (i.e., the metric), but it is close: they come from the fields that are related by supersymmetry transformations to gravity.

Finding a phenomenologically interesting theory in four dimensions from $11D$ supergravity proved difficult historically. For example, the $SO(8)$ gauge group obtained from round sphere compactification of $11D$ supergravity on $M_4 \times S^7$ [CJ79] is not large enough to contain the Standard Model gauge group. One needed a way to generate larger non-abelian gauge symmetry, but smooth 7-manifolds with large isometry groups, and preserving minimal susy, cannot give rise to a chiral theory of fermions in SM representations. It turns out that many phenomenologically interesting spaces yielding large gauge symmetry are those with small

or trivial isometry groups. But the idea behind this would require string theory.

There is an argument that simplicity alone is not sufficient for explaining the absence of additional terms in a microscopic Lagrangian (see e.g. [W96b]). That is, if one leaves out particular terms consistent with the low energy symmetries of nature, one must explain why those particular terms are not present. For example, there are higher order interaction terms one can add to the Einstein-Hilbert action that preserve the general coordinate invariance of spacetime. String theory is an example of such a generalization of spacetime quantum field theory: one can tentatively view it as an infinite expansion in higher order interaction terms of the spacetime curvature and other fields, whose explicit form is not known. Obtaining a proper effective field theory from a string theory (by integrating out states with string-scale masses) appears to be difficult. However, supergravity appears as the *truncation* of the field expansion of string theory to the *massless string states* at the *string tree level*. In this sense, supergravity theories are generically the low energy approximation of superstring theories.

In 1984, Green and Schwarz showed that the gravitational anomalies of perturbative superstring theory could cancel, leading to a consistent quantum theory of interacting strings [GS84, GS85a, GS85b]. Furthermore, due to the nature of string interactions in spacetime, perturbative string theory seems to be finite at each order in the topological expansion in powers of the string coupling α'.[2] Thus, the perturbative superstring theory became a serious candidate to describe a quantum theory of gravitation, as well as of matter and the other interactions. It was soon shown that there were only five distinct perturbative superstring theories, which were consistent only in ten spacetime dimensions (consistent in the sense that all of the possibly troublesome anomalies in local symmetries vanished).

To get from $10D$ string theory to $4D$ semi-realistic scenarios, it was originally noted that the $10D$ ground state spacetime could be $\mathcal{M}^4 \times Y$, where Y is a complex 3-manifold with vanishing first Chern class (i.e., with holonomy $SU(3)$). Such a manifold Y is called a Calabi-Yau manifold, and is chosen so that $\mathcal{N} = 1$ supersymmetry is preserved in the low energy $4D$ theory. Such a compactification of the heterotic superstring theory with $E_8 \times E_8$ gauge group led to semi-realistic models in that, for example, one could find a Standard Model gauge group, and one could show how to get several generations of matter particles as a topological property of the internal space [W85, CHSW85, DHVW85, DHVW86]. However, there remain a number of unresolved issues, and the full Standard Model has not been obtained. In addition, the four dimensional Newtonian constant G_N comes out orders of magnitude too large (assuming Y is isotropic and the volume is $V \sim M_{GUT}^{-6}$, the prediction

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[2]However, the full perturbation series diverges; it is not Borel summable.

is $G_N \sim 10^{-35} GeV^{-1}$ compared to the observed $G_N \sim 10^{-38} GeV^{-1}$).[3]

Of course, perturbative S-matrices are not all there is to physics; there is non-perturbative phenomena that cannot be captured by perturbative calculations. (There have been some results in rigid supersymmetric theories regarding the extraction of non-perturbative physics from perturbative calculations, though [DV02].) Classical general relativity, and presumably any extension involving higher order interactions, is a background-field-independent theory: given a suitable topology of spacetime, one solves a set of equations to find the metric on this space. Perturbative string theory does not yet satisfactorily address this issue, though work in string field theory has made serious in-roads [S89, S90a, S90b, S90c, S93]. It would be surprising if some generic results from the quantum geometry program [A02, A03] were not mirrored in a proper non-perturbative formulation (or generalization) of string theory. These results are, afterall, simply a consequence of constructing a background-field-independent quantum theory of spacetime geometry. One notable feature of string theory is that it has provided a way of introducing topological and geometric transitions as well as allowing particular classes of singular spacetime geometries. String theory does seem to have the feature that background geometry is in the eye of the beholder: using strings as probes allows for interpretation in terms of different geometries. This is reflected in early studies of string dualities [DHVW85, DHVW86].

While perturbative string theories are consistent only in ten dimensions, there is a (unique) $11D$ supergravity theory. It turns out that the strong coupling limit of some superstring theories leads to an eleven dimensional theory, M-theory, whose weakly coupled low energy description is the $11D$ supergravity [HW96a, HW96b, W96a] (the other string theories have each other as strongly coupled limits). Below we provide some of the weak-strong coupling duality relationships between the heterotic $E_8 \times E_8$ string theory and M-theory by listing the internal manifolds that lead to dual theories. For reviews, see [S97a, M99].

Heterotic	M-theory
X	$X \times S^1/\mathbb{Z}_2$
T^n	$K3 \times T^{n-3}$
$T^3 \times Q$	$K3 \times Q$

Table 1.2: Partial list of heterotic/M-theoretic dualities; X is a d-manifold preserving minimal susy; $3 \leq n \leq 6$; and Q is a 3-manifold.

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[3]As pointed out in [W95], one can take Y to be anisotropic, with d dimensions each of size characterized by the string coupling ($\sim \sqrt{\alpha'}$), and $6 - d$ dimensions each of size M_{GUT}^{-1} to get the observed GUT scale symmetry breaking, but the best one can do is still too large by an order of magnitude.

As previously mentioned, it was shown that if gauge coupling unification was assumed and supersymmetry was present at some higher energy, the prediction of the weak mixing angle, in the form of $\sin^2 \theta_W$, was closer to the experimental value than in non-supersymmetric models [AdBF91, GKL91, EKN91, LL91]:

Experiment	0.23161 ± 0.00018
Non-Susy SU(5) GUT	$0.214 \pm 0.003 \pm 0.006 \ln[\frac{0.16 GeV}{\Lambda_{\overline{MS}}}]$
Susy SU(5) GUT	0.236 ± 0.003
Heterotic String	$\star$

Table 1.3: Values of $\sin^2 \theta_W$

From the second to third line, the predicted value of $\sin^2 \theta_W$ gets better, while the scale of unification, M_U, is pushed up toward the scale at which quantum gravitational effects become non-negligible (which we will call M_{st} whether or not string theory is actually involved). This hints that perhaps a unification of all interactions should be sought. Afterall, there is much extrapolation in traversing the desert between the electroweak scale $M_{ew} \sim \mathcal{O}(100) GeV$, and a conjectured unification scale M_U. But a framework that incorporates gravitation must be chosen before this issue can be addressed.

As we have mentioned, the heterotic superstring theory with $E_8 \times E_8$ gauge group is the most direct route to string phenomenology, and involves a quantum theory of gravity (more exactly, a modification of general relativity by higher order, but suppressed, interactions on a classical background spacetime). We've denoted the prediction of $\sin^2 \theta_W$ for this theory by $\star$ in table (1.3) since the situation is complicated by the fact that we are now dealing with a string theory, which naturally has a single parameter. As a result, unification of gauge and gravitational couplings is a generic feature, and the scale of this unification is predicted to be the string scale M_{st}. A naive field theory calculation (up to two loop order) shows that the prediction of the weak mixing angle is far off from the measured value. (In other words, the MSSM calculation predicts that $M_U << M_{st}$, while the tree string calculation predicts unification at M_{st}.) At this point there are various paths to take (see [D97a] for a review), of which we'll mention a few. (1) One can simply abandon the requirement that gravitational and SM gauge couplings unify at a single scale, and instead allow the usual GUT scenario at an intermediate scale[4] (in which case, all couplings will still ultimately unify at the string scale). (2) Sring theory allows one to abandon a GUT scenario altogether by changing the Kac-Moody algebra of the string worldsheet so that all couplings unify at

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[4]By intermediate, we mean an energy scale between the electroweak scale M_{ew} and the scale M_{st}.

the string scale, but without a GUT group. (3) Look for gauge and gravitational coupling unification at a single energy by including heavy string and Wilson line corrections to the renormalization group flow. We take the approach of the third path: as more corrections are considered, the prediction for $\sin^2 \theta_W$ can move into closer agreement with measurement, while the gauge coupling unification scale is pushed up even closer to the perturbative string scale [NS95, NS97].

The work of Hořava and Witten [HW96a, HW96b, W96a] showed that the territory of weakly coupled string theories was only part of the string story. They showed that the strong coupling limit of the $10D$ heterotic string theory with $E_8 \times E_8$ gauge group has a description as a weakly coupled $11D$ theory (M-theory) on $M_{10} \times S^1/\mathbb{Z}_2$, which is isomorphic to a spacetime $\mathcal{M}_{10} \times \mathcal{I}$, where $\mathcal{I}$ is an interval. Further compactification on a Calabi-Yau 3-fold Y yields a theory on $\mathcal{M}_4 \times S^1/\mathbb{Z}_2$ with gauge fields that have support only on the boundaries. Due to the ground state product structure of the spacetime, the size of the spaces can be adjusted independently; setting the size of $S^1/\mathbb{Z}_2$ to be much larger than Y, we obtain an effective $5D$ theory at some intermediate energy scale. As a result, the running of the gravitational coupling starting from $4D$ is pushed up at the compactification scale M_c (the inverse size R^{-1} of $S^1/\mathbb{Z}_2$), and all four couplings can meet in the newly unveiled $5D$ theory. The running of the gauge couplings is unaffected by the fifth dimension since the gauge fields are *confined* to the $4D$ boundary. This allows a complete unification of couplings in the framework of string theory, without ruining the original minimal supersymmetric SM predictions of $\sin^2 \theta_W$ (this ignores the contributions due to heavy string states). In other words, the string scale is pushed down toward the usual MSSM prediction of M_U. Furthermore, such scenarios predict a $4D$ Newtonian constant that can have a physically correct order of magnitude, in contrast to compactifications of weakly coupled heterotic string theory.

The work of Hořava and Witten deals with only one particular phase of M-theory: one in which there is an unbroken E_8 gauge group at the $10D$ boundaries of the $11D$ spacetime. But M-theory can sit on a myriad of backgrounds with $4D$ low energy effective behavior giving various gauged supergravity theories coupled to matter. Some of these have a $5D$ universe at an intermediate energy scale. The details of these theories are hard to come by since the internal 7-manifold must have G_2 structure [AW03], and such manifolds are not as well-known as Calabi-Yau manifolds.

Taking a step back, *field-theoretic* model building has a long history of scenarios in which the universe appears higher-dimensional above some energy scale. As previously mentioned, the original Kaluza-Klein scenarios involved compactification on S^1 or tori, and attempts were made in obtaining four-dimensional gauge and gravitational symmetries from the higher

dimensional gravitational theory (or supergravity in later versions). In [F83], it was pointed out that the estimated scale of strong-electroweak unification, $M_{sew} \sim 10^{16} GeV$, was around the energy scale where a Kaluza-Klein type universe may not be able to be approximated by a $4D$ theory, in which case grand unification would occur in higher dimensions. Earlier, the authors of [M79, CM81] had constructed models in which one has a pure GUT gauge theory in higher dimensions, leading to a low energy theory with Standard Model gauge group and Higgs sector. Nevertheless, these Kaluza-Klein theories did not yield the Standard Model for reasons depending on the scenarios.

In [A90], followed by [AADD98, ADPQ99], it was suggested that the size of an extra dimension could be much larger (TeV scale) within the framework of perturbative string theory (one of the motivations was to tie this scale to the $\mathcal{N} = 1$ supersymmetry breaking scale). Subsequently, the Hořava-Witten (HW) scenario [HW96a, HW96b, W96a, LOSW98] and Randall-Sundrum (RS) scenarios [RS99a, RS99b] served as the most recent revival of the idea that there could be a five-dimensional universe at some *intermediate* energy scale, but not via compactification on S^1 (or tori in higher dimensional versions), but rather via the "orbifold" S^1/Γ (in the "upstairs picture") [K00, K01a, K01b, K01c, AF01, HMR01a, KY02, HMN02, DM02, HN01], which corresponds to a manifold with boundaries (in the "downstairs picture").[5] More precisely, the points of $\mathcal{M}_4 \times S^1$ that are invariant under the action of the discrete group Γ are isomorphic to four-dimensional boundaries. The interior of the $5D$ spacetime is referred to as the "bulk". In contrast to the HW and RS scenarios, in these theories the SM gauge fields propagate in the full five dimensions, and gauge coupling unification can be explored there.

From a ground-up perspective, spacetimes such as these can resolve a number of issues in the supersymmetric Standard Model and supersymmetric GUTs, as well as move closer to realizing the goals of the earlier Kaluza-Klein scenarios. The GUTs on these spacetimes, which we refer to as orbifold-GUTs, can have suppressed proton decay by first eliminating dimension five operators responsible for too-rapid decay rates; and second by giving large masses to the new fields involved in the decay processes. Additionally, undesirable scalars in Higgs multiplets of GUT theories can receive large masses, leaving only massless weak doublets. These scenarios provide an alternative to $4D$ GUTs that have been ruled out, such as the minimal supersymmetric $SU(5)$ theory [MP01]. The presence of boundaries can also perform some or all of the breaking of the GUT group to the Standard Model gauge group, removing the need for an extended Higgs sector that is needed in $4D$ GUTs. However, since

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[5]See chapter 4 for use of the term "orbifold" in this thesis.

the gauge fields now live in $5D$, the presence of the fifth dimension affects the running of the gauge couplings, in contrast to the HW and RS scenarios. Therefore, we should be concerned about the prediction of $\sin^2 \theta_W$ [DDG98, DDG99]. However, in $5D$ GUT theories, the fifth dimension is the only extra dimension that is "large", which means the modifications are generically not as drastic (this can be seen in the calculations of gauge coupling running throughout the orbifold-GUT literature).

There is another price to pay for going from four to five dimensions: not only is the gravitational sector non-renormalizable, but the entire theory is non-renormalizable from dimensional arguments, and the infinite tower of massive states can lead to catastrophic UV effects [DT82a, DT82b]. One must introduce a cutoff or consider a UV completion via a UV fixed point [S96b] or a new theory like superstring/M-theory. Of course, the latter theories are consistent only in ten and eleven dimensions, respectively, and so we must pay the price of additional dimensions if we are to use these as UV completions. Even if one cuts off the theory so that there is a finite number of massive states, the presence of these states may still be problematic. The orbifold-GUT literature is filled with calculations arguing that gauge coupling unification can still be acheived in such circumstances.

If these theories are to be low energy effective descriptions of superstring theory, there is a technical feature that makes unification scenarios nicer in $5D$ orbifold theories. To break a GUT group to the Standard Model gauge group solely via a Higgs mechanism requires fields in the adjoint, or larger, representation of the GUT group (as in the model in [NSW01] described below). Such a state does not exist below the string scale in (weakly coupled) heterotic string theory if the gauge group is based on a level one Kac-Moody algebra defined on the worldsheet [DL89, FIQ90]. Thus, breaking the unified gauge group with a Higgs mechanism at a $4D$ boundary requires a string theory whose $4D$ gauge group is based on a higher level underlying Kac-Moody algebra. This is not as economical since a large number of additional states, which appear unnecessary, are introduced.[6] In contrast, one can spontaneously break a *partially* unified gauge group with a level one spectrum, so all that would remain is to explicitly break the unified group to such a partially unified gauge group using boundary conditions. (*Partially* unified gauge group refers to non-simple groups such as $SU(5) \times U(1)$, $SU(4) \times SU(2) \times SU(2)$, or $SU(3) \times SU(3) \times SU(3)$.) Such a mixed scenario has been considered in [KR02]. This is one of the benefits of orbifold-GUT scenarios (from the point of view of string theory).

Since an orbifold spacetime is singular, field theories are not well-defined on it, which

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[6]There are alternative ways to obtain adjoint reps without extra massless matter [F95] but these models seem to require a complicated setup.

requires some further interpretation (in the downstairs picture, the boundary is sharp). Supergravity admits solitonic solutions that could ultimately be interpreted as the boundaries of these theories; these solutions are domain walls with some thickness, smoothing out the singular nature of sharp boundaries. Alternatively, since supergravity is generally a low energy approximation of string/M-theory, we can rely on the singularity-resolution that takes place in those theories [DHVW85, DHVW86, M86, A94]. Anyway, in light of this introduction so far, a natural next step from $5D$ supersymmetric orbifold-GUTs is to embed them into $5D$ $\mathcal{N} = 2$ supergravity. General features of $5D$ supergravity orbifolds (without reference to string/M-theory) have been considered in the literature, with couplings to vector and hypermultiplets [YL03, ZGAZ04, DGKL04].

Embedding $5D$ supersymmetric GUTs into supergravity is not a trivial incorporation, as $\mathcal{N} = 2$ supergravity places restrictive relationships between gaugings and matter content. It helps to review the situation in the case of *four-dimensional* theories. In the case of rigidly supersymmetric theories, the set of allowed spin-1/2 multiplet couplings is in one-to-one correspondence with all Kähler manifolds in the case of $\mathcal{N} = 1$ supersymmetry [Z79, AGF80]; and hyper-Kähler manifolds in the case of $\mathcal{N} = 2$ supersymmetry [AGF81]. Local supersymmetry (supergravity) imposes additional restrictions such that the set of allowed spin-1/2 multiplet couplings is in one-to-one correspondence with Hodge manifolds, which are special cases of Kähler manifolds, in the case of $\mathcal{N} = 1$ supersymmetry [CJSFGvN79, BW82]; and one-to-one correspondence with quaternionic manifolds in the case of $\mathcal{N} = 2$ supersymmetry [BW83]. Thus, in the case of $\mathcal{N} = 1$ supersymmetry, only a subset of the possible matter couplings with rigid supersymetry may be directly coupled to supergravity; and in the case of $\mathcal{N} = 2$ supersymmetry, none of the possible matter couplings with rigid supersymmetry may be directly coupled to supergravity. The latter result follows from the fact that quaternionic manifolds have non-zero curvature, while hyper-Kähler manifolds are flat.

Despite the inability to directly couple $\mathcal{N} = 2$ supersymmetric theories to supergravity, we can obtain one theory from the other using a particular mapping. The mapping is between quaternionic and hyper-Kähler manifolds as the curvature is taken to zero, corresponding to the decoupling of supergravity (the "rigid limit"). See [ABC97, BDF98] for the discussion of rigid limits in $4D$ $\mathcal{N} = 2$ supergravity. It has not been proven that a rigid limit always exists. In fact, it has been shown that there is no rigid limit for supergravity coupled to particular compact scalar manifolds. If the rigid limit exists, we may take a supergravity theory with compact gauge group coupled to a particular quaternionic scalar manifold, and obtain a particular super-Yang-Mills theory coupled to a hyper-Kähler scalar manifold. Though the

nature of the hypermultiplet couplings change in this limit, the gauge group representations assigned to the hypermultiplets remain unchanged.

The layout of the thesis is as follows. Chapter 2 is a review five-dimensional $\mathcal{N} = 2$ Yang-Mills-Einstein supergravity theories (YMEGTs) coupled to tensor- and hyper-multiplets. In chapter 3, which is based on [M05a], we discuss the phenomenologically interesting GUT field content admitted by YMESGTs in the form of hypermultiplets, as well as discuss the possibilities of embedding this content in tensor multiplets or gauge multiplets of non-compact gauge symmetries. Chapter 4, which is based on [GMZ05a, GMZ05b, M05b], introduces $5D$ YMESGTs on the a spacetime that is topologically $\mathcal{M}_4 \times S^1 / \Gamma$, where $\mathcal{M}_4$ is a 4-manifold and Γ is a discrete group acting non-freely on the circle. The spectrum of boundary-propagating supermultiplets, as determined by orbfifold parity assignments of bulk fields, is given. Furthermore, the required parity assignments of objects other than fields are listed. In chapter 5, which is based on [M05c], we discuss the form of the symmetries arising from local symmetries of the YMESGT on the above spacetime and the anomalies that may break these local symmetries. Chapter 6, which is also based on [M05c], covers a few phenomenological issues arising from a YMESGT on the above spacetime, such as the presence of QCD-type axions with possiblility of cosmologically allowed coupling strength and Yukawa terms. Finally in chapter 7, which will appear in [M05d], we discuss possible M-theoretic origins of these theories, and contrast with previous string/M-phenomenology. Appendix 1 contains a few conventions that we follow in the thesis. Appendix 2 covers more details of the orbifold parity assignments discussed in chapter 4.

Chapter 2

$5D$ $\mathcal{N} = 2$ Yang-Mills-Einstein Supergravity

2.1 Conventions

Let us lay out the conventions used in this thesis. We use the mostly plus signature $\eta_{\hat{m}\hat{n}} = \text{diag}(-1, +1, +1, +1, +1)$ with $\hat{m} = 0, 1, 2, 3, 5$. For gamma matrices, we take

$$\Gamma^m = \begin{pmatrix} 0 & \sigma^m \\ -\sigma^m & 0 \end{pmatrix} \quad \Gamma^5 = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$$

where σ^m are the $4D$ spacetime Pauli matrices, and $m = 0, 1, 2, 3$ is a tangent spacetime index. We use the convention where $\Gamma^{m_1 \cdots m_\wp} \equiv \Gamma^{[m_1} \cdots \Gamma^{m_\wp]}$. The charge conjugation matrix is taken to be

$$C = \begin{pmatrix} e & 0 \\ 0 & -e \end{pmatrix} \quad \text{where} \quad e = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}.$$

The charge conjugation matrix therefore satisfies

$$C^T = -C = C^{-1} \quad \text{and} \quad C\Gamma^m C^{-1} = (\Gamma^m)^T.$$

In five spacetime dimensions, there are a minimum of eight supercharges so that there is a global $SU(2)_R$ symmetry (the automorphism group of the superalgebra). It is therefore convenient to use symplectic-Majorana spinors, which form an explicit $SU(2)_R$ doublet. Given a pair of 4-component spinors λ^i, the Dirac conjugate is defined by

$$\bar{\lambda}^i = (\lambda_i)^\dagger \Gamma^0,$$

where i is an $SU(2)_R$ index, which is raised and lowered according to

$$\lambda^i = \epsilon^{ij} \lambda_j \quad \lambda_j = \lambda^i \epsilon_{ij},$$

with $\epsilon_{12} = \epsilon^{12} = 1$. Then a symplectic-Majorana spinor is one that satisfies

$$\bar{\lambda}^i = \lambda^{iT} C.$$

We will take the following form for our Majorana spinors showing the 2-component spinor content:

$$\lambda^1 = \begin{pmatrix} \xi \\ e\zeta^* \end{pmatrix} \quad \lambda^2 = \begin{pmatrix} \zeta \\ -e\xi^* \end{pmatrix}.$$

2.2 $5D$ MESGTs

We follow the formulation of Maxwell-Einstein supergravity theories of Günaydin, Sierra, and Townsend [GST84a] and their promotion to Yang-Mills-Einstein supergravity theories [GST85a, GST85b, EGZ01]. An $\mathcal{N} = 2$ $5D$ Maxwell-Einstein supergravity theory (MESGT) describes the coupling of a minimal supergravity multiplet to n_V vector supermultiplets. The total field content is

$$\{\hat{e}_{\hat{\mu}}^{\hat{m}}, \Psi_{\hat{\mu}}^i, A_{\hat{\mu}}^I, \lambda^{i\tilde{p}}, \phi^{\tilde{x}}\},$$

where the $5D$ curved spacetime index is $\hat{\mu} = \dot{0}, \dot{1}, \dot{2}, \dot{3}, \dot{5}$; the $5D$ tangent spacetime index is $\hat{m} = 0, 1, 2, 3, 5$; the index $I = (0, 1, ..., n_V)$ labels the "bare graviphoton" and vector fields from the n_V vector multiplets; $i = (1, 2)$ is an $SU(2)_R$ index; and $\tilde{p} = (1, ..., n_V)$ and $\tilde{x} = (1, ..., n_V)$ label the fermions and scalars from the n_V vector multiplets. The scalar fields parametrize an n_V-dimensional real Riemannian manifold $\mathcal{M}_R$, so the indices $\tilde{p}, \tilde{q}, \dots$ and $\tilde{x}, \tilde{y}, \dots$ may also be viewed as flat and curved indices of $\mathcal{M}_R$, respectively.

Introducing $(n_V + 1)$ parameters $\xi^I(\phi)$ depending on the scalar fields, we define a cubic polynomial

$$\mathcal{V}(\xi) = C_{IJK}\xi^I\xi^J\xi^K,$$

where C_{IJK} is a constant rank-3 symmetric tensor. This polynomial can be used to define a symmetric rank-2 tensor

$$a_{IJ}(\xi) = -\frac{1}{3}\frac{\partial}{\partial\xi^I}\frac{\partial}{\partial\xi^J}\ln\mathcal{V}(\xi).$$

The parameters ξ^I can be interpreted as coordinate functions for an (n_V+1)-manifold, which we call the "ambient space". The tensor a_{IJ}, which may have indefinite signature, defines a metric on this space. However, the coordinates are restricted via $\mathcal{V}(\xi) > 0$ so that the metric

is positive definite, which means that the manifold is Riemannian. The equation $\mathcal{V}(\xi) = k$ ($k \in \mathbb{R}$) defines a family of real hypersurfaces, and in particular

$$\mathcal{V}(\xi) = 1$$

defines a real n_V-manifold. As was shown in [GST84a], this manifold can be identified with the scalar manifold $\mathcal{M}_R$ of the $5D$ MESGT. We can denote the restriction of the ambient space metric to $\mathcal{M}_R$ as:

$$\overset{\circ}{a}_{IJ} = a_{IJ}|_{\mathcal{V}=1}.$$

The metric of the scalar manifold is the pullback of the restricted ambient space metric to $\mathcal{M}_R$:

$$g_{\tilde{x}\tilde{y}} = \frac{3}{2\hat{\kappa}^2} \overset{\circ}{a}_{IJ} h^I_{,\tilde{x}} h^J_{,\tilde{y}},$$

where $\hat{\kappa}$ is the $5D$ gravitational coupling (with units of inverse energy); and the function h^I is directly proportional to $\xi^I|_{\mathcal{V}=1}$:

$$h^I(\phi) = \sqrt{\frac{2}{3}} \xi^I(\phi)|_{\mathcal{V}=1},$$

so that the h^I are essentially embedding coordinates of $\mathcal{M}_R$ in the ambient space. Both of the metrics $\overset{\circ}{a}_{IJ}$ and $g_{\tilde{x}\tilde{y}}$ are positive definite due to the constraint $\mathcal{V} > 0$ that was imposed. The vielbein $f^{\tilde{p}}_{\tilde{x}}$ relate curved and flat scalar manifold indices. In particular, the $\mathcal{M}_R$ and tangent space metrics are related by $g_{\tilde{x}\tilde{y}} = f^{\tilde{p}}_{\tilde{x}} f^{\tilde{q}}_{\tilde{y}} \eta_{\tilde{p}\tilde{q}}$.

Up to four-fermion terms, the five-dimensional MESGT Lagrangian is [GST84a] (hats denote $5D$ quantities)

$$\begin{aligned}
\hat{e}^{-1}\mathcal{L}_{(5)} = &- \frac{1}{2\hat{\kappa}^2}\hat{R} - \frac{1}{4}\overset{\circ}{a}_{IJ} F^I_{\hat{\mu}\hat{\nu}}F^{J\,\hat{\mu}\hat{\nu}} - \frac{1}{2}g_{\tilde{x}\tilde{y}}\partial_{\hat{\mu}}\phi^{\tilde{x}}\partial^{\hat{\mu}}\phi^{\tilde{y}} \\
&- \frac{1}{2\hat{\kappa}^2}\bar{\Psi}^i_{\hat{\mu}}\Gamma^{\hat{\mu}\hat{\nu}\hat{\rho}}\nabla_{\hat{\nu}}\Psi_{\hat{\rho}\,i} - \frac{1}{2}\bar{\lambda}^{i\tilde{p}}\left(\Gamma^{\hat{\mu}}\nabla_{\hat{\mu}}\delta^{\tilde{p}\tilde{q}} + \Omega^{\tilde{p}\tilde{q}}_{\tilde{x}}\Gamma^{\hat{\mu}}\partial_{\hat{\mu}}\phi^{\tilde{x}}\right)\lambda^{\tilde{q}}_i \\
&- \frac{i}{2}\bar{\lambda}^{i\tilde{p}}\Gamma^{\hat{\mu}}\Gamma^{\hat{\nu}}\Psi_{\hat{\mu}\,i}f^{\tilde{p}}_{\tilde{x}}\partial_{\hat{\nu}}\phi^{\tilde{x}} + \frac{1}{4}h^{\tilde{p}}_I\bar{\lambda}^{i\tilde{p}}\Gamma^{\hat{\mu}}\Gamma^{\hat{\nu}\hat{\rho}}\Psi_{\hat{\mu}\,i}F^I_{\hat{\nu}\hat{\rho}} \\
&+ \frac{i\hat{\kappa}}{2\sqrt{6}}\left(\frac{1}{4}\delta_{\tilde{p}\tilde{q}}h_I + T_{\tilde{p}\tilde{q}\tilde{r}}h^{\tilde{r}}_I\right)\bar{\lambda}^{i\tilde{p}}\Gamma^{\hat{\mu}\hat{\nu}}\lambda^{\tilde{q}}_i F^I_{\hat{\mu}\hat{\nu}} \\
&- \frac{3i}{8\sqrt{6}\hat{\kappa}}h_I\left(\bar{\Psi}^i_{\hat{\mu}}\Gamma^{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}}\Psi_{\hat{\nu}\,i}F^I_{\hat{\rho}\hat{\sigma}} + 2\bar{\Psi}^{\hat{\mu}\,i}\Psi^{\hat{\nu}}_i F^I_{\hat{\mu}\hat{\nu}}\right) \\
&+ \frac{\hat{\kappa}\hat{e}^{-1}}{6\sqrt{6}}C_{IJK}\epsilon^{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}\hat{\lambda}}F^I_{\hat{\mu}\hat{\nu}}F^J_{\hat{\rho}\hat{\sigma}}A^K_{\hat{\lambda}},
\end{aligned} \tag{2.1}$$

where $\hat{e}^{-1}$ is the inverse of the fünfbein determinant; $\hat{\kappa}$ is the $5D$ gravitational coupling; R is the spacetime Ricci curvature scalar; $\nabla_{\hat{\mu}}$ is the spacetime covariant derivative based on the spacetime spin connection; and $F^I_{\mu\nu} = 2\partial_{[\mu}A^I_{\nu]}$ are abelian fieldstrengths.

The functions h^I and $h^I_{\tilde{x}}$ satisfy a set of relations, which follow from supersymmetry [GST84a]:

$$h^I h_I = 1 \qquad h^I_{\tilde{x}} h_I = 0$$
$$\overset{\circ}{a}_{IJ} = h_I h_J + h^{\tilde{x}}_I h^{\tilde{y}}_J g_{\tilde{x}\tilde{y}} \qquad g_{\tilde{x}\tilde{y}} = \overset{\circ}{a}_{IJ} h^I_{\tilde{x}} h^J_{\tilde{y}}$$
$$C_{IJK} = \frac{5}{2} h_I h_J h_K - \frac{3}{2} \overset{\circ}{a}_{(IJ} h_{K)} + T_{\tilde{x}\tilde{y}\tilde{z}} h^{\tilde{x}}_I h^{\tilde{y}}_J h^{\tilde{z}}_K$$
$$h^I_{\tilde{x}} = -\sqrt{\frac{3}{2\hat{\kappa}^2}} h^I_{,\tilde{x}} \qquad h_{I\tilde{x}} = \sqrt{\frac{3}{2\hat{\kappa}^2}} h_{I,\tilde{x}}.$$

$$(2.2)$$

As a result, we have

$$h_I = \overset{\circ}{a}_{IJ} h^J \qquad h^I_{\tilde{x}} = g_{\tilde{x}\tilde{y}} h^{I\tilde{y}}$$
$$T_{\tilde{x}\tilde{y}\tilde{z}} = C_{IJK} h^I_{\tilde{x}} h^J_{\tilde{y}} h^K_{\tilde{z}}$$
$$C_{IJK} h^K = h_I h_J - \frac{1}{2} h^{\tilde{p}}_I h^{\tilde{p}}_J.$$

Note that the canonical relationship $g_{\tilde{x}\tilde{y}} = h^I_{\tilde{x}} h^J_{\tilde{y}} \overset{\circ}{a}_{IJ}$ is interpreted as the pullback of the ambient space restriction to the scalar manifold upon use of (2.2), which involves $\hat{\kappa}$.

The C_{IJK} tensor completely determines the MESGT Lagrangian [GST84a]. Therefore, the global symmetry group of the Lagrangian is given by the symmetry group, G, of this tensor, along with automorphisms of the $\mathcal{N} = 2$ superalgebra: $G \times SU(2)_R$. Since G consists of symmetries of the full Lagrangian, they are symmetries of the scalar sector in particular, and therefore isometries of the scalar manifold $\mathcal{M}_R$: $G \subset Iso(\mathcal{M}_R)$ (the $SU(2)_R$ action is trivial on the scalars). The full Lagrangian, however, is not necessarily invariant under the full group $Iso(\mathcal{M}_R)$. The action of G on the elements of the ambient space is $\xi^I \rightarrow M^I_J \xi^J$, with the C-tensor invariance condition

$$M^{I'}_{(I} C_{JK)I'} = 0.$$

A sometimes convenient ("canonical") basis can be chosen such that the C-tensor takes the form [GST84a]

$$C_{000} = 1, \;\; C_{0ij} = -\frac{1}{2}\delta_{ij}, \;\; C_{00i} = 0, \;\; C_{ijk} = \text{arbitrary}, \tag{2.3}$$

where we have made the index split $I = (0, i)$ with $i = 1, \ldots, n_V$.

2.3 Gauged supergravity theories

A gauged supergravity theory can be obtained from a MESGT by promoting the vector fields of the theory to gauge fields of a subgroup of the global symmetry group of the Lagrangian. We use the terminology of [GST85a] when the following groups, K, are gauged:

$$K \subset G \qquad \text{Yang-Mills-Einstein supergravity theory (YMESGT)}$$
$$K \subset SU(2)_R \qquad \text{Gauged Maxwell-Einstein supergravity theory}$$
$$K \subset G \times SU(2)_R \qquad \text{Gauged YMESGT,}$$

where, in the latter case, a (non-trivial) subgroup of both factors is gauged. *We are primarily interested in YMESGTs in this thesis.*

Obviously, a necessary condition for a subgroup of the global symmetry group $K \subset G$ to be gauged is that there be enough vector multiplets (i.e., $n_V + 1 \geq \dim[K]$). The symmetry group of the Lagangian is broken to K by the minimal couplings that are introduced. The $n_V + 1$ vector fields of the theory decompose into K-reps

$$(\mathbf{n_V + 1}) = \mathbf{adj(K)} \oplus \mathbf{non\text{-}singlets(K)} \oplus \mathbf{singlets(K)}. \tag{2.4}$$

Gauging a *compact* semi-simple group K yields at least one K-singlet spectator vector field (which can be identified as the graviphoton). However, if there is an abelian factor in K, the graviphoton can be identified as the corresponding gauge field.

2.3.1 Pure YMESGTs

Let us for now assume that there aren't any non-singlets in the decomposition above. The bosonic fields transform as follows under infinitesimal K-transformations parametrized by $\alpha^I(x)$:

$$\delta_\alpha A_{\hat\mu}^I = -\frac{1}{g}\partial_{\hat\mu}\alpha^I + \alpha^J f_{JK}^I A_{\hat\mu}^K$$
$$\delta_\alpha \phi^{\tilde x} = \alpha^I K_I^{\tilde x}(\phi) \tag{2.5}$$
$$\delta_\alpha \lambda^{i\tilde p} = \alpha^I L_I^{\tilde p \tilde q}(\phi)\lambda^{\tilde q},$$

where $K_I^{\tilde{x}}(\phi)$ are a set of $n_V + 1$ Killing vectors on the scalar manifold parametrized by the $\phi^{\tilde{x}}$ (furnishing a representation of K); $L_I^{\tilde{p}\tilde{q}}$ are a set of $n_V + 1$ scalar-dependent K-representation matrices;[1] and f_{JK}^I and α^I vanish if any index corresponds to a spectator (gauge singlet) vector field. *Now the C_{IJK} must be a rank-3 symmetric invariant of K.*

To obtain the YMESGT Lagrangian and supersymmetry transformations, one makes the following replacements in the MESGT Lagrangian

$$F_{\hat{\mu}\hat{\nu}}^I \longrightarrow \mathcal{F}_{\hat{\mu}\hat{\nu}}^I = F_{\hat{\mu}\hat{\nu}}^I + g f_{JK}^I A_{\hat{\mu}}^J A_{\hat{\nu}}^K$$
$$\partial_{\hat{\mu}}\phi^{\tilde{x}} \longrightarrow D_{\hat{\mu}}\phi^{\tilde{x}} = \partial_{\hat{\mu}}\phi^{\tilde{x}} + g A_{\hat{\mu}}^I K_I^{\tilde{x}}(\phi)$$
$$\nabla_{\hat{\mu}}\lambda^{i\tilde{p}} \longrightarrow D_{\hat{\mu}}\lambda^{i\tilde{p}} + g A_{\hat{\mu}}^I L_I^{\tilde{p}\tilde{q}}(\phi)\lambda^{i\tilde{q}},$$

with the exception of the Chern-Simons term.[2] Instead, we must (in general) replace the term

$$\frac{\hat{\kappa}\,\hat{e}^{-1}}{6\sqrt{6}}C_{IJK}\epsilon^{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}\hat{\lambda}}F_{\hat{\mu}\hat{\nu}}^I F_{\hat{\rho}\hat{\sigma}}^J A_{\hat{\lambda}}^K$$

with

$$\frac{\hat{\kappa}\,\hat{e}^{-1}}{6\sqrt{6}}C_{IJK}\,\epsilon^{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}\hat{\lambda}}\{F_{\hat{\mu}\hat{\nu}}^I F_{\hat{\rho}\hat{\sigma}}^J A_{\hat{\lambda}}^K + \frac{3}{2}g F_{\hat{\mu}\hat{\nu}}^I A_{\hat{\rho}}^J (f_{LM}^K A_{\hat{\sigma}}^L A_{\hat{\lambda}}^M) \tag{2.6}$$
$$+ \frac{3}{5}g^2 (f_{GH}^J A_{\hat{\nu}}^G A_{\hat{\rho}}^H)(f_{LF}^K A_{\hat{\sigma}}^L A_{\hat{\lambda}}^F)\,A_{\hat{\mu}}^I\}.$$

Furthermore, to preserve supersymmetry, one must add the Yukawa term

$$\hat{e}^{-1}\Delta\mathcal{L} = -\frac{i}{2}g\bar{\lambda}^{i\tilde{p}}\lambda_i^{\tilde{q}}K_{I[\tilde{p}}h_{\tilde{q}]}^I.$$

Finally, the supersymmetry transformations do not require corrections in the gauging procedure.

A pure $5D$ YMESGT does not have scalar potential, so that the bulk spacetime will be flat.

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[1]The transformation matrix fields are related to the Killing vectors by $L_I^{\tilde{p}\tilde{q}} = K_I^{\tilde{p};\tilde{q}} - \Omega_{\tilde{x}}^{\tilde{p}\tilde{q}}K_I^{\tilde{x}}$, where Ω is the $\mathcal{M}_R$ spin connection.

[2]By "Chern-Simons term", we do not mean the $5D$ Chern-Simons form, in general. This will be the case when C_{IJK} is an irreducible tensor.

2.3.2 YMESGTs coupled to tensor multiplets

Let's return to the fact that, in general, the $n_V + 1$ vector fields of a MESGT decompose into K-reps as

$$\mathbf{n_V + 1 = adj(K) \oplus non\text{-}singlets(K) \oplus singlets(K)}.$$

There is a conflict for non-singlet vector fields, as they must transform non-trivially under a non-abelian group as well as have an associated abelian symmetry (being Maxwell fields). The former will break the latter so that the vectors are not protected from becoming massive at the quantum level. But if these vectors become massive, supersymmetry will be broken due to a mismatch in bosonic and fermionic degrees of freedom. The simplest requirement is for the non-singlet vector fields to be dualized to anti-symmetric tensor fields [GZ99] satisfying a field equation that serves as a "self-duality constraint" (thus keeping the degrees of freedom the same) [TPvN84].[3] For a single uncoupled tensor in five dimensions, this is of the form

$$B_{\mu\nu} = ic\,\epsilon_{\mu\nu}{}^{\rho\sigma\lambda}\,\partial_{[\rho}B_{\sigma\lambda]}, \tag{2.7}$$

where c has dimensions of inverse mass. The factor of i is required in odd spacetime dimensions so that the tensors can be written in a complex form, implying that there is an even number n_T of tensors. At the end of the day, one may write the tensors in a real basis. However, since the tensors in a YMESGT will be K-non-singlets, the complex structure actually implies that one must have a symplectic structure, so that *only gaugings in which the non-singlets appearing in the decomposition form a symplectic representation are allowed.* This solves the problem since a tensor field does not require an associated abelian invariance to remain massless. *Now, if K is compact, there can be tensor fields transforming in non-singlet representations of this group iff at least one abelian isometry from $\mathcal{M}_R$ is gauged in K.* (The tensor fields must then at least be charged under the abelian factor, but can transform non-trivially under the other factors as well, according to the decomposition in (2.4).) It is important to note that, after gauging, g-dependent terms in the Lagrangian prevent Hodge-dualization from tensors back to vectors.

Tensor supermultiplets are of the same form as the vector multiplets from which they came. In fact, in the MESGT prior to gauging, the dualization from vectors $A_{\hat{\mu}}^M$ to tensors $B_{\hat{\mu}\hat{\nu}}^M$ does not modify the scalar and fermionic sectors. The scalar and fermionic fields of the

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[3]In the case of maximal $\mathcal{N} = 8$ supergravity in five dimensions, the AdS superalgebra already admits such tensor couplings transforming non-trivially under the bosonic subalgebra [GRW85, GRW86, PPvN85, GM85].

tensor sector are therefore intertwined with those of the vector sector of the YMESGT. We can define

$$\mathcal{H}_{\hat\mu\hat\nu}^{\tilde I} := \begin{pmatrix} \mathcal{F}_{\hat\mu\hat\nu}^I \\ B_{\hat\mu\hat\nu}^M \end{pmatrix},$$

where $\tilde I = (I, M)$, and write the vector and tensor multiplets as[4]

$$\{\mathcal{H}_{\hat\mu\hat\nu}^{\tilde I}, \lambda^{\tilde p\, i}, \phi^{\tilde x}\}.$$

However, to be consistent with the gauge symmetry, the components of the C-tensor are constrained to have components:

$$C_{IMN} = \frac{\sqrt 6}{2}\Omega_{NP}\Lambda_{IM}^P$$

$$C_{MNP} = 0 \quad C_{MIJ} = 0, \tag{2.8}$$

where Ω_{NP} is the K-invariant symplectic metric on the space spanned by the $B_{\mu\nu}^M$; and Λ_{IM}^P are symplectic K-representation matrices appearing in the K-transformation of the tensor fields: $\delta_\alpha B_{\mu\nu}^M = \alpha^I \Lambda_{IN}^M B_{\mu\nu}^N$. Furthermore, C_{IJK} must be a rank-three symmetric K-invariant tensor as in the pure YMESGT case. *Note: We are assuming a gauge group of the form $K_{semi-simple} \times K_{abelian}$; see [BCdWGVvP04] for more general couplings where $C_{MIJ} \neq 0$.*

The terms in the bosonic $5D$ Lagrangian involving tensors are [GZ99]

$$\hat e^{-1}\mathcal{L}_T = -\frac{1}{4}\,\overset{\circ}{a}_{MN}\, B_{\hat\mu\hat\nu}^M\, B_{\hat\rho\hat\sigma}^N\, \hat g^{\hat\mu\hat\rho}\hat g^{\hat\nu\hat\sigma} - \frac{1}{2}\,\overset{\circ}{a}_{IM}\, \mathcal{F}_{\hat\mu\hat\nu}^I\, B_{\hat\rho\hat\sigma}^M\, \hat g^{\hat\mu\hat\rho}\hat g^{\hat\nu\hat\sigma}$$

$$+\frac{\hat e^{-1}}{4g}\epsilon^{\hat\mu\hat\nu\hat\rho\hat\sigma\hat\lambda}\, \Omega_{MN} B_{\hat\mu\hat\nu}^M\, \nabla_{\hat\rho} B_{\hat\sigma\hat\lambda}^N + \frac{\hat e^{-1}}{2\sqrt 6}C_{MNI}\, \epsilon^{\hat\mu\hat\nu\hat\rho\hat\sigma\hat\lambda} B_{\hat\mu\hat\nu}^M\, B_{\hat\rho\hat\sigma}^N\, A_{\hat\lambda}^I.$$

The $5D$ field equations for the $B_{\hat\mu\hat\nu}^M$ (in form notation) are

$$\star DB^M = g\Omega^{MN}\,\overset{\circ}{a}_{M\tilde I}\,\mathcal{H}^{\tilde I}.$$

---

[4]It is understood here that the 2-forms $\mathcal{F}^I$ satisfy the Bianchi identity $\mathcal{F}^I = DA^I$, so that the 1-forms A^I are the fundamental fields.

2.3.3 Coupling to hypermultiplets

An $\mathcal{N} = 2$ $5D$ hypermultiplet consists of four real scalars and four helicity $1/2$ states. A collection of n_H hypermultiplets is then

$$\{\zeta^{iA}, q^{\tilde{X}}\}.$$

The scalars $q^{\tilde{X}}$ ($\tilde{X} = 1, \ldots, 4n_H$) of n_H hypermultiplets parametrize a $4n_H$-real-dimensional quaternionic scalar manifold $\mathcal{M}_Q$ with tangent space group $Usp(2n_H) \times Usp(2)$ [BW83]. The $4n_H$-bein $f_{iA}^{\tilde{X}}$ relate scalar manifold and tangent space metrics

$$g_{\tilde{X}\tilde{Y}} f_{iA}^{\tilde{X}} f_{jB}^{\tilde{Y}} = \epsilon_{ij} C_{AB},$$

where $\tilde{X}, \tilde{Y}$ are curved indices; $i, j = 1, 2$ are $Usp(2)$ indices; and $A, B = 1, \ldots, 2n_H$ are $Usp(2n_H)$ indices.

The total scalar manifold of a MESGT coupled to hypermultiplets is

$$\mathcal{M} \equiv \mathcal{M}_R \times \mathcal{M}_Q,$$

with isometry group $Iso(\mathcal{M}) \simeq Iso(\mathcal{M}_R) \times Iso(\mathcal{M}_Q)$ [S85]. Once again, one can gauge a subgroup $K \subset G \subset Iso(\mathcal{M})$. In particular, since we want non-trivially charged hypermultiplets, $K \subset Iso(\mathcal{M}_R) \times Iso(\mathcal{M}_Q)$, where K is generally $K_{semi} \times K_{abel}$. For the semi-simple part, $K_{semi} \subset G_1 \times G_2 \subset Iso(\mathcal{M})$ such that K_{semi} is isomorphic to both a subgroup of $G_1 \subset Iso(\mathcal{M}_R)$ and a subgroup of $G_2 \subset Iso(\mathcal{M}_Q)$.

If there are non-trivial isometries of $\mathcal{M}_Q$, the Killing fields $K_I^{\tilde{X}}$, and the related representation matrices L_I^{AB} using the $\mathcal{M}_Q$ spin connection, act as

$$\begin{aligned}
\delta_\alpha q^{\tilde{X}} &= \alpha^I K_I^{\tilde{X}}(q) \\
\delta_\alpha \zeta^A &= \alpha^I L_I^{AB}(q)
\end{aligned} \tag{2.9}$$

The susy transformations are

$$\begin{aligned}
\delta q^{\tilde{X}} &= -i\bar{\epsilon}^i \zeta^A f_{iA}^{\tilde{X}} \\
\delta \zeta^A &= -\frac{1}{2}\Gamma^{\hat{m}} \epsilon^i D_{\hat{m}} q^{\tilde{X}} f_{i\tilde{X}}^A + \frac{\sqrt{6}}{4} g \epsilon^i f_{\tilde{X}i}^A K_I^{\tilde{X}} h^I(\phi) + \cdots,
\end{aligned} \tag{2.10}$$

where dots in the fermion transformation indicate terms with fermionic fields. The hyper-

multiplet Lagrangian (coupled to a YMESGT), without all-fermionic terms, is

$$
\begin{aligned}
\hat{e}^{-1}\mathcal{L}_{hyper} = &-\frac{1}{2}g_{\tilde{X}\tilde{Y}}D_{\hat{\mu}}q^{\tilde{X}}D^{\hat{\mu}}q^{\tilde{Y}} - \bar{\zeta}^A\Gamma^{\hat{\mu}}D_{\hat{\mu}}\zeta_A - \frac{2\hat{g}^2}{\hat{\kappa}^2}V_{iA}V^{iA} \\
&+ i\bar{\zeta}_A\Gamma^{\hat{m}}\Gamma^{\hat{\mu}}D_{\hat{\mu}}q^{\tilde{X}}\Psi^i_{\hat{m}}f^A_{i\tilde{X}} + \frac{\sqrt{6}}{8}i\hat{\kappa}h_I\bar{\zeta}_A\Gamma^{\hat{m}\hat{n}}\mathcal{H}^I_{\hat{m}\hat{n}}\zeta^A \\
&+ \hat{g}[-\frac{2}{\hat{\kappa}^2}V_i^A\bar{\zeta}_A\Gamma^{\hat{m}}\Psi^i_{\hat{m}} - \sqrt{\frac{3}{2\hat{\kappa}^2}}ih^It^A_{IB}\bar{\zeta}_A\zeta^B + 2iK_I^{\tilde{X}}f^A_{i\tilde{X}}h^I_x\bar{\zeta}_A\lambda^{ix}],
\end{aligned}
\tag{2.11}
$$

where the t^A_{IB} are $n_V + 1$ gauge group representation matrices for the fermions, which are determined by the Killing fields $K_I^{\tilde{X}}$ of the quaternionic scalar manifold. Smooth manifolds, such as the quaternionic manifolds parametrized by the $q^{\tilde{X}}$, have an affine connection $\Gamma^{\tilde{X}}_{\tilde{Y}\tilde{Z}}$. But in the vielbein formulation, there are local fields $f^{\tilde{X}}_{iA}$ (the vielbein), and local connections $\gamma^{\tilde{X}}_{AB}$ determined by them. Therefore, letting $\mathcal{D}_{\tilde{X}}K_I^{\tilde{Y}} \equiv \partial_{\tilde{X}}K_I^{\tilde{Y}} + \Gamma^{\tilde{Y}}_{\tilde{X}\tilde{Z}}K_I^{\tilde{Z}}$

$$
t^A_{IB} \equiv \frac{1}{2}f^{\tilde{X}}_{iB}\mathcal{D}_{\tilde{X}}K_J^{\tilde{Y}}f^{iA}_{\tilde{Y}}.
$$

Also,

$$
V^{iA} \equiv \frac{\sqrt{6}}{4}h^I K_I^{\tilde{X}}f^{Ai}_{\tilde{X}}
\tag{2.12}
$$

and the K-covariant derivatives act as

$$
\begin{aligned}
D_{\hat{\mu}}q^{\tilde{X}} &= \mathcal{D}_{\hat{\mu}}q^{\tilde{X}} + gA^I_{\hat{\mu}}K_I^{\tilde{X}}(q) \\
D_{\hat{\mu}}\zeta^A &= \mathcal{D}_{\hat{\mu}}\zeta^A + gA^I_{\hat{\mu}}\omega^A_{IB}(q)\zeta^B
\end{aligned}
\tag{2.13}
$$

with $\mathcal{D}_\mu$ the covariant derivative based on the tangent space Lorentz and $Usp(2n_H) \times SU(2)$ connections. *The scalar potential admits supersymmetric AdS vacua.*

Chapter 3

Options for Phenomenological Field Content in $5D$

In orbifold-GUTs, the five-dimensional field content for a given gauge group is not arbitrary, but depends on the scalar manifold $\mathcal{M}_V \times \mathcal{M}_Q$ chosen. In this chapter, we describe some of the phenomenologically interesting vector, tensor, and hypermultiplets that can be coupled to supergravity with gauge groups $SU(5)$, $SO(10)$ and E_6. The material in this chapter is based on [M05a].

3.1 Options for $5D$ hypermultiplets

In orbifold theories, the minimal supersymmetric Standard Model (MSSM) Higgs sector can be taken to lie in $5D$ hypermultiplets. Furthermore, if we do not wish to deal with ad hoc additions of SM matter on the boundaries of the spacetime, we can incorporate these fields in bulk hypermultiplets as well. The goal of this section is to then examine the possibilities for coupling hypermultiplets that are charged under a GUT group $SU(5)$, $SO(10)$, or E_6 within the framework of $5D$ $\mathcal{N} = 2$ supergravity. Although there may be further mention of orbifold field theories in this chapter, we will not need to know any of the details until the next chapter.

Due to supersymmetry, the scalars in $5D$ hypermultiplets must generally form the

$$2(\Sigma_i \mathbf{n_i}) \oplus 2\Sigma_\alpha(\mathbf{n}_\alpha \oplus \bar{\mathbf{n}}_\alpha)$$

representation of any gauge group we consider, where i labels pseudoreal irreps and α labels real and complex irreps. Notice the factor of two (not four) in front of the pseudoreal terms; this is due to the fact that such $5D$ hypermultiplets can be split into two $4D$ chiral multiplets each in a pseudoreal irrep of a gauge group, which is a reflection of the fact that they are self-conjugate. The main groups discussed here ($SU(5)$, $SO(10)$, and E_6) do not

carry pseudoreal representations. However, E_7 has the **56**, which will play a role later on.

Let's set our notation. A $5D$ $\mathcal{N} = 2$ hypermultiplet contains four real scalars. If $4m$ scalars form the real or complex representations $\mathbf{m}_{\mathbb{H}} \equiv 2(\mathbf{m} \oplus \bar{\mathbf{m}})$, we say that the hypermultiplet is in the $\mathbf{m}$ (we do not need to distinguish from $\bar{\mathbf{m}}$). If the $4m$ scalars form the pseudoreal $\mathbf{m}_H = 2[\mathbf{2m}]$, we will denote the hypermultiplet as being in the $\mathbf{2m}$. A $4D$ $\mathcal{N} = 1$ spin-1/2 multiplet contains two real scalars. If $2m$ scalars form the real or complex $\mathbf{m}_{\mathbb{C}} \equiv \mathbf{m} \oplus \overline{\mathbf{m}}$, we say that the spin-1/2 multiplet is in the $\mathbf{m}$ (or equivalently, there is a chiral plet in the $\mathbf{m}$ and its CPT conjugate); while if the $2m$ scalars form the $\mathbf{2m}$, we say the spin-1/2 multiplet is in the pseudoreal $\mathbf{2m}$.

The $4D$ minimal supersymmetric Standard Model requires a minimum Higgs supermultiplet content of two chiral multiplets forming the $\mathbf{2} \oplus \bar{\mathbf{2}}$ of $SU(2)$, along with their CPT conjugate supermultiplets. In fact, this minimum number is preferred by predictions of $\sin^2 \theta_W$ [DG81, AdBF91, GKL91, EKN91, LL91]. However, we do not need to assume such minimality in general; in fact, some constructions outside the framework of orbifold-GUTs prefer non-minimal Higgs coupling [BL99, BKL01, BL04].

A $5D$ hypermultiplet $\mathcal{H}$ consists of four scalars and two spin-1/2 fields, which would form a pair of $4D$ $\mathcal{N} = 1$ chiral multiplets $\{H, H^c\}$ and their CPT conjugates. However, orbifold parity assignments restrict the boundary propagating modes to be either the H or H^c chiral multiplets. Therefore, in $SU(5)$ orbifold models the Higgs scalars can be minimally taken to come from $5D$ hypermultiplets in the $\mathbf{5} \oplus \mathbf{5}$ of $SU(5)$. In $SO(10)$ orbifold models, the scalars are taken to sit in a $5D$ hypermultiplet in the $\mathbf{10}$ of $SO(10)$.

The hypermultiplet content necessary to contain Standard Model matter in five dimensions is also a simple extension of the matter content in $4D$ supersymmetric GUTs. In the supersymmetric GUTs (in four dimensions), matter fields lie in chiral multiplets. For $SU(5)$ GUTs, the left-handed quarks and leptons sit in left-chiral supermultiplets in the $\bar{\mathbf{5}} \oplus \mathbf{10}$; for $SO(10)$ GUTs, the left-chiral multiplets, including an (unobserved) additional susy Standard Model singlet, are in the spinor irrep $\mathbf{16}$. We have saved the case of E_6 GUTs until now since SM Higgs, quarks, leptons, and the additional SM singlet all fit into the fundamental irrep $\mathbf{27}$.

3.1.1 Simple classes of YMESGTs without tensors or spectators

There are many YMESGTs one can consider, characterized by various choices of C_{IJK}, and the gauging generally requires a number of spectator vector fields and/or the presence of charged tensors (with the latter requiring an abelian gauge factor). Throughout, we will be

interested in relatively large non-abelian gaugings of the form $K_{simple} \times U(1)$ for the purposes of discussing five-dimensional GUTs, and in which there is minimal additional field content. In this section, we list several forms of the C-tensor that allow gaugings not involving tensors or "extra" spectators (that is, other than the graviphoton in the case of compact gaugings). This is to allow us to first focus on the coupling of charged hypermultiplets containing matter and/or Higgs fields.

(1) Consider the simple theory with n_V vector multiplets and

$$C_{ijk} = 0.$$

The scalar manifold for this choice of theories is in general non-homogeneous, and the Lagrangian is invariant under the maximum possible group $G = SO(n_V)$ (as is clear from the form of C_{ijk}). The vector fields decompose as $\mathbf{n_V} \oplus \mathbf{1}$ under G. Thus, the vector fields other than the graviphoton transform in the fundamental representation of $SO(n_V)$; the graviphoton is a spectator vector field.

Remark: any choice of $C_{ijk} \neq 0$ breaks the global symmetry group of the Lagrangian to a subgroup of $SO(n_V)$, and the n_V vector fields will no longer necessarily form an irrep of this new symmetry group.

The adjoint representation of any compact group K can always be embedded in the fundamental representation of $SO(n_V)$ with $n_V \geq \dim(K)$; the $n_V - \dim(K)$ vectors join the graviphoton as spectators.

It follows that the adjoint representation of K can be exactly embedded into the $\mathbf{n_V}$ of $SO(n_V)$ without additional fields (i.e., $n_V = \dim(K)$). For example, consider $n_V = 24$ and $K = SU(5)$; $n_V = 45$ and $K = SO(10)$; or $n_V = 78$ and $K = E_6$. In this way, we can obtain an $SU(5)$, $SO(10)$, or E_6 YMESGT with singlet graviphoton. Of course, one may consider other compact gaugings similarly.

(2) One may split the index $i = (a, \alpha)$, and take C_{ijk} to be

$$C_{abc} = bd_{abc} \qquad C_{\alpha\beta\gamma} = 0,$$

where d_{abc} are the d-symbols of $SU(n) \subset SO(n_V)$; and $b \geq 0$ is a real parameter. The group action preserving the C-tensor is consequently reduced to a subgroup $SU(n) \subset SO(n_V)$. Now, $K \subset SU(n)$ can be gauged, with the remaining $n_V - \dim(K)$ vector fields outside the adjoint representation being spectators. Again, if we are interested in minimal field content, then we demand that $n_V = \dim(K)$, which means that we must restrict out attention to

$SU(n)$ gaugings with $\dim[SU(n)] = n_V$. There are then no vector fields with α indices (i.e., no singlets).

Remark: In contrast to the case of $b = 0$, for $b \neq 0$ we have a single parameter family of theories. The theories are of the same form, since the C-tensor determines couplings in the theory, but there is a single adjustable parameter affecting the strength of those couplings.

(3) There is a class of theories called **unified** YMESGTs[1] in which all vector fields of the theory, *including the graviphoton*, furnish the adjoint representation of a simple gauge group. The known $5D$ **unified** YMESGTs are those in which the C_{IJK} are the d-symbols of the "reduced" Lorentzian Jordan algebras $J_{(1,N)0}^{\mathbb{C}}$ ($N > 2$), which can be realized as traceless $(N + 1) \times (N + 1)$ matrices with complex elements that are hermitian with respect to an $(N + 1)$-dimensional Minkwoski metric. In a **unified** YMESGT, the gauge group is the invariance group of the C_{IJK}, which in the known cases is the automorphism group of the full Lorentzian Jordan algebra $J_{(1,N)}^{\mathbb{C}}$, which is $SU(N, 1)$ (this is the group under which the Jordan algebra is reducible: $\{e\} \oplus J_{(1,N)0}^{\mathbb{C}}$, where $\{e\}$ are elements proportional to the identity of the full Jordan algebra).

While the above pure YMESGTs are minimal in their additional field content, we should note that there can be important implications in four dimensions (either in the dimensional reduction or orbifold effective theory) based on one's choice of C_{IJK} components. In particular, this is responsible for determining the part of the $4D$ scalar manifold arising from the MESGT sector.

3.1.2 Theories based on homogeneous quaternionic scalar manifolds

To list the hypermultiplet content allowed by supergravity, one starts with a particular quaternionic scalar manifold admitting the desired gauge group, and corresponding to some total number of hypermultiplets in representations of $Iso(\mathcal{M}_Q)$. The representations of the (real) scalars break down under the global symmetry group of the Lagrangian, $G \subset Iso(\mathcal{M}_R) \times Iso(\mathcal{M}_Q)$. Finally, under the group we wish to gauge, $K \subset G$, the scalars decompose further giving the spectrum of hypermultiplet representations in the theory.

It is a simple exercise to write down the list of possible matter representations given a gauge group and quaternionic scalar manifold. We first list the hypermultiplets that appear in theories based on homogeneous/symmetric spaces, followed by a brief discussion

---

[1]We use boldface to distinguish this from grand unification. We will discuss this more later

of those based on homogeneous/non-symmetric quaternionic scalar manifolds. We then make comments on non-homogeneous quaternionic scalar manifolds in section (3.1.4).

Homogeneous spaces are characterized by the fact that the isometry group acts transitively on the space $\mathcal{M}$. Such spaces are isomorphic to $Iso(\mathcal{M})/H$, where H is the isotropy group of $\mathcal{M}$. Generally, though not always, homogeneous spaces are the ones admitting large isometry groups, which can then admit large gauge groups. A listing of the homogeneous "special" quaternionic manifolds appearing in the coupling to supergravity (in $3, 4$, and 5 spacetime dimensions) can be found in [dWVP95]; the spaces we are interested in are given in table (3.1). For these theories, $Iso(\mathcal{M}_Q)$ is the symmetry group G of the Lagrangian.

After reducing the scalars of these theories down to reps of E_6, $SO(10)$, and $SU(5)$, we find the possible hypermultiplet representations charged under those gauge groups; they are given in tables (3.2), (3.3) and (3.4), respectively. We have listed those cases with the lowest irrep dimensions (with the exception of the case $L(0, 74)$).

Type	Scalar Manifold	$\dim_{\mathbb{H}}(\mathcal{M}_Q)$	H_Q-rep of scalars
$L(0,P)$	$\frac{SO(P+4,4)}{SO(P+4)\times SO(4)}$	$P+4$	$(\mathbf{P+4},\mathbf{4})$
$L(2,1)$	$\frac{E_6}{SU(6)\times SU(2)}$	10	$(\mathbf{20},\mathbf{2})$
$L(4,1)$	$\frac{E_7}{SO(12)\times SU(2)}$	16	$(\mathbf{32'},\mathbf{2})$
$L(8,1)$	$\frac{E_8}{E_7\times SU(2)}$	28	$(\mathbf{56},\mathbf{2})$
$L(-3,P)$	$\frac{USp(2P+2,2)}{USp(2P+2)\times SU(2)}$	$P+1$	$(\mathbf{2P+2},\mathbf{2})$
$L(-2,P)$	$\frac{SU(P+2,2)}{SU(P+2)\times SU(2)\times U(1)}$	$P+2$	$(\mathbf{P+2},\mathbf{2})\oplus(\overline{\mathbf{P+2}},\mathbf{2})$
$L(q,P)$	Discussed in text		

Table 3.1: Homogeneous quaternionic scalar manifolds. The "type" of space is the classification name as in [dWVP95]; and H_Q is the isotropy group of $\mathcal{M}_Q$.

Type	$\dim_{\mathbb{H}}(\mathcal{M}_Q)$	K-rep of hypermultiplets
$L(-2,P)$	$P + 2 = 27n$	$n(\mathbf{27})$
$L(0,P)$	$P + 4 = 54$	$\mathbf{27} \oplus \mathbf{27}$
	78	$\mathbf{78}$
$L(8,1)$	28	$\mathbf{1} \oplus \mathbf{27}$

Table 3.2: List of hypermultiplets in lowest dimensional representations when gauging E_6, where $n = 1, 2, \ldots$.

Type	$\dim_{\mathbb{H}}(\mathcal{M}_Q)$	K-rep of hypermultiplets
$L(-2,P)$	$P + 2 = 27n$	$n(\mathbf{1} \oplus \mathbf{10} \oplus \mathbf{16})$
$L(0,P)$	$P + 4 = 10n$	$n(\mathbf{10})$
	78	$\mathbf{1} \oplus 2(\mathbf{16}) \oplus \mathbf{45}$
$L(4,1)$	16	$\mathbf{16}$
$L(8,1)$	28	$2(\mathbf{1}) \oplus \mathbf{10} \oplus \mathbf{16}$

Table 3.3: List of hypermultiplets in lowest dimensional representations when gauging $SO(10)$, where $n = 1, 2, \ldots$.

Type	$\dim_{\mathbb{H}}(\mathcal{M}_Q)$	K-rep of hypermultiplets
$L(-3,P)$	$P + 1 = 5n$	$n(\mathbf{5})$
$L(-2,P)$	$P + 2 = 5n$	$n(\mathbf{5})$
	$27n$	$2n(\mathbf{1}) \oplus 3n(\mathbf{5}) \oplus n(\mathbf{10})$
$L(0,P)$	$P + 4 = 10n$	$2n(\mathbf{5})$
	78	$4(\mathbf{1}) \oplus 2(\mathbf{5}) \oplus 4(\mathbf{10}) \oplus \mathbf{24}$
$L(2,1)$	10	$\mathbf{10}$
$L(4,1)$	16	$\mathbf{1} \oplus \mathbf{5} \oplus \mathbf{10}$
$L(8,1)$	28	$3(\mathbf{1}) \oplus 3(\mathbf{5}) \oplus \mathbf{10}$

Table 3.4: List of hypermultiplets in lowest dimensional representations when gauging $SU(5)$, where $n = 1, 2, \ldots$.

3.1.3 Homogeneous, non-symmetric spaces

Other $L(q, P)$:

We will now go through the type of hypermultiplet content obtained by coupling homogeneous/non-symmetric quaternionic scalar manifolds to supergravity. There will generically be a number of gauge singlets in addition to non-trivial irreps. The isotropy group for quaternionic scalar manifolds $L(q, P)$ that are homogeneous, but non-symmetric is

$$H = SO(q + 3) \times SU(2) \times S_q(P, \dot{P}),$$

where $S_q(P, \dot{P})$ is given in table 10 of [dWVP95]. The quaternionic dimension of the manifold is

$$n + 1 = 4 + q + (P + \dot{P})D_{q+1}.$$

The isometry algebra has a three-grading with respect to a generator ϵ':

$$V = V_0 \oplus V_1 \oplus V_2$$
$$V_0 = \epsilon' \oplus so(q + 3, 3) \oplus s_q(P, \dot{P})$$
$$V_1 = (1, spinor, vector)$$
$$V_2 = (2, vector, 0),$$

where *spinor* is the spinor representation of $so(q + 3, 3)$, which is dimension $4D_{q+1}$.

- **$SU(5) \subset H$**

  The only way this can arise is when $S_q \equiv U(5)$; in turn, this occurs for:

q	D_{q+1}	P	$dim_{\mathbb{H}}(\mathcal{M}_Q)$
2	4	5	26
6	16	5	90

  When we gauge $SU(5)$, the scalars will then have the following representation under the gauge group:

  $$[\mathbf{1} \oplus (9 + 3q)\mathbf{1}] \oplus [2D_{q+1}(\mathbf{5} \oplus \bar{\mathbf{5}})] \oplus [(q + 6)\mathbf{1}].$$

- **$SU(6) \subset H$**

  Again, the only way to get this is to have $S_q \equiv U(6)$. This case is then similar to the

above, with $P = 6$. We will get vectors and singlets of $SU(6)$, and therefore **5**s and singlets of $SU(5)$.

- **SO(10) $\subset$ H**
  A. S$_q$ $\equiv$ SO(10)
  The choices are then

q	D_{q+1}	P	$dim_{\mathbb{H}}(\mathcal{M}_Q)$
-1	1	10	13
1	2	10	25
7	16	10	171

The scalars in these cases form the following representation under $SO(10)$:

$$[\mathbf{1} \oplus (3q + 9)\mathbf{1})] \oplus [(4D_{q+1})\mathbf{10}] \oplus [(q + 6)\mathbf{1}]$$

B. SO(q + 3) $\equiv$ SO(10) ($q = 7$, $D_{q+1} = 16$, $P =$ arbitrary)
These spaces have quaternionc dimension $11 + 16P$. However, under an $SO(10)$ subgroup of the isotropy group, the scalars form a set of representations inconsistent with supersymmetry, as they do not form quaternions that can sit in $5D$ hypermultiplets. Thus, neither $SO(10)$ nor its $SU(5)$ subgroup can be consistently gauged.

- **Sp(10) $\subset$ H** This case is similar to the above and will not be discussed.

3.1.4 Comments on theories based on non-homogeneous spaces

Non-homogeneous real and quaternionic scalar manifolds are relevant in string compactifications. For example, it has been shown that, in the special case of the universal hypermultiplet of string compactifications, the quaternionic scalar manifold generally becomes non-homogeneous after string corrections are considered [S97b]. In string theory on a Calabi-Yau manifold, 1-loop effects can show up in $11D$ supergravity on a Calabi-Yau, and thus can appear in compactifications to $5D$. However, non-homogeneous quaternionic manifolds have not been generally classified; in particular those admitting relatively large isometry groups (suitable for obtaining large gauge groups with charged hypermultiplets in phenomenologically interesting representations). We can list three methods from the literature for obtaining (non-compact) non-homogeneous quaternionic manifolds.
(i) In $5D$, it has been shown that there are Maxwell-Einstein supergravity theories with large

isometry groups based on real non-homogeneous manifolds parametrized by scalars from the vector multiplets [EGZ01, GZ03]. By dimensionally reducing these five-dimensional theories to three dimensions, we can obtain non-homogeneous quaternionic scalar manifolds with large isometry groups, which we can couple to $5D$ supergravity. Such a reduction was done for the theories with a special class of symmetric scalar manifolds [GST83], and later an analysis for more general homogeneous spaces appeared [dWVP95]. An analysis of the isometries of the non-homogeneous quaternionic spaces arising from the theories in [GZ03] is a work in progress by the authors of [GMZ05a].

(ii) One may construct $4n + 4$-dimensional non-homogeneous quaternionic manifolds $\hat{\mathcal{M}}$ by fibering over an arbitrary $4n$-dimensional quaternionic base manifold $\mathcal{M}$ with isometry group $Iso(\mathcal{M})$, as discussed in [PP86]. The isometry group is locally $Iso(\mathcal{M}) \times SU(2)$.

(iii) In [KG87], it was shown how $4n$-dimensional generalizations of the four-dimensional non-homogeneous quaternionic space of Pedersen [HP87] (originally considered by Hitchin) could be constructed. These spaces have $SU(n) \times SU(2) \times U(1)$ isometries, and seem to be non-homogeneous forms of the space $L(-2, P)$ in table (3.1). Aside from the spaces that are cosets of exceptional groups, most of the infinite families of quaternionic manifolds classified to date are quaternionic quotients by quaternionic isometries of the quaternionic projective space $\mathbb{H}H^n = Sp(2n + 2)/Sp(2n) \times SU(2)$, or non-compact or pseudo-quaternionic forms thereof. The spaces in [KG87] are of this type, and are presumably pseudo-quaternionic analogues of the spaces $L(-2, P)$ in table (3.1).

We will not attempt to discuss the possible roles of these theories within this thesis.

3.1.5 Summary and discussion: YMESGTs coupled to hypermultiplets

Just as all compact gaugings are possible in pure MESGTs, all compact gaugings are possible when coupling to non-trivially charged hypermultiplets. For example, the theories of type $L(0, P)$ admit any compact gauge group K, since the adjoint representation of K can always be embedded in the fundamental representation of $SO(P + 4)$ if $P + 4 \geq \dim(K)$. However, the resulting hypermultiplet content may be undesirable. This then restricts the number of theories in which we can obtain both the gauge group and hypermultiplet content desired (a theory being uniquely determined by the scalar manifold up to possible arbitrary parameters).

The restriction is even more severe than this. It is not guaranteed that an arbitrary set of hypermultiplets can be obtained by finding a suitable *quaternionic* manifold admiting the

desired gauging. This is clear at least within the set of quaternionic scalar manifolds that are homogeneous, as discussed in this thesis. For example, one may not obtain an arbitrary number of hypermultiplets in the **10** of an $SU(5)$ gauge group. It should be noted that one may *not* get around this restriction by simply coupling two quaternionic scalar manifolds $\mathcal{M}_1$ and $\mathcal{M}_2$ such that $\mathcal{M}_Q = \mathcal{M}_1 \times \mathcal{M}_2$, since these are no longer *quaternionic* manifolds. A quaternionic structure is necessary for coupling to supergravity [BW83].

However, there is a way to construct new quaternionic manifolds from a pair $(\mathcal{M}_1, \mathcal{M}_2)$ of quaternionic manifolds, as discussed in [S91]. The construction relies on the fact that a $4n$-dimensional hyper-Kähler manifold ($Sp(2n)$ holonomy) can be constructed as a bundle over a $(4n-4)$-dimensional quaternionic manifold ($Sp(2n-2) \times SU(2)$ holonomy). Let $\mathcal{M}_1$, $\mathcal{M}_2$ be quaternionic scalar manifolds of dimension $4n_1$ and $4n_2$, respectively. Then there exists a $4n_1 + 4n_2 + 4$-dimensional quaternionic manifold $\mathcal{J}(\mathcal{M}_1, \mathcal{M}_2)$ called the "quaternionic join". Let U_1, U_2 be the hyper-Kähler bundles with base manifolds $\mathcal{M}_1$ and $\mathcal{M}_2$, respectively. The hyper-Kähler manifold $U_1 \times U_2$ is then the bundle over $\mathcal{J}(\mathcal{M}_1, \mathcal{M}_2)$. Locally, the manifold $\mathcal{J}(\mathcal{M}_1, \mathcal{M}_2)$ is a $\mathbb{Z}_2$ quotient of $\mathcal{M}_1 \times \mathcal{M}_2$. The construction requires hyper-Kähler manifolds admitting a hyper-Kähler potential (a Kähler potential for each of the three complex structures). The global manifold does not need to carry the isometries of $\mathcal{M}_1$ or $\mathcal{M}_2$, which may ruin the options for gauging. Even if a particular gauge group is still allowed, the local structure does not necessarily admit the representation $R[\mathcal{M}_1] \oplus R[\mathcal{M}_2] \oplus \mathbf{1}$, where $R[\mathcal{M}]$ is the representation of the scalars (parametrizing $\mathcal{M}$) under the gauge group. This is in contrast with the case of vector and tensor scalars, which locally form a product structure $\mathcal{M}_V \times \mathcal{M}_T$ so that in any neighborhood, scalars can always be divided up into $R[\mathcal{M}_V] \oplus R[\mathcal{M}_T]$; i.e., representations of a "vector sector" and reps of a "tensor sector".

Theories with only bulk Higgs coupling

In many phenomenological models, the $5D$ theory is super-Yang-Mills coupled to $5D$ Higgs hypermultiplets in particular representations of the gauge group (this is sometimes referred to as "the field theory in the bulk"). It is simple to obtain such field content in supergravity. In the case of $SU(5)$ gauging, one can couple any number n of hypermultiplets in the **5** by coupling the scalar manifolds $L(-3, 5n - 1)$ or $L(-2, 5n + 2)$. In the case of $SO(10)$ gauging, one can couple any number n of hypermultiplets in the **10** by coupling the scalar manifold $L(0, 10n - 4)$. In gauging E_6, any number n of hypermultiplets in the **27** may be obtained by coupling to the scalar manifold $L(-2, 27n - 2)$.

Theories with bulk matter

As an interesting illustration of the options that supergravity allows (within the homo-

geneous quaternionic cases), it appears that to obtain a generation of bulk matter hypermultiplets in the $\mathbf{5} \oplus \mathbf{10}$ of $SU(5)$ one *must* include a gauge singlet hypermultiplet (see table (3.4)). But the corresponding theory of type $L(4,1)$ naturally allows gauging of $SO(10)$, under which the hypermultiplets form the irreducible $\mathbf{16}$ (see table (3.3)). One can then orbifold one of these theories as desired.

If a Higgs sector and single generation of matter hypermultiplets is to be coupled in the bulk $SU(5)$ theory, one *must* add two or three additional singlets, which corresponds to the coupling of a different scalar manifold: $L(-2, 25)$ or $L(8, 1)$, respectively (see table (3.4)). Now the field content can sit in smaller set of reps if we gauge E_6 instead, under which the hypermultiplets form a single $\mathbf{27}$ or $\mathbf{27} \oplus \mathbf{1}$, respectively (see table (3.2)). Again, one may then orbifold one of these theories as desired. One can go further. The hypermultiplets in the $\mathbf{27} \oplus \mathbf{1}$ in the $L(8, 1)$ theory can form the $\mathbf{56}$ pseudoreal irrep if we gauge the E_7 allowed by that space. Upon orbifolding the E_6 theory, we would get a chiral multiplet in the $\mathbf{27} \oplus \mathbf{1}$ and its CP conjugate. Orbifolding the E_7 theory yields a spin-1/2 multiplet in the self-conjugate $\mathbf{56}$, which is not good phenomenologically.

Finally, suppose one desires three generations of matter and Higgs in an $SU(5)$ theory. Once again, one must couple two additional singlet hypermultiplets for each generation, corresponding to the scalar manifold $L(-2, 79)$. This begs the question why we shouldn't gauge E_6 instead such that the fields form three generations of $\mathbf{27}$.

One might instead envision the breaking *in five dimensions* of $SO(10)$ to $SU(5)$ gauge group in the theory with scalar manifold $L(4, 1)$. If spontaneous, this breaking would require an additional $5D$ Higgs sector, which would require a coupling of a *different* quaternionic scalar manifold. For example, one would have to couple to $L(-2, 25)$ or $L(8, 1)$, which introduce additional gauge singlets. Note that if the group is broken to $SU(5) \times U(1)$, the $U(1)$ factor consists of mixture of gauged real and quaternionic manifold isometries (since all of the $SO(10)$ gauge symmetries are gauged isometries of *both* the real and quaternionic scalar manifolds). However, tensors charged under this $U(1)$ are not required since there are massive vector multiplets from the Higgs mechanism that take their place.

Similarly, we could evisage the spontaneous breaking of the above E_6 theory to $SO(10)$. We'd need to couple a new scalar manifold admitting a Higgs sector (the simplest being $L(-2, 52)$ or $L(0, 50)$ with two $\mathbf{27}s$), but this would make the field content more complicated in the end.

Alternatively, the $5D$ breaking could be performed by Wilson lines of, e.g., the $U(1)$ factor in the subgroup $SU(5) \times U(1) \subset SO(10)$ or of $SO(10) \times U(1) \subset E_6$. Some details of such breaking and the relation to parity assignments is discussed in [M05c]; for related issues

of boundary conditions and Wilson lines in S^1/Γ orbifold field theory scenarios, see [HMN02, HHHK03, BR04].

Partial unification in four dimensions

Most of our discussion is in the context of orbifold-GUTs wherein the Standard Model gauge group remains at one of the fixed points. If one wants to break to a partially unified group at one of the fixed points, the Higgs content in the bulk must of course be enlarged (suppose that we do not wish to turn to string theory and its twisted sector states).

$\mathbf{SO(10) \to SU(4) \times SU(2)_L \times SU(2)_R}$

In four dimensions, we would need the $\mathbf{16} \oplus \overline{\mathbf{16}}$ of $SO(10)$ to break the Pati-Salam gauge group (PS) to that of the SM (we need the $(4, 1, 2)$ of PS and its conjugate to perform the breaking), as well as the electroweak breaking. Since each hypermultiplet can provide a $4D$ left-chiral $\mathbf{16}$ *or* $\overline{\mathbf{16}}$, we will need to have 2 five-dimensional hypermultiplets in the $\mathbf{16}$. In addition, a single generation of matter is in the $\mathbf{16}$ of $SO(10)$. The minimal way to get multiple $\mathbf{16}$s in the bulk is by coupling an $SO(10)$ YMESGT to hypermultiplets whose scalars parametrize the manifold $L(-2, 27n - 2)$; that is, we must have n hypers in the $\mathbf{1} \oplus \mathbf{10} \oplus \mathbf{16}$. Therefore, extra fields must come along (n copies of $(2, 2, 1) \oplus (1, 1, 6) \oplus (1, 1, 1)$ of $SU(2) \times SU(2) \times SU(4)$). These extra states will show up as color triplets and extra weak doublets and singlets, and must be made massive via boundary conditions. This in turn will affect the gauge coupling running, possibly adversely. Anyway, the necessary additions beg the question: why not gauge E_6 instead so that the fields form an irrep?

$\mathbf{E_6 \to SU(3)_c \times SU(3)_L \times SU(3)_R}$

To round out the discussion, suppose we wish to have an $SU(3)^3$ trinification [G84, LPS93] scenario at the fixed points of the orbifold. In four dimensions, we would need the $\mathbf{27} \oplus \overline{\mathbf{27}}$ of E_6 to break the trinification group (TG) to SM, and then down to the visible $SU(3)_c \times U(1)_{em}$. Since the color singlets inside the $\mathbf{27}$s are used to perform gauge symmetry breaking, we must add an additional $\mathbf{27}$ for each generation of matter. We can obtain such a model by coupling an E_6 YMESGT to n hypermultiplets whose scalars once again parametrize the manifold $L(-2, 27n - 2)$.

3.2 Options for YMESGTs coupled to tensor multiplets

As mentioned previously, we may gauge any compact group in the framework of $5D$ $\mathcal{N} = 2$ supergravity, but we cannot presume any charged field content we like. A real Riemannian scalar manifold must be specified for which $K \subset G \subset Iso(\mathcal{M}_R)$. In [TPvN84], it was shown that tensor multiplets carrying non-trivial representations of a group G and satisfying a self-duality condition are required to come in complex conjugate pairs $\Sigma_i \mathbf{m}_i \oplus \Sigma_i \bar{\mathbf{m}}_i$. However, it was shown in [GZ99] that, when gauging K, the complex K-representations should be of "quaternionic type"[2]; that is, symplectic representations. Tensor multiplets charged under a gauge group K therefore arise when there are symplectic, non-singlet representations of K in the decomposition appearing in (2.4). It is sufficient to find complex representations of a compact gauge group $K_{semi-simple} \times U(1)$ coming in pairs $\mathbf{m} \oplus \bar{\mathbf{m}}$.

We may now look for gauge theories we are interested in that admit tensor couplings. We will ignore any coupling to hypermultiplets here so that the scalar manifold is $\mathcal{M}_R$, and we can gauge $K \subset G \subset Iso(\mathcal{M}_R)$, where G consists of isometries that are symmetries of the Lagrangian. Recall that the rank-3 symmetric tensor C_{IJK} uniquely determines the form of a MESGT (up to possible arbitrary reparametrizations) and that the non-trivial invariance group of this tensor is precisely G. An algebraic analysis of the form of this tensor is useful for understanding the vector and tensor field content in a YMESGT, and for gauging purposes is equivalent to the geometric analysis of the form of the scalar manifold and its isometry group.

It was shown in [GZ99] that for theories based on *homogeneous, symmetric* real scalar manifolds, there aren't any large non-abelian gauge groups (such as the typical GUT groups) with charged tensors.

3.2.1 Homogeneous, non-symmetric scalar manifolds

These scalar manifolds are described in [dWVP95]. The presence of tensor fields in these theories was addressed in [GZ99], though we perform a slightly different analysis.

---

[2]"Quaternionic type" meaning that there exists a K-invariant anti-symmetric bilinear form on the vector space.

The isometry algebra has a one-grading with respect to a generator λ:

$$\chi = \chi_0 \oplus \chi_{3/2}$$
$$\chi_0 = \lambda \oplus so(q+1,1) \oplus s_q(P,\dot{P})$$
$$\chi_{3/2} = (spinor, vector),$$

where *spinor* denotes the spinor representation of $SO(q+1,1)$ of dimension D_{q+1}, and the groups corresponding to the algebras $s_q(P)$ are listed in [dWVP95]. The isotropy group is

$$H = SO(q+1) \times S_q(P,\dot{P}),$$

where $S_q(P,\dot{P})$ is the group corresponding to the algebra $s_q(P,\dot{P})$. The dimension of the real Riemannian scalar manifold is $(n-1) = 2 + q + D_{q+1}P$. The vector multiplets form the following representation under H:

$$(\mathbf{1},\mathbf{1}) \oplus (\mathbf{q+1},\mathbf{1}) \oplus (\mathbf{D_{q+1}},\mathbf{P}). \tag{3.1}$$

The condition for gauging a group K is that the adjoint representation of K should appear in the decomposition into K-reps (see (2.4)).

- $K \subset SO(q+1)$ and $q \geq 9$, $D_{q+1} \geq 32$, $P =$arbitrary

  $\mathbf{K} = \mathbf{SU(5)}$
  (i) If $10 \leq q+1 \leq 23$, then the only possibility for the adjoint rep of K to exist is if the spinor rep of $SO(q+1,1)$ contains it in the decomposition. The spinor rep is of dimension $32 \leq D_{q+1} \leq 4096$ in our special case. It seems that the largest representation dimension decomposing from the spinor representation of $SO(q+1,1)$ is the $\mathbf{16}$ of $SO(10)$ so that we cannot gauge $SU(5)$.
  (ii) If $q+1 \geq 24$, any value of q in this range allows gauging of $SU(5)$, with $(q+1)-24$ spectator fields. The remaining vector fields are P copies of the decomposition of $spinor[SO(q+1,1)]$ into $SU(5)$-reps. This yields a large number of tensor fields ($>$ 4096).

  $\mathbf{K} = \mathbf{SO(10)}$
  (i) If $10 \leq q+1 \leq 44$, the only possibility for the adjoint rep of K to exist is if the spinor rep of $SO(q+1,1)$ contains it in the decomposition. The spinor rep is of dimension $32 \leq D_{q+1} \leq 2^{23}$, but the largest irrep in the decomposition into $SO(10)$ representations is the $\mathbf{16}$. Therefore, $SO(10)$ cannot be gauged for these values of q.

(ii) If $q+1 \geq 45$, we can gauge $SO(10)$ for all values of q in this range. The $(q+1) - 45$ vector fields not involved in this gauging are spectators. The remaining vector fields are in P copies of the decomposition of $spinor[SO(q+1,1)]$ into $SO(10)$-reps. This yields a large number of tensor fields ($> 2^{23}$) in the $\mathbf{16} \oplus \overline{\mathbf{16}}$.

- $K \subset S_q(P) \equiv SO(P)$ with $P \geq 10$

  We require that $\mathrm{adj}(K) \subset vector[S_q(P)]$. The adjoint of any compact Lie group sits in the $\mathbf{n}$ of $SO(n)$ if $n \geq \dim[\mathrm{adj}(K)]$. There will then be $n - \dim[\mathrm{adj}(K)]$ spectator vector fields in addition to the graviphoton. Therefore, no tensors charged under a non-abelian gauge group appear.

- $K \subset S_q(P) \equiv U(P)$ with $P \geq 5$

  The values of q are restricted to $q = 2 \mod(8)$ and $6 \mod(8)$. Again, we require $\mathrm{adj}(K) \subset vector[S_q(P)]$. The vector fields form the $\mathbf{1} \oplus (q+1)\mathbf{1} \oplus (D_{q+1})\mathbf{P}$ of $S_q(P)$. For the adjoint of $U(5)$ to appear, we require $P \geq 25$. However, there aren't any tensors to be charged with respect to the $U(1)$ factor. This applies for other non-abelian cases.

3.2.2 Non-homogeneous scalar manifolds

- Theory with $C_{ijk} = 0$

  This is the choice in case **1** of section (3.1.1). It is clear that these theories do not admit tensor multiplets charged under non-abelian gauge groups.

A set of interesting theories are based on Lorentzian Jordan algebras [GZ03][3] $J^{\mathbb{A}}_{(1,N)}$ of degree $(N+1)$, where $\mathbb{A} = \mathbb{R}, \mathbb{C}, \mathbb{H}$; there is also the exceptional theory based on $J^{\mathbb{O}}_{(1,2)}$. These theories are listed below, where G denotes the invariance group of the Lagrangian. In constrast to the theories with homogeneous scalar manifolds, these theories admit GUT groups coupled to non-trivially charged tensor multiplets. For phenomenological reasons, though, we focus on $SU(5) \times U(1)$ gauging.

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[3]These were originally called "Minkowski" Jordan algebras in that work.

J.A.	$\dim(\mathcal{M}_R)$	G
$J^{\mathbb{R}}_{(1,N)}$	$N(N+3)/2 - 1$	$SO(N,1)$
$J^{\mathbb{C}}_{(1,N)}$	$N(N+2) - 1$	$SU(N,1)$
$J^{\mathbb{H}}_{(1,N)}$	$N(2N+3) - 1$	$USp(2N,2)$
$J^{\mathbb{O}}_{(1,2)}$	25	$F_{(4,-20)}$

These theories are all examples of "**unified**" MESGTs, in the sense that there is a continuous symmetry connecting every field of the theory:[4]

$$
\begin{array}{ccc}
\leftrightarrow & \leftrightarrow & \leftrightarrow \\
\{e_\mu^m & \Psi_\mu & A_\mu^0\} \\
& \Updownarrow & \\
\{A_\mu^i & \lambda^a & \phi^x\} \\
\leftrightarrow & \leftrightarrow & \leftrightarrow
\end{array}
\tag{3.2}
$$

where horizontal arrows represent (local) supersymmetry action, and the vertical arrow represents the action of a *simple* global symmetry group G, like those listed in the above theories. (For more on **unified** MESGTs, see [GST84a, GZ03].)

A general discussion of the tensor couplings in these theories can be found in [GZ03], which we use to write down the theories of interest to us here.

- $J^{\mathbb{R}}_{(1,N)}$

  One can gauge $SU(n) \times U(1) \subset SO(2n,1)$ (with $N = 2n$), obtaining tensors in

  $$
  \left(\frac{n(n+1)}{2} \oplus \overline{\frac{n(n+1)}{2}} \right) \oplus (n \oplus \bar{n}).
  $$

  In particular, gauging $SU(5) \times U(1)$ ($n = 5$), we get tensors in $(\mathbf{15} \oplus \overline{\mathbf{15}}) \oplus (\mathbf{5} \oplus \bar{\mathbf{5}})$.

- $J^{\mathbb{C}}_{(1,N)}$

  If we gauge $SU(N) \times U(1) \subset SU(N,1)$, we get tensors in the $\mathbf{N} \oplus \overline{\mathbf{N}}$. In particular, taking $N = 5$, we get $\mathbf{5} \oplus \bar{\mathbf{5}}$ tensors. More generally, consider gauging $SU(5) \times U(1) \subset SU(5n,1)$; we get n sets of tensors in the $(\mathbf{5} \oplus \mathbf{5})$ of $SU(5)$.

  Next consider $N = 27$ and gauge $E_6 \times U(1) \subset SU(27) \times U(1) \subset SU(27,1)$. This yields 704 fields outside of the adjoint representation.

- $J^{\mathbb{H}}_{(1,N)}$

---

[4]We will use boldface to distinguish this from any other sense of "unified".

Gauging $SU(N,1) \subset USp(2N,2)$, we get tensors in $\frac{N(N+1)}{2} \oplus \overline{\frac{N(N+1)}{2}}$ Choosing $N = 5$, we get $\mathbf{15} \oplus \overline{\mathbf{15}}$. Under $SU(5) \times U(1)$, this becomes $(\overline{\mathbf{5}} \oplus \mathbf{10}) \oplus (\mathbf{5} \oplus \overline{\mathbf{10}})$.

- $J^{\mathbb{O}}_{(1,2)}$
  The symmetry group of the Lagrangian is too small to gauge a GUT group.

We summarize the theories with reasonable numbers of tensor couplings in table (3.5). They are all of $SU(5) \times U(1)$ type gauging.

J.A.	$\dim(\mathcal{M}_R)$	Tensor K-reps
$J^{\mathbb{R}}_{(1,10)}$	64	$(\mathbf{15} \oplus \overline{\mathbf{15}}) \oplus (\mathbf{5} \oplus \overline{\mathbf{5}})$
$J^{\mathbb{C}}_{(1,5n)}$	$5n(5n+2) - 1$	$n(\mathbf{5} \oplus \overline{\mathbf{5}})$
$J^{\mathbb{H}}_{(1,5)}$	64	$(\overline{\mathbf{5}} \oplus \mathbf{10}) \oplus (\mathbf{5} \oplus \overline{\mathbf{10}})$

Table 3.5: Summary of theories admitting $SU(5) \times U(1)$ gauging with tensor couplings (and with smallest field content).

3.2.3 Summary and discussion: YMESGTs coupled to tensors

Within the class of theories discussed in this thesis, the only ones admitting reasonable numbers of tensor multiplets of interest in GUTs are the those based on Lorentzian Jordan algebras discussed in the previous section. Table (3.5) lists the theories for the case of $SU(5) \times U(1)$ gauge group (they were discussed in [GZ03]). Generically, the gauging GUT groups starting from a given MESGT is associated with large numbers of tensors and spectator vector fields. We have not systematically considered $E_6 \times U(1)$ gauging with tensor multiplets, though it appears that these theories also have large numbers of tensors and singlets. A large number of unwanted tensor multiplets can be troublesome in orbifold-GUT models since one cannot get rid of all of the field content in these multiplets via orbifold boundary conditions (this is shown in [M05b]).

There are other families of tensor couplings with non-homogeneous scalar manifolds that we have not discussed in this thesis. Although the geometry of these scalar manifolds is not understood, an algebraic discussion of such theories can be found in [EGZ01].

Non-trivially charged tensor multiplets offer a possible way to introduce scalar fields in non-trivial gauge group representations in a more economical way; for every hypermultiplet introduced, there are four real scalars, whereas for every tensor muliplet introduced, there is only one real scalar. Such a $5D$ GUT model is considered in [DGKL01]. The authors

consider a field content that consists of gauge multiplets for $SU(5)$, 10 tensor multiplets (in the $\mathbf{5} \oplus \bar{\mathbf{5}}$ of $SU(5)$), a spectator vector multiplet, and a graviton multiplet. The theory is not obtained from an explicit scalar manifold, though and appears to be obtained by decomposition of $SU(5) \subset G$, with $G = SU(6)$ or a non-compact form thereof. However, one must recall that, if tensor mulitplets non-trivially charged under a compact gauge group K, there must be at least one abelian factor (corresponding to an isometry of the real scalar manifold).

Example of gauging with tensor couplings

Within the framework of supergravity that we have reviewed throughout, the above model can be obtained from the theory based on Lorentzian Jordan algebra $J_{(1,5)}^{\mathbb{C}}$. Gauging $SU(5) \times U(1) \subset SU(5,1)$ yields a theory with the 35 vector fields decomposing into $\mathbf{1} \oplus \mathbf{5} \oplus \bar{\mathbf{5}} \oplus \mathbf{24}$ under $U(5)$. The $\mathbf{5} \oplus \bar{\mathbf{5}}$ vectors must be dualized to tensor fields, and the singlet gauging the $U(1)$ is the graviphoton. Under particular boundary conditions of the theory, one can obtain massless chiral multiplets in the $\mathbf{5} \oplus \bar{\mathbf{5}}$ along with their CPT conjugates, which can therefore potentially serve as a Higgs sector. Otherwise, these will lead to massive vector multiplets in the $\mathbf{5} \oplus \bar{\mathbf{5}}$. More details of this will be shown in [M05b], which decribes the options for parity assignments in $5D$ $\mathcal{N} = 2$ orbifold supergravity theories. *Note that the graviphoton is required in the gauging.* This theory comes from the gauging of a **unified** MESGT, where all of the fields had been connected by a continuous global symmetry; the gauging of $SU(5) \times U(1)$ then disconnects the gravity and vector supermultiplets.

Obtaining a simple unifying group appears difficult starting from a theory with $K_{simple} \times K_{abel}$ and charged tensors. One could be satisfied with a partially unified group like the "flipped SU(5)" model with $SU(5) \times U(1)$ gauge group [B82, DRGG80, AEHN87]. But suppose we wish to embed the above theory into one with a simple compact gauge group. There will be tensors at each stage of the embedding, requiring a U(1) factor, until the tensor reps lie in the adjoint of the simple group. Starting from $SU(5) \times U(1)$, that group is E_8. However, starting with such a gauge group, there must be a mechanism for breaking it down the line to $SU(5) \times U(1)$; a Higgs mechanism will yield massive vectors, not tensors. So we cannot embed the tensor coupled theory in this way. Anyway, we don't know of a real scalar manifold admitting an E_8 gauging unless we take the theory defined by $C_{ijk} = 0$. But tensor couplings require non-trivial C-tensor components (see section (2.3.2)). It could be that tensor multiplets have a natural home in higher dimensional unification scenarios.

There is an alternative scenario in $5D$ that allows tensor coupings with simple groups: *non-compact* gaugings. This is the topic of the next section.

3.3 Non-compact gaugings and unified YMESGTs

In contrast to their rigid limits, $5D$ $\mathcal{N} = 2$ supergravity theories coupled to vector multiplets admit non-compact non-abelian gauge groups while remaining unitary. Such gaugings have been considered since the 1980's [GST83, GZ03]. The ground states of these theories preserve at most the maximal compact subgroup as a symmetry group, with the non-compact gauge multiplets becoming $5D$ BPS massive:

$$\{A_\mu^M, \lambda^{mi}\},$$

where M is the index for the non-compact generators of the gauge group (and m is the tangent space index for the scalar manifold coresponding to the directions of the non-compact gauged isometries).

Example of Higgs sector from non-compact gauging

Consider the infinite family of YMESGTs based on the Lorentzian Jordan algebras $J^{\mathbb{C}}_{(1,N)}$ with gauge group $SU(N, 1)$. This family of YMESGTs are known as **unified** in the sense that there is a continuous (and in this case local) symmetry relating every field in the theory; see (3.2), where horizontal arrows represent (local) supersymmetry transformations, and vertical arrows now represent the action of a *simple* gauge group involving *all* the vector fields of the theory (For more on **unified** YMESGTs, see [GST83, GZ03, GMZ05a].) The ground state of these theories preserves at most an $SU(N) \times U(1)$ symmetry group, while the $2N$ non-compact gauge fields transforming in the $\mathbf{N} \oplus \bar{\mathbf{N}}$ become BPS massive.

In particular, we may consider the $N = 5$ case: we have an $SU(5, 1)$ gauge group with 35 gauge fields. The ground state of the theory can have at most $SU(5) \times U(1)$ gauge group with the remaining 10 vector multiplets in the $\mathbf{5} \oplus \bar{\mathbf{5}}$ becoming BPS massive. This could be a unified theory into which flipped $SU(5)$ is embedded, since $U(5)$ is not simple, but $SU(5, 1)$ is. Note that the scalars in the non-compact gauge multiplets are eaten by the vector partners, and so we cannot obtain $5D$ Higgs scalars from these theories. Upon dimensional reduction, a $5D$ $\mathcal{N} = 2$ BPS vector multiplet yields a $4D$ $\mathcal{N} = 2$ BPS vector multiplet:

$$\{A_\mu^M, \lambda^{mi}, A^M\},$$

where the real scalar fields A^M come from the reduction of $5D$ vectors. Truncating to $\mathcal{N} = 1$, we would expect a massive $4D$ $\mathcal{N} = 1$ vector multiplet; we would not obtain $4D$ massless Higgs scalars. However, just as in the case of tensor multiplets on the orbifold, one can assign parities appropriately so that instead of a massive vector multiplet, we are left with

massless $4D$ $\mathcal{N} = 1$ chiral multiplets in the $\mathbf{5} \oplus \bar{\mathbf{5}}$ along with their CPT conjugate multiplets (see chapter 4). One can use orbifold parity conditions to obtain a subgroup of $SU(5) \times U(1)$ and/or use a vev on the boundaries to perform the breaking, as is usually done in orbifold models.

Example of non-compact gauging with tensors

One can also now have tensor multiplets charged under a simple gauge group. For example, one can gauge $SU(N, 1)$ in the theory based on the Lorentzian Jordan algebra $J^{\mathbb{H}}_{(1,N)}$, with charged tensor multiplets in the $\frac{(\mathbf{N+1})}{\mathbf{2}} \oplus \overline{\frac{(\mathbf{N+1})}{\mathbf{2}}}$ (see section (3.2.2)). The theory is a **unified** YMESGT coupled to tensors. The original MESGT is a **unified** theory, but when gauged, the tensor sector is "cut off". In particular, consider the case of $N = 5$; we get the **unified** gauge group $SU(5, 1)$ again, but now coupled to tensors in the $\mathbf{15} \oplus \overline{\mathbf{15}}$. The ground state has at most $SU(5) \times U(1)$ gauge group, with $\mathbf{5} \oplus \bar{\mathbf{5}}$ BPS massive vector multiplets, and tensor multiplets in the $(\bar{\mathbf{5}} \oplus \mathbf{10}) \oplus (\mathbf{5} \oplus \overline{\mathbf{10}})$. This theory can then be orbifolded [M05b].

Though one may obtain non-compact gauged supergravity theories from "compactification" of M- or string theory on non-compact hyperboloidal manifolds [CGP04], it is not clear that this is the only way to obtain such gaugings from a more fundamental theory in higher dimensions.

3.4 Summary of $5D$ options

We have pointed out that, within the classification of homogeneous quaternionic scalar manifolds, one cannot choose any pairing of gauge group and hypermultiplet representations one likes. However, we have shown how orbifold models considered in the literature can be constructed in supergravity via these manifolds (without the introduction of boundary localized fields).

- Any number n of $5D$ Higgs hypermultiplets in the $\mathbf{5}$ or $\mathbf{10}$ can be embedded in an $SU(5)$ or $SO(10)$ (resp.) gauge theory by coupling the relevant homogeneous scalar manifolds in tables (3.4) and (3.3) (resp.).

- If a single generation of matter in the $\mathbf{5} \oplus \mathbf{10}$ of an $SU(5)$ theory is desired, an additional singlet must be coupled (see table (3.4)). The theory is characterized by the coupling

of the scalar manifold (see table (3.1))

$$\mathcal{M}_R \times \frac{E_7}{SO(12) \times SU(2)}.$$

But this theory admits an $SO(10)$ gauging under which the matter forms the **16** (table (3.3)).

- If n generations of bulk matter *and* Higgs multiplets in the

$$(\bar{\mathbf{5}} \oplus \mathbf{10}) \oplus (\mathbf{5} \oplus \bar{\mathbf{5}})$$

of the gauge group $SU(5)$ are desired, two additional singlets must be added (see table (3.4)). This corresponds to the coupling of the scalar manifold (see table (3.1))

$$\mathcal{M}_R \times \frac{SU(27n, 2)}{SU(27n) \times SU(2) \times U(1)}.$$

But this theory admits an E_6 gauging under which each generation of fields form the **27** (table (3.2)).

Since there are no known quaternionic scalar manifolds with compact E_8 isometries, there isn't a unification of generations into an irreducible representation within the framework of $5D$ supergravity. However, multiple generations arise in Calabi-Yau compactifications of string or M-theory, and are therefore expected to appear in the supergravity approximations in four and five dimensions, respectively.

In all of the above cases, $\mathcal{M}_R$ is whatever scalar manifold one has chosen, with the following constraints:

(i) The isometry group must admit the gauge group as a subgroup, and
(ii) If the decomposition of the $n_V + 1$ vector fields under the compact gauge group has non-singlet (not including the adjoint) representations, the corresponding vectors must be dualized to tensors and at least one abelian isometry must be gauged.

For minimality, one may take the spaces described in section (3.1.1) in which all of the vector fields gauge the group of interest, except the graviphoton, which is a spectator in the compact gaugings of scalar manifold isometries. However, it should be emphasized that this choice is not motivated by any other compelling phenomenological reason.

As an alternative scenario to hypermultiplets in the bulk, we may attempt to put the Higgs scalars or matter fields in tensor multiplets; however, gauging a compact subgroup of

$Iso(\mathcal{M}_R)$ with charged tensors requires the group to have at least one abelian factor. For example, we may consider the $\mathbf{5} \oplus \bar{\mathbf{5}}$ tensor multiplets of the $SU(5) \times U(1)$ gauge theory based on the Lorentzian Jordan algebra $J_{(1,5)}^{\mathbb{C}}$. This is a more economical approach, in the sense that one uses only 10 scalars as opposed to the 40 (in this example). Even though theories with tensor couplings do not involve a simple gauge group, there is the partial unification model based on $SU(5) \times U(1)$ ("flipped $SU(5)$"). Furthermore, a generation of matter can sit in a bulk $(\bar{\mathbf{5}} \oplus \mathbf{10}) \oplus (\mathbf{5} \oplus \overline{\mathbf{10}})$ by coupling the real scalar manifold based on the Lorentzian Jordan algebra $J_{(1,5)}^{\mathbb{H}}$. We will thus be left with a non-chiral theory.

Non-compact gauge groups in $5D$ supergravity are another novel way to get $4D$ massless chiral multiplets in interesting representations of the gauge group, which allows for an extension of previously studied gauge-Higgs unifications via compact groups [HNS02, BN03, HS03]. We have mentioned the example of a **unified** $SU(5,1)$ YMESGT based on the Lorentzian Jordan algebra $J_{(1,5)}^{\mathbb{C}}$, though there is an infinite family of such non-compact gaugings; the theories based on $J_{(1,N)}^{C}$ admit **unified** $SU(N,1)$ YMESGTS. **Unified** here, is in the sense that all fields of the YMESGT are connected by a combination of supersymmetry and gauge transformations; therefore, this is in some sense in between the ideas of Grand Unification and gauge/gravity coupling unification.

We have not discussed the case of a "gauged YMESGT" (where a subgroup of $SU(2)_R$ is gauged), which admits AdS or flat supersymmetric vacua, depending on the linear combination of vector fields used to gauge this factor (coupling to tensors then results in novel supersymmetric vacua) [GST84b, GST85a, GST85b, GZ99, GZ00a, GZ01a].

Chapter 4

Supergravity on S^1/Γ

In modeling five-dimensional spacetimes with four-dimensional boundaries, we can choose a particular construction using a spacetime of the form $M_4 \times S^1/\Gamma$, where Γ is a discrete group that acts non-freely on the circle. Following the physics literature, we call S^1/Γ an "orbifold", and $M_4 \times S^1/\Gamma$ an "orbifold spacetime". However, we note here that the correct definition of an n-orbifold X is a singular n-manifold, whose singularities are locally isomorphic to $\mathbb{R}^n/\Gamma$ where $\Gamma \subset GL(n, \mathbb{R})$, such that the orbifold fixed surfaces (under action of Γ) are at least codimension 2. Clearly, S^1/Γ is not an orbifold by this definition; it has codimension 1 fixed points. While true orbifolds such as $T^2/\mathbb{Z}_2$ have properties in common with manifolds, the "orbifold" $S^1/\mathbb{Z}_2$ does not benefit from such similarities. Nevertheless, we will refer to this as an orbifold, dropping the quotes from now on.

Orbifold constructions have been performed many times in the literature for both rigidly and locally supersymmetric field theories; for examples of the former, see [K00, K01a, K01b, K01c, AF01, HMR01a, KY02, HMN02]; for the latter, see [ABN01, BB03, YL03, ZGAZ04]. The generic results of the construction are two $4D$ boundary theories that preserve $\mathcal{N} = 1$ supesymmetry and support broken gauge groups. However, a systematic classification of the types of boundary theories available via parity assignments has not been performed for Yang-Mills-Einstein supergravity theories (YMESGTs) coupled to vector, hyper-, and tensor multiplets. In this chapter, based on [M05b], we hope to provide a more complete list of options for the low energy spectrum via parity assignments in the simple case of the $S^1/\mathbb{Z}_2$ orbifold, and the extension to the case $S^1/(\mathbb{Z}_2 \times \mathbb{Z}_2)$. Some of the results are generic to theories with super-Yang-Mills coupled to hypermultiplets, while others are unique to supergravity. While orbifold-GUTs are a main motivation, the results are not restricted to these scenarios. As a novel example of $5D$ GUT in the framework of supergravity, we illustrate some of the parity assignments using an $SU(5, 1)$ gauging.

Before continuing, let's review the supermultiplet structure of $4D$ $\mathcal{N} = 1$ theories, which has susy automorphism group $U(1)_R$.

- The supergravity multiplet consists of the graviton and gravitino fields

$$\{g_{\mu\nu}, \Psi_\mu\}$$

- A vector multiplet consists of a vector field and left/right helicities of a spin-1/2 field

$$\{A_\mu, \lambda\}$$

- A massive vector multiplet consists of helicity +1,0,-1 states (forming a massive vector field), two $\pm 1/2$ helicity states (forming two massive spin-1/2 fields), and a real (massive) scalar field

$$\{A_\mu, \lambda, \phi\}$$

- A left (right) chiral multiplet consists of a +1/2 (resp. -1/2) helicity field and a real scalar field

$$\{\lambda_{L\,(R)}, \phi\}$$

A pair of left and right chiral supermultiplets are charge conjugates (the two scalar fields are a complex scalar and its complex conjugate).

4.1 $\Gamma = \mathbb{Z}_2$ Orbifold field theory

A groundstate spacetime $\mathcal{M}_4 \times \mathcal{I}$, where $\mathcal{I}$ is an interval, is isomorphic to the orbifold $\mathcal{M}_4 \times S^1/\Gamma$, where Γ is a discrete subgroup of the $U(1)$ isometry group of the circle. Instead of considering a $5D$ theory with a boundary (downstairs picture), it is often convenient to compactify the $5D$ theory on S^1, followed by assignment of Γ-parities to quantities in the theory (upstairs picture). The choice of Γ reflects different classes of boundary conditions from the downstairs point of view. We will first consider the simplest case $\Gamma = \mathbb{Z}_2$, which results in a theory with equivalent spectra and interactions at the two fixed points.

The choice of the way $\mathbb{Z}_2$ acts on quantities in the theory reflects a particular set of consistent boundary conditions. First of all, $\mathbb{Z}_2$ cannot have a free action on S^1, so there will be fixed points. In particular, it acts as reflections on the S^1 covering space $[-\pi R, \pi R]$ (where $-\pi R \equiv \pi R$), with fixed points at $\{0\}, \{\pi R\}$. However, when fields carry internal quantum numbers, they are sections of a fiber bundle, with spacetime being the base space. In such a situation, it makes sense for the action of $\mathbb{Z}_2$ to be lifted from the base space to the total space [DHVW85, DHVW86]. There are a number of ways to perform this

lift, corresponding to various classes of boundary conditions. Just as the $\mathbb{Z}_2$ action on the covering space S^1 results in a singular space $S^1/\mathbb{Z}_2$, the $\mathbb{Z}_2$ action on the total space will, in general, change the structure of the fibers over the base space.

In particular, we are interested in gauge theories, so there will be a gauge bundle. Objects other than fields that appear in the Lagrangian carry representation indices of the gauge group. For example, in a YMESGT where I, J are adjoint indices, $\overset{\circ}{a}_{IJ}(\phi)$ must be a rank-2 symmetric invariant (locally δ_{IJ}), and C_{IJK} is a rank-three symmetric invariant of the gauge group (in the case of $SU(N)$ gauge groups, these are proportional to the d-symbols). Such quantities are structures appearing in the gauge bundle, and are therefore generally affected by modifications of the gauge bundle resulting after $\mathbb{Z}_2$ action. This gives meaning to assigning these objects $\mathbb{Z}_2$ parities.

Although physical states on $\mathcal{M}^4 \times S^1/\mathbb{Z}_2$ must be even under $\mathbb{Z}_2$-action, the field operators can carry even or odd parity. A field on $\mathcal{M} \times S^1$ can be expressed as a sum over Kaluza-Klein modes; but under $\mathbb{Z}_2$ action, the spacetime becomes singular. The general expansion of an odd parity field will have nth term of the form

$$\Phi_-^{(n)}(x^\mu, x^5) = A_n \Phi_-^{(n)}(x^\mu)\sin(nx^5/R) + B_n \Phi_-^{(n)}(x^\mu)\epsilon(x^5)\cos(nx^5/R), \qquad (4.1)$$

where $\epsilon(x^5)$ is $+1$ for $(-\pi R, 0)$ and -1 for $(0, \pi R)$ (see figure (4.1)); the $\Phi_-^{(n)}(x^\mu)$ are even; and A_n, B_n are normalization factors.

The equations of motion for bosonic fields are 2nd order differential equations, so these fields cannot have $\epsilon(x^5)$ factors (otherwise, there will be δ' and δ^2 factors in the equations of motion, with $\delta(x^5)$ being the Dirac distribution). Therefore, we impose the condition $B_n = 0$ for odd bosonic fields; it's clear, then, that odd bosonic fields $\Phi(x^\mu, x^5)$ vanish on the orbifold fixed planes.

On the other hand, the equations of motion for fermionic fields are 1st order differential equations, so $\epsilon(x_5)$ factors are allowed (they will give rise to $\delta(x_5)$ factors in the equations of motion). Therefore, fermionic fields on $S^1/\mathbb{Z}_2$ do not necessarily have well-defined limits in the upstairs picture.

Field-independent objects $C_{J_1...J_n}^{I_1...I_n}$ carrying gauge indices that are assigned odd parity are redefined, where allowed, by $\epsilon(x^5)C_{J_1...J_n}^{I_1...I_n}$, with $C_{J_1...J_n}^{I_1...I_n}$ now being parity even. However,
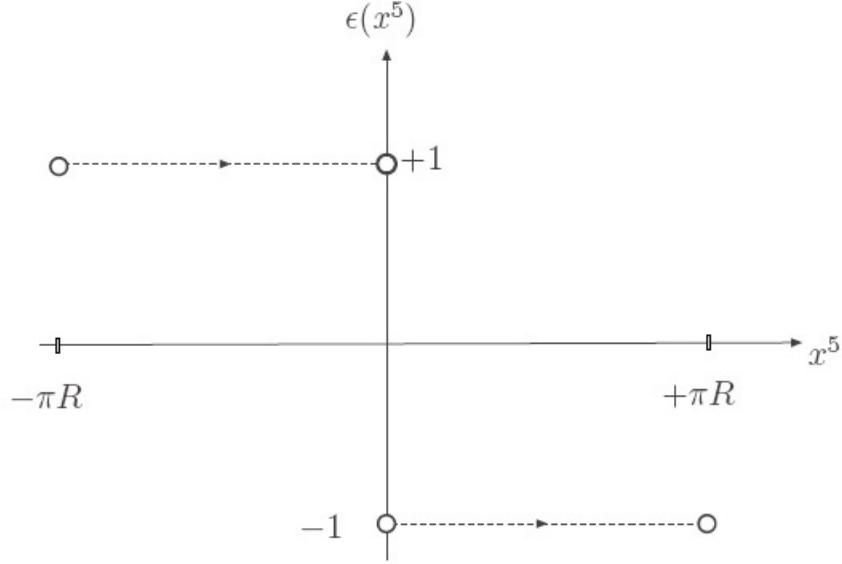
Figure 4.1: The function $\epsilon(x^5)$ vs. x^5 where $\{-\pi R\} \equiv \{+\pi R\}$.

some such objects are required to be redefined as $\kappa(x^5)C^{I_1...I_n}_{J_1...J_n}$ for consistency, where

$$\kappa(x^5) = \begin{cases} 0 & \text{for } x^5 = -\pi R \\ -1 & \text{for } -\pi R < x^5 < 0 \\ 0 & \text{for } x^5 = 0 \\ +1 & \text{for } 0 < x^5 < \pi R \end{cases} \qquad (4.2)$$

See figure (4.2).

To leave the space $\mathcal{M}_4 \times S^1/\mathbb{Z}_2$ invariant under the $\mathbb{Z}_2$ action, the coordinate functions, basis vectors, basis 1-forms, and metric components have

$$P(x^\mu; \, \partial_\mu; \, dx^\mu) = +1 \quad P(x^5; \, \partial_5; \, dx^5) = -1$$
$$P(\hat{g}_{\mu\nu}; \, \hat{g}_{55}) = +1 \quad P(\hat{g}_{\mu 5}) = -1,$$

where $P(\Phi)$ denotes the $\mathbb{Z}_2$ parity of the object Φ. Since the S^1 measure is odd on the orbifold, the fixed planes $\{\pi R \equiv -\pi R\}$ and $\{0\}$ must be non-orientable for the action S to be invariant under $\mathbb{Z}_2$ reflections. It is natural to take the integration path over x^5 to always be in the orientation of the dx^5 form; we can use the following prescription: in the region $[-\pi R, 0]$, one can integrate from $\{-\pi R\}$ to $\{0\}$ (taking dx^5 to be positively oriented
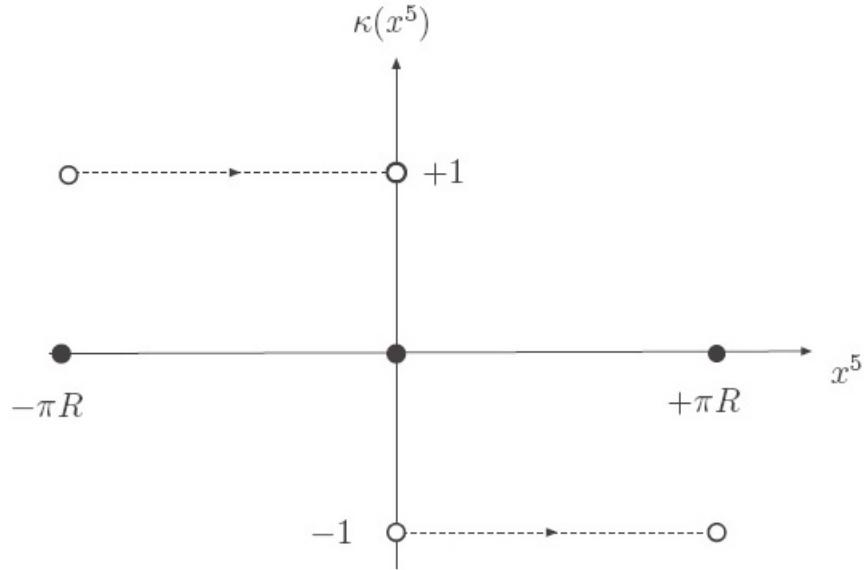
Figure 4.2: The function $\kappa(x^5)$ is plotted vs. x^5, where $\{-\pi R\} \equiv \{+\pi R\}$.

in x^5), while in the region $[0, \pi R]$, one can integrate from $\{\pi R\}$ to $\{0\}$ (since dx^5 is negatively oriented in x^5). In the downstairs picture, we will have two boundaries that are oppositely oriented. For the assignment of parities, it follow that we must require the $\mathbb{Z}_2$-action to leave the Lagrangian invariant. This puts constraints on the relative parities of the fields. There are further constraints imposed by the consistency of local coordinate transformations, supersymmetry transformations and gauge transformations.

4.2 $5D$ $\mathcal{N} = 2$ **Yang-Mills-Einstein Supergravity**

Recall that the total field content for a pure $5D$ $\mathcal{N} = 2$ YMESGT consists of a supergravity multiplet coupled to n_V gauge multiplets:

$$\{g_{\hat{\mu}\hat{\nu}}, \Psi^i_{\hat{\mu}}, A^I_{\hat{\mu}}, \lambda^{\tilde{p}\,i}, \phi^{\tilde{x}}\}.$$

The supersymmetry parameters ϵ^i, the gravitini $\Psi^i_{\hat{\mu}}$, and the spin-1/2 fields $\lambda^{\tilde{p}\,i}$ are $5D$ symplectic-Majorana spinors (see (2.1)), which can be written as

$$\epsilon^1 = \begin{pmatrix} \eta \\ e\zeta^* \end{pmatrix} \qquad \epsilon^2 = \begin{pmatrix} \zeta \\ -e\eta^* \end{pmatrix}$$

$$\Psi_\mu^1 = \begin{pmatrix} \alpha_\mu \\ e\beta_\mu^* \end{pmatrix} \quad \Psi_\mu^2 = \begin{pmatrix} \beta_\mu \\ -e\alpha_\mu^* \end{pmatrix} \tag{4.3}$$

$$\lambda^{\tilde{p}\,1} = \begin{pmatrix} \delta^{\tilde{p}} \\ e\gamma^{\tilde{p}\,*} \end{pmatrix} \quad \lambda^{\tilde{p}\,2} = \begin{pmatrix} \gamma^{\tilde{p}} \\ -e\delta^{\tilde{p}\,*} \end{pmatrix}.$$

The $5D$ bosonic YMESGT Lagrangian is

$$
\begin{aligned}
\hat{e}^{-1}\mathcal{L}_{bos} = & -\frac{1}{2\hat{\kappa}^2}\hat{R} - \frac{1}{4}\overset{\circ}{a}_{IJ}\,\mathcal{F}_{\hat{\mu}\hat{\nu}}^I \mathcal{F}^{J\,\hat{\mu}\hat{\nu}} - \frac{1}{2}g_{\tilde{x}\tilde{y}}\,D_{\hat{\mu}}\phi^{\tilde{x}}D^{\hat{\mu}}\phi^{\tilde{y}} \\
& + \frac{\hat{\kappa}\hat{e}^{-1}}{6\sqrt{6}}C_{IJK}\,\epsilon^{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}\hat{\lambda}}\Big\{ F_{\hat{\mu}\hat{\nu}}^I F_{\hat{\rho}\hat{\sigma}}^J A_{\hat{\lambda}}^K + \frac{3}{2}gF_{\hat{\mu}\hat{\nu}}^I A_{\hat{\rho}}^J (f_{LM}^K A_{\hat{\sigma}}^L A_{\hat{\lambda}}^M) \\
& \qquad\qquad\qquad\quad + \frac{3}{5}g^2(f_{GH}^J A_{\hat{\nu}}^G A_{\hat{\rho}}^H)(f_{LF}^K A_{\hat{\sigma}}^L A_{\hat{\lambda}}^F)\,A_{\hat{\mu}}^I \Big\},
\end{aligned}
\tag{4.4}
$$

where hats indicate five-dimensional quantities; $\hat{e}$ is the determinant of the fünfbein; and

$$
\begin{aligned}
F_{\hat{\mu}\hat{\nu}}^I &= 2\partial_{[\hat{\mu}}A_{\hat{\nu}]}^I \\
\mathcal{F}_{\hat{\mu}\hat{\nu}}^I &= F_{\hat{\mu}\hat{\nu}}^I + 2gA_{[\hat{\mu}}^I A_{\hat{\nu}]}^J \\
D_{\hat{\mu}}\phi^{\tilde{x}} &= \partial_{\hat{\mu}}\phi^{\tilde{x}} + gK_I^{\tilde{x}}A_{\hat{\mu}}^I.
\end{aligned}
$$

The supersymmetry transformations are

$$
\begin{aligned}
\delta\hat{e}_{\hat{\mu}}^{\hat{m}} &= \frac{1}{2}\bar{\epsilon}^i\Gamma^{\hat{m}}\Psi_{\hat{\mu}\,i} \\
\delta\Psi_{\hat{\mu}\,i} &= D_{\hat{\mu}}\epsilon_i + \frac{i}{4\sqrt{6}}h_I\{\Gamma_{\hat{\mu}}^{\hat{\nu}\hat{\rho}} - 4\delta_{\hat{\mu}}^{\hat{\nu}}\Gamma^{\hat{\rho}}\}F_{\hat{\nu}\hat{\rho}}^I\epsilon_i + \cdots \\
\delta A_{\hat{\mu}}^I &= -\frac{1}{2}h_{\tilde{p}}^I\bar{\epsilon}^i\Gamma_{\hat{\mu}}\lambda_i^{\tilde{p}} + \frac{i\sqrt{6}}{4}h^I\bar{\hat{\Psi}}_{\hat{\mu}}^i\epsilon_i \\
\delta\lambda_i^{\tilde{p}} &= -\frac{i}{2}f_{\tilde{x}}^{\tilde{p}}\Gamma^{\hat{\mu}}(\partial_{\hat{\mu}}\phi^{\tilde{x}})\epsilon_i + \frac{1}{4}h_I^{\tilde{p}}\Gamma^{\hat{\mu}\hat{\nu}}\epsilon_i F_{\hat{\mu}\hat{\nu}}^I \\
\delta\phi^{\tilde{x}} &= \frac{i}{2}f_{\tilde{p}}^{\tilde{x}}\bar{\epsilon}^i\lambda_i^{\tilde{p}},
\end{aligned}
\tag{4.5}
$$

where dots indicate terms with fermionic fields.

4.2.1 Reduction of $5D$ $\mathcal{N}=2$ YMESGT on S^1

In the "upstairs" orbifold construction, one starts with a $5D$ theory, and compactifies on S^1. It is sufficient for our purposes to use the dimensionally reduced theory, consisting of those

fields satisfying $\partial_5\Phi = 0$. This captures the zero modes of the theory on $\mathcal{M}_4 \times S^1/\mathbb{Z}_2$. The dimensional reduction of the theory (as well as the orbifold) breaks the $5D$ local Lorentz invariance to a $4D$ local Lorentz invariance. The four local symmetries that are broken can be used to fix four degrees of freedom in the fünfbein. Splitting $\hat\mu = (\mu, 5)$, we choose the parametrization for the fünfbein to be [GST84a]

$$\hat{e}_{\hat\mu}^{\hat m} = \begin{pmatrix} e^{-\frac{\sigma}{2}} e_\mu^m & 2e^\sigma C_\mu \\ 0 & e^\sigma \end{pmatrix}.$$

Since $\hat{g}_{\hat\mu\hat\nu} = \hat{e}_{\hat\mu}^{\hat m} \hat{e}_{\hat\nu}^{\hat n} \eta_{\hat m\hat n}$, we find that

$$\begin{aligned} \hat{g}_{\mu\nu} &= e^{-\sigma} g_{\mu\nu} + 4e^{2\sigma} C_\mu C_\nu \\ \hat{g}_{55} &= e^{2\sigma} \\ \hat{g}_{\mu 5} &= 2e^{2\sigma} C_\mu. \end{aligned} \tag{4.6}$$

Furthermore, let

$$A_{\hat\mu}^I = (A_\mu^I,\ A^I).$$

Under infinitesimal local coordinate transformations of the compact coordinate parameterized by $\xi^5(x^\mu)$, the $4D$ fields A_μ^I and C_μ transform as[1]

$$\begin{aligned} \delta_{\xi^5} A_\mu^I &= -\partial_\mu \xi^5 A^I & (4.7) \\ \delta_{\xi^5} C_\mu &= -2\partial_\mu \xi^5, & (4.8) \end{aligned}$$

with the remaining four dimensional bosonic fields being invariant. One can interpret $\xi^5(x^\mu)$ as a parameter for local $U(1)$ transformations, for which C_μ is a gauge field. Note that the vector fields A_μ^I transform non-trivially under these $U(1)$ transformations. In order to obtain $U(1)$(or KK)-invariant fields, we make the local field redefinition

$$A_\mu^I \to A_\mu^I + 2C_\mu A^I,$$

such that the new A_μ^I satisfies $\delta_{\xi^5} A_\mu^I = 0$.

In terms of these KK-invariant vector fields A_μ^I, the dimensionally reduced bosonic La-

---

[1]For $(D + d)$ reductions where $d > 1$, there are also *global* $SL(d, R)$ transformations coming from the $(D + d)$-dimensional local coordinate transformations [C81].

grangian becomes [GMZ05a]

$$
\begin{aligned}
e^{-1}\mathcal{L}_{DR} = &-\frac{1}{2\hat{\kappa}^2}R - \frac{3}{4\hat{\kappa}^2}\,\mathring{a}_{IJ}\,D_\mu\tilde{h}^I D^\mu\tilde{h}^J - \frac{1}{2}e^{-2\sigma}\,\mathring{a}_{IJ}\,D_\mu A^I D^\mu A^J \\
&- (\frac{1}{2\hat{\kappa}^2}e^{3\sigma} + e^\sigma\,\mathring{a}_{IJ}\,A^I A^J)C_{\mu\nu}C^{\mu\nu} \\
&- \frac{1}{4}e^\sigma\,\mathring{a}_{IJ}\,\mathcal{F}^I_{\mu\nu}\mathcal{F}^{\mu\nu\,J} - e^\sigma\,\mathring{a}_{IJ}\,A^I \mathcal{F}^J_{\mu\nu}C^{\mu\nu} \\
&+ \frac{\hat{\kappa}e^{-1}}{2\sqrt{6}}C_{IJK}\epsilon^{\mu\nu\rho\sigma}(A^I\mathcal{F}^J_{\mu\nu}\mathcal{F}^K_{\rho\sigma} + 2A^I A^J \mathcal{F}^K_{\mu\nu}C_{\rho\sigma} + \frac{4}{3}A^I A^J A^K C_{\mu\nu}C_{\rho\sigma}) \\
&- \frac{g^2}{\hat{\kappa}^2}P,
\end{aligned}
\tag{4.9}
$$

where $\tilde{h}^I = e^\sigma h^I$, and

$$
P = \frac{3}{4}e^{-3\sigma}\,\mathring{a}_{IJ}\,(A^K f^I_{KL}h^L)(A^M f^J_{MN}h^N),
\tag{4.10}
$$

and

$$
\begin{aligned}
D_\mu A^I &\equiv \partial_\mu A^I + g A^J_\mu f^I_{JK}A^K \tag{4.11} \\
D_\mu\tilde{h}^I &\equiv \partial_\mu\tilde{h}^I + g A^J_\mu f^I_{JK}\tilde{h}^K \tag{4.12} \\
C_{\mu\nu} &\equiv 2\partial_{[\mu}C_{\nu]}. \tag{4.13}
\end{aligned}
$$

Just as A^I_μ was redefined to be KK-invariant, we make the further redefinitions

$$
\begin{aligned}
\Psi^i_\mu &\rightarrow \Psi^i_\mu + \Psi^i_{\dot{5}}C_\mu \tag{4.14} \\
\Gamma_\mu &\rightarrow \Gamma_\mu + \Gamma_{\dot{5}}C_\mu, \tag{4.15}
\end{aligned}
$$

so that Ψ^i_μ and Γ_μ are now KK-invariant. The dimensionally reduced susy transformations are then

$$
\begin{aligned}
\delta' e^m_\mu &= \frac{1}{2}\bar{\epsilon}^i\Gamma^m\Psi^{(4)}_{\mu i} \\
\delta\rho &= \frac{1}{2}\bar{\epsilon}^i\Gamma^5\psi_i \\
\delta C_\mu &= \frac{1}{2}\rho^{-3/2}\bar{\epsilon}^i\Gamma^5\Psi_{\mu i} \\
\delta\phi^x &= \frac{i}{2}f^x_a\bar{\epsilon}^i\lambda^a_i,
\end{aligned}
\tag{4.16}
$$

where δ' denotes the "bare" susy transformation from five dimensions plus a local Lorentz transformation to maintain the condition $\hat{e}_5^m = 0$; and we have identified the four-dimensional gravitini to be

$$\Psi_{\mu i}^{(4)} \equiv e_\mu^n \{ \Psi_{ni} + \frac{1}{2}(\Gamma^n)^{-1}\Gamma^5 \Psi_{5i} \}. \tag{4.17}$$

The kinetic energy term and potential of the scalar fields[2] can be written as [GST84a, GMZ05a]

$$e^{-1}\mathcal{L}_S = -\frac{1}{2}\left(\frac{1}{\hat{\kappa}^2}\tilde{a}_{IJ}D_\mu\tilde{h}^I D^\mu\tilde{h}^J + \frac{2}{3}\tilde{a}_{IJ}D_\mu A^I D^\mu A^J \right)$$

and

$$P = \frac{1}{2}e^{-3\sigma}\tilde{a}_{IJ}(A^K f_{KL}^I \tilde{h}^L)(A^M f_{MN}^J \tilde{h}^N)$$

where we have defined

$$\tilde{a}_{IJ} = \frac{3}{2}e^{-2\sigma}\,\overset{\circ}{a}_{IJ}\,.$$

The condition $\mathcal{V}(h) = 1$ becomes

$$\mathcal{V}(\tilde{h}) = C_{IJK}\tilde{h}^I\tilde{h}^J\tilde{h}^K = e^{3\sigma} > 0. \tag{4.18}$$

Defining

$$\mathcal{F}_{\mu\nu}^A = \begin{pmatrix} C_{\mu\nu} \\ F_{\mu\nu}^I \end{pmatrix},$$

we can express the vector part of the Lagrangian concisely as

$$e^{-1}\mathcal{L}_V = -\frac{1}{4}(\mathcal{F}^T)_{\mu\nu}^A M_{AB}\mathcal{F}^{B\,\mu\nu} + \frac{e^{-1}}{2\sqrt{6}}\epsilon^{\mu\nu\rho\sigma}(\mathcal{F}^T)_{\mu\nu}^A N_{AB}\mathcal{F}_{\rho\sigma}^B,$$

where

$$M_{AB} = \begin{pmatrix} \frac{2}{\hat{\kappa}^2}e^{3\sigma} + 4e^\sigma\,\overset{\circ}{a}_{IJ}\,A^I A^J & 2e^\sigma\,\overset{\circ}{a}_{IJ}\,A^I \\ \\ 2e^\sigma\,\overset{\circ}{a}_{IJ}\,A^J & e^\sigma\,\overset{\circ}{a}_{IJ} \end{pmatrix}$$

and

$$N_{AB} = \begin{pmatrix} \frac{4\hat{\kappa}}{3}C_{IJK}A^I A^J A^K & \hat{\kappa}C_{IJK}A^I A^K \\ \\ \hat{\kappa}C_{IJK}A^J A^K & \hat{\kappa}C_{IJK}A^K \end{pmatrix}.$$

The scalar manifolds of four dimensional MESGTs are Kähler [CJSFGvN79, BW82], so their

---

[2]σ and h^I have zero mass dimension.

metrics are determined locally by a Kähler potential F as

$$g_{I\bar{J}} = \partial_I \partial_{\bar{J}} F.$$

The Kähler potentials of scalar manifolds of the $4D$ MESGTs obtained by dimensional reduction from five dimensions are given by a cubic form defined by C_{IJK} [GST84a]. The corresponding Kähler geometry is called "very special". In terms of the complex combinations of the scalar fields

$$z^I = \sqrt{\frac{1}{2}} \left(\sqrt{\frac{2}{3}} A^I + \frac{i}{\hat{\kappa}} \tilde{h}^I \right),$$

the kinetic energy term of the scalar manifolds in four dimensions can be written as [GST84a]

$$e^{-1} \mathcal{L}_{\mathrm{S}} = -g_{I\bar{J}} \partial_\mu z^I \partial^\mu \bar{z}^{\bar{J}},$$

where

$$g_{I\bar{J}} = \tilde{a}_{IJ}(z - \bar{z}) = -\frac{1}{2} \partial_I \partial_{\bar{J}} \ln \mathcal{V}(z - \bar{z}),$$

with

$$\mathcal{V}(z - \bar{z}) = C_{IJK}(z - \bar{z})^I (z - \bar{z})^J (z - \bar{z})^K$$

satisfying $\mathcal{V}(Im(z)) > 0$ (see (4.18)).

4.2.2   YMESGT sector parity assignments

Let's split the index $I = (0, \alpha, a)$; $\tilde{x} = (x, \chi)$; and $\tilde{p} = (p, \rho)$. At the fixed points (upstairs picture), the fermionic fields generally satisfy jumping conditions and so don't have a well-defined limit. In the downstairs picture, the fermions will have a well-defined limit at the boundaries (see [ABN01, BB03] e.g.). Thus, in the downstairs picture, the fermions in (4.3) can be written at the boundaries either as left-chiral fermions with their right-chiral conjugates

$$\lambda^{\tilde{p}\,1} = \begin{pmatrix} \delta^{\tilde{p}} \\ 0 \end{pmatrix} \quad \lambda^{\tilde{p}\,2} = \begin{pmatrix} 0 \\ -e\delta^{\tilde{p}\,*} \end{pmatrix},$$

or right-chiral fermions with their left-chiral conjugates

$$\tilde{\lambda}^{\tilde{p}\,1} = \begin{pmatrix} 0 \\ e\gamma^{\tilde{p}\,*} \end{pmatrix} \quad \tilde{\lambda}^{\tilde{p}\,2} = \begin{pmatrix} \gamma^{\tilde{p}} \\ 0 \end{pmatrix}.$$

In particular, it is clear from appendix B that the action of $\mathbb{Z}_2$ on the supersymmetry spinors ϵ^i necessarily requires half of the components to be odd, so that the original eight supersymmetry currents will be broken to four on the boundaries. The boundary theories therefore have at most $\mathcal{N} = 1$ susy. In terms of symplectic-Majorana spinors ϵ^i, the $\mathbb{Z}_2$ action is represented as

$$-i\Gamma^5\epsilon^1 \quad \text{and} \quad i\Gamma^5\epsilon^2.$$

Note: The 4-component *eigenspinors* of the $\mathbb{Z}_2$ action are the two (Dirac spinor) linear combinations of the two symplectic-Majorana spinors.

The general set of consistent parity assignments allows for the following boundary propagating multiplets

Multiplet	Representation	Type
$\{g_{\mu\nu},\ \Psi_\mu\}$	$K_{(\alpha)}$ singlet	
$\{A_\mu^\alpha,\ \lambda^{\rho i}\}$	$\text{adj}[K_{(\alpha)}]$	Real
$\{\tilde{\lambda}^{p i},\ z^a\}$	$\mathbf{R_V}[K/K_{(\alpha)}]$	Real
$\{\Psi_{\dot{5}},\ z^0\}$	$K_{(\alpha)}$ singlet	

where the value of n' in $\alpha = 1, \ldots, n'$ and $a = (n' + 1), \ldots, (n_V + 1)$ is arbitrary. We have denoted the surviving gauge group on the boundaries as $K_{(\alpha)}$. The second to last multiplet consists of a chiral multiplet in a real representation and its CPT conjugate. The case where there are $5D$ spectator vector multiplets should be clear.

What happens when a non-compact group is gauged? If the non-compact gauge fields were assigned *even* parity, then a non-compact gauge group would appear in the $4D$ theory. However, there would not be the proper degrees of freedom to give a ground state with compact gauge symmetry since the scalar degrees of freedom A^I needed to form massive $\mathcal{N} = 1$ vector multiplets must have odd parity. Therefore, *the non-compact gauge fields must be assigned odd parity*. We will then get $\mathcal{N} = 1$ chiral multiplets in the coset K/H, with H the maximal compact subgroup of K. Since these multiplets furnish representations of the non-compact isometries of the scalar manifold, *there are non-vanishing Killing fields appearing in the scalar potential (5.6)*. This is a novel way of obtaining a $4D$ Higgs sector, along the lines of previous Higgs-gauge unifications in higher dimensions [HNS02, BN03, HS03].

4.3 Hypermultiplet sector

As discussed in section (2.3.3), hypermultiplets in five dimensions consist of $2n_H$ fermions and $4n_H$ real scalars, the latter parametrizing a quaternionic n_H-manifold $\mathcal{M}_Q$ with tangent

space group $USp(2n_H) \times SU(2)_R$. We write the multiplets as

$$\{\zeta^A, q^{\tilde{X}}\},$$

where $\tilde{X} = 1, \ldots, 4n_H$ are the curved indices of $\mathcal{M}_Q$; and $A = 1, \ldots, 2n_H$ are flat, $USp(2n_H)$ indices. The $4n_H$-bein $f_{iA}^{\tilde{X}}$ relate scalar manifold curved and flat space metrics

$$g_{\tilde{X}\tilde{Y}} f_{iA}^{\tilde{X}} f_{j\tilde{B}}^{\tilde{Y}} = \epsilon_{ij} C_{A\tilde{B}},$$

where $i, j = 1, 2$ are $SU(2)_R$ indices. Note that, in contrast to the case of vector multiplets, the scalars form $2n_H$ $SU(2)_R$-doublets, while the $2n_H$ fermions are $SU(2)_R$-singlets.[3] In 2-component spinor notation, we write the fermions as

$$\zeta^A = \begin{pmatrix} \zeta_1^A \\ \zeta_2^A \end{pmatrix}.$$

4.3.1 Hypermultiplet parity assignments

Let's split the index $\tilde{X} = (X, \chi)$, with $X = 1, \ldots, 2n_H$ and $\chi = 2n_H + 1, \ldots, 4n_H$. We let q^X be the even parity fields, and q^χ the odd fields. Similarly, we spit the index $A = (n, \tilde{n})$ with $n = 1, \ldots, n_H$ and $\tilde{n} = n_H + 1, \ldots, 2n_H$. If we couple a 5D YMESGT to hypermultiplets in the quaternionic $\mathbf{R_H}[K]$ of the gauge group, the multiplets with boundary propagating modes will be

Multiplet	Representation	Type
$\{\zeta_1^n, q^{X_1}\}$	$\mathbf{R_H}[K_\alpha]$	Real or Complex
$\{\zeta_2^{\tilde{n}}, q^{X_2}\}$	$\overline{\mathbf{R_H}}[K_\alpha]$	Real or Complex

where we have further split $X = (X_1, X_2)$ with $X_1 = 1, \ldots, n_H$ and $X_2 = n_H + 1, \ldots, 2n_H$. That is, we get a left-chiral multiplet and its CPT conjugate. Here $\mathbf{R_H}[K_\alpha]$ is the decomposition of $\mathbf{R_H}[K]$ under the group $K_\alpha \subset K$.

Example

Consider the **unified** MESGT with $SU(5,1)$ global symmetry group (see section (3.3)), whose vector fields are in 1-1 correspondence with the traceless elements of the Lorentzian Jordan algebra $J_{(1,5)}^{\mathbb{C}}$ [GZ03]. All of the vector fields of the 5D theory (including the bare

---

[3]Recall that, in this thesis, we are not considering gaugings of $SU(2)_R$ or its subgroups.

graviphoton) furnish the adj$[SU(5,1)]$. The C_{IJK} tensor is a rank-3 symmetric invariant of the global symmetry group, so its components are proportional to the d-symbols of $SU(5,1)$.

As in section (3.1.2), we can now couple hypermultiplets whose scalars parametrize the quaternionic manifold

$$\mathcal{M}_Q = \frac{E_7}{SO(12) \times SU(2)} \tag{4.19}$$

to the MESGT based on $J^{\mathbb{C}}_{(1,5)}$, gauging the common $SU(5,1)$ subgroup. As pointed out in [GST84c, GZ03], the dimensionless ratio g^3/κ is quantized, where g is the non-abelian gauge coupling and κ is the gravitational coupling. Then the five-dimensional ground state would have at most an $SU(5) \times U(1)$ gauge group coupled to hypermultiplets in the $\mathbf{1} \oplus \mathbf{5} \oplus \mathbf{10}$. We may then make the following assignments (in terms of $SU(5)$ reps)

α	a	0	n	$\tilde{n}$
adj$[SM]$	$SU(5)/SM \oplus \mathbf{5} \oplus \bar{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{1} \oplus \bar{\mathbf{5}} \oplus \mathbf{10}$	$c.c.$

The $4D$ low energy effective theory (LEET) will have an $\mathcal{N} = 1$ supergravity multiplet; SM gauge multiplets; weak doublet and color triplet chiral multiplets both with a scalar potential term; and left-chiral matter multiplets (including sterile fermion multiplet) along with their right-chiral conjugates. There are also the generic singlet left and right-chiral multiplet coming from the $5D$ supergravity multiplet, and chiral multiplets in the symmetric space $SU(5)/SM$.

4.4   Tensor multiplet couplings

When a MESGT with n_V abelian vector multiplets is gauged, the symmetry group of the Lagrangian is broken to the gauge group $K \subset G$ and the $n_V + 1$ vector fields decompose into K-reps

$$\mathbf{n_V + 1} = \mathrm{adj}(\mathbf{K}) \oplus \textbf{non-singlets}(\mathbf{K}) \oplus \textbf{singlets}(\mathbf{K}).$$

As was discussed in section (2.3.2), such a gauging requires the non-singlet vector fields to be dualized to anti-symmetric tensor fields satisfying a field equation that serves as a "self-duality" constraint [TPvN84, GRW85, GRW86, PPvN85, GM85, GZ99] (thus keeping the degrees of freedom the same):

$$B^M_{\mu\nu} = c^M_N \, \epsilon_{\mu\nu}{}^{\rho\sigma\lambda} \, \partial_{[\rho} B^N_{\sigma\lambda]} + \cdots , \tag{4.20}$$

where c^M_N has dimensions of inverse mass, and dots denote terms involving other fields.

We have already discussed the scalar sector of a pure $5D$ YMESGT. When tensor multiplets are coupled the scalar manifold is again a real Riemannian space, but which cannot be decomposed globally as a product of "vector" and "tensor" parts. We can, of course, identify an orthogonal frame of scalars at each point of the manifold: the vector multiplets are associated with the combination $h_{\tilde{x}}^I \phi^{\tilde{x}}$ at a given point, while the tensor multiplets are associated with the independent combination $h_{\tilde{x}}^M \phi^{\tilde{x}}$. Similarly, the combination of fermions $h_{\tilde{p}}^I \lambda^{\tilde{p} i}$ are associated with vector multiplets, while $h_{\tilde{p}}^M \lambda^{\tilde{p} i}$ with tensor multiplets. (In contrast to vector multiplet scalars, the tensor multiplet scalars have a potential term in the Lagrangian - see (4.22).) We will write $\phi^{\tilde{x}}$ and $\phi^{\tilde{m}}$ to denote the scalar partners of the vector and tensors, respectively, at any given point of the scalar manifold. Similarly, we write $\lambda^{\tilde{p} i}$ and $\lambda^{\tilde{\ell} i}$ as the fermionic partners of the vector and tensor fields, respectively. It is then implicitly understood that the meaning of this notation is given by the above discussion.

When tensors are present, we will use indices I, J, K for $5D$ vector fields and M, N, P for $5D$ tensor fields. We write the tensor multiplets as

$$\{B_{\hat{\mu}\hat{\nu}}^M, \lambda^{\tilde{\ell} i}, \phi^{\tilde{m}}\}.$$

To be consistent with the gauge symmetry, the components of the C-tensor are constrained to be:

$$C_{IMN} = \frac{\sqrt{6}}{2}\Omega_{NP}\Lambda_{IM}^P, \quad C_{MNP} = 0, \quad C_{MIJ} = 0, \tag{4.21}$$

where Ω_{NP} is the antisymmetric symplectic metric on the vector space spanned by the $(B^{\mathcal{M}}, B^{\bar{\mathcal{M}}})$ and Λ_{IM}^P are symplectic K-representation matrices appearing in the K-transformation of the tensor fields:

$$\delta_\alpha B_{\mu\nu}^M = \alpha^I \Lambda_{IN}^M B_{\mu\nu}^N.$$

Furthermore, C_{IJK} must be a rank-three symmetric K-invariant tensor. Note: We are assuming the most general gauging is $K = K_{semi-simple} \times K_{abelian}$; see [BCdWGVvP04] for more general couplings where $C_{MIJ} \neq 0$.

The terms in the bosonic $5D$ Lagrangian involving tensors are [GZ99]

$$\hat{e}^{-1}\mathcal{L}_T = -\frac{1}{4}\overset{\circ}{a}_{MN} B_{\hat{\mu}\hat{\nu}}^M B_{\hat{\rho}\hat{\sigma}}^N \hat{g}^{\hat{\mu}\hat{\rho}}\hat{g}^{\hat{\nu}\hat{\sigma}} - \frac{1}{2}\overset{\circ}{a}_{IM} \mathcal{F}_{\hat{\mu}\hat{\nu}}^I B_{\hat{\rho}\hat{\sigma}}^M \hat{g}^{\hat{\mu}\hat{\rho}}\hat{g}^{\hat{\nu}\hat{\sigma}}$$
$$+\frac{\hat{e}^{-1}}{4g}\epsilon^{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}\hat{\lambda}}\Omega_{MN}B_{\hat{\mu}\hat{\nu}}^M \partial_{\hat{\rho}}B_{\hat{\sigma}\hat{\lambda}}^N + \frac{\hat{e}^{-1}}{2\sqrt{6}}C_{MNI}\epsilon^{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}\hat{\lambda}}B_{\hat{\mu}\hat{\nu}}^M B_{\hat{\rho}\hat{\sigma}}^N A_{\hat{\lambda}}^I.$$

The $5D$ field equations for the $B^M_{\hat{\mu}\hat{\nu}}$ are

$$\star DB^M = g\Omega^{MN}\, \mathring{a}_{MI}\, \hat{\mathcal{H}}^I,$$

where

$$\mathcal{H}^{\tilde{I}} = \begin{pmatrix} \mathcal{F}^I \\ B^M \end{pmatrix}.$$

The presence of non-trivially charged tensors introduces a scalar potential $P^{(T)}$ that was not present in the case of pure YMESGTs. The term in the Lagrangian is

$$\hat{e}^{-1}\mathcal{L}_{P^{(T)}} = -2g^2\, W^{\tilde{p}}W^{\tilde{p}}$$

$$\text{with}\quad W^{\tilde{p}} = -\frac{\sqrt{6}}{8}h^{\tilde{p}}_M\Omega^{MN}h_N. \tag{4.22}$$

In the dimensional reduction, we parametrize the tensor field as

$$B^M_{\hat{\mu}\hat{\nu}} = \begin{pmatrix} 0 & -\tilde{A}^M_\nu \\ \tilde{A}^M_\mu & B^M_{\mu\nu} \end{pmatrix},$$

where tildes have been used to help distinguish from vector fields coming from $5D$ vectors. Consider the ξ^5 transformation of the dimensionally reduced fields $B^M_{\mu\nu}$ and $\tilde{A}^M_\mu$:

$$\begin{aligned} \delta_{\xi^5}B^M_{\mu\nu} &= \partial_\mu\xi^5\tilde{A}^M_\nu - \partial_\nu\xi^5\tilde{A}^M_\mu \\ \delta_{\xi^5}\tilde{A}^M_\mu &= 0. \end{aligned} \tag{4.23}$$

Just as for vector fields A^I_μ, we must make a field redefinition

$$B^M_{\mu\nu} \to B^M_{\mu\nu} - 4C_{[\mu}\tilde{A}^M_{\nu]}$$

such that the $B^M_{\mu\nu}$ are now KK-invariant.

The "naive" dimensionally reduced Lagrangian is then [GMZ05b]

$$
\begin{aligned}
e^{-1}\mathcal{L}_{DR} &= -\frac{1}{2\hat{\kappa}^2}R - \frac{3}{4\hat{\kappa}^2}\overset{\circ}{a}_{\tilde{I}\tilde{J}}(\mathcal{D}_\mu\tilde{h}^{\tilde{I}})(\mathcal{D}^\mu\tilde{h}^{\tilde{J}}) - \frac{1}{2}e^{-2\sigma}\overset{\circ}{a}_{IJ}(\mathcal{D}_\mu A^I)(\mathcal{D}^\mu A^J) \\
&\quad -e^{-2\sigma}\overset{\circ}{a}_{IM}(\mathcal{D}_\mu A^I)A^{\mu M} - \frac{1}{2}e^{-2\sigma}\overset{\circ}{a}_{MN}A_\mu^M A^{\mu N} \\
&\quad +\frac{e^{-1}}{g}\epsilon^{\mu\nu\rho\sigma}\Omega_{MN}B_{\mu\nu}^M(\partial_\rho A_\sigma^N + gA_\rho^I\Lambda_{IP}^N A_\sigma^P) \\
&\quad +\frac{e^{-1}}{g}\epsilon^{\mu\nu\rho\sigma}\Omega_{MN}C_{\mu\nu}A_\rho^M A_\sigma^N + \frac{e^{-1}}{2\sqrt{6}}C_{MNI}\epsilon^{\mu\nu\rho\sigma}B_{\mu\nu}^M B_{\rho\sigma}^N A^I \\
&\quad -\frac{1}{4}e^\sigma\overset{\circ}{a}_{MN}B_{\mu\nu}^M B^{N\mu\nu} - \frac{1}{2}e^\sigma\overset{\circ}{a}_{IM}(\mathcal{F}_{\mu\nu}^I + 2C_{\mu\nu}A^I)B^{M\mu\nu} \\
&\quad -\frac{1}{4}e^\sigma\overset{\circ}{a}_{IJ}(\mathcal{F}_{\mu\nu}^I + 2C_{\mu\nu}A^I)(\mathcal{F}^{J\mu\nu} + 2C^{\mu\nu}A^J) - \frac{1}{2}e^{3\sigma}C_{\mu\nu}C^{\mu\nu} \\
&\quad +\frac{e^{-1}\hat{\kappa}}{2\sqrt{6}}C_{IJK}\epsilon^{\mu\nu\rho\sigma}\left\{\mathcal{F}_{\mu\nu}^I\mathcal{F}_{\rho\sigma}^J A^K + 2\mathcal{F}_{\mu\nu}^I C_{\rho\sigma}A^J A^K + \frac{4}{3}C_{\mu\nu}C_{\rho\sigma}A^I A^J A^K\right\} \\
&\quad -\frac{g^2}{\hat{\kappa}^2}P,
\end{aligned}
\tag{4.24}
$$

where

$$
\begin{aligned}
\mathcal{D}_\mu A^I &\equiv \partial_\mu A^I + gA_\mu^J f_{JK}^I A^K \tag{4.25} \\
\mathcal{F}_{\mu\nu}^I &\equiv 2\partial_{[\mu}A_{\nu]}^I + gf_{JK}^I A_\mu^J A_\nu^K \tag{4.26} \\
\mathcal{D}_\mu\tilde{h}^{\tilde{I}} &\equiv \partial_\mu\tilde{h}^{\tilde{I}} + gA_\mu^J M_{J\tilde{K}}^{\tilde{I}}\tilde{h}^{\tilde{K}}, \tag{4.27}
\end{aligned}
$$

where

$$
M_{J\tilde{K}}^{\tilde{I}} = \begin{pmatrix} f_{JK}^I & 0 \\ 0 & \Lambda_{JN}^M \end{pmatrix}
$$

and the total scalar potential, P, is given by

$$
P = 2e^{-\sigma}W^{\tilde{p}}W^{\tilde{p}} + e^{-3\sigma}\overset{\circ}{a}_{\tilde{I}\tilde{J}}U^{\tilde{I}}U^{\tilde{J}}, \tag{4.28}
$$

where $W^{\tilde{p}}$ is given in (4.22), while

$$
U^{\tilde{I}} := \frac{\sqrt{3}}{2}A^I M_{I\tilde{K}}^{\tilde{I}}h^{\tilde{K}}.
$$

4.4.1 Parity assignments for tensor-coupled theories

Since (4.23) are only true for transformations connected to the identity, the $\tilde{A}_\mu^M$ are not necessarily even under $\mathbb{Z}_2$ action. However, these expressions do lead to the constraint

$$P(\tilde{A}_\mu^M) = -P(B_{\mu\nu}^M),$$

componentwise. These two fields do not describe independent propagating degrees of freedom since they are related by a field equation (coming from the fact that the $5D$ tensors satisfied a "self-duality" field equation reducing the number of propagating modes):

$$B_{\mu\nu}^M = c_N^M(^\star D\tilde{A}^N)_{\mu\nu} + \cdots , \qquad (4.29)$$

where c_N^M is proportional to $\Omega^{MP}\, \overset{\circ}{a}_{PN}$; $\star$ is the Hodge operator; and the dots indicate terms involving other fields. There are two classes of assignments we can make, characterized by the parity of the symplectic form Ω_{MN} on the vector space spanned by the $5D$ tensors.

Odd Parity $\Omega_{\mathbf{MN}}$

Let $P(\tilde{A}_\mu^M) = +1$. In the field equations for the vector fields $\tilde{A}_\mu^M$, the mass squared matrix is proportional to $c_M^P c_{PN}$; if the self-duality relation is used to express all tensor fields in terms of the vectors $\tilde{A}_\mu^M$, the mass of the $\tilde{A}_\mu^M$ is non-vanishing at the orbifold fixed points. However, there are insufficient fermionic degrees of freedom to form massive $\mathcal{N} = 1$ vector multiplets. Therefore, we must use the field equation to write $\tilde{A}_\mu^M \to B_{\mu\nu}^M$. Now, these fields can be Hodge dualized to scalars B^M by adding a term of the form

$$\epsilon^{\mu\nu\rho\sigma}\Omega_{MN}\mathcal{B}_{\mu\nu\rho}^M D_\sigma B^N \qquad (4.30)$$

to the Lagrangian, where D_ρ is the gauge covariant derivative acting on the scalars, and $\mathcal{B}_{\mu\nu\rho}^M = 3!\, \partial_{[\mu}B_{\nu\rho]}^M$. From this term, it is clear that the B^M will have even parity. We'll then get massive spin-1/2 multiplets if we assign $\mathbf{P(h^M) = +1}$.

Remark: it is inconsistent to try to write the Lagrangian as a mixture of $A_\mu^{M'}$ and $B_{\mu\nu}^{M''}$ by splitting the index M since the field equation relating the vectors and tensors mixes the two types of indices; we must choose one type of field to appear in the Lagrangian.

The multiplets that will propagate on the fixed planes are

Multiplet	Representation	Type
$\{\tilde{\lambda}^{pi},\, z^M\}$	$\mathbf{N} \oplus \overline{\mathbf{N}}$	Real

This multiplet consists of a chiral multiplet in a real representation and its CPT conjugate.

Example

Consider, again, the **unified** $5D$ MESGT based on the Lorentzian Jordan algebra $J_{(1,5)}^{\mathbb{C}}$, whose global symmetry group is $SU(5,1)$ [GZ03]. We can couple this theory to hypermultiplets whose scalars parametrize the particular scalar manifold (4.19). If we gauge the common $SU(5) \times U(1) \subset SU(5,1)$ subgroup, we will get $SU(5) \times U(1)$ gauge multiplets, along with tensor multiplets in the $\mathbf{5} \oplus \bar{\mathbf{5}}$ and hypermultiplets in the $\mathbf{1} \oplus \mathbf{5} \oplus \mathbf{10}$. This is then similar to the ground state theory in the $SU(5,1)$ gauging example of section(4.3), but with some important differences, one of which being the scalar potential in this case is of the form $P^{(T)}$ in (4.22). We can make the assignments

α	a	0	M	n	$\tilde{n}$
adj[SM]	$SU(5)/SM$	$\mathbf{1}$	$\mathbf{5} \oplus \bar{\mathbf{5}}$	$\mathbf{1} \oplus \bar{\mathbf{5}} \oplus \mathbf{10}$	$c.c.$

The propagating modes along the fixed planes will be $SU(3) \times SU(2) \times U(1)$ gauge fields; weak doublet (Higgs) chiral multiplets; color triplet chiral multiplets; and left-chiral matter multiplets (including a sterile fermion multiplet) with their CPT conjugates. Again, there is also the generic singlet spin-1/2 multiplet coming from the $5D$ supergravity multiplet, and chiral multiplets in the symmetric space $SU(5)/SM$. All of these multiplets are tree-level massless, while the scalars in the $\mathbf{5} \oplus \bar{\mathbf{5}}$ have a potential term.

Even Parity for $\Omega_{\mathbf{MN}}$

The multiplets with boundary propagating modes will be

Multiplet	Representation	Type
$\{A^{\mathcal{M}},\ \lambda^{\rho i},\ h^{\mathcal{M}}\}$	$\Re(\mathbf{N})$	Real

The notation for the representation means that the gauge group at the fixed points must support a *real* $\mathbf{N}$. Let's illustrate this with an example.

Example

The minimal example in which one is left with a group containing SM is where the $5D$ gauge group is $SU(10) \times U(1)$. Starting with the **unified** MESGT defined by the Lorentzian Jordan algebra $J_{(1,10)}^{\mathbb{C}}$ and with $SU(N,1)$ global symmetry of the Lagrangian, we can gauge the $SU(10) \times U(1)$ subgroup, yielding tensors in the $\mathbf{10} \oplus \overline{\mathbf{10}}$. If the symplectic form has even parity, then the orbifold conditions require the group to be broken to at least $SO(10) \times U(1)$, under which we have massive vector multiplets in the (real) $\mathbf{10}$. There are also chiral multiplets from the broken gauge multiplets forming the $\mathbf{54}$, along with their CPT conjugates.

4.5 Objects other than fields

There are field-dependent and independent objects that appear in the Lagrangian and super-symmetry transformations that carry $\mathbb{Z}_2$ parity. In particular, the field independent objects are the C_{IJK} tensor defining the MESGT that exists prior to gauging; the structure constants f_{JK}^I and transformation parameters $\alpha^I(x)$ of the $5D$ gauge group; and the symplectic tensor Ω_{MN} and transformation matrices Λ_{IN}^M in the tensor coupled theory. These contain a jumping function implicitly when assigned odd parity. The field dependent objects are the restricted ambient space metric $\overset{\circ}{a}_{IJ}(\phi)$ and scalar manifold metrics $g_{xy}(\phi)$ and $g_{XY}(q)$; the $h_p^I(\phi)$; the scalar vielbein $f_x^p(\phi)$ and $f_{iA}^{\tilde{X}}(q)$; the Killing vectors on the scalar manifold $K_x^I(\phi)$ and $K_{\tilde{X}}^I(q)$. These vanish when assigned odd parity.

Pure YMESGT

Recall from equation (2.3) that the C_{IJK} defining a MESGT may be put in a "canonical" basis satisfying the positivity of $\mathcal{V} = C_{IJK}\zeta^I\zeta^J\zeta^K$:

$$C_{000} = 1, \ \ C_{0ij} = -\frac{1}{2}\delta_{ij}, \ \ C_{00i} = 0, \ \ C_{ijk} = \text{arbitrary}. \tag{4.31}$$

The parity assignments of the components are determined by requiring the polynomial $\mathcal{V}$ to be invariant under $\mathbb{Z}_2$ action. *There is freedom in choosing $\epsilon(x^5)$ or $\kappa(x^5)$ as the jumping function for odd components.* However, we will choose the former for reasons to be discussed later. Splitting $i = (\alpha, a)$, we have

Even	Odd
$C_{abc} \ \ C_{a\alpha\beta}$	$C_{\alpha\beta\gamma} \ \ C_{\alpha ab}$

Before moving on, let us make some brief remarks. In the upstairs picture, we can effect odd parity for components of C_{IJK} by redefining them as $\epsilon(x_5)C_{IJK}$, where the C_{IJK} are now even, as we did in the above example. Such components are not well-defined at the fixed points, while the polynomial $\mathcal{V}$ characterizing the real scalar manifold *is*. However, one may instead redefine the C_{IJK} to be $\kappa(x^5)C_{IJK}$ (see (4.2)). In the downstairs picture, one may interpret this to mean the components vanish at the boundaries due to degenerations in the gauge bundle over the boundary points. If they are taken to vanish *everywhere*, the $5D$ theory one starts with is restricted in the form of its vector and tensor sector from the beginning. As an aside: if the $5D$ theory arises from compactification of $11D$ supergravity on a Calabi-Yau space, collapsing CY cycles lead to vanishing intersection numbers, which are the components of the C_{IJK} tensor. In particular, these degenerations can occur over the

singular points of the $5D$ spacetime (i.e., over the orbifold fixed points). In that case, there will generally be massless states coming from membranes wrapping CY 2-cycles, localized at the orbifold fixed points (this is in addition to any brane fields that have support there). See [MZ01, LMZ03] for a discussion of a purely $5D$ supergravity description of collapsing CY cycles.

Consistency of the infinitesimal gauge transformations (2.5) require parity assignments for the f_{JK}^I and $\alpha^I(x)$ to be

Even	Odd
$f_{\beta\gamma}^\alpha \quad f_{ab}^\alpha$	$f_{bc}^a \quad f_{\alpha\beta}^a$
$f_{a\beta}^0 \quad f_{\alpha 0}^0$	$f_{\alpha\beta}^0$
α^β	$\alpha^0 \quad \alpha^b$

where f_{JK}^I vanishes if any of the indices correspond to $5D$ spectator vector fields;[4] and permutations of the indices have the same parity. The gauge transformation parameters are subject to an expansion on $S^1/\mathbb{Z}_2$, and consequently, odd parameters are not well-defined at the orbifold fixed points. Consistency of the algebra requires that *odd f_{JK}^I be redefined as* $\epsilon(x^5)f_{JK}^I$.

The components of the restricted ambient space metric and scalar manifold metric have parities determined by the requirement that the line elements of those spaces be preserved.

Even	Odd
$\overset{\circ}{a}_{\alpha\beta} \quad \overset{\circ}{a}_{ab}$	$\overset{\circ}{a}_{a\beta} \quad \overset{\circ}{a}_{0\alpha}$
$\overset{\circ}{a}_{00} \quad \overset{\circ}{a}_{0a}$	
$g_{xy} \quad g_{\chi\psi}$	$g_{\chi y}$

Consistency of the gauge transformations (2.5) determine the parities of the Killing vectors

Even	Odd
K_α^x	$K_0^x \quad K_a^x$
$K_0^\chi \quad K_a^\chi$	K_α^χ

Note that the non-zero components K_α^x on the fixed planes are Killing fields of the scalar manifold parametrized by those $\phi^{\tilde{x}}$ that are fixed-plane propagating. (There will also be new Killing vectors, which are not involved in the gauging, associated with the A^I). The K_α^x are sections of the tangent bundle over the scalar manifold. There are also non-zero components K_a^χ, which are a set of sections of the normal bundle over the $4D$ scalar manifold. In fact,

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[4]This will be true, e.g., for the "bare graviphoton" A_μ^0 if the $5D$ gauge group is compact.

these normal vector fields determine the form of the scalar potential involving the ϕ^x and A^a at the fixed points (c.f. (4.10)):

$$g_{\chi\psi}\, K_a^\chi\, K_b^\psi A^a A^b, \tag{4.32}$$

where $g_{\chi\psi}$ is the metric determined by the normal bundle connection.

Finally, the functions h^I and h_p^I; the vielbein f_x^p; and the functions $h_x^I = h_p^I f_x^p$ are required to satisfy

Even	Odd
$h^0 \;\; h^a$	h^α
$h_p^0 \;\; h_p^a \;\; h_\rho^\alpha$	$h_\rho^a \;\; h_p^\alpha$
$f_x^p \;\; f_\chi^\rho$	$f_\chi^p \;\; f_x^\rho$
$h_x^0 \;\; h_x^a \;\; h_\chi^\alpha$	$h_\chi^a \;\; h_x^\alpha$

Tensor couplings

In the tensor-coupled theory, the parities of the additional C-tensor components are

Even	Odd
$C_{\mathcal{M}\bar{\mathcal{N}}\alpha}$	$C_{\mathcal{M}\mathcal{N}\alpha} \;\; C_{\bar{\mathcal{M}}\bar{\mathcal{N}}\alpha}$
$C_{\mathcal{M}\mathcal{N}a} \;\; C_{\bar{\mathcal{M}}\bar{\mathcal{N}}a}$	$C_{\mathcal{M}\bar{\mathcal{N}}a}$
$P(\Omega_{MN}) = +\Omega_{MN}$	

Even	Odd
C_{MNa}	$C_{MN\alpha}$
$P(\Omega_{MN}) = -\Omega_{MN}$	

Remarks

Let us again briefly consider the higher dimensional origins of the tensor-coupled theory. String or M-theory can be consistent in singular spaces associated with collapsing Calabi-Yau cycles, whose intersections provide the components of C_{IJK} in a Yang-Mills-Einstein supergravity theory. We have shown that odd components of C_{IMN} appearing in the coupling of tensors must vanish at the orbifold fixed points when Ω_{MN} has even parity. From a higher dimensional point of view ($11D$ supergravity on a Calabi-Yau space), we cannot ignore the associated collapsing cycles since membranes wrapping CY 2-cycles appear as massless states,and should appear in the supergravity description. Since the collapsing of the cycles occurs over the $5D$ orbifold fixed points, the new massless states will have support there.

Consistency of the gauge transformations require the representation matrices to satisfy

Even	Odd
$\Lambda_{\alpha\mathcal{N}}^{\mathcal{M}}\;\; \Lambda_{\alpha\bar{\mathcal{N}}}^{\bar{\mathcal{M}}}$	$\Lambda_{\alpha\mathcal{N}}^{\bar{\mathcal{M}}}\;\; \Lambda_{\alpha\bar{\mathcal{N}}}^{\mathcal{M}}$
$\Lambda_{0\mathcal{N}}^{\bar{\mathcal{M}}}\;\; \Lambda_{0\bar{\mathcal{N}}}^{\mathcal{M}}$	$\Lambda_{0\mathcal{N}}^{\bar{\mathcal{M}}}\;\; \Lambda_{0\mathcal{N}}^{\mathcal{M}}$
$\Lambda_{a\mathcal{N}}^{\bar{\mathcal{M}}}\;\; \Lambda_{a\bar{\mathcal{N}}}^{\mathcal{M}}$	$\Lambda_{a\bar{\mathcal{N}}}^{\bar{\mathcal{M}}}\;\; \Lambda_{a\mathcal{N}}^{\mathcal{M}}$

$$P(\Omega_{MN}) = +\Omega_{MN}$$

Even	Odd
$\Lambda_{\alpha N}^{M}$	$\Lambda_{0N}^{M}\;\; \Lambda_{aN}^{M}$

$$P(\Omega_{MN}) = -\Omega_{MN}$$

As in the pure YMESGT case, the ambient space and scalar manifold line elements should be preserved under the $\mathbb{Z}_2$ action so that

Even	Odd
$\mathring{a}_{\mathcal{MN}}\;\; \mathring{a}_{\bar{\mathcal{M}}\bar{\mathcal{N}}}$	$\mathring{a}_{\mathcal{M}\bar{\mathcal{N}}}$
$\mathring{a}_{\bar{\mathcal{M}}\alpha}\;\; \mathring{a}_{\mathcal{M}a}$	$\mathring{a}_{\mathcal{M}\alpha}\;\; \mathring{a}_{\bar{\mathcal{M}}a}$
$g_{mn}\;\; g_{\bar{m}\bar{n}}$	$g_{\bar{m}n}$
$g_{xm}\;\; g_{\chi\bar{m}}$	$g_{x\bar{m}}\;\; g_{\chi m}$

$$P(\Omega_{MN}) = +\Omega_{MN}$$

Even	Odd
$\mathring{a}_{aM}$	$\mathring{a}_{\alpha M}$
$g_{xm}\;\; g_{mn}$	$g_{\chi m}$

$$P(\Omega_{MN}) = -\Omega_{MN}$$

Finally, the functions $h^M(\phi)$ and h_{ℓ}^M; the vielbein $f_m^{\tilde{\ell}}$; and the functions $h_x^I = f_x^{\tilde{\ell}}h_{\tilde{\ell}}^I$ satisfy (where we have split $\tilde{\ell} = (\ell,\bar{\ell})$ and $\tilde{m} = (m,\bar{m})$)

Even	Odd
$h^{\mathcal{M}}(\phi)$	$h^{\bar{\mathcal{M}}}(\phi)$
$h_m^{\mathcal{M}}\;\; h_{\bar{m}}^{\bar{\mathcal{M}}}$	$h_{\bar{m}}^{\mathcal{M}}\;\; h_m^{\bar{\mathcal{M}}}$
$h_{\ell}^{\mathcal{M}}\;\; h_{\tilde{\ell}}^{\bar{\mathcal{M}}}$	$h_{\tilde{\ell}}^{\mathcal{M}}\;\; h_{\ell}^{\bar{\mathcal{M}}}$

$$P(\Omega_{MN}) = +\Omega_{MN}$$

Even	Odd
$h^M(\phi)$	
h_m^M	
h_{ℓ}^M	$h_{\tilde{\ell}}^M$

$$P(\Omega_{MN}) = -\Omega_{MN}$$

Hypermultiplet couplings

The parity assignments for the Killing vectors and vielbein of the quaternionic scalar manifold are required to be

Even	Odd
$K_\alpha^X\;\; K_a^\Omega$	$K_a^X\;\; K_\alpha^\Omega$
$f_{1n}^X\;\; f_{2\bar{n}}^X$	$f_{2n}^X\;\; f_{1\bar{n}}^X$
$f_{2n}^\Omega\;\; f_{1\bar{n}}^\Omega$	$f_{1n}^\Omega\;\; f_{2\bar{n}}^\Omega$

4.5.1 Discussion

We previously expressed, and note again with the parity assignments above, that there appears to be a notational complication arising from the fact that the scalar and fermionic partners of the vector vs. tensor fields are generally different linear combinations of the manifest scalars and fermions appearing in the Lagrangian, depending on the point of the

scalar manifold at which the theory lives. So far, it has been understood that the indices x (resp. m) and p (resp. ℓ) appearing in the above partity assignments are representative of the partner fields of the vectors (resp. tensors). What we have assumed is that, in practice, one looks at combinations like $h_{\tilde{x}}^{\alpha}\phi^{\tilde{x}}$ appearing in the "manifest basis" of scalars that appear in the Lagrangian. This combination should vanish so that the supersymmetry transformations are consistent (since they can't be $4D$ $\mathcal{N}=1$ superpartners of any fields). Thus, in our notation, we simply say that "h_x^{α} and ϕ^{χ} are $\mathbb{Z}_2$-odd, while h_{χ}^{α} and ϕ^x are $\mathbb{Z}_2$ even". But in the cases of symmetric "very special" scalar manifolds, as well as in the case where $C_{ijk}=0$, this is in fact correct notation, since one can consistently assign parities to the $\phi^{\tilde{x}}$. It is perhaps not apparent at this point whether or not setting some set of scalars $\phi^{\chi}=0$ at the fixed points is the correct truncation in general, though, so let's illustrate with some examples.

Let's begin with a MESGT that's in the "generic Jordan" family, which have symmetric scalar manifolds [GST84a]

$$\mathcal{M}_R = SO(1,1) \times \frac{SO(n_V-1,1)}{SO(n_V-1)}.$$

The cubic polynomial for the theory in the absence of an orbifold is $\mathcal{V}=C_{IJK}\xi^I\xi^J\xi^K$, where

$$C_{000}=1,\ \ C_{00i}=0,\ \ C_{0ij}=-\frac{1}{2},\ \ C_{111}=\frac{1}{\sqrt{2}},\ \ C_{1ab}=+\frac{1}{\sqrt{2}}\delta_{ab},$$

with $a,b=2,\ldots,n_V$.

However, on $\mathcal{M}_4 \times S^1/\mathbb{Z}_2$, the C_{IJK} can have odd components satisfying jumping conditions (upstairs picture). There is a caveat: at least one h^I must have even parity so that it doesn't vanish at the fixed points. Otherwise, the polynomial would vanish leading to an ill-defined theory. In the canonical basis, it is natural for h^0 to have even parity so that we may only give odd parity to the C_{ijk}. Let us give odd parity to h^1 and C_{1ij} in the current example by redefining $C_{1ij} \to \epsilon(x^5)C_{1ij}$. Then the polynomial is

$$\mathcal{V} = \left\{ (h^0)^3 - \frac{3}{2}h^0\delta_{ij}h^ih^j - \frac{\epsilon(x^5)}{\sqrt{2}}(h^1)^3 + \frac{3\epsilon(x^5)}{\sqrt{2}}h^1[(h^2)^2 + \cdots + (h^{n_V})^2] \right\}.$$

At the orbifold fixed points, the terms with h^1 vanish so that

$$\mathcal{V}|_{fp} = h^0\left\{ (h^0)^2 - \frac{3}{2}\delta_{ab}h^ah^b \right\}.$$

That is, this has the affect of restricting the $4D$ complex scalar manifold to the $\text{Im}(z^1) = 0$ surface, with the h^I still satisfying the condition on the "bulk polynomial" $\mathcal{V} = 1$. In general, $4D$ $\mathcal{N} = 1$ supergravity theories are in 1-1 correspondence with Hodge manifolds. The $4D$ $\mathcal{N} = 1$ supergravity theory we obtain from orbifolding is of a special class based on a (not necessarily irreducible) cubic polynomial satisfying $\mathcal{V}_{fp}(\text{Im}(z)) = e^{3\sigma} > 0$ (for similar discussion in the $\mathcal{N} = 2$ case via dimensional reduction, see [GST84a, GMZ05a]).

The solution to the condition $\mathcal{V} = 1$ in this example is $h^0 \propto 1/|\phi|^2$ and $h^i \propto \phi^i$ ($i = 1, \ldots, n_V$), where $|\phi|^2$ is the "Minkowski" norm with signature $(+ - \cdots -)$. Clearly, the assignment of parity to the ϕ^i is straightforward in this case, and the vacua of the theory will follow the $\langle \phi^1 \rangle = 0$ flow. Let's contrast this with a different example: the non-Jordan family with cubic polynomial

$$\mathcal{V} = \sqrt{2}\check{h}^0(\check{h}^1)^2 - \check{h}^1 \sum_{\jmath}(\check{h}^{\jmath})^2,$$

with solution to $\mathcal{V} = 1$:

$$\check{h}^0 = \frac{1}{\sqrt{2}(\phi^1)^2} + \frac{1}{\sqrt{2}}\phi^1 \sum_{\jmath}(\phi^{\jmath})^2$$
$$\check{h}^1 = \phi^1$$
$$\check{h}^{\jmath} = \phi^1 \phi^{\jmath}$$

where $\jmath = 2, \ldots, n_V$ and $\check{h}^I$ is not in the canonical basis. Clearly, *there is now a restriction that $\check{h}^0, \check{h}^1$ be even (which means two vectors at least must be projected out)*, while there is freedom in parity assignments in the remaining $\check{h}^{\jmath}$. In this case, the requirement that $\check{h}^{\alpha}_{\tilde{x}}\phi^{\tilde{x}}$ be odd (and therefore vanish at the fixed points) allows for an infinite family of ground state flows in which the vev for all scalars can be non-zero. For example, if A^2_{μ} has even parity, then $\check{h}^2_{\tilde{x}}\phi^{\tilde{x}}$ must have odd parity so that there is a collection of flows, with the direction normal to the flows being $\langle \phi^1 \rangle \phi^2 + \langle \phi^2 \rangle \phi^1$ (since this is the direction in which the propagating scalar is truncated). However, as $\check{h}^2$ must also be odd, this requires ϕ^2 to be odd so that it vanishes at the fixed points. Therefore, the theory is restricted to lie along the flow $\langle \phi^2 \rangle = 0$ connected to the basepoint.

In fact, this is a general result: If some $h^I(\phi)$ are non-linear polynomials in $\phi^{\tilde{x}}$, truncation of the scalar combination $h^{\alpha}_{\tilde{x}}\phi^{\tilde{x}}$ allows for an infinite family of vacua generated by the Killing vectors K^x_{α} (one for each remaining gauge symmetry). However, the h^{α}, which are polynomials in the $\phi^{\tilde{x}}$, must be odd, which implies that some $\phi^{\tilde{x}}$ are necessarily odd, and

so vanish at the orbifold fixed points. Therefore, we are always restricted to some set of $\langle \phi^{\tilde{x}} \rangle = 0$ flows, which are connected to the basepoint. These scalars are what we have been calling ϕ^{χ}. We now have our justification for the way in which we've been assigning parities to objects with scalar manifold indices like the $\phi^{\tilde{x}}$: although the combination $h_{\tilde{x}}^{\alpha} \phi^{\tilde{x}}$ appears as the $5D$ partner of the vector component A_{μ}^{α}, only the trivial set of solutions (the $\langle \phi^{\chi} \rangle = 0$ flow) is truncated.

There is an additional subtlety, which the above "non-Jordan" family illustrates. In that example, the C-tensor was written in a non-canonical basis in which it was clear that $\check{h}^0$ and $\check{h}^1$ needed to have even parity. However, if we write the C-tensor in the canonical basis, all of the h^i can be assigned parity freely. In the absence of the orbifold, the two bases are related by a linear transformation $\check{h}^I = M_J^I h^J$, and the theories described by them are the same. In the presence of the orbifold, however, the two bases are no longer related by a *symmetry* transformation. The transformation, involving jump functions, takes us between two different theories, with different sets of parity assignments. It is not clear in general if there is always a basis in which there aren't constraints on the parity assignments of the vector sector scalars. Regardless, for the generic non-Jordan family, generic Jordan family, magical Jordan theories, and the $C_{ijk} = 0$ theories, one can always work in such a basis.

4.6 Extension to $\Gamma = \mathbb{Z}_2 \times \mathbb{Z}_2$

There are a couple of phenomenological issues that make the $S^1/\mathbb{Z}_2$ orbifold models too simplistic. First, there are always massless chiral multiplets in real representations when a gauge group is broken at the orbifold fixed planes (though these may contain MSSM Higgs fields). Second, all chiral multiplets come in complete representations of the $5D$ gauge group, which can lead to unwanted fields charged under the Standard Model gauge group. The boundary conditions described by an $S^1/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ [BHN01] construction are for the most part capable of resolving these issues.

An exception is the tensor sector: although there is a choice in assignment of parity for the symplectic form Ω_{MN}, we cannot assign $(+-)$ parity under $\mathbb{Z}_2 \times \mathbb{Z}_2$ action (it leads to inconsistencies of assignments for the fields). Furthermore, given a choice of Ω_{MN} parity, there wasn't a choice of parity assignments in the $\Gamma = \mathbb{Z}_2$ case since supersymmetry dictated the results (see appendix for details). Therefore, the situation with tensors is no different in the $S^1/\mathbb{Z}_2 \times \mathbb{Z}_2$ construction. *This means that, e.g., tensor multiplets do not allow a doublet-triplet resolution via parity assignments* (see the example with odd Ω_{MN} parity in section (4.4)).

An expansion of $\Phi(x, x^5)$ on $S^1/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ will take the form

$$\Phi^{(++)}(x, x^5) = \sum_n \Phi^{(n)}(x) \cos[\frac{2nx^5}{R}]$$

$$\Phi^{(+-)}(x, x^5) = \sum_n \Phi^{(n)}(x)(A_{(n)} \cos[\frac{(2n+1)x^5}{R}] + B_{(n)}\epsilon(x^5) \sin[\frac{(2n+1)x^5}{R}])$$

$$\Phi^{(-+)}(x, x^5) = \sum_n \Phi^{(n)}(x)(C_{(n)} \sin[\frac{(2n+1)x^5}{R}] + D_{(n)}\epsilon(x^5) \cos[\frac{(2n+1)x^5}{R}])$$

$$\Phi^{(--)}(x, x^5) = \sum_n \Phi^{(n)}(x)(E_{(n)} \sin[\frac{2nx^5}{R}] + F_{(n)}\epsilon(x^5) \cos[\frac{2nx^5}{R}])$$

so that $\Phi^{(+-)}(x, x^5)$ vanishes at $x^5 = 0$ and $\Phi^{(-+)}(x, x^5)$ vanishes at $x^5 = \pi R/2$. Once again, bosonic fields cannot have ϵ factors since the equations of motion would involve δ' or δ^2, where δ is the Dirac distribution. For those, we must set $B_{(n)} = D_{(n)} = F_{(n)} = 0$. But fermionic fields are allowed these terms in the expansion so that they are generally not well-defined at the fixed points.

Let $P(\Phi)$ be the parity of Φ under the first $\mathbb{Z}_2$ factor, and $P'(\Phi)$ denote the parity under the second factor. Taking the covering space to be $[-\pi R, \pi R]$ (with $\{-\pi R\} \equiv \{\pi R\}$) as before, the orbifold now has fixed points at $\{0\}, \{\pi R/2\}$.

4.6.1   Vector sector

In the previous sections, we made an index split for quantities with ± 1 parity under the single $\mathbb{Z}_2$. We will make a further index splitting for quantities with the four possible values $\{\pm 1, \pm 1\}$ for the parity $\{P(\Phi), P'(\Phi)\}$:

$$i = (\alpha, \alpha', a, a') \quad \tilde{p} = (\rho, \rho', p, p').$$

A given assignment of $\mathbb{Z}_2 \times \mathbb{Z}_2$ parity to an object will consist of the union of two assignments in the $S^1/\mathbb{Z}_2$ construction.

Fields from the $5D$ vector multiplets will have the following assignments:

++	+−	−+	−−
A_μ^α	$A_\mu^{\alpha'}$	A_μ^a	$A_\mu^{a'}$
$A^{a'}$	A^a	$A^{\alpha'}$	A^α
$h^{a'}$	h^a	$h^{\alpha'}$	h^α
δ^ρ	$\delta^{\rho'}$	δ^p	$\delta^{p'}$
$\gamma^{p'}$	γ^p	$\gamma^{\rho'}$	γ^ρ

Note: the bare graviphoton A_μ^0 always has $(--)$ parity (so A^0 has $(++)$ parity). The range of $\wp_1$, $\wp_2$, and $\wp_3$ in $\alpha = 1, \ldots, \wp_1$; $\alpha' = \wp_1 + 1, \ldots, \wp_2$; $a = \wp_2 + 1, \ldots, \wp_3$; and $a' = \wp_3 + 1, \ldots, n_V$, are arbitrary.

The fields with $(+-)$ or $(-+)$ eigenvalues have massive $n = 0$ modes on the fixed planes for the same reason that any Kaluza-Klein field does: there is excitation in the x^5 direction. In the low energy effective theory, such fields will fall into massive $\mathcal{N} = 1$ multiplets in four dimensions due to terms in the Lagrangian with $\partial_5 \Phi^{+-}$ or $\partial_5 \Phi^{-+}$.

In contrast to the $S^1/\mathbb{Z}_2$ construction, we can now remove all massless chiral multiplets in real representations by choosing there to be no a', p' indices. In that case, no fields from the $5D$ vector multiplets are completely projected out of the boundary spectra of propagating modes. Alternatively, we can keep a subset of those massless chiral multiplets (in a real representation) *such that they no longer furnish complete K-representations*. We can summarize the results in a table. We have decomposed the representation $\mathbf{R_V}[K] = \text{adj}[K_\alpha] \oplus \mathbf{R}_V^1[K_\alpha] \oplus \mathbf{R}_V^2[K_\alpha] \oplus \mathbf{R}_V^3[K_\alpha]$.

Multiplet	Representation	Type	Boundary	Tree-level Mass
$\{A_\mu^\alpha, \lambda^{\rho\,i}\}$	$\text{adj}[K_\alpha]$	Real	Both	Massless
$\{\tilde{\lambda}^{p'\,i}, z^{a'}\}$	$\mathbf{R}_V^1[K_\alpha]$	Real	Both	Massless
$\{\Psi_5^i, z^0\}$	K_α-singlet	Real	Both	Massless
$\{A_\mu^{\alpha'}, \lambda^{\rho'\,i}\}$	$\mathbf{R}_V^2[K_\alpha]$	Real	$y = 0$	$\mathcal{O}(1/R)$
$\{\tilde{\lambda}^{p\,i}, z^a\}$	$\mathbf{R}_V^3[K_\alpha]$	Real	$y = 0$	$\mathcal{O}(1/R)$
$\{A_\mu^a, \lambda^{p\,i}\}$	$\mathbf{R}_V^3[K_\alpha]$	Real	$y = \pi R/2$	$\mathcal{O}(1/R)$
$\{\tilde{\lambda}^{\rho'\,i}, z^{\alpha'}\}$	$\mathbf{R}_V^2[K_\alpha]$	Real	$y = \pi R/2$	$\mathcal{O}(1/R)$

Example

Let's revisit the $SU(5,1)$ example based on the Lorentzian Jordan algebra $J_{(1,5)}^\mathbb{C}$. We can obtain chiral multiplets (with a scalar potential) in the $(\mathbf{1}, \mathbf{2}) \oplus (\mathbf{1}, \bar{\mathbf{2}})$ of $SU(3) \times SU(2) \times U(1)$ (along with a spin-1/2 gauge singlet multiplet). Let the indices correspond to:

| I	α	a'	α'	a	0
$SU(5,1)$	$SU(3) \times SU(2) \times U(1)$	$(1,2) \oplus (1,\bar{2})$	$(3,1) \oplus (\bar{3},1)$ $\oplus (3,2) \oplus (\bar{3},2)$	N/A	(1,1)
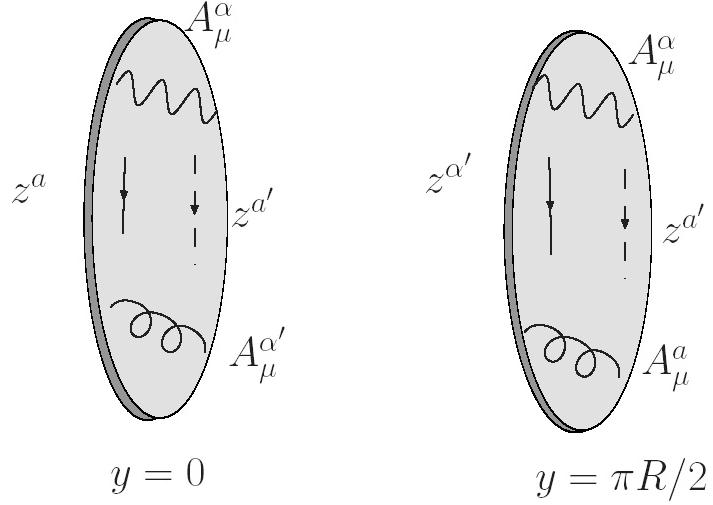
Figure 4.3: Schematic of massive and massless vector and scalar fields with boundary propagating modes.

The A_μ^α correspond to Standard Model gauge fields propagating on both fixed planes; the remaining vector fields either sit in massive multiplets, or are simply projected out. In particular, we take the $A_\mu^{\alpha'}$ to be the $(3,2) \oplus (\bar{3}, 2)$ vectors (X, Y bosons) and color triplet vectors $(3,1) \oplus (\bar{3}, 1)$, which will propagate in massive supermultiplets in the effective theory of the $y = 0$ plane. This implies that massive spin-1/2 multiplets in the $((3,2) \oplus (3,1)) \oplus (c.c.)$ will propagate in the effective theory of the $y = \pi R$ plane. Next, let the $A_\mu^{a'}$ denote the vectors in the $(1,2) \oplus (1, \bar{2})$, which means there will be chiral multiplets in this representation at both fixed planes (with scalar potential terms). Finally, we get conjugate pairs of massless chiral gauge singlet multiplets from the $5D$ supergravity multiplet. There are no fields with index a in this example.

4.6.2 Hypermultiplet sector

So far, we have not been able to obtain massless chiral multiplets in *complex* representations of the boundary gauge group even in the $\Gamma = \mathbb{Z}_2 \times \mathbb{Z}_2$ construction. Once again, the only way to do this (starting only from a $5D$ bulk theory) is to couple $5D$ hypermultiplets. We can make an index split as in the previous cases:

$$\tilde{X} = (X, X', \Omega, \Omega') \quad A = (n, n', \tilde{n}, \tilde{n}'),$$

where the fields have the following parity assignments under $\mathbb{Z}_2 \times \mathbb{Z}_2$:

$++$	$+-$	$-+$	$--$
q^X	$q^{X'}$	q^Ω	$q^{\Omega'}$
ξ_1^n	$\xi_1^{n'}$	$\xi_1^{\tilde{n}}$	$\xi_1^{\tilde{n}'}$
$\xi_2^{\tilde{n}}$	$\xi_2^{\tilde{n}'}$	ξ_2^n	$\xi_2^{n'}$

The fields with $(+-)$ and $(-+)$ eigenvalues have massive $n = 0$ modes, and so should fall into massive spin-1/2 multiplets. Therefore, *the indices n and $\tilde{n}$ are required to be in 1-1 correspondence as are the indices n' and $\tilde{n}'$.* However, there is no constraint between unprimed and primed indices, and each pair has an arbitrary range. That is, $n = 1, \ldots, \wp$ and $n' = \wp + 1, \ldots, n_H$; while $\tilde{n} = n_H + 1, \ldots, (n_H + \wp)$ and $\tilde{n}' = (n_H + \wp) + 1, \ldots, 2n_H$, where $\wp$ is variable.

If K is the 5D gauge group, and K_α is the boundary gauge group, the K_α-representations of the massless chiral multiplets at the boundaries *no longer need to form complete K-representations.*

We can summarize the results for the hypermultiplets in a table. Start with n_H 5D hypermultiplets in the $\mathbf{R_H}[K]$ of the 5D gauge group K. Let the gauge group at the orbifold fixed points be K_α so that under this group, the $\mathbf{R_H}[K]$ decomposes into the representation

$$\mathbf{R_H}[K_\alpha] = \mathbf{R}_H^1[K_\alpha] \oplus \mathbf{R}_H^2[K_\alpha],$$

where the indices 1 and 2 denote the splitting of $\tilde{X}$ into (X, X'). At the fixed points, the hypermultiplets split into chiral multiplets with indices split into $(X, \Omega'; X', \Omega)$, and are in the representations $\mathbf{R}_H^i[K_\alpha]$ or $\mathbf{R}_H^i[K_\alpha] \oplus \overline{\mathbf{R}}_H^i[K_\alpha]$ as listed here:

Multiplet	Representation	Type	Boundary	Tree-level Mass
$\{\xi^n, q^{X_1}\}$	$\mathbf{R}_H^1[K_\alpha]$	Real or Complex	Both	Massless
$\{\xi^{\tilde{n}}, q^{X_2}\}$	$\overline{\mathbf{R}}_H^1[K_\alpha]$	Real or Complex	Both	Massless
$\{\xi^{A'}, q^{X'}\}$	$\mathbf{R}_H^2[K_\alpha] \oplus \overline{\mathbf{R}}_H^2[K_\alpha]$	Real	$y = 0$	$\mathcal{O}(1/R)$
$\{\xi^{A'}, q^\Omega\}$	$\mathbf{R}_H^2[K_\alpha] \oplus \overline{\mathbf{R}}_H^2[K_\alpha]$	Real	$y = \pi R/2$	$\mathcal{O}(1/R)$

We have split $X = (X_1, X_2)$ such that $X_1 = 1, \ldots, n_H$ and $X_2 = n_H + 1, \ldots, 2n_H$. Also, $A' = (n', \tilde{n}')$ is a $USp(2m)$ index.

Example

Consider the $SU(5)$ YMESGT with C_{IJK} as in (2.3) (where C_{ijk} are the d-symbols of $SU(5)$), coupled to the minimal amount of Higgs and matter content in the bulk. From

section (3.1.2), this can be realized by coupling the YMESGT to hypermultiplets whose scalars parametrize the quaternionic manifold[5]

$$\mathcal{M}_Q = \frac{SU(27n, 2)}{SU(27n) \times SU(2) \times U(1)},$$

resulting in a coupling of n sets of hypermultiplets in the $\mathbf{1} \oplus 3(\mathbf{5}) \oplus \mathbf{10}$ of $SU(5)$.

Suppose we are going to break $SU(5) \to SU(3) \times SU(2) \times U(1)$; focussing on the hyper-multiplet sector, we can make the following assignments

n	$\tilde{n}$	n'	$\tilde{n}'$
$Matter \oplus (1,2) \oplus (1,\bar{2})$	$c.c.$	$(3,1) \oplus (\bar{3},1)$	$c.c.$

This will result in a low energy effective theory at both boundaries with Standard Model chiral matter multiplets; a pair of left-chiral Higgs doublets and their CPT conjugates; and a pair of massive spin-1/2 color triplet multiplets, all at both boundaries.

4.7 Summary of parity assignments

We have found parity assignments for fields and other objects in five-dimensional Yang-Mills-Einstein supergravity coupled to tensor and hypermultiplets. This is useful for understanding the theory at the boundaries as well as the low energy effective theory. We have not considered boundary localized fields, which would arise from p-branes located there, or states becoming massless due to membranes wrapping cycles that collapse there. We have allowed for general gauge symmetry breaking $K \to K_\alpha$, which can arise via boundary conditions, or in special cases via Wilson lines looping between boundaries.

Five-dimensional tensor multiplets can yield $4D$ $\mathcal{N} = 1$ chiral multiplets with a scalar potential (admitting supersymmetric Minkowski ground states). Since they appear in real representations, they are potential Higgs multiplets. However, we are stuck with complete K-representations. Therefore, we cannot use boundary conditions to make unwanted representations massive (which was one of the original motivations for field theory orbifold models).

A novel feature of supergravity theories is that one can gauge a non-compact group. The vectors representing the non-compact generators must be given odd parity, yielding a $4D$ compact gauge group and chiral multiplets in real representations. There is a potential

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[5]By allowing an additional singlet hypermultiplet, we can instead couple the exceptional scalar manifold $\frac{E_8}{SU(6) \times SU(2)}$.

serving as a mass term for the scalars in these chiral multiplets (admitting supersymmetric Minkowksi ground states).

Bulk hypermultiplets can lead to chiral multiplets in complex representations. The bulk minimal coupling to gauge fields yields another potential for scalars in the YMESGT sector. In the case of $5D$ non-compact gauging, this provides another mass term for these scalars. Furthermore, there is a potential for the scalars from the hypermultiplets; in the case of non-compact gauging, there is a mass term for these scalars as well.

Non-compact gauging in orbifold-GUTs thus leads to Higgs and matter masses in a new way. Of course, one is left with the usual problem of breaking supersymmetry while fixing an appropriate vev for the Higgs scalars.

Chapter 5

Symmetries and Anomalies

It is well known that the *manifest* global symmetry algebra of a dimensionally reduced five-dimensional $\mathcal{N} = 2$ MESGT Lagrangian (up to topological terms) is

$$\mathfrak{g}_4 = (\mathfrak{g}_5 \oplus \beta) \, \text{\textcircled{S}} \, \mathfrak{t},$$

where $\mathfrak{g}_5$ is the global symmetry algebra of the $5D$ Lagrangian; β are rescalings present in any dimensionally reduced supergravity theory; and $\mathfrak{t}$ act as translations of the $4D$ scalars A^I arising from the $5D$ vector fields [GST84a, dWVP95]. (The translation algebra also acts non-trivially on the fieldstrengths of the $4D$ theory, but this action does not carry over to the orbifold theory.) If $k_5 \subset \mathfrak{g}_5$ is gauged in the $5D$ theory, then the $4D$ gauge algebra is $k_5 \oplus \mathfrak{u}(1)$,[1] and the global translations are no longer necessarily symmetries due to the minimal coupling terms and scalar potential. With that in mind, let's look at the case of an *orbifold* reduction.

5.1   Symmetries at the orbifold fixed points

The gauge transformations of the A_μ^β and A^b *restricted to the fixed points* are

$$\delta A_\mu^\beta|_{fp} = -\frac{1}{g} \sum_n B_{(n)}^1 \partial_\mu \alpha_{(n)}^\beta + \sum_{n,m} B_{(n)}^1 B_{(m)}^2 \alpha_{(n)}^\delta f_{\delta\gamma}^\beta A_{\mu\ (m)}^\gamma + \cdots$$

$$\delta A^a|_{fp} = -\frac{1}{g} \sum_n \left(\frac{n}{R}\right) G_{(n)} \alpha_{(n)}^a - \frac{1}{g} \sum_n H_{(n)} \alpha_{(n)}^a \delta(x^5)$$
$$+ \sum_{n,m} B_{(n)}^1 B_{(m)}^2 \alpha_{(n)}^\beta f_{\beta\,c}^a A_{(m)}^c + \cdots,$$

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[1] In constrast to k_5, the $\mathfrak{u}(1)$ local symmetry is not part a YMESGT gauging since nothing is charged under it.

where dots indicate terms involving $\epsilon(x^5)$, and the $B_{(n)}$ and $G_{(n)}, H_{(n)}$ are expansion constants. However,

$$\int \partial_\mu \alpha^a(x, x^5)|_{fp} dx^5 = 0$$

so that α^a *is constant on the fixed planes.*[2] The first set of transformations are simply the local action of the non-abelian gauge group on the fixed-plane-propagating vector fields. KK modes are mixed when we have non-abelian gauging; however, for $n > 0$ vector fields, the transformations are not symmetries of the equations of motion, so that only the zero mode components of the first expression are symmetries of the on-shell theory. The second set of transformations are constant shifts on all A^a together with the action of the gauge group on the $A^{\hat{a}}$; that is, the "Poincare group" acting on the non-singlet scalars. Thus, the symmetry algebra generating the $5D$ gauge group K is broken, with the fields furnishing a representation of a resulting algebra with structure constants $f^\alpha_{\beta\gamma}$, f^α_{ab}, $f^a_{b\alpha}$. These structure constants indicate that the orbifold forces a symmetric structure decomposition for the algebra

$$(k_{(\alpha)} \,\circledS\, t^{(\hat{a})}) \oplus t^{(\tilde{I})}, \tag{5.1}$$

where $\circledS$ is the semi-direct sum; $k_{(\alpha)}$ is the Lie algebra for a compact subgroup $K_{(\alpha)} \subset K$ (the new gauge group); $t^{(\hat{a})}$ correspond to $K_{(\alpha)}$-*non-singlets* in the coset $K/K_{(\alpha)}$; and $\tilde{I} = (0, \tilde{a})$ runs over *singlets* of $K_{(\alpha)}$. This then implies that

$$C_{abc} \propto \{C_{\tilde{a}} C_{\tilde{b}} C_{\tilde{c}}, \ C_{\tilde{a}} \delta_{\hat{b}\hat{c}}\} \tag{5.2}$$

$$C_{a\alpha\beta} \propto C_{\tilde{a}} \, \delta_{\alpha\beta}, \tag{5.3}$$

where $C_{\tilde{a}}$ are real-valued constants.

Consider breaking the simple gauge algebra $\mathfrak{su}(N, 1)$. The algebra admits the 3-graded form (with respect to the $\mathfrak{u}(1)$ generator)

$$\overline{N}_- \oplus [\mathfrak{su}(N) \oplus \mathfrak{u}(1)] \oplus N_+.$$

If $k = \mathfrak{su}(N, 1)$ and $k_{(\alpha)} = \mathfrak{su}(N) \oplus \mathfrak{u}(1)$, then $k_{(\alpha)}$ is the gauge symmetry algebra at the orbifold fixed points, with scalars in the $\overline{\mathbf{N}}_- \oplus \mathbf{N}_+$ propagating there. The vectors A^α_μ, scalars

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[2]In the upstairs picture, $\partial_\mu \alpha^a|_{fp}$ is proportional to $\epsilon(x^5)$ so that the expression is only well-defined as an integral over the fifth dimension. In the downstairs picture, on the other hand, the boundary conditions are $\alpha^a(x, y)|_{bdry} = 0$ and $\partial_\mu \alpha^a(x, y)|_{bdry} = 0$.

$A^{\hat{a}}$, and scalar A^0 therefore together furnish a representation of

$$(\mathfrak{su}(N) \textcircled{s} \, t^{2N}) \oplus t^0. \tag{5.4}$$

From a four-dimensional point of view, this consists of gauge transformations and shifts of the A^a scalars, while from a five-dimensional point of view, this algebraic structure is simply a reflection of the on- and off-boundary gauge interactions at the fixed points. In this example, there aren't any non-zero C_{abc} or $C_{a\alpha\beta}$ terms in the $4D$ Lagrangian.

If we want $k_{(\alpha)} = \mathfrak{su}(N)$ instead, the structure of the symmetry algebra at the fixed points is

$$(\mathfrak{su}(N) \textcircled{s} \, t^{2N}) \oplus t^{(\tilde{I})}$$

despite the fact that the $A^{\hat{a}}$ were charged with respect to the manifest $U(1)$ in the previous example. Now both the scalar A^0 and the scalar from the $U(1)$ factor furnish a representation of $t^{(\tilde{I})}$. In this case, C_{abc} and $C_{a\alpha\beta}$ are described by a single parameter, and may therefore be non-zero.

The dimensionally reduced Lagrangian restricted to the orbifold fixed points has the terms

$$e^{-1}\mathcal{L}_V|_{fp} = -\frac{e^\sigma \, C_{\tilde{I}} h^{\tilde{I}}}{4} \delta_{\alpha\beta} \mathcal{F}^\alpha_{\mu\nu} \, \mathcal{F}^{\beta\,\mu\nu} + \frac{\hat{\kappa} C_{\tilde{I}} A^{\tilde{I}}}{2\sqrt{6}} e^{-1} \epsilon^{\mu\nu\rho\sigma} \delta_{\alpha\beta} \mathcal{F}^\alpha_{\mu\nu} \, \mathcal{F}^\beta_{\rho\sigma}, \tag{5.5}$$

where $\tilde{I} = (0, \tilde{a})$. These terms will describe the fields that will be massless in the $4D$ effective theory of a given boundary. As a result, the shifts of the singlet scalar $C_{\tilde{I}} A^{\tilde{I}}$ are not symmetries of the Lagrangian due to the topological shifts they induce. The shifts of the $A^{\hat{a}}$ are also not symmetries of the Lagrangian due to the minimal couplings to the the gauge fields A^α_μ.

The scalar potential restricted to the fixed points is

$$V|_{fp} = g_{\chi\psi} \, K^\chi_a \, K^\psi_b A^a A^b. \tag{5.6}$$

In general, this potential can contribute to the breaking of the possible shift symmetries. In particular, shift symmetries of the $A^{\tilde{I}}$ can be broken by this term. If the $5D$ gauge group K is compact, then it is a subgroup of the isotropy group of the scalar manifold $\mathcal{M}_V$. In that case, there always exists a point on $\mathcal{M}_V$ that is invariant under K-action: $\delta\phi^{\tilde{x}}_0 = 0$, so that $K^I_{\tilde{x}}|_0 = 0$ for all I and $\tilde{x}$. This holds upon dimensional reduction, where now the subset $K^\chi_a|_0 = 0$ so that there exists a K-invariant critical point of the above scalar potential. In the canonical basis, the critical point with vanishing potential is the canonical basepoint

$h^0 = 1, h^a = 0$.

While sitting at the basepoint of the $5D$ theory is sufficient for the existence of $t^{(\tilde{I})}$ symmetries of the potential (5.6), we can relax this requirement after rewriting the potential as:

$$e^{-1}\mathcal{L}_{pot} = -\frac{3\hat{g}^2}{4\hat{\kappa}^2}e^{-3\sigma}\,\overset{\circ}{a}_{IJ}\,(A^K f^I_{KL}h^L)(A^M f^J_{MN}h^N).\tag{5.7}$$

The fields are still not manifestly written in terms of $C_{\tilde{I}}A^{\tilde{I}}$ since the metric contracts with the structure constants. But we may look at the form of this potential at the fixed points of the orbifold and write

$$\left(\overset{\circ}{a}_{\alpha\beta}\,f^\alpha_{ab}f^\beta_{cd} + \epsilon(x^5)^2\,\overset{\circ}{a}_{eg}\,f^e_{ab}f^g_{cd}\right)A^a h^b A^c h^d.\tag{5.8}$$

The indices $a, b, \ldots$ can be split into non-singlet and singlet indices $\hat{a}, \hat{b}, \ldots$ and $\tilde{a}, \tilde{b}, \ldots$. For general vacua, the above potential contributes to the breaking of any possible shift symmetries, with the exception of scalars arising from K-singlet vectors in $A^I_{\hat{\mu}}$ (which don't appear in the above scalar potential).

Note that, while the infinitesimal gauge transformations for A^β and A^b_μ vanish at the orbifold fixed points, two gauge transformations of these fields yields a fixed point localized term due to the odd structure functions $\epsilon(x^5)f^I_{JK}$. This is similar to the fact that these fields do appear in fixed-point localized terms (i.e., with delta functions). In the presence of the fixed points, there appears an algebraic element $f^\alpha_{a\beta}A^a\alpha^\beta$ and $f^a_{b\beta}\alpha^\beta A^b_\mu$ parametrized by the gauge coupling at the boundary. Thus, there is a structure in addition to that of (5.1). Since the fields A^β and A^b_μ appear as auxilliary fields in the upstairs picture, this structure that they close into is an "auxilliary algebra", which will not appear as a physical symmetry algebra of the low energy effective theory of a given boundary.

In the $S^1/\mathbb{Z}_2$ construction, it is clear that the α^β parametrize gauge symmetries of the Lagrangian as well as of the equations of motion for $n = 0$ fields. Let us now look at the $S^1/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ construction. Here, the Γ action on the gauge bundle gives fields four possible parities $(\pm\pm)$. The situation for $(++)$ and $(--)$ fields is the same as before. But $(+-)$ fields have propagating modes only on the $x^5 = 0$ fixed plane, while $(-+)$ has propagating modes only on the $x^5 = \pi R/2$ plane. Let's consider a $(+-)$ vector field $A^I_\mu(x, x^5)$; it has the form

$$\phi^{(+-)}(x, x^5) = \sum_n \phi^{(n)}(x)\cos[(2n+1)x^5/R] + \cdots,$$

where dots indicate additional terms that are even under the first $\mathbb{Z}_2$ action and odd under

the second $\mathbb{Z}_2$ action. The $5D$ Lagrangian yields the equations of motion for the $A_\mu^I(x, x^5)$:

$$\partial^\mu F_{\mu\nu}^{I\,(n)} - \left(\frac{(2n+1)}{R}\right)^2 A_\mu^{I\,(n)} + \cdots = 0,$$

where dots indicate a gauge invariant topological term $(F^I \wedge F^J)$. Therefore, for all n, the equations of motion at $x^5 = 0$ do not respect the local symmetries of the Lagrangian restricted to $x^5 = 0$, so that these $(+-)$ vector fields are not gauge fields from the point of view of a low energy observer at $x^5 = 0$. The off-shell theory, of course, is invariant under $(++)$ and $(+-)$ parameters at $x^5 = 0$ so that, e.g., coupling terms in the Lagrangian respect the larger set of symmetries at the fixed point. Due to these off-shell symmetries, however, there may be residual *global* symmetries even in the on-shell or effective theory. The arguments extend to all local symmetries of the $5D$ theory for $n \neq 0$ vector fields (KK excited modes): the restricted Lagrangian satisfies the symmetries, while the equations of motion (and effective theory) do not.

Aside from the symmetries arising from $5D$ gauge symmetry, we should check whether supersymmetry is preserved in the theory on the orbifold. In particular, $\mathcal{N} = 2$ susy should be unbroken in the bulk, while $\mathcal{N} = 1$ unbroken on the boundaries. Despite the fact that we have assigned parities consistent with the susy transformations, the fermions of the theory involve jumping functions so that the susy transformations will involve Dirac distributions with support at one or both orbifold fixed points. For example, from the susy transformation $\delta \Psi_{\hat{\mu}\,i}$, we have the component

$$\delta \Psi_{5\,i} = D_5 \epsilon_i \cdots$$

where

$$\epsilon^1 = \begin{pmatrix} \eta \\ \epsilon(x^5) e \xi^* \end{pmatrix} \quad \epsilon^2 = \begin{pmatrix} \epsilon(x^5)\xi \\ -e\eta^* \end{pmatrix}.$$

At first sight, it appears that this can break the supersymmetry of the entire theory. If this is so, then fixed point localized terms must be added (see [ABN01, BB03] e.g.). In addition to the delta factors in the susy transformations, there are fixed point localized fermion terms in the Lagrangian such as in the kinetic term $\bar{\Psi}_{\hat{\mu}}^i \Gamma^{\hat{\mu}\hat{\nu}\hat{\rho}} D_{\hat{\nu}} \Psi_{\hat{\rho}\,i}$.

5.2 Symmetry breaking via Wilson lines

Much of the literature on orbifold theories focuses on the symmetry breaking that is associated with the presence of Wilson lines on the spacetime $S^1/\mathbb{Z}_2$ or $\mathbb{R}/(\mathbb{Z} \ltimes \mathbb{Z}_2)$: since the

Wilson line is a mapping to an element of the original symmetry algebra, its presence restricts the surviving gauge algebra to be the maximal one commuting with this element. In the first case, the rank is not reduced, while in the second it is reduced by one. However, an orbifold spacetime breaks supersymmetry, which can be interpreted as a boundary condition on the Killing spinors. Similarly, we can view the presence of Wilson lines as be generated consistently with the boundary conditions of the vector fields. For example, the $5D$ graviphoton must be projected out of the boundary spectrum if the boundary is isomorphic to an orbifold fixed plane; the boundary condition allows the field to form a Wilson line stretching between boundaries. Thus, it is the very nature of the orbifold spacetime to allow for general gauge symmetry breaking by a set of allowed boundary conditions [HMR01b, M05b]. Wilson lines, in turn, can be associated with vacuum expectation values of scalar fields on a boundary.

Let's first recall the notion of a Wilson line. A gauge field A_μ can have a vacuum expectation value, but the local Lorentz symmetries will be broken. Therefore, one generally does not allow such vevs for vector indices in the observed $4D$ spacetime. A gauge field background can be locally gauge-transformed to zero (that is, the fieldstrength is locally exact $F = dA$); if it can be so transformed everywhere, then the original vev is not physical (it is "pure gauge"). One way to measure the non-triviality of a given gauge field background is with the topological quantity $\nu \equiv \int_M F$, with M the spacetime manifold. Since F is closed (the Bianchi identity for the fieldstrength), the value ν labels a particular cohomology class of the field configuration on a given manifold M. Now suppose one has a 1-form gauge field on a circle; it is either zero everywhere, or it can wrap the circle any integral number of times. But in this case, there is no fieldstrength to be defined, and anyway one cannot integrate over anything more than 1-forms since the manifold is 1-dimensional. In this case, it is natural to consider the quantity $\nu \equiv \int_{S^1} A$, where $A = A_\mu dx^\mu$. Now we can consider 1-form gauge fields that are closed, $dA \equiv F = 0$, so that there is no background fieldstrength. Then ν labels the cohomology class of the gauge field A, which again depends on the topology of the manifold M. This generalizes to field configurations on any n-manifold M: the quantity $\nu \equiv \int_P A$, where P is a closed path in M, depends on the topology of the manifold into which the path is embedded. There can be non-zero ν if $\pi_1(M)$ is not finite; for example, if it contains $\mathbb{Z}$. Otherwise, all of the closed paths are homotopic to a point. The exponentiation of ν, $\exp(i \int_P A_\mu dx^\mu)$, is called a Wilson loop. However, we will use the term "Wilson loop" to refer to $\int_P A$, which is common in the literature. Since a Wilson loop $\int_P A$ is a mapping from the n-manifold M to an element of the gauge algebra, the exponentiation is a mapping to an element of the gauge group K. Conformally mapping points at infinity to a single point, the n-manifold can be mapped to S^n so that the exponentiated Wilson loop is an element

of the n-th homotopy group $\pi_n(K)$. Therefore, if a manifold M has a non-finite $\pi_1(M)$, and if the gauge group K is such that $\pi_n(K)$ is also non-finite, the vacuum breaks the gauge group to the centralizer of the K-element ν. For example, a gauge field on a hyper-cylinder that is topologically $\mathbb{R}^2 \times S^1$, can have non-zero winding number ν, while $\pi_3(K) = \mathbb{Z}$ for any compact connected simple K. What about for a manifold of the form $M_4 \times S^1/\Gamma$? Due to the Γ action, the manifold is simply connected (i.e., $\pi_1(M)$ is finite) so that all closed loops are contractable to a point. But if we start on the covering space $M_4 \times S^1$, we can have Wilson loops, and after Γ-identification, we have a path stretching from one fixed plane to the other. This situation inspires us to generalize the notion of a Wilson loop to a Wilson line, simply by allowing the path P to have distinct endpoints. Now, if the path is not contractable to a point, such as in the case in which the endpoints are on distinct fixed planes, we have a topologically nontrivial path. Therefore, we generalize our conditions for gauge group breaking to be: if $\pi_1(M)$ is non-finite for either M being the spacetime, or its covering space in the case of orbifolds, and if $\pi_n(K)$ is also non-finite, then the gauge group may be broken by the presence of Wilson loops/lines. In the case of $5D$ spacetimes, $\pi_5(K) = \mathbb{Z}$ for $K = SU(n)$, $n \geq 3$ and for $K = SO(n)$, $n \geq 7$. For $K = USp(2n)$, $n \geq 1$, $\pi_5(K) = \mathbb{Z}_2$. The exceptional groups all have $\pi_5(K) = 0$ ($\pi_3(K) = \mathbb{Z}$ for the exceptional groups, while E_6 has $\pi_9(E_6) = \mathbb{Z}$).[3]

With this in mind, let's consider the presence of Wilson lines in the upstairs picture, where we work on the covering space $[-\pi R, \pi R]$ (with $\{-\pi R\} \equiv \{\pi R\}$). A Wilson line can thus begin at $\{-\pi R\}$ and end at $\{\pi R\}$; we call this path P. Due to the $\mathbb{Z}_2$ action, $dx^5 \to -dx^5$ about $x^5 = 0$. Therefore, the orientation of the line will flip at $x^5 = 0$:

$$\left\langle A_{\hat{\mu}}^a \right\rangle|_P = \frac{1}{2} \sum_n \int_{-\pi R}^0 A_{(n)}^a(x) C_{(n)} \cos\left(\frac{n x^5}{R}\right) dx^5$$
$$- \frac{1}{2} \sum_n \int_0^{\pi R} A_{(n)}^a(x) C_{(n)} \cos\left(\frac{n x^5}{R}\right) dx^5,$$

where the factor of $1/2$ is due to the $\mathbb{Z}_2$ identification to obtain the spacetime $M_4 \times S^1/\mathbb{Z}_2$. The result is then

$$\left\langle A_{\hat{\mu}}^a \right\rangle|_P = \pi R \, C_{(0)} A_{(0)}^a(x), \tag{5.9}$$

where $C_{(0)}$ is simply an $\mathcal{O}(1)$ expansion constant, and $A_{(0)}^a$ is a vacuum expectation value. In the downstairs picture, where the fifth dimension is parametrized by $y \in [0, \pi R]$, there

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[3]Note that strictly using Wilson lines in a more general spacetime with boundaries (i.e., not isomorphic to an orbifold) to break gauge symmetries would exclude an exceptional GUT scenario in five dimensions.
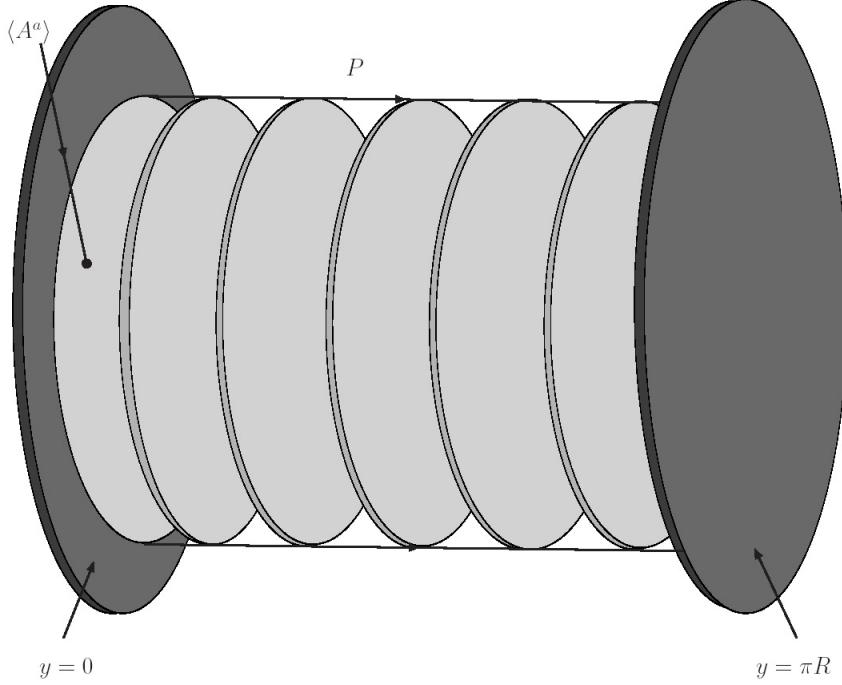
Figure 5.1: $4D$ slices of spacetime showing the strength of the potential $A_{\hat{\mu}}^a$ by shading. On the boundaries $y = 0$ and $y = \pi R$, the scalar field A^a has a vev.

is one copy of a Wilson line that stretches from one boundary to the other. Figure 5.1 is a schematic of this Wilson line, which is seen as a scalar vev for A^a on the boundaries. Similarly, for the $A_{\hat{\mu}}^\alpha$

$$\langle A_{\hat{\mu}}^\alpha \rangle |_P = -2 \sum_{n \text{ odd}} \left(\frac{R}{n} \right) C_n A_{(n)}^\alpha (x). \tag{5.10}$$

See figure 5.2 for a schematic of this Wilson line. The strength vanishes on the boundaries so that there is no vev for the scalar A^α that is projected out of the boundary-propagating spectrum. If we wrap around the covering circle N times, then there are N Wilson lines stretching between the boundaries in the downstairs picture. The total strength is then $N\pi R \langle A \rangle$.

The Wilson lines are such that, for very large energies (large number of levels n excited), the full group K can be unbroken in the presence of (5.9) and (5.10). For lower energies (lower levels n excited), the group is broken to the centralizer of the element of K associated with (5.10); the resulting five-dimensional gauge group therefore depends on the range of α (i.e., the parity assignments of the A_μ^I) and, ultimately, which Wilson lines are turned
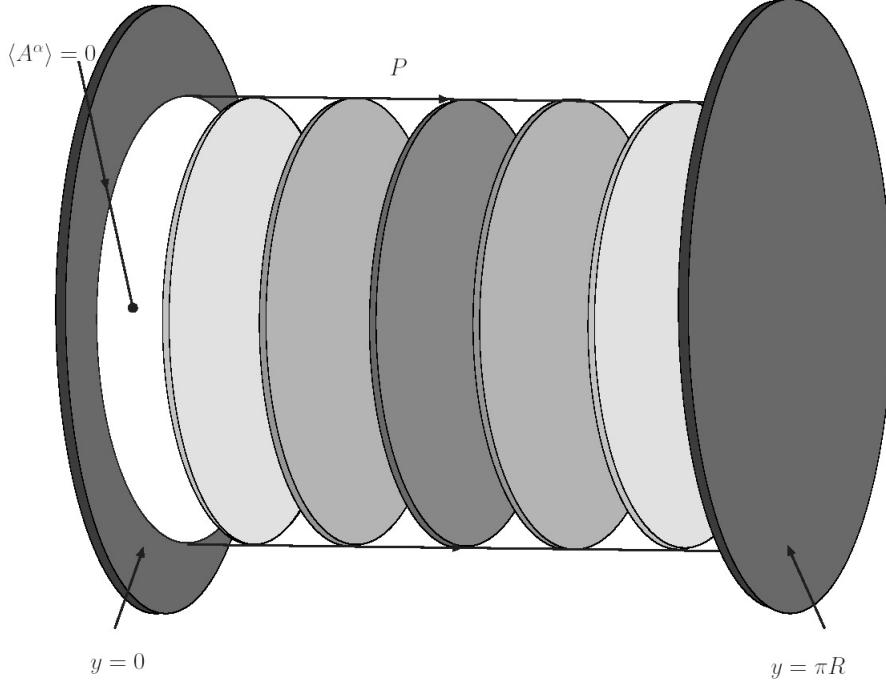
Figure 5.2: $4D$ slices of spacetime showing the strength of the potential $A_{\hat{\mu}}^{\alpha}$ by shading. On the boundaries $y = 0$ and $y = \pi R$, the scalar field and its vev A^{α} vanish.

on. The $4D$ low energy effective theory of a given boundary (below the scale, M_c, of the compact dimension) is dominated by the $n = 0$ level and has an unbroken gauge group as determined by the parity assignments for the A_{μ}^I (boundary conditions in the downstairs picture). Finally, at even lower energies, the $4D$ scalar sector, in part determined by the form of (5.9), may break the gauge group further (e.g. electroweak breaking).

Although we have been discussing Wilson lines on $S^1/\mathbb{Z}_2$, we could consider non-abelian lines on $\mathbb{R}/(\mathbb{Z} \ltimes \mathbb{Z}_2)$, which allows the rank to be reduced by one (where $\mathbb{Z}$ acts on elements of $\mathbb{R}$ as addition by $2\pi n$, $n = 0, 1, 2, \dots$).

Let's extend the discussion to the case of $S^1/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orbifolds. The parity assignments are consistent with an inhomogeneous Wilson line that drops in strength from one boundary to the next. The result for $(+-)$ fields is now

$$\langle A_{\hat{\mu}}^a \rangle_P = \sum_n \int_{-\pi R/2}^0 A^a(x) \cos \frac{(2n+1)x^5}{R}$$
$$= \sum_n A^a(x) \frac{R}{2n+1}.$$
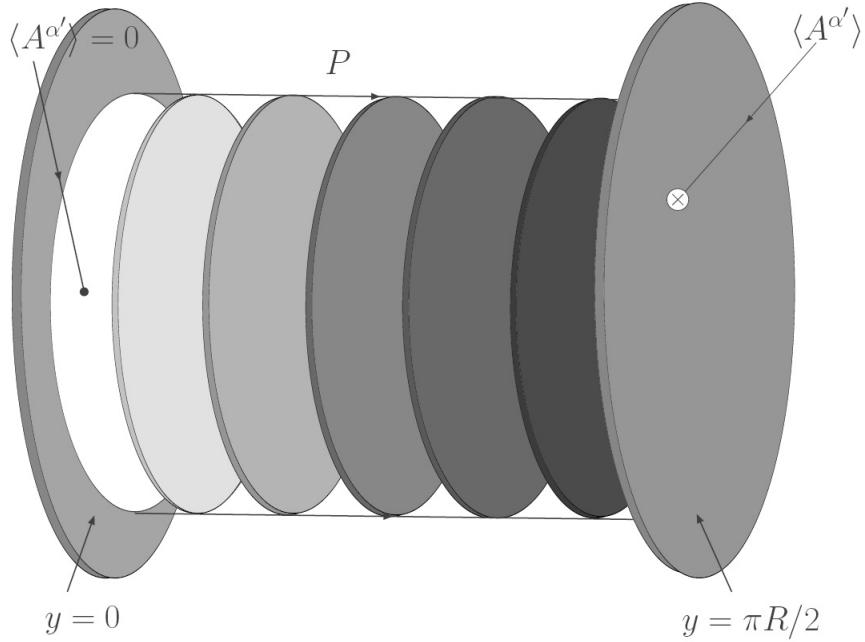
Figure 5.3: $4D$ slices of spacetime showing the strength of the potential $A_{\hat{\mu}}^{\alpha'}$ by shading. On the boundary $y = 0$, the vev for the scalar $A^{\alpha'}$ vanishes, while on $y = \pi R/2$, it is non-zero.

Figure 5.3 scehmatially illustrates the downstairs picture, where the fifth dimension is parametrized by $y \in [0, \pi R/2]$; the strength of the Wilson line schematically goes from zero at the $y = 0$ boundary to some non-zero value at the $y = \pi R/2$ boundary (really, there are diminishing contributions from a countably infinite set of excited modes of the $5D$ field $A_{\hat{\mu}}^{\alpha'}$).

Again, for large energies (where large n modes are excited), the gauge group is K, while for intermediate energies the above Wilson lines, along with Wilson lines from $(++)$ modes (of the same form as (5.10)), break the group to $K_{(\alpha)}$ in five dimensions. However, the $x^5 = 0$ fixed point does not experience the breaking until the energy scale goes below M_c since the Wilson line strength drops to zero there. Below the scale M_c, both fixed planes will have gauge symmetry $K_{(\alpha)}$.

In light of the discussion in section (3.1.2), it is natural to have $SO(10)$ with the $L(4,1)$ space (or E_6 with the $L(8,1)$ space) for large energies, whereas for intermediate energies the theory is still *five-dimensional* and based on the $L(4,1)$ space (resp. $L(8,1)$) but with $SU(5)$ (resp. $SO(10)$) gauge subgroup.

5.3   Anomalies

Previous work on anomalies in orbifold theories can be found in [AHCG01, SSSZ02, PR02, BCCRS02, L02, L03, vGQ03, SS04]. We will present here an independent analysis of the anomalies present in $5D$ supergravity on $M_4 \times S^1/\Gamma$, which is based on [M05c].

We would like to consider the minimal field additions for anomaly cancelation. There is nothing wrong with adding more multiplets localized at boundaries consistent with the symmetries and anomaly cancellation, but one is less compelled to make these additions. In the framework of string theory, the entire low energy field content would follow, in principle, from the particular compactification considered. These states would then survive in the supergravity approximation. In the bottom-up approach, by contrast, massless supermultiplets can be added by hand after constructing the orbifold version of the $5D$ theory.

Five-dimensional orbifold theories can have pure gauge or mixed anomalies due to the presence of $4D$ chiral fermions in complex representations of the gauge group. However, charged chiral multiplets coming from $5D$ vector or tensor multiplets appear in real representations of the boundary gauge group. Furthermore, if the $5D$ gauge group is compact, the chiral multiplet coming from the $5D$ supergravity multiplet is a gauge singlet. If noncompact, this chiral multiplet joins the other chiral multiplets to form a real representation. The $4D$ spin-3/2 fermion is in the $4D$ supergravity multiplet and will not have anomalous gauge couplings (we are not gauging R-symmetries). Therefore, the only fermions that can have anomalous gauge couplings are the chiral multiplets coming from $5D$ hypermultiplets charged under the gauge group. Note that R-symmetries are not gauged, so there aren't any Fayet-Illiopoulos terms.

One can always express a non-zero variation of a $4D$ action as an integral over a 4-form; with the Wess-Zumino consistency condition, we can express it as $\delta_\Lambda \int_\Sigma I_5$, which in turn can be expressed in terms of a gauge invariant 6-form I_6 such that $I_6 = dI_5$ for transformations parametrized by Λ that are connected to the identity. The gauge invariant 6-forms that serve as the gauge and mixed anomaly polynomials will be of the form $\mathrm{tr}F^3$ and $\mathrm{tr}R^2F$. These anomaly terms are associated with three external gauge boson and two graviton/one gauge boson triangle diagrams, respectively. If there are polynomials I_6 that are not reducible (cannot be written in the form $I_m I_{6-m}$ for $m \neq 0$), the massless spectrum of the $4D$ theory must be modified by the addition of suitable multiplets with support only on the boundary.

In addition to these familiar $4D$ anomalies, $5D$ theories with bulk Chern-Simons (CS) terms can contribute to reducible and irreducible anomalies at $4D$ boundaries due to a nonzero classical gauge variation of these terms, which is interpreted as an influx of charge

due to the bulk gauge current [CH85]. In the class of orbifold theories where none of the bulk vector fields propagate at the fixed points, there can be a contribution when the bulk fieldstrength is related to gauge fields whose support is *only* at the fixed points, as occurs in [HW96a, HW96b], where the bulk theory is required to couple to a localized boundary gauge theory. On the other hand, in orbifold GUTs, the $5D$ CS terms involve gauge fields that are directly involved in any anomalous couplings with chiral fermions at the fixed points.

The presence of Chern-Simons terms and associated anomaly inflow has been discussed in the literature mostly in the context of rigidly supersymmetric gauge theories. On the other hand, where M-theory or supergravity is considered, there isn't any inflow from bulk fields since the models there are typically of the HW or RS type (where none of the bulk vectors propagate on the boundaries of the spacetime). In the case of rigidly supersymmetric theories, there is no *a priori* reason to have a $5D$ CS term. In fact, it is often noted [SSSZ02] for orbifold theories that a CS term is not invariant under orbifold parities (i.e., it's odd). It has been suggested in such cases that one can couple an odd field to the CS term, rendering it invariant; the auxilliary field could then obtain vevs, serving as a dynamically determined coefficient of the CS term [SSSZ02]. Alternatively, it has been shown [L02, L03] that a $5D$ $U(1)$ gauge theory minimally coupled to a single fermion on an orbifold can be given an infinite "jumping" (or kink) mass, giving rise to a fermion zero mode quantum anomaly at the fixed points along with a residual $5D$ CS term (whose classical gauge variation cancels the quantum anomaly).

However, in $5D$ supergravity, Chern-Simons terms are part of the classical theory. Furthermore, these terms are consistent with the orbifold symmetries: they have the tensor C_{IJK} as a coefficient, which carries representation indices of the global symmetry group, G, in the case of a MESGT, and those of the gauge group, K, in the case of a YMESGT. Therefore, as the Γ action is lifted to the gauge or flat G-bundle (that is, with trivial connection), respectively, it can act non-trivially on C_{IJK}.

The action for a five-dimensional Maxwell-Einstein supergravity theory has a Chern-Simons term

$$S_{CS} = \int_{\mathcal{M}_5} \frac{5!}{6\sqrt{6}} C_{IJK} F^I \wedge F^J \wedge A^K,$$

where C_{IJK} is a rank-3 symmetric invariant of the global symmetry group G of the Lagrangian. In Minkowski spacetime, the full action is invariant under local abelian transformations $\delta_\alpha A^I = d\alpha^I$.[4] Now we are considering a spacetime with boundary (downstairs

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[4]These are not proper "gauged supergravities", which arise when R-symmetries or scalar isometries are gauged.

picture). In showing that the bulk variations vanish, there will be boundary localized terms. Consider the variation of the action under the abelian transformations above. The action is clearly invariant except for the CS term, which gives

$$\delta_\alpha S_{CS} = \frac{5!}{6\sqrt{6}} C_{\alpha\beta\gamma} \int_{\partial\mathcal{M}_5} (F^\alpha \wedge F^\beta)\alpha^\gamma,$$

where α, β, γ label vector fields that propagate along the boundary; and $C_{\alpha\beta\gamma}$ is a rank-3 symmetric invariant of the global symmetry subgroup $G_\alpha \subset G$ of the boundary theories. Since the boundaries are oppositely oriented, the flux of charged current coming from one boundary is received by the other (i.e., the anomaly globally cancels). However, there is a classical inflow anomaly at the individual boundaries, so that the corresponding local abelian symmetries of a $5D$ MESGT are broken. This inflow must be compensated locally if the associated $5D$ symmetry is to be preserved.

However, the classical Lagrangian is otherwise invariant under the local abelian transformations we are discussing. The fermions propagating on the boundaries will not have a chiral anomaly contribution since they are not charged with respect to any of the abelian fields. Therefore, in dealing with MESGTs in the presence of boundaries, only the theories with $C_{\alpha\beta\gamma} = 0$ are invariant under the full set of local abelian transformations. To compensate for the inflow present in theories with $C_{\alpha\beta\gamma} \neq 0$, *we can add a boundary localized set of terms* involving fermions and minimal coupling to the propagating vectors.

We are more interested in gauged $5D$ supergravity theories here. If we gauge abelian isometries of the scalar manifold, the anomaly inflow will be of the same form as above, but now $C_{\alpha\beta\gamma} = C_\alpha C_\beta C_\gamma$ so that the anomaly associated with the set of local abelian transformations parametrized by α^β is proportional to $(C_\alpha F^\alpha) \wedge (C_\beta F^\beta)(C_\gamma \alpha^\gamma)$. That is, we have a $U(1) - U(1) - U(1)$ anomaly parametrized by $C_\gamma \alpha^\gamma$. Again, we can consider fermions chirally coupled to the gauge fields propagating on the boundaries in such a way that the quantum anomaly contribution compensates the inflow. In a pure YMESGT, there aren't any chiral couplings on the boundaries. *The only way to obtain them is to either start with a different 5D theory coupled to hypermultiplets, or to consider fermionic fields with support only at the boundaries (with minimal coupling to the gauge fields from the 5D theory).*

Finally, let's move to the case of primary concern: orbifold-GUTs. In spacetimes *without* boundary, a YMESGT based on a *reducible* C-tensor simply has the gauge invariant term $\mathcal{F}^I \wedge \mathcal{F}^J \wedge A^K$, where the bare A^K can always be made to be one of the singlet vectors that must be present in such theories [GST84c]. In the presence of boundaries, however, a non-abelian YMESGT must generally have a full "Chern-Simons" extension, and the reducibility

of the C-tensor is no longer a sufficient criterion. If C_{IJK} is reducible, there is again at least one singlet vector field (the graviphoton) of any non-abelian group that is to be gauged, and it necessarily appears in every FFA term of the MESGT. If all such singlet vectors do not have propagating modes on the boundaries (the $5D$ graviphoton never does), then we may write the FFA terms of the MESGT such that the singlets appear only as the bare A. Then, promoting the theory to a YMESGT is simple: replace the abelian fieldstrengths with non-abelian ones, including in the FFA term. The $5D$ infinitesimal gauge transformation of the $\mathcal{F}\mathcal{F}A$ vanishes, so that there is no anomaly contribution in this case. On the other hand, if there are any boundary-propagating spectator vectors, the full Chern-Simons extension must be considered in the YMESGT. The inflow anomaly, however, will be reducible of the form $U(1) - K_{(\alpha)} - K_{(\alpha)}$ so that the addition of boundary-supported fields is not required.

As an example, consider a MESGT of the "generic Jordan" family [GST84a] with C_{IJK} tensor being the norm form of the cubic Euclidean Jordan algebra $J^2 \oplus \mathbb{R}$: sQ, where Q is a quadradic "Minkowski norm" with signature $(+ - \cdots -)$. The scalar manifold is

$$SO(1,1) \times \frac{SO(n_V - 1, 1)}{SO(n_V - 1)},$$

where n_V is the number of vector multiplets coupled to the theory. We can gauge $SU(n) \subset SO(n_V - 1)$ such that $\dim[SU(n)] = n_V - 1$, in which case we'll be left with two spectator vectors: the "bare" graviphoton $A^0_{\hat{\mu}}$ and a vector multiplet field $A^1_{\hat{\mu}}$. Although the C-tensor is reducible, if we allow the $4D$ vector component A^1_μ to propagate on the boundaries, promoting the MESGT to a YMESGT will require the full CS form, and there will consequently be a reducible anomaly inflow.

More interestingly, there will be an *irreducible* anomaly inflow when the C-tensor is irreducible. This, in turn occurs for

(A) YMESGTs in which some components $C_{ijk} \neq 0$. When promoting the MESGT to a YMESGT, we must replace the FFA term by (2.6)

(B) Unified YMESGTs (in which all of the vector fields in the theory form the adjoint of a simple group).

Case A

This will occur for non-abelian gaugings containing a subgroup that has a cubic symmetric invariant $d_{\alpha\beta\gamma}$ (i.e., $SU(n)$ $n > 2$ type); and which, in addition, have $D \neq 0$ for $C_{\alpha\beta\gamma} = Dd_{\alpha\beta\gamma}$. Phenomenologically, the interesting gauge groups in the low energy effective orbifold theories do have cubic symmetric invariants, but we can make life simpler by considering

the class of those gauge theories defined by $C_{\alpha\beta\gamma} = 0$. Consider the example in which the MESGT C_{ijk} are the d-symbols of $SU(n)$, under which the $n_V + 1$ vector fields form the $\mathbf{1} \oplus \mathrm{adj}[SU(n)]$ (that is, we have $n_V = \dim[SU(n)]$ vector multiplets). Then we can promote the MESGT to a YMESGT with gauge group $K = SU(n)$ under which all of the fields in the vector multiplets form the adjoint representation, while the graviphoton is a spectator. When promoting the MESGT to a YMESGT, let's simplify the replacement of the FFA term by its gauge invariant form. We can split the $C_{IJK}F^I \wedge F^J \wedge F^K$ terms in the Lagrangian as

$$\left(C_{000}F^0 \wedge F^0 \wedge A^0 + 2C_{0ij}F^0 \wedge F^i \wedge A^j + C_{0ij}F^i \wedge F^j \wedge A^0 \right)$$
$$+ C_{ijk}F^i \wedge F^j \wedge A^k,$$

with first three terms being the "reducible part", and the fourth being the "irreducible part". Integration by parts allows us to re-express the reducible part in the form

$$\int_{M_5} C_{0IJ}F^I \wedge F^J \wedge A^0 \tag{5.11}$$

(since the vector A^0_μ is necessary removed from the boundary spectrum of propagating modes). To promote this theory to a YMESGT, we must make the replacement (2.6) for the irreducible part involving C_{ijk}, while we simply replace the abelian fieldstrengths in (5.11) with non-abelian ones. Therefore

$$\frac{5!}{6\sqrt{6}}C_{ijk}\left[F^i \wedge F^j \wedge A^k + \frac{3}{2}gf^k_{lm}\left(F^i \wedge A^j \wedge A^l \wedge A^m \right) \right.$$
$$\left. + \frac{3}{5}g^2 f^j_{gh}f^k_{lf}(A^g \wedge A^h \wedge A^l \wedge A^f \wedge A^i) \right] \tag{5.12}$$
$$+ \frac{5!}{6\sqrt{6}}C_{0ij}\left(\mathcal{F}^i \wedge \mathcal{F}^j \wedge A^0 \right).$$

The last term is invariant under $5D$ non-abelian gauge transformations, and we pick up no boundary term. The only contribution to the inflow anomaly comes from the C_{ijk} terms. We can rewrite this in terms of the Chern-Simons form (with constant prefactors)

$$\mathcal{L}_{CS} = (D\hat{\kappa}/g^3)\mathrm{Tr}[F \wedge F \wedge A + \frac{3}{2}F \wedge A \wedge [A, A] + \frac{3}{5}A \wedge [A, A] \wedge [A, A]],$$

where $A = g t_i A^i$; $[A, A] = g\, t_i f^i_{jk}\, A^j \wedge A^k$; and we have used the fact that the C-tensor is proportional to the d-symbols of the gauge group, with constant of proportionality being

D. Under global gauge transformations, this term generally transforms as a non-trivial element of $\pi_5(K)$. However, under transformations connected to the identity, it transforms with a trivial element: $\delta_\alpha L_{CS} = dL_{FF}(\alpha)$. Therefore, the irreducible anomaly inflow is $\delta_\alpha S_{CS}^{5D} = S_{FF}^{(4D)}(\alpha)$, with

$$S_{FF}^{(4D)}(\alpha) = (D\kappa/g^3) \int_{\partial M_5} \mathrm{Tr}[(\mathcal{F} \wedge \mathcal{F})\,\alpha], \tag{5.13}$$

where $\mathcal{F} = g\, t_\beta\, \mathcal{F}^\beta$ and $\alpha = g\, t_\beta\, \alpha^\beta$.

Case B

Consider the unified YMESGTs based on Lorentzian Jordan algebras $J_{(1,N)}^{\mathbb{C}}$ (represented as matrices over the complex numbers that are Hermitian with respect to a Minkowski metric). In this case, the C_{IJK} are the d-symbols of the $SU(N,1)$ gauge group and all FFA terms of the original MESGT must be replaced by (2.6). Then the form of the anomaly inflow is the same as in (5.13), where the gauge group $K_{(\alpha)}$ is at most $SU(N)$. Again, the irreducible anomaly will require chiral couplings from bulk hypermultiplets or boundary localized fields.

Extension to $\mathbf{S^1/(\mathbb{Z}_2 \times \mathbb{Z}_2)}$

We may now extend to the $S^1/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ case. As mentioned in a previous section, these spacetimes are consistent with inhomogeneous Wilson lines, whose strength varies from one boundary to the other. This asymmetric background allows the anomaly flow from one boundary to the other not to be conserved (that is, there is a global anomaly). However, due to the nature of $(+-)$ and $(-+)$ type symmetries at the orbifold fixed points, there can't be a local anomaly contribution associated with this excess charge flow; the low energy theory doesn't have these as gauge symmetries. Therefore, the corresponding inflow contribution can only be anomalous for rigid symmetries of the theory arising from the broken local symmetries.

Let's consider an example of a $5D$ YMESGT on $S^1/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ with the assignments as in section (4.6.1). First, let's note the following: $C_{\alpha\beta\gamma'}$ is rank-2 with adjoint indices of $SU(3) \times SU(2) \times U(1)$, and so is $\propto \delta_{\alpha\beta} C_{\gamma'}$. But it also carries an index in $SU(5)/SM$, so can't be a singlet of $SU(5)$. Therefore, $C_{\alpha\beta\gamma'} = 0$. Finally $C_{\alpha'\beta'\gamma'} = 0$ since there isn't a rank-3 symmetric singlet of $SU(5)/SM$. Therefore, a gauge variation of the downstairs action yields the *global* inflow contribution

$$\delta_\alpha S = \int_{y=0} C_{\alpha'\beta'\gamma}\Big(\mathcal{F}^{\alpha'} \wedge \mathcal{F}^{\beta'} \alpha^\gamma + 2\mathcal{F}^{\alpha'} \wedge \mathcal{F}^\beta \alpha^{\gamma'}\Big).$$

Again, these contributions do not have to be cancelled locally on the $y = 0$ boundary, but rather represent a breaking of rigid symmetries. The $A_\mu^{\alpha'}$ will be massive fields in the low energy effective theory of the $y = 0$ boundary.

Chapter 6

Phenomenology

6.1 The QCD Axion

In QCD, there is a chiral anomaly in the current $J_\mu = \sum_f \bar{q}_f \gamma_\mu \gamma_5 q_f$: $\partial_\mu J^\mu = \frac{g^2}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} \mathrm{Tr}[F_{\mu\nu} F_{\rho\sigma}]$, where Tr is the trace in the adjoint representation. Therefore, the amount of charge lost is

$$\Delta Q_A \equiv \int vol \ \partial_\mu J^\mu$$

$$= \frac{g^2}{32\pi^2} \int vol \ \epsilon^{\mu\nu\rho\sigma} \mathrm{Tr}[F_{\mu\nu} F_{\rho\sigma}] = \nu$$

where ν is the winding integer for the non-trivial field configuration. Therefore, there is a set of degenerate vacua corresponding to a set of states $\{|\nu>\}$, and the effective Lagrangian will contain a term

$$\Delta\mathcal{L} = \theta \frac{g^2}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} \mathrm{Tr}[F_{\mu\nu} F_{\rho\sigma}], \tag{6.1}$$

where θ parametrizes the vacuum state (which is a superposition of states with winding numbers in $\{\nu\}$).

The anomaly in J_μ is directly related to the $U(1)$ chiral anomaly of QCD: making the transformation $\delta q = e^{i\theta\gamma_5}$ on a quark field q, with θ parametrizing $U(1)$, the Lagrangian shifts by $\delta\mathcal{L} = i\theta\partial^\mu J_\mu$, where J_μ is the V-A current above. Since $\int vol \ \delta\mathcal{L} = i\theta\nu$, we see that the anomalous current J_μ is in direct correspondence with the breaking of chiral $U(1)$ symmetry, and that θ parametrizes the vaccum that couples to the current. In the path integral of the theory, we will have $e^{i\theta\nu} e^{iS_{QCD}}$, where S_{QCD} is the $SU(3)$ Yang-Mills action with minimal coupling to quarks.

When the minimally coupled quark are massless, it is clear that we can make a chiral transformation such that the term $\Delta\mathcal{L}$ in the effective Lagrangian vanishes, without any further consequences. Therefore, although the chiral transformations are not quantum symmetries, they interpolate between degenerate vacua labeled by θ that are physically in-

distinguishable. (So really, the chiral transformations are quantum symmetries in this sense). However, this symmetry is broken once we minimally couple *massive* fermions to the gauge fields. This is due to the fact that the mass of the fermions sitting in an instanton vacuum depends on the vacuum state labeled by θ; for example the up and down quarks u and d with masses m_u and m_d have the term

$$\mathcal{L}_{CP} = i\theta \frac{m_u m_d}{m_u + m_d}(\bar{u}\gamma_5 u + \bar{d}\gamma_5 d).$$

Thus, θ can be measured, since a quantum field calculation tells us that the dipole moments of composite particles will be changed by this term. The neutron dipole moment is the cleanest to measure, and it is found that $\theta \leq 10^{-10}$ [B79, CdVVW79]. Since (6.1) is odd under the action of charge and parity operations (CP), the question of why this dimensionless parameter is so small (or zero) for the strong interactions is called the strong-CP problem.

Peccei and Quinn [PQ77a, PQ77b] pointed out that one could construct a true chiral global symmetry $U(1)_{PQ}$ of the theory $\mathcal{L}_{QCD} + \mathcal{L}_{EW}$, so that the θ could be rotated to zero without affecting any observables, including the masses of the fermions after the spontaneous breakdown of the electroweak symmetry. However, Weinberg [SW77] and Wilczek [FW77] then pointed out that $U(1)_{PQ}$, while capable of being a classical symmetry, was only an approximate quantum symmetry due to the presence of instantons, which as we described above, are associated with the presence of chiral anomalies. Furthermore, they pointed out that this approximate symmetry will be spontaneously broken since the spontaneous electroweak breakdown gives masses to the fermions of the theory (which are manifestly not $U(1)_{PQ}$ invariant). As a result of a spontaneously broken *exact* symmetry, there is a massless Goldstone scalar field. But the spontaneous breakdown of an *approximate* global symmetry results in a vacuum with a (spin-0) pseudo-Goldstone particle, which has a small mass. This particle is the QCD axion.

Let us then summarize the axion resolution to the strong-CP problem: there is an approximate quantum chiral symmetry $U(1)_{PQ}$ that is ultimately spontaneously broken, if not above, then at the electroweak breaking scale. There is then a light scalar axion $\mathcal{A}$ appearing in the effective Lagrangian below this scale

$$\mathcal{L}_{\mathcal{A}} \propto \frac{\mathcal{A}}{M_{PQ}}\epsilon^{\mu\nu\rho\sigma}\text{Tr}[F_{\mu\nu}F_{\rho\sigma}], \tag{6.2}$$

which joins the effective QCD term (6.1). Here, $\mathcal{A}$ has mass dimension 1 and M_{PQ} is the energy scale at which $U(1)_{PQ}$ is broken. The vacuum minimum is given by the effective θ

parameter $\theta_{eff} \equiv \langle \mathcal{A} \rangle + \theta = 0$, so that at the end of the day, there aren't any CP-violating terms in QCD. The axion is the dynamical, quantum realization of Peccei and Quinn's classical proposal.

An immediate question that arises is: if the axion exists, why haven't we seen it? First of all, it is a spin-0 SM gauge singlet, and so can only charged under the gravitational interaction, which is quite weak at the scales we probe. However, (6.2) generally involves all gauge fields of the theory at hand, so that it allows for magnetic field-induced decays of axions to photons, which we can look for experimentally [S83]. Furthermore, the effective Lagrangian of QCD coupled to an axion will involve axions coupled to quark-gluon composites, like the pion. The decay rate of the axions into them depends on M_{PQ} in (6.2). All of these decays contribute to the energy content of the universe and therefore to its temperature. However, from cosmological observations and constraints on reheating arising from these decays, there are limits on the lifetime of a SM axion; the coupling strength can be $10^{10} GeV < f_{\mathcal{A}} < 10^{12} GeV$ [T90, R90, PWW83, AS83, DF83]. In the SM scenario, this scale f_A would be the $U(1)_{PQ}$ breaking scale M_{PQ}, and therefore, the natural scale of axion vevs $\langle \mathcal{A} \rangle$. This clearly rules out an electroweak breaking of the PQ symmetry, since that is on the order of $10^2 GeV$. So a SM axion solution to the strong-CP problem requires the introduction of a new, intermediate, scale.[1] This isn't so bad, since there are other intermediate scales in "beyond the Standard Model" scenarios. The problem comes when there are a number of distinct intermediate scales for various new physics, such as axions and strong-electroweak unification.

We should point out that, in supersymmetric axion scenarios, the constraints on $f_{\mathcal{A}}$ are model-dependent, since one must examine the cosmological implications of axions with their coupling to supersymmetric particles [BDG03, BDFG03].

Fields denoted as "axions" appear generically in compactifications of supergravity and string/M-theory. These axions are not necessarily QCD-type axions capable of solving the strong-CP problem; rather they are generally just scalars that parametrize a $U(1)$. (They do appear in P/CP violating terms like (6.2), so they are sometimes called pseudoscalars.) The axions capable of resolving the strong-CP problem in these scenarios generically have a large coupling strength $f_A \sim 10^{16} - 10^{18} GeV$ [BD97]. This is why arguments for larger values of f_A in superymmetric theories, such as in [BDG03, BDFG03], are important. Since supergravity is a low energy approximation to string/M-theory, we should find axions there as well. The graviphoton in a five dimensional MESGT is $h_I F^I_{\mu\nu}$, a combination that depends

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[1]The presence of this intermediate scale wouldn't spoil the usual predictions of gauge coupling unification since the axion is a gauge-singlet.

on the background $\langle \phi^{\tilde{x}} \rangle$. Upon dimensional reduction, the "axion/dilaton" form a complex scalar field $h_I z^I$ sitting in a $4D$ $\mathcal{N} = 2$ vector multiplet, and parametrize a scalar manifold $SU(1,1)_G/U(1)$ [MO98, GMZ05a]. The "axion" is the scalar arising from the $5D$ graviphoton of the form $h_I A^I \equiv \Re(h_I z^I)$. However, in $5D$ YMESGTs on an orbifold spacetime, a QCD-type axion appears generically, which is not the above scalar.

There has been previous work in field-theoretic models in which a QCD-type axion arises from a $5D$ field. In [ADD98, AADD98, CTY00a, CTY00b], models were constructed in which the strong CP axion comes from a higher dimensional *scalar field*. In [DDG00] it was shown that doing so decouples the PQ breaking scale f_{PQ} from the mass of the axion, and that $m_{\mathcal{A}} \leq \mathcal{O}(R^{-1})$. On the other hand, the authors of [FHM05] (which followed [ACCR03a, ACCR03b]) consider a $U(1)$ rigidly supersymmetric orbifold gauge theory in which the gauge field $A_{\hat{\mu}}$ is given odd parity. Then, according to [ACCR03a, ACCR03b, FHM05] $\langle A_{\hat{\mu}} \rangle (x)$ can be treated as a field in the effective $4D$ theory, assuming that $A_5(x)$ is slowly varying on the fixed planes. The situation described in this thesis are distinct from these scenarios.

Looking at the FF term (5.5), we immediately see that, while the singlet from the $5D$ graviphoton is $h_I A^I$, the candidate axion is $C_{\tilde{I}} A^{\tilde{I}}$; in the case where $n_V = 0$ (simple supergravity), these two coincide. However, the scalars $A^{\tilde{a}}$ appear in the potential (5.8) (while A^0 and any scalar arising from K-singlet vectors in $A_{\hat{\mu}}^I$ don't). Usually, for $\mathcal{A}$ to be a QCD-type axion we would have to require the ground state to be such that terms in the potential containing $A^{\tilde{a}}$ vanish. This can be satisfied, for example, when $\langle h^{\tilde{a}} \rangle = 0$ or $\langle A^{\tilde{a}} \rangle = 0$. However, we will see that the mass of the axion (including any contributions from the scalar potential) is decoupled from the strength of coupling to matter fermions.

We can re-express everything in terms of the scalar $\mathcal{A}$ using the relations (see (2.2) and [GST84a])

$$h_I = C_{IJK} h^J h^K \tag{6.3}$$

$$\overset{\circ}{a}_{IJ} = \{3C_{IKL} C_{JMN} h^K h^L h^M h^N - 2C_{IJK} h^K\} \tag{6.4}$$

$$T_{\tilde{p}\tilde{q}\tilde{r}} h_I^{\tilde{r}} = C_{IJK} \{\frac{1}{2} \eta_{\tilde{p}\tilde{q}} h^J h^K + h_{\tilde{p}}^J h_{\tilde{q}}^K\} \tag{6.5}$$

$$\Phi_{I\tilde{p}\tilde{q}} = \sqrt{\frac{2}{3}} C_{IJK} \{\frac{3}{4} \eta_{\tilde{p}\tilde{q}} h^J h^K + h_{\tilde{p}}^J h_{\tilde{q}}^K\}. \tag{6.6}$$

In the canonical basis,

$$C_{000} = 1, \quad C_{00i} = 0, \quad C_{0ij} = -\frac{1}{2} \delta_{ij}, \quad C_{ijk} = \text{rank-3 symmetric invariant.}$$

However, this is not the natural basis in which to write the scalar $C_{\tilde{I}}A^{\tilde{I}}$. With index splitting $I = (0, \tilde{a}, \hat{a}, \alpha)$, we can perform a linear transformation

$$h^0 \to h^0 + h^{\tilde{a}}C_{\tilde{a}}.$$

In the new basis, the first three components above become

$$C_{000} = (C_0)^3 = 1, \quad C_{0\tilde{a}\tilde{b}} = C_0 C_{\tilde{a}}C_{\tilde{b}}, \quad C_{00\tilde{a}} = (C_0)^2 C_{\tilde{a}},$$

with $C_0 = 1$. Therefore, the only non-zero components of C_{IJK} in the YMESGT are

$$\begin{aligned}
C_{\tilde{I}\alpha\beta} &= -\frac{1}{2}C_{\tilde{I}}\delta_{\alpha\beta} \\
C_{\tilde{I}\hat{a}\hat{b}} &= -\frac{1}{2}C_{\tilde{I}}\delta_{\hat{a}\hat{b}} \\
C_{\tilde{I}\tilde{J}\tilde{K}} &= C_{\tilde{I}}C_{\tilde{J}}C_{\tilde{K}} \\
C_{\alpha\beta\gamma} &= D d_{\alpha\beta\gamma},
\end{aligned} \tag{6.7}$$

where $\tilde{I} = (0, \tilde{a})$; $C_0 = 1$; $d_{\alpha\beta\gamma}$ are symmetric invariants of $K_{(\alpha)}$ (d-symbols if the gauge group is $K_{(\alpha)} = SU(N)$); and D is an arbitrary parameter.

Let $\mathcal{A} \equiv C_{\tilde{I}}A^{\tilde{I}}$, $\mathfrak{h} \equiv C_{\tilde{I}}h^{\tilde{I}}$ and $\hat{h}^2 \equiv \delta_{\hat{a}\hat{b}}h^{\hat{a}}h^{\hat{b}}$. Then the Lagrangian at the orbifold fixed points for $\mathcal{A}$ becomes

$$\begin{aligned}
e^{-1}\mathcal{L}_{\mathcal{A}}|_{fp} = &-\frac{e^{-2\sigma}}{2}[\frac{3}{4}(\mathfrak{h}^2 + \hat{h}^2)^2 + \mathfrak{h}]\,\partial_\mu\mathcal{A}\partial^\mu\mathcal{A} \\
&-\frac{e^{-1}\,\hat{\kappa}}{4\sqrt{6}\,\hat{g}^2}\,\mathcal{A}\,\epsilon^{\mu\nu\rho\sigma}\,\mathrm{Tr}[\mathcal{F}_{\mu\nu}\mathcal{F}_{\rho\sigma}] \\
&-e^{-2\sigma}[\frac{3}{2}(\mathfrak{h}^2 + \hat{h}^2)\mathfrak{h} + 1]\,\delta_{\hat{a}\hat{b}}h^{\hat{a}}\partial_\mu A^{\hat{b}}\partial^\mu\mathcal{A} + pot + \cdots,
\end{aligned}$$

where $\mathcal{F}_{\mu\nu} \equiv g\,t_\alpha\mathcal{F}^\alpha_{\mu\nu}$ with $t_\alpha \in k_{(\alpha)}$ (i.e., elements of the gauge algebra on the boundary); "pot" denotes the potential contribution from (5.8); dots indicate *derivative* couplings to YMESGT fermions. Due to the parity assignments of section (4.5), the scalars $A^{\tilde{I}}$ (and

therefore $\mathcal{A}$) do not appear in the scalar potential. The fermionic coupling terms are

$$e^{-1}\mathcal{L}_{f\mathcal{A}}|_{fp} = \frac{\sqrt{2}}{2\sqrt{3}}e^{-\sigma}\partial_\mu\mathcal{A}\left(\frac{3}{4}\eta_{\tilde{p}\tilde{q}}\{\mathfrak{h}^2 - \frac{1}{2}\hat{h}^2\}\bar{\lambda}^{\tilde{p}i}\Gamma^\mu\lambda_i^{\tilde{q}}\right.$$
$$+\{\mathfrak{h}_p\mathfrak{h}_q - \frac{1}{2}\delta_{\hat{a}\hat{b}}h_p^{\hat{a}}h_q^{\hat{b}}\}\bar{\lambda}^{pi}\Gamma^\mu\lambda_i^q \tag{6.8}$$
$$\left.-\{\frac{1}{2}\delta_{\alpha\beta}h_\rho^\alpha h_\sigma^\beta\}\bar{\lambda}^{\rho i}\Gamma^\mu\lambda_i^\sigma\right) + \cdots ,$$

where dots indicate 4-fermion couplings.

After integrating the zero mode Lagrangian over x^5, we pick up a factor of R, the size of the fifth dimension. Furthermore, $\hat{g} = g\sqrt{R}$ and $\hat{\kappa} = \kappa\sqrt{R}$. Therefore,

$$1/M_{PQ} = \frac{\kappa}{g^3}, \tag{6.9}$$

which is clearly independent of the compactification energy scale. At first site, this reproduces the usual difficulty with axions in string/M-theory: the coupling is far too small for the Sandard Model axion (though for supersymmetric axions, one may have larger M_{PQ}).

However, M_{PQ} above is not the scale at which a global $U(1)_{PQ}$ symmetry is broken. The global symmetry, parametrized by $C_{\tilde{I}}\alpha^{\tilde{I}}$, is defined on the boundaries of the spacetime so that the natural energy scale is $M_c \sim 1/R$. It arises from local symmetries of the bulk theory. As discussed in the last section, Wilson lines $\langle A_{\hat{\mu}}^a \rangle_P$ stretching between boundaries of the spacetime give rise to boundary vevs for scalar fields A^a. But $\langle A_{\hat{\mu}}^a \rangle_P$ is a dimensionless object that equals $CA^a(x)/M_c$, where C is a constant, and A^a is the dimension 1 scalar field that appears in the $4D$ effective Lagrangian. Therefore, $\langle A^a \rangle = M_c \langle A^{(5)} \rangle /C$, where M_c is the compactification energy scale ($M_c \sim 1/R$), and $\langle A^{(5)} \rangle \sim \mathcal{O}(1)$ is expected generically. The dimensionless coefficient in the axion term is then

$$1/F_{PQ} = \frac{\kappa M_c}{g^3},$$

which for $M_c << 10^{18}GeV$ is a small number. This reflects the fact that, although $M_{PQ} \sim 10^{16}GeV$ is the dimensionful coefficient in (6.2), the characteristic axion scale can be much lower. That is, under the global $U(1)$ shift parametrized by $\tilde{\alpha} \equiv C_{\tilde{I}}\alpha^{\tilde{I}}$ the axion shifts as $\mathcal{A} \rightarrow \mathcal{A} + M_c\tilde{\alpha}$.

More precisely, the first thing to notice is that the dimensionless parameter $\theta + \frac{1}{F_{PQ}}$ is the physically relevant one, as it is this parameter which must be $<< 10^{-10}$. As for the Standard Model case, the QCD instanton potential will be minimized for $\theta + 1/F_{PQ} = 0$.

The difference in the orbifold scenario is that this generally no longer has $\mathcal{O}(1)$ solutions for θ; it can have very small θ solutions, depending on the size of M_c relative to $10^{18} GeV$ (i.e., κ^{-1}).

Second, to find the decay rate of axions into physically observable composite particles like pions, one must find the relevant terms in the effective Lagrangian. To do this, we should be coupling to SM matter, which can sit in $5D$ hypermultiplets. (The fermion couplings in (6.8) are those of a pure YMESGT.) At first site, the $A^{\tilde{a}}$ may appear in the minimal coupling terms involving the ζ^A and $q^{\tilde{X}}$ (see (2.11) and (2.13)). Recall that the surviving hyper-scalars are the q^X so that the 5th component of the covariant derivative for the scalars becomes

$$D_5 q^X |_{fp} = \left(\mathcal{D}_5 q^X + g A^a K_a^X \right) |_{fp}.$$

But $K_a^X(q)$ has odd parity, and vanishes at the fixed points. So there won't be any A^a (and therefore $\mathcal{A}$) factors in terms coming from $5D$ $D_{\hat{\mu}} q^{\tilde{X}}$ terms. As for terms with $D_{\hat{\mu}} \zeta^A$, we must check whether there are bare $A^{\tilde{I}}$ factors arising from the ω_{IB}^A couplings. In general, the K_a^Ω have even orbifold parity (section (4.5)), which means the components ω_{aB}^A determined by them will not generally vanish at the orbifold fixed points. However, we can show that the $K_{\tilde{I}}^\Omega(q)$ Killing vectors of this set do vanish, just as in the analogous case of the $K_{\tilde{I}}^X(\phi)$ in the pure YMESGT sector. We will argue this in a different way, though. From [BCdWGVvP04], the Killing vectors satisfy

$$R_{\tilde{X}\tilde{Y}\ j}^i K_I^{\tilde{X}} K_J^{\tilde{Y}} + \frac{1}{2} f_{IJ}^K P_{Kj}^i = 0, \tag{6.10}$$

where $R_{\tilde{X}\tilde{Y}\ j}^i$ is the $SU(2)_R$ curvature of the quaternionic manifold (i, j are $SU(2)_R$ indices):

$$R_{\tilde{X}\tilde{Y}\ ij} = \hat{\kappa} \left(f_{\tilde{X}iA} f_{\tilde{Y}j}^A - f_{\tilde{Y}iA} f_{\tilde{X}j}^A \right), \tag{6.11}$$

and P_{Kj}^i is the Killing prepotential of the scalar manifold, whose explicit form is irrelevant here. From section (5.1), we know that the structure constants

$$f_{\tilde{a}\tilde{b}}^K \rightarrow \epsilon(x^5) f_{\tilde{a}\tilde{b}}^K.$$

Then (6.10) makes sense as an integral over the fifth dimension, so that it becomes

$$R_{\tilde{X}\tilde{Y}\ j}^i K_{\tilde{I}}^{\tilde{X}} K_{\tilde{J}}^{\tilde{Y}} = 0. \tag{6.12}$$

But this cannot have a non-trivial solution for the Killing fields since this would impose a non-trivial constraint on the geometry encoded in the curvature tensor, which is determined

when one chooses the $5D$ hypermultiplet couplings (and the $SU(2)_R$ curvature does not vanish, which is apparent by looking at the fixed-point form of (6.11)). Therefore, $K_{\tilde{I}}^{\tilde{X}}$, where $\tilde{I}$ are $K_{(\alpha)}$-singlet indices, must vanish. As a result, scalars $A^{\tilde{I}}$ do not appear in the scalar potential of (2.11).

Finally, the K-representation matrix is

$$\omega_{IB}^A = K_{I\tilde{X};\tilde{Y}} f_i^{\tilde{X}A} f_B^{\tilde{Y}i},$$

where the semi-colon denotes the covariant derivative on $\mathcal{M}_Q$. Since the $K_{\tilde{I}}^{\tilde{X}}$ vanish at the orbifold fixed points, so must their covariant derivatives, and therefore, the components $\omega_{\tilde{I}B}^A$ vanish there as well. Thus, the scalars $A^{\tilde{I}}$ also do not appear in the minimal coupling terms in (2.11) of the form $\bar{\zeta}^A \Gamma^5 D_5 \zeta_A$.

We have shown that the candidate axion $\mathcal{A} = C_{\tilde{I}} A^{\tilde{I}}$ is not directly coupled to hypermultiplet fields. On the other hand, there is a derivative coupling of the $A^{\tilde{I}}$ to the hyper-fermions, just as in the YMESGT sector. From

$$\hat{e}^{-1} \mathcal{L}_{\zeta\zeta F} = \frac{\sqrt{6}i}{8} \hat{\kappa} h_I \bar{\zeta}_A \Gamma^{\hat{\mu}\hat{\nu}} \mathcal{F}_{\hat{\mu}\hat{\nu}}^I \zeta^A,$$

we find the fixed point contribution

$$e^{-1} \mathcal{L}_{\zeta\zeta F} = \frac{\sqrt{6}i}{4} \frac{\hat{\kappa}}{\hat{g}^3} e^{-2\sigma} \partial_\mu \mathcal{A} g^{\mu\nu} \{ -\frac{1}{2}\hat{h}^2 + \mathfrak{h}^2 \} \text{tr} \left(\bar{\zeta} \Gamma_{\nu\dot{5}} \zeta \right),$$

where $\mathcal{A} \equiv gC t_{\tilde{I}} A^{\tilde{I}}$ and $\text{tr}(\bar{\zeta}\Gamma_{\mu 5}\zeta) \equiv \bar{\zeta}^A C_{AB} \Gamma_{\mu 5} \zeta^B$ (tr is the trace in the representation of the fermions). After integrating over x^5, the coefficient is

$$\frac{e^{-2\sigma}}{M_{PQ}} \{ -\frac{1}{2}\hat{h}^2 + \mathfrak{h}^2 \},$$

where M_{PQ} is defined in (6.9).

We can compare this with the SM analysis in which the coupling coefficient of a quark and axion is f_q/M_{PQ}, where f_q is a dimensionless coupling expected to be order unity, depending on the quark species. In our case, $f_q = e^{-2\sigma}\{-\frac{1}{2}\hat{h}^2 + \mathfrak{h}^2\}$.

As far as this thesis is concerned, we end the analysis of axions here, as the situation becomes a bit complicated. While the decay rate of the axion may be calculated, there are couplings to supersymmetric particles, like the axion superpartner, the saxion, which may itself be the more demanding particle in terms of cosmological constraints [BD97, BDG03,

BDFG03]. The axion and saxion have general couplings in the Lagrangian so that the analysis depends on which $5D$ YMESGT one chooses (that is, the choice of scalar manifold $\mathcal{M}_\mathcal{R} \times \mathcal{M}_Q$ of the vector and hypermultiplet sector). However, the analysis here has illustrated the generic features of the axion in orbifold YMESGTs, with the most important result being that the model is quite flexible in allowing a range of axion decay constants, depending on one's taste for cosmological constraints.

As a final note, there is a concern that the axion coupling strength is adversely affected by quantum gravitational interactions. However, in the scenario described here: (1) the vev of the axion comes from a Wilson line in the bulk, and (2) the global shift symmetry that is broken by the $\mathcal{F} \wedge \mathcal{F}$ coupling comes from a gauge symmetry in five dimensions. Therefore, we expect that the strength of the axion couplings to hadrons and the axion mass will be protected from large corrections due to quantum gravitational interactions.

6.2   Yukawa Couplings

Yukawa couplings are, in general situations, terms in a Lagrangian involving scalar and fermionic fields. In the Standard Model, the only (conjectured) scalars are the Higgs fields, so the Yukawa couplings in the SM are the Higgs couplings to the matter fields. These terms are important in model building for several reasons. The first and foremost is that these become mass terms for the fermionic fields after electroweak symmetry breaking. Therefore, the form of the coupling matrix and the magnitudes of the entries in this matrix are fixed by experiment. Second, in GUTs, the couplings for the leptons seem to unify upon renormalization group flow to higher energies (just as the gauge coupling seem to approach each other). It is an open problem in GUT model building as to why this is so, and in addition, why not for the quarks?

Higgs from $5D$ gauge multiplets

First, consider the possibility of putting the MSSM Higgs fields in $5D$ non-compact gauge multiplets. The exmple we have focussed on in this thesis from time to time is the $SU(5,1)$ YMESGT associated with the Lorentzian Jordan algebra $J^{\mathbb{C}}_{(1,5)}$. This theory is a **unified** YMESGT in the sense that all of the $5D$ vector fields of the theory belong to the adjoint representation [GST85a, GST85b]. The non-compact gauge symmetries as well as the $U(1)$ gauged by the graviphoton must be broken at the orbifold fixed points, leaving at most an $SU(5)$ gauge theory there; the non-compact gauge multiplets are projected to left chiral multiplets in the $\mathbf{5} \oplus \bar{\mathbf{5}}$, along with their CPT conjugates. There is a scalar potential at the

fixed points of the form

$$g_{\chi\psi}(\phi)K_a^\chi K_b^\psi A^a A^b,$$

(or, see (5.7)) which is non-negative.

Once we couple charged hypermultiplets to the theory, the Yukawa couplings between the non-singlet fields $A^{\hat{a}}$ and the hypermultiplet fermions are of the form

$$\hat{g}A^{\hat{a}}\zeta^A \omega_{\hat{a}\,AB}(q)\zeta^B,$$

where $\omega_{\hat{a}\,AB}$ are components of the spin connection on the quaternionic scalar manifold parametrized by the $q^{\tilde{X}}$. Since the vev of the scalars $A^{\hat{a}}$ on the boundaries should arise due to the fact that the $5D$ vector A_μ^a has a Wilson line stretching between boundaries, then we should be concerned about the energy scale involved in the Yukawa couplings, which serve as mass terms for matter fermions after electroweak symmetry breaking. Although the Wilson line has associated energy scale M_c, the Yukawa couplings are given by $\lambda_{AB}(q) = gA^{\hat{a}}\omega_{a\,AB}$; therefore, the couplings depend on the geometry of the quaternionic scalar manifold (and therefore, on the particular background of the theory). After integration over x^5, we find the coefficient with mass dimension 1 is

$$\frac{\left\langle A^{\hat{a}} \right\rangle \omega(\langle q \rangle)}{g^2}.$$

Since the vev for $A^{\hat{a}}$ is of the order of M_c, we generally require either $M_c \sim 10^2 GeV$ or $\omega(\langle q \rangle)$ to be quite small.

Higgs from $5D$ hypermultiplets

Next, let's consider the most popular scenario in orbifold-GUTs: placing Higgs fields in hypermultiplets. In chapter 3, we discussed the possibilities for the hypermultiplet content in YMESGTs coupled to homogeneous quaternionic manifolds. For concreteness, let's consider an $SU(5)$ $S^1/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orbifold YMESGT coupled to hypermultiplets in the $\mathbf{5} \oplus \mathbf{5}$.[2] We may assign parities as in section (4.6.2), where we can get the MSSM Higgs content and massive color triplet chiral multiplets. The scalars q^Ω in the $4D$ Higgs multiplets have a potential inherited from the potential in five dimensions (see (2.11) and (2.12)) as well as from the pure $5D$ YMESGT sector via minimal couplings to $5D$ vectors A_μ^a. The Yukawa terms now come from the scalar-fermion couplings of the $5D$ hypermultiplet Lagrangian (2.11).

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[2]We need two copies since the MSSM has a minimum of two left-chiral multiplets (and their CPT conjugates), while orbifold parity assignments project out half of a hypermultiplet's degrees of freedom.

The scalar potential for the q^Ω at the fixed points is

$$e^{-1}\mathcal{L}_{q-pot}|_{fp} = -\frac{\hat{g}^2}{\hat{\kappa}^2}\{V^{1\,\tilde{n}}V_{1\,\tilde{n}} + V^{2\,n}V_{2\,n}\}|_{fp},$$

where

$$V^{iA}|_{fp} = \frac{\sqrt{6}}{4}e^{-\sigma}h^{\hat{a}}K^{\Omega}_{\hat{a}}f^{iA}_{\Omega}.$$

We might have expected a scalar potential arising from the minimal couplings between the $A^a_{\hat{\mu}}$ and q^X in five dimensions. However, in the previous section, it was pointed out that D_5q^X vanishes at the fixed points, so that such a potential is not present. The above is the only contribution to the potential for the q^X. It is well-known that the scalar potential in $5D$ hypermultiplet-coupled YMESGTs admits supersymmetric AdS vacua so that a tree-level negative potential for the candidate Higgs scalars in the set q^X is possible. Of course, supersymmetry should be spontaneously broken at least the same scale as the electroweak breaking performed by the Higgs. It is possible that (i) there is a non-supersymmetric, stable ground state with $SU(3)_c \times U(1)_{em}$ local symmetry, (ii) corrections to the classical, supergravity approximation introduce new contributions to the scalar potential leading to such a stable ground state, or (iii) supersymmetry is dynamically broken at some larger scale so that the candidate electroweak breaking scalar potential need only admit stable ground states with $SU(3)_c \times U(1)_{em}$ local symmetry.

The Yukawa terms now involve the ϕ- and q-dependent matrix $\hat{g}h^I(\phi)t^A_{IB}(q)$, where the ϕ-dependence is clearly universal for all fermion species. Once again, Yukawa unification in a GUT scenario can be acheived in a similar fashion as in $4D$ models [FNS79, GJ79]. For example, in an $SU(5)$ GUT, the matrix coupling to quarks and leptons contained in $5D$ hypermultiplets is in the reducible $\mathbf{5} \oplus \mathbf{10}$. We can look for a theory such that, in a particular ground state of the q^X, $h^It^5_{I\,5}$ is proportional to the identity (unification of Yukawa couplings for the leptons), while $h^It^{10}_{I\,10}$ isn't (no such unification for the quarks). Once again, the t-matrices depend on (a) the form of the hypermultiplet couplings (i.e., the quaternionic scalar manifold that is coupled to the YMESGT in five dimensions) and (b) the ground state of the q^X scalars. Furthermore, to check whether the couplings become the phenomenologically correct values, one must perform a renormalization group flow down to lower scales. In contrast to standard supersymmetric GUTs, the scalars determining the couplings are contained in the matter hypermultiplets so that a large Higgs sector is not needed for phenomenological mass relations as in [FNS79, GJ79].

We leave these issues to future studies.

Chapter 7

Some thoughts on M-theoretic origins

In the 1980s and early 1990s, the (weakly coupled) $E_8 \times E_8$ heterotic string theory was the most phenomenologically interesting of the string theories. The six extra dimensions were presumed to be extremely small, around the Planck scale $\mathcal{O}(10^{-18})GeV^{-1}$, which was the scale at which stringy effects would be important. The 6D internal manifold was taken to be a smooth Calabi-Yau space, which preserves $\mathcal{N} = 1$ supersymmetry. String phenomenology sought to find gauge coupling unification (GUTs) as well as unification of gauge and gravitational couplings within this framework.

However, M-theory entered the scene as the 11D theory appearing in the strong coupling regime of the (ten-dimensional) heterotic $E_8 \times E_8$ [HW96a, HW96b, W96a] and type IIA string theories [W95]. This gave new meaning for the 11D supergravity theory that had been the great interest of the late 1970s and much of the 1980s: as the tree level, massless approximation of M-theory. Some immediately interesting consequences of strongly coupled heterotic string, which can be described as M-theory (or 11D sugra) on $M_{10} \times S^1/\mathbb{Z}_2$, are (i) Newton's constant is allowed to have a physically correct order of magnitude, and (ii) the string (energy) scale can be orders of magnitude less than in the 10D string theories. Thus, while the string models had urged a merger of gravitational and gauge couplings by pushing the usual GUT scale $10^{16}GeV$ to the usual $10^{18}GeV$ string scale, the Horava-Witten theory reversed this.

Consider a calculation of some observable in perturbative string theory. A result that arises beyond string tree level is a "stringy effect" in that it does not appear in the supergravity approximation. The strongly coupled theory should capture the perturbative effects (as well as non-perturbative effects) without doing "loop calculations", which arise in perturbative descriptions. But in string theory, the strong coupling limit is equivalent to either a different, weakly coupled, string theory; or equivalent to weakly coupled M-theory. In the latter case, the strongly coupled physics of the original string theory, including the 1-loop calculation, should therefore be captured by the $11D$ supergravity theory.

As an example, consider the compactification of heterotic string theory on a smooth Calabi-Yau 3-fold Y. The result of the low energy approximation is a $4D$ supergravity theory based on a cubic polynomial that's reducible $\mathcal{V} = C_{IJK}h^I h^J h^K \equiv sQ$, where Q is a quadratic polynomial and $s \in \mathbb{R}$. The supergravity approxiation does not carry the $E_8 \times E_8$ gauge group of the string theory. A 1-loop string calculation, however, modifies the cubic polynomial so that it's irreducible $\mathcal{V} = C_{IJK}h^I h^J h^K$. Now, consider the M-theoretic limit of the theory. The low energy theory should still be four dimensional with the compactifying space having G_2 structure and an unbroken E_8 gauge phase. In the example at hand, the internal space is $Y \times S^1/\mathbb{Z}_2$ so that the E_8 gauge theory lying at each boundary of the space arises from the $E_8 \times E_8$ gauge group of the weakly coupled theory. One can first compactify M-theory on a Calabi-Yau 3-fold Y, resulting in a theory with a generically irreducible cubic polynomial $\mathcal{V}$, but in five dimensions. One can then compactify on $S^1/\mathbb{Z}_2$ to get a $4D$ theory that can be compared to the pure string compactification. The sugra approximation of the M-theory compactification will now carry information about the perturbative stringy corrections to the cubic polynomial.

As another example of recent interest, consider flux compactifications. For example, the compactification of type II string theory on $AdS_5 \times S^5$, which involves infinite towers of Kaluza-Klein excitations. The non-trivial Kaluza-Klein modes do not lie in the supergravity truncation, so that this is the domain of stringy effects. In attempting to obtain super-symmetric Randall-Sundrum scenarios, it has been shown that a so-called "breathing mode" coresponding to a non-trivial KK-excitation must be allowed to have a non-zero fieldstrength. Therefore the effect is not captured in the sugra truncation (and thus the same is true for the minimal, $\mathcal{N} = 2$ truncation). The role of this flux is to generate a scalar potential in the low energy theory, which can lift the degeneracy in the vacua. In light of the previous discussion, we might expect that an M-compactification can include this information in the supergravity approximation. Indeed this is true. However, we cannot directly compactify M-theory to type IIB string theory (type IIB is dual to type IIA, and the strong coupling limit of type IIA is equivalent to M-theory on S^1). As a first step, suppose that, instead of compactifying type IIB string theory (and its sugra approximation) on $AdS_5 \times S^5$, we compactify M-theory *a lá* [BG00], obtaining a theory with AdS_5 ground states and $\mathcal{N} = 2$ supersymmetry. The non-zero flux is now a supergravity-level field. From the point of view of the $5D$ supergravity approximation, the gauge theory is precisely that of a YMESGT coupled to hypermultiplets, yielding a particular scalar manifold. Therefore, supergravity is capable of dealing with this situation, and there is no "stringy" physics needed.

There are phases of M-theory with large unbroken gauge groups other than those men-

tioned so far, obtained by compactification on more general singular spaces, both of orbifold type as well as those with worse-than-orbifold singularities [ES98, AW01, AW03]. It is a known feature, for example, that compactification of string theories with gauge symmetry G on smooth d-manifolds can be dual to M-theory (which has no vector gauge symmetry *a priori*) on a singular $d+1$-manifold. The singularities in these cases must be worse than orbifold singularities; that is, they aren't just locally of the form $\mathcal{M}/G$, where G is a discrete group that acts non-freely on a smooth manifold $\mathcal{M}$. The sugra approximation of string theories doesn't necessarily carry the gauge symmetry, while the sugra theory that is the low energy approximation of M-theories (including the known limits of the string theories) can carry the gauge symmetry, though not in the case where Y is a smooth CY manifold. While supergravity is capable of doing what string theories can in many cases, it is, of course, not all of M-theory. Ultimately the corrections to the supergravity approximation must be considered. Unfortunately, M-theory is not fully known, so only higher order corrections such as R^4 spacetime curvature terms can be considered (in the framework of weakly coupled field theory), along with the known features of 2- and 5-branes.

We wish to describe compactifications of M-theory leading to an effective description of supergravity on $\mathcal{M}_4 \times S^1/\Gamma$. While the action of Γ on the fields of the effective theory involves a choice of lifting from the base space to total space of the gauge bundle, the parity assignments will follow from a choice of internal manifold and discrete group action. The $11D$ supergravity theory that serves as the low energy approximation of M-theory is a relatively simple framework for assigning consistent parities. An example of such singular spaces is the compactification of M-theory on the five-dimensional space $(K3 \times S^1)/\mathbb{Z}_2$, where the $\mathbb{Z}_2$ acts non-trivially on the entire $K3 \times S^1$ space, and is discussed in [S96a].

Let us be more precise with the conditions of the compactification space we are looking for. First of all, the 7-manifold should have G_2 holonomy so that we end up with a $4D$ $\mathcal{N}=1$ theory. Second, the space should admit a limit in which it is isomorphic to $\mathcal{M}_4 \times S^1/\Gamma$. The only (connected) Lie subgroups of G_2 that can be the holonomy group of a Riemannian metric on a 7-manifold are $\{1\}$, $SU(2)$, $SU(3)$ and G_2. If M is a compact manifold with G_2 structure, then the holonomy group is G_2 iff $\pi_1(M)$ is a finite group. But when M is compact and the metric is Ricci flat, then $\pi_1(M)$ is finite iff M is simply connected. If, however, M is isomorphic to a product of a simply connected space and T^n, then the holonomy group is $SU(2)$ or $SU(3)$ when $n=3$ or $n=1$, respectively. For example, $Y \times S^1$, where Y is a Calabi-Yau manifold, has $SU(3)$ holonomy, while $Y \times S^1/\mathbb{Z}_2$ has G_2 holonomy. Since a product metric $g_1 \times g_2$ has a product holonomy group $Hol(g_1) \times Hol(g_2)$, and since M should have G_2 holonomy, the internal compactification space can't be geometrically a product space. In

the case of a space with boundaries, the boundary metric can be reduced to a product and the holonomy group broken. In summary, the compactifications we are interested in involve irreducible spaces are naively of the form $(Y \times S^1)/\Gamma$, where Γ is some discrete group and Y is a Calabi-Yau manifold. However, since the internal space must be simply connected to have G_2 holonomy, the singularities must admit resolutions to such a space.

Depending on the choice of compactification space, there may or may not be magnetic sources in the boundary theories, as there is in the Horava-Witten construction (in the form of magnetic 5-branes). There are two conditions for magnetic sources to be required: (i) the action of the discrete group Γ must be such that some components of the fieldstrength G_{ABCD} of the $11D$ supergravity 3-form are Γ-odd, and (ii) G_{ABCD} must be in a non-trivial cohomology class of the space. These are necessarily true in the HW construction. There, condition (i) follows immediately from the compactification space $Y \times S^1/\mathbb{Z}_2$, while condition (ii) follows from anomaly cancellation. But we are interested in more general compactification spaces, and a new anomaly analysis must be performed [W01]. This in turn depends on the generalized twisted sector, which must be understood to obtain the full boundary-localized spectrum of the theory. In the $5D$ supergravity approximation, we have already seen that there can be some requirements on boundary-localized fields due to anomaly inflow via a $5D$ Chern-Simons term, but there isn't much help beyond this. For anomaly analysis in the presence of p-branes and string/M-theory, see [FLO99, SS99a, SS99b, W01]

In summary, the motivations for looking for the orbifold theories of this thesis in M-theory rather than string theories are:

(i) The cubic polynomial defining the vector/tensor-part of the supergravity approximation can be irreducible

(ii) The $4D$ Newtonian constant G_N can have the correct order of magnitude

(iii) The appearance of one large extra dimension (relative to the other dimensions) is a dynamical feature of the theory

Appendix A

Notational Conventions

We use the mostly plus spacetime metric $\eta_{\mu\nu} = \text{diag}(-+\cdots+)$. Four-dimensional curved spacetime indices are μ, ν; flat spacetime indices are m, n. The index range is taken as $0, 1, 2, 3$ for these. Hats often denote five-dimensional quantities, such as $\hat{\mu}$, $\hat{\nu}$ and $\hat{m}$, $\hat{n}$. The index range is now $0, 1, 2, 3, 5$. Where it is important to distinguish, we will sometimes use $\dot{5}$ to denote the fifth dimension curved index, and 5 to denote the flat index.

We will use the representation notation where $\mathbf{m}_\mathbb{C} \equiv \mathbf{m} \oplus \bar{\mathbf{m}}$ is an element of a 2m-dimensional real representation space. For example, the "Higgs doublet" consists of $\mathbf{2}_\mathbb{C} \equiv \mathbf{2} \oplus \bar{\mathbf{2}}$. The MSSM minimal Higgs content is two left-chiral supermultiplets forming the $\mathbf{2} \oplus \mathbf{2}$ and two right-chiral supermultiplets forming the $\bar{\mathbf{2}} \oplus \bar{\mathbf{2}}$; each left-right CPT pair of chiral multiplets is then in the $\mathbf{2}_\mathbb{C} \equiv \mathbf{2} \oplus \bar{\mathbf{2}}$. These four chiral multiplets can form an $\mathcal{N} = 2$ hypermultiplet. We use the notation where $\mathbf{m}_\mathbb{H} \equiv \mathbf{m}_\mathbb{C} \oplus \mathbf{m}_\mathbb{C}$. Embedding the MSSM Higgs content into an $\mathcal{N} = 2$ theory, the hypermultiplet would then be in the $\mathbf{2}_\mathbb{H}$; we will assume this is understood when we simply say "the hypermultiplet is in the $\mathbf{2}$".

We will use the notation where

$$T_{[i_1 \cdots i_n]} = \frac{1}{n!} \sum_{\sigma \in S_n} sign(\sigma) T_{i_{\sigma(1)} \cdots i_{\sigma(n)}},$$

where S_n is the group of permutations of $\{1, \ldots, n\}$; and $sign(\sigma)$ is +1 if σ is even and -1 if σ is odd. The convention in the case of $T_{[i_1} \cdots T_{i_n]}$ is analogous.

Appendix B

Parity Assignments

B.1 Bosonic fields

B.1.1 Generic assignments

To leave the space $\mathcal{M}_4 \times S^1/\mathbb{Z}_2$ invariant under the $\mathbb{Z}_2$ action, the coordinate functions, basis vectors, basis 1-forms, and metric components have

$$P(x^\mu;\ \partial_\mu;\ dx^\mu) = +1 \quad P(x^5;\ \partial_5;\ dx^5) = -1$$
$$P(\hat{g}_{\mu\nu};\ \hat{g}_{55}) = +1 \quad P(\hat{g}_{\mu 5}) = -1,$$

where $P(\Phi)$ denotes the $\mathbb{Z}_2$ parity of the object Φ. The local coordinate tranformation parameters ξ^μ and ξ^5 must transform the same as their corresponding coordinate functions, so $P(\xi^\mu) = +1$ and $P(\xi^5) = -1$.

Equations (4.6) require $P(C_\mu) = -1$ and $P(e^\sigma) = +1$. That C_μ does not propagate along the fixed planes is a reflection of the fact that it is a Kaluza-Klein gauge field for the theory on $\mathcal{M}_4 \times S^1$. The local coordinate tranformation parameters ξ^μ and ξ^5 must transform the same as their corresponding coordinate functions, so $P(\xi^\mu) = +1$ and $P(\xi^5) = -1$. From (4.8), it follows that

$$P(A_\mu^I) = -P(A^I). \tag{B.1}$$

B.1.2 Vector sector

The dimensionally reduced Maxwell-Einstein Lagrangian is invariant under the parity assignments discussed so far. However, there is remaining freedom in the assignments of the vector fields, which we will now discuss. There are two terms in the reduced Lagrangian of

the form

$$\epsilon^{\mu\nu\rho\sigma} C_{\mu\nu} C_{\rho\sigma} (C_{IJK} A^I A^J A^K)$$
$$\epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}^I F_{\rho\sigma}^J (C_{IJK} A^K). \tag{B.2}$$

From the C_{000} terms, it follows that $P(A^0) = +1$, which means $P(A_\mu^0) = -1$, so that the bare graviphoton does not have propagating modes along the fixed planes.

Consider the splitting of the index $i = (\alpha, a)$. We can assign parities to the vector fields (A_μ^α, A_μ^a) and scalars (A^α, A^a) such that

$$P(A_\mu^a) = -P(A_\mu^\alpha) \qquad P(A^a) = -P(A^\alpha),$$

keeping in mind that we still must satisfy (B.1). Let us now make a particular choice (since α, a are dummy indices, it doesn't matter which choice we make):

$$P(A^a) = +1 \qquad P(A_\mu^a) = -1$$
$$P(A^\alpha) = -1 \qquad P(A_\mu^\alpha) = +1. \tag{B.3}$$

From these parities and the C_{IJK} terms in (B.2), it follows that the components of C_{IJK} in the canonical basis satisfy

$$C_{000} = 1 \qquad C_{00i} = 0$$
$$C_{0ab} = -\frac{1}{2}\delta_{ab} \qquad C_{0\alpha\beta} = -\frac{1}{2}\delta_{\alpha\beta} \tag{B.4}$$
$$P(C_{\alpha\beta\gamma}) = -1 \qquad P(C_{\alpha ab}) = -1$$
$$P(C_{abc}) = +1 \qquad P(C_{a\alpha\beta}) = +1.$$

Once we gauge isometries of the scalar manifold, we can write down the parity assignments of the additional objects appearing in the Lagrangian. Unless otherwise noted, assume throughout that we are gauging a compact group K for simplicity of discussion, and that all vectors (other than the bare graviphoton) are gauge fields so that i, j, k are adjoint representation indices of K. Note that now the C_{ijk} are no longer arbitrary, but must be rank-3 symmetric invariant tensors of K.

From the infinitesimal gauge transformations

$$
\begin{aligned}
\delta_\alpha A_{\hat\mu}^I &= -\frac{1}{g}\partial_{\hat\mu}\alpha^I + \alpha^J f_{JK}^I A_{\hat\mu}^K \\
\delta_\alpha \phi^{\tilde x} &= \alpha^I K_I^{\tilde x},
\end{aligned}
\tag{B.5}
$$

we require that

$$
P(\alpha^0) = -1 \qquad P(\alpha^b) = -1 \qquad P(\alpha^\beta) = +1
$$
$$
P(f_{\beta\gamma}^\alpha) = +1 \qquad P(f_{bc}^a) = -1 \qquad P(f_{\alpha\beta}^a) = -1 \qquad P(f_{ab}^\alpha) = +1,
$$

where f_{JK}^I vanishes if any of the indices correspond to $5D$ spectator vector fields[1]; and a structure constant f_{JK}^I with indices permuted has the same parity assignment.

B.1.3 Scalar sector

The cubic polynomial $\mathcal{V}(\zeta) = C_{IJK}\zeta^I\zeta^J\zeta^K$ is a centerpiece of $5D$ supergravity since the C_{IJK} determine a MESGT completely (and in particular, $\mathcal{V} = 1$ characterizes the real scalar manifold). Unitarity of a YMESGT requires $\mathcal{V} > 0$ [GST84a]. To maintain this positivity, we require $P(\zeta^I)P(\zeta^J)P(\zeta^K) = P(C_{IJK})$, so that

$$
P(\zeta^0) = +1, \quad P(\zeta^\alpha) = -1, \quad P(\zeta^a) = +1.
$$

The ζ^I are directly proportional to the embedding coordinates, h^I, of the scalar manifold when restricted to $\mathcal{V} = 1$, so that they satisfy $P(h^I) = P(\zeta^I)$.

We can split the index $I = (0, \alpha, a)$ as we did for the vector sector of the theory, where α is an index for vector fields with even parity, and a is an index for vectors with odd parity. From the parity assignments of the C_{IJK} in the previous subsection, we demand that $P(h^\alpha) = -1$ and $P(h^0) = +1$, where h^α are functions of a subset of vector multiplet scalar fields, ϕ^χ. The remaining h^a are functions of the remaining scalars, ϕ^x (we are making the split $\tilde x = (x, \chi)$). The supersymmetry transformations for the $5D$ scalars will require that these functions satisfy $P(h^a) = +1$ (that is, fermions will need the scalars ϕ^x as superpartners). The parities of the tensors $\overset{\circ}{a}_{IJ}$ and g_{xy} are now fixed (see section (4.5)).

---

[1] We emphasize that for compact gaugings, there is at least one gauge singlet that can be identified as the physical graviphoton; for non-compact gaugings, the graviphoton is one of the gauge fields (i.e., it is no longer a spectator).

The $5D$ gauge transformations of the $\phi^{\tilde{x}}$ in (B.5) require

$$
\begin{aligned}
P(K_0^x) = -1 \quad & P(K_a^x) = -1 \quad & P(K_\alpha^x) = +1 \\
P(K_0^\chi) = +1 \quad & P(K_a^\chi) = +1 \quad & P(K_\alpha^\chi) = -1.
\end{aligned} \tag{B.6}
$$

There are functions $h_{\tilde{x}}^I$ appearing implicitly in the Lagrangian and supersymmetry trans-
formation laws. They are directly proportional to $h_{,\tilde{x}}^I$, so that $P(h_{\tilde{x}}^I) = P(h^I)P(\phi^{\tilde{x}})$. We
will also need to determine the parity assignments of the $h_{\tilde{p}}^{\tilde{I}}$ to find the assignments for
the components of the fermionic $\lambda^{i\tilde{p}}$ later ($\tilde{p}$ is the *flat* index of the real scalar manifold).
We have already determined the parity assignments for the scalar functions $h_{\tilde{x}}^I$ (see section
(4.5)). The two sets of functions are related by the n_V-bein of the scalar manifold:

$$
h_{\tilde{x}}^I = h_{\tilde{p}}^I f_{\tilde{x}}^{\tilde{p}},
$$

where the vielbein satisfy

$$
f_{\tilde{x}}^{\tilde{p}} f_{\tilde{y}}^{\tilde{q}} \, \delta_{\tilde{p}\tilde{q}} = g_{\tilde{x}\tilde{y}}. \tag{B.7}
$$

Making the index split for the flat scalar manifold indices $\tilde{p} = (p, \rho)$, we find

$$
P(f_x^p) = -P(f_\chi^p) \quad P(f_x^\rho) = -P(f_\chi^\rho), \quad \text{for all } x, \chi.
$$

We select even parity assignment for the f_x^p, which will contribute to the vielbein for the
fixed plane theories' scalar manifold. The supersymmetry transformations then require that
f_χ^ρ have even parity as well. Consequently, we get the parity assignments for the $f_{\tilde{x}}^{\tilde{p}}$ and $h_{\tilde{x}}^I$
in section (4.5).

B.1.4 Hypermultiplet sector

For a discussion of hypermultiplet coupling in $5D$ supergravity, see [**?**] and appendix E. The
scalars from n_H hypermultiplets parametrize a quaternionic manifold of real dimension $4n_H$,
with tangent space group $Usp(2n_H) \times Usp(2)$. The supersymmetry transformations for the
scalars and fermions in the hypermultiplets are given in (2.10). The assignments for the
fermions will require that half of the hypermultiplet field content is generically projected
out of the boundary spectrum. Therefore, let us split the index $\tilde{X} = (X, \Omega)$, where $X =
1, \ldots, 2n_H$ and $\Omega = 1, \ldots, 2n_H$. Since these are dummy indices, we can assign even parity

to either subset of $q^{\tilde{X}}$. Let's choose

$$P(q^X) = +1 \qquad P(q^\Omega) = -1.$$

From the gauge transformations of the scalars in (E.1), we find the parity assignments for the quaternionic manifold Killing vectors (see section (4.5)), where $K_I^{\tilde{x}} = 0$ for K-singlet values of I.

B.2 Fermionic fields

From the (naive) dimensionally reduced supersymmetry transformations

$$\delta e_\mu^m = \frac{1}{2}\bar{\epsilon}^i \Gamma^m \tilde{\Psi}_{\mu\,i}$$

$$\delta C_\mu = \frac{1}{2}\bar{\epsilon}^i e^{-\sigma}\Gamma^5(\Psi_{\mu\,i} - \Psi_{\dot{5}\,i}C_\mu),$$

where

$$\tilde{\Psi}_{\mu\,i} = e^{\sigma/2}\Psi_{\mu\,i} + \frac{e^{-\sigma}}{2}(\Gamma^m)^{-1}\Gamma^5\Psi_{\dot{5}\,i}e_\mu^m,$$

we find that

$$P(\bar{\epsilon}^i\,\Gamma^m\Psi_{\mu\,i}) = +1 \quad \text{and} \quad P(\bar{\epsilon}^i\,\Gamma^5\Psi_{\mu\,i}) = -1,$$

respectively. Written out explicitly, these constraints are

$$P(-\eta^T e\sigma^m e\alpha^* - \zeta^\dagger\sigma^m\beta_\mu - \zeta^T e\sigma^m e\beta_\mu^* - \eta^\dagger\sigma^m\alpha_\mu) = +1$$

$$P(i\eta^T e\beta_\mu + i\zeta^\dagger e\alpha_\mu^* - i\zeta^T e\alpha_\mu - i\eta^\dagger e\beta_\mu^*) = -1.$$

Together, these imply that

$$P(\eta) = P(\{\alpha_\mu^*\}) = -P(\{\beta_\mu\}) = -P(\zeta^*)$$

$$P(\zeta) = P(\{\beta_\mu^*\}) = -P(\{\alpha_\mu\}) = -P(\eta^*).$$

This means that there are two classes of fermionic parity assignments, which we will take to be determined by the choice of assignments for the supersymmetry parameters ϵ^i. However, one of the classes yields two helicity $1/2$ (or $-1/2$) states whose Dirac conjugates do not have support at the orbifold fixed points. This violates the CPT theorem. The other class of assignments describes a helicity $1/2$ state and its helicity $-1/2$ CPT conjugate. The

assignments for the components of ϵ^i and Ψ^i_μ are listed in table (B.1).

From the dimensionally reduced susy transformation

$$\delta e^\sigma = \frac{1}{2}\bar{\epsilon}^i \Gamma^5 \Psi_{\dot{5}\,i},$$

we find $P(\bar{\epsilon}^i \Gamma^5 \Psi_{5\,i}) = +1$. Writing this out in terms of 2-component spinors, we find

$$P(\beta_5) = P(\eta) \quad P(\alpha_5^*) = P(\zeta^*) \quad P(\alpha_5) = P(\zeta) \quad P(\beta_5^*) = P(\eta^*),$$

so that the components of $\Psi^{5\,i}$ have the parity assignments in table (B.1).

From the dimensionally reduced susy transformation

$$\delta A^I_\mu = -\frac{1}{2}h^I_p \bar{\epsilon}^i \Gamma_\mu \lambda^p_i + \frac{i\sqrt{6}}{2}\bar{\Psi}^i_\mu \epsilon_i h^I,$$

we find

$$P(h^\alpha_{\tilde{p}} \bar{\epsilon}^i \Gamma_\mu \lambda^{\tilde{p}}_i) = +1 \quad \text{and} \quad P(h^a_{\tilde{p}} \bar{\epsilon}^i \Gamma_\mu \lambda^{\tilde{p}}_i) = -1.$$

From $\delta \phi^x = \frac{1}{2}i f^x_a \bar{\epsilon}^i \lambda^a_i$, we find that

$$P(f^x_{\tilde{p}} \bar{\epsilon}^i \lambda^{\tilde{p}}_i) = +1 \quad \text{and} \quad P(f^\chi_{\tilde{p}} \bar{\epsilon}^i \lambda^{\tilde{p}}_i) = -1.$$

We have determined the parity assignments for the functions $h^{\tilde{I}}_{\tilde{p}}$ and $f^{\tilde{p}}_{\tilde{x}}$ (see section (4.5)), so that we arrive at the following constraints

$$P(\{\delta^{\rho\,*}\}) = P(\{\gamma^p\}) = P(\eta) = -P(\{\delta^{p\,*}\}) = -P(\{\gamma^\rho\}) = -P(\zeta^*) \qquad \text{(B.8)}$$

$$P(\{\delta^p\}) = P(\{\gamma^{\rho\,*}\}) = P(\zeta) = -P(\{\gamma^{p\,*}\}) = -P(\{\delta^\rho\}) = -P(\eta^*). \qquad \text{(B.9)}$$

Consequently, we find the assignments for the components of the $\lambda^{\tilde{p}\,i}$ as in table (B.1). Now consider the hypermultiplet sector. From

$$\delta q^{\tilde{X}} = -i\bar{\epsilon}^i \zeta^A f^{\tilde{X}}_{iA},$$

we find that

$$P(\eta^T e\,\xi^n_1) = P(\zeta^\dagger \xi^n_2) \quad P(\zeta^T e\,\xi^n_1) = P(\eta^\dagger \xi^n_2)$$

$$P(\eta^T e\,\xi^{\tilde{n}}_1) = P(\zeta^\dagger \xi^{\tilde{n}_2}) \quad P(\zeta^T e\,\xi^{\tilde{n}}_1) = P(\eta^\dagger \xi^{\tilde{n}_2}).$$

This implies that

$$P(\xi_1^n) = -P(\xi_2^n) \qquad P(\xi_1^{\tilde{n}}) = -P(\xi_2^{\tilde{n}}),$$

so that half of the fermionic degrees of freedom from the hypermultiplets are projected out at the fixed plane. It then follows that

$$P(\bar{\epsilon}^1 \xi^n) = P(f_{1n}^X) \qquad P(\bar{\epsilon}^1 \xi^{\tilde{n}}) = P(f_{1\tilde{n}}^X)$$
$$P(\bar{\epsilon}^2 \xi^n) = P(f_{2n}^X) \qquad P(\bar{\epsilon}^2 \xi^{\tilde{n}}) = P(f_{2\tilde{n}}^X),$$

for all X; and opposite signs for $X \to \Omega$. The fermionic assignments are only consistent if

$$P(f_{1n}^X) = -P(f_{2n}^X) \qquad P(f_{1\tilde{n}}^X) = -P(f_{2\tilde{n}}^X),$$

for all X. Therefore

$$P(\zeta_1^n) = -P(\zeta_2^n) \quad P(\zeta_1^{\tilde{n}}) = -P(\zeta^{\tilde{n}_2}).$$

In fact, it turns out that only the parity assignments for the $f_{iA}^{\tilde{X}}$ in section (4.5) and assignments for the ζ^A in table (B.1) are consistent.

Even		Odd	
α_μ	α_μ^*	β_μ	β_μ^*
$\beta_{\dot{5}}$	$\beta_{\dot{5}}^*$	$\alpha_{\dot{5}}$	$\alpha_{\dot{5}}^*$
γ^p	γ^{p*}	δ^p	δ^{p*}
δ^ρ	$\delta^{\rho*}$	γ^ρ	$\gamma^{\rho*}$
η	η^*	ζ	ζ^*
ζ_1^n	$\zeta_2^{\tilde{n}}$	ζ_2^n	$\zeta_1^{\tilde{n}}$

Table B.1: Parity assignments for fermions

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Vita

Born in Orange, California (USA), Sean McReynolds received the Bachelor of Science degree in Physics from the University of California, Irvine, in June 1999. He enrolled in the Ph.D. program at the Pennsylvania State University in August 1999.