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# Entropy of black holes with multiple horizons

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**Abstract**

We examine the entropy of black holes in de Sitter space and black holes surrounded by quintessence. These black holes have multiple horizons, including at least the black hole event horizon and a horizon outside it (cosmological horizon for de Sitter black holes and "quintessence horizon" for the black holes surrounded by quintessence). Based on the consideration that the two horizons are not independent each other, we conjecture that the total entropy of these black holes should not be simply the sum of entropies of the two horizons, but should have an extra term coming from the correlations between the two horizons. Different from our previous works, in this paper we consider the cosmological constant as the variable and employ an effective method to derive the explicit form of the entropy. We also try to discuss the thermodynamic stabilities of these black holes according to the entropy and the effective temperature.


## 1. Introduction

Accelerating expansion of the universe can be accounted by a positive cosmological constant or other exotic matter fields such as quintessence, phantom, etc. When black holes exist in these backgrounds, they may have multiple horizons. When Schwarzschild black hole or Reissner–Nordström black hole is embedded into de Sitter space, one has the Schwarzschild–de Sitter

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(SdS) or Reissner–Nordström–de Sitter (RNdS) black holes. The dynamical and thermodynamic properties of these black holes have been studied extensively [1–13]. In addition, if quintessences exist, black holes surrounded by them may also be modified. Since Kiselev first studied black holes surrounded by quintessence (BHQ) [14], quasinormal modes [15,16], thermodynamic properties [17–20] and $P-V$ criticality [21,22] of BHQ were extensively studied.

Taking RNdS black hole as an example, the temperatures at the black hole horizon and cosmological horizon are not equal in general. Thus, the whole RNdS system cannot be in equilibrium thermodynamically. To overcome this problem, one way is to analyze the two horizons separately and independently. For instance, one can analyze one horizon and take another one as the boundary or separate the two horizons by a thermally opaque membrane or wall [8,23–25]. Besides, one can also take a global view to construct the globally effective temperature and other effective thermodynamic quantities by analogy with the first law of thermodynamics [26–29]. However, there are two special cases for RNdS black hole, in which the temperatures at the both horizons are the same. One case is the so-called Nariai black hole, the other is the lukewarm black hole [30–33].

In this paper we are concerned with the entropy of these black holes with multi-horizons. As is well known, separately, Schwarzschild black hole and de Sitter space both have the entropy proportional to the horizon area, exactly $S_b = A_b/4$ and $S_c = A_c/4$ with $A_b$ the black hole event horizon, $A_c$ the cosmological horizon. What is the entropy of the whole SdS black hole? As far as we know, in [23,32–34] the authors used several different methods to verify the area law of the total entropy for de Sitter black holes. Namely, they think that the total entropy of SdS black hole is $S = S_b + S_c = A_b/4 + A_c/4$. However, we have different opinion. We believe that the entropy of the whole de Sitter black hole may be not simply the sum of entropy of the black hole horizon and that of the cosmological horizon due to two reasons. Firstly, as mentioned above, black holes with multiple horizons are in fact in non-equilibrium thermodynamic states. Thus the equilibrium thermodynamics may not apply. There may be an extra entropy developed internally in the system as a result of being out of equilibrium [35]. Secondly, entropy is usually related to the numbers of microscopic states. Although the microscopic origin of black hole entropy is still unclear, it should exist. For black holes with multiple horizons, such as SdS black hole, the black hole horizon and the cosmological horizon are in fact not independent. There may exist some correlations between them. The size of black hole horizon is closely related to the size of the cosmological horizon, and the evolution of black hole horizon will lead to the evolution of the cosmological horizon. Taking the correlations between the horizons into account, the total numbers of microscopic states are not simply the product of those of the two horizons (it will be, if the two horizons are isolated). Therefore, the total entropy is no longer the sum of the entropies of the two horizons, but should include an extra term coming from the contribution of the correlations of the two horizons. Until now, we still do not understand either non-equilibrium thermodynamics or microscopic origin of black hole entropy completely. We cannot quantitatively derive the explicit form of the corrected term in the total entropy from the first principle. Below we will take an effective method to derive an result.

The paper is arranged as follows. In Section 2, we study the entropy of de Sitter black holes including RNdS black hole and SdS black hole. And we analyze their thermodynamic stability based on the entropy. In Section 3 we will analyze the entropy of black holes surrounded by quintessence. We make some concluding remarks in Section 4. We take the units $G = \hbar = k_B = 1$.

## 2. Entropy of de Sitter black holes

### 2.1. RNdS black hole

The line element of the RNdS black hole is given by

$$ds^2 = -h(r)dt^2 + h(r)^{-1}dr^2 + r^2 d\Omega^2, \tag{1}$$

where

$$h(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda}{3}r^2. \tag{2}$$

There are three positive real roots for $h(r) = 0$. The smallest one $r_-$ is the inner/Cauchy horizon, the intermediate one $r_+$ is the event horizon of black hole and the largest one is the cosmological horizon. Here $\Lambda = 3/l^2$ with $l$ the de Sitter radius.

The cosmological constant $\Lambda$ can be viewed as a thermodynamic variable in de Sitter black holes [36–42]. Thus, the first law of thermodynamics can be established respectively for the black hole horizon and the cosmological horizon [43],

$$dM = T_+ dS_+ + \Phi_+ dQ + V_+ d\Lambda, \quad dM = -T_c dS_c + \Phi_c dQ + V_c d\Lambda. \tag{3}$$

The minus sign before $T_c$ is due to the negative surface gravity of the cosmological horizon.

As we mentioned above, the two horizons are not independent. The evolution of each horizon will lead to the evolution of the other. Especially, the three quantities $M,\ Q,\ \Lambda$ are common to the two horizons. This means that thermodynamic quantities on the both horizons depend on the same variables $M,\ Q,\ \Lambda$. This further indicates the relations between the two horizons.

Considering the connection between the black hole horizon and the cosmological horizon, we write the total entropy of RNdS black hole in the form

$$S = S_+ + S_c + S_{ex} = \pi r_c^2 \left[1 + x^2 + f(x)\right], \tag{4}$$

where $x = r_+/r_c \le 1$ and $f(x)$ represents the extra entropy coming from the correlation between the two horizons. When $x = 0$, the black hole event horizon vanishes, so in this case $f(x) = 0$. It should be noted that RNdS black hole will not return to the SdS black hole or dS space in the $x \to 0$ limit due to the existence of the charge $Q$.

Employing Eq. (4) and combining the two equations in Eq. (3), we can derive the effective first law of black hole thermodynamics:

$$dM = T_{eff} dS + \phi_{eff} dQ + V_{eff} d\Lambda, \tag{5}$$

where $T_{eff},\ \Phi_{eff},\ V_{eff}$ are the corresponding effective thermodynamic quantities.

All thermodynamic quantities can be expressed according to $r_+,\ r_c,\ Q$. The mass $M$ and the cosmological constant $\Lambda$ can be written into

$$M = \frac{(x+1)\left(x^2 r_c^2 + Q^2 x^2 + Q^2\right)}{2x\left(x^2 + x + 1\right) r_c}, \quad \Lambda = \frac{3\left(x r_c^2 - Q^2\right)}{x\left(x^2 + x + 1\right) r_c^4} \tag{6}$$

The effective temperature can be derived from Eqs. (4)–(6),

$$T_{eff}=\left.\frac{\partial M}{\partial S}\right|_{Q,\Lambda}=\left.\frac{\partial(M,\Lambda)}{\partial(S,\Lambda)}\right|_{Q}=\left.\frac{\partial(M,\Lambda)}{\partial(x,r_c)}\right|_{Q}\Big/\left.\frac{\partial(S,\Lambda)}{\partial(x,r_c)}\right|_{Q}$$
$$=\frac{(x-1)\left[Q^2\left(x^2+2x+3\right)-x(x+2)r_c^2\right]\left[x^2(2x+1)r_c^2-Q^2\left(3x^2+2x+1\right)\right]}{A(x)}, \tag{7}$$

where

$$A(x)=4\pi x\left(x^2+x+1\right)r_c^3\left[x^2\left(x^2+x+1\right)r_c^2-2Q^2x\left(x^2+x+1\right)\right]f'(x)$$
$$+4\pi x\left(x^2+x+1\right)r_c^3\left[Q^2\left(3x^2+2x+1\right)-x^2(2x+1)r_c^2\right]f(x)$$
$$+4\pi x\left(x^2+x+1\right)r_c^3\left[x^4r_c^2-x^2r_c^2-Q^2(x-1)(x+1)^3\right]. \tag{8}$$

Now it is time to determine the function $f(x)$. For RNdS black holes, the temperature on the black hole horizon and that on the cosmological horizon are not the same in general, thus the globally effective temperature $T_{eff}$ cannot be compared with them. However, in the case of lukewarm black hole, the two horizons have the same temperature. We conjecture that in this special case the effective temperature should also take the same value. In this way, we can obtain the information on the $f(x)$.

In the lukewarm case, for the RNdS black hole there are [31,33,44]

$$M^2=Q^2=\frac{x^2r_c^2}{(x+1)^2}. \tag{9}$$

Pluging the $Q$ into Eq. (7), we can obtain

$$T_{eff}=\frac{(1-x)x}{\pi(x+1)^2r_c\left[\left(x^2+1\right)f'(x)-2xf(x)\right]}. \tag{10}$$

We also know that for the lukewarm RNdS black hole, the temperature is

$$T_+=T_c=\frac{1-x}{2\pi\,(x+1)^2\,r_c}. \tag{11}$$

Equating the two temperatures in Eq. (10) and Eq. (11), we can obtain

$$f'(x)-\frac{2x}{1+x^2}f(x)=\frac{4x}{x^3+x^2+x+1}. \tag{12}$$

This differential equation has an exact solution:

$$f(x)=-\left(x^2+1\right)\ln(x+1)+\frac{1}{2}\left(x^2+1\right)\ln\left(x^2+1\right)+x(x+1), \tag{13}$$

where we have taken the boundary condition $f(0)=0$. Therefore the entropy of RNdS black hole should be

$$S=\pi r_c^2\left[2x^2+x+1-\left(x^2+1\right)\ln(x+1)+\frac{1}{2}\left(x^2+1\right)\ln\left(x^2+1\right)\right]. \tag{14}$$

As is depicted in Fig. 1, $f(x)$ is always positive and increases monotonically. This is expectable. Because the correlations between the black hole horizon and the cosmological horizon

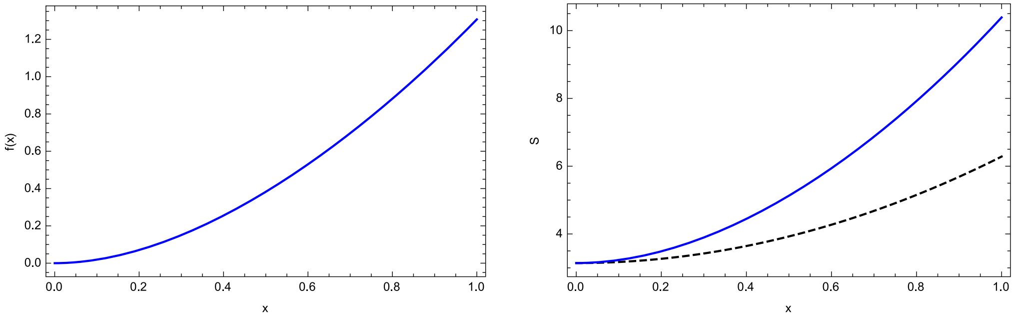


Fig. 1. The left panel: $f(x)$ as the function of $x$ for RNdS black hole. The right panel: plots of entropy as functions of $x$ for RNdS black hole. The dashed (black) line represents the sum of the two horizon entropy and the solid (blue) curve depicts the result in Eq. (14). We have set $r_c = 1$. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

make the total microscopic states greater than isolated black hole horizon and cosmological horizon. Moreover, one can imagine that as the two horizons approach, the correlations between them should be more stronger.

We can also derive the effective temperature by substituting $f(x)$ into Eq. (7). Its complete expression is lengthy and it is unnecessary to show its exact form here. We find that the effective temperature $T_{eff}$ always diverges at $x = 0$ and tends to zero at $x = 1$. In the range $0 < x < 1$, $T_{eff}$ can have three different behaviors for different values of $Q$. As is shown in Fig. 2(a), $T_{eff}$ is finite when $Q$ takes values in the region $0.092 < Q < 0.707$. $T_{eff}$ has one divergent point when $Q > 0.707$ and has two divergent points when $Q < 0.092$. In either case, the $T_{eff}$ cannot be always positive. In Figs. 2(b)–2(d), we plot the $T_{eff} - x$ curves with different charge $Q$, respectively.

We will mainly focus on the finite temperature, namely the case with $Q = 0.5$ shown in Fig. 2(d). $T_{eff}$ has a minimum (A) and a maximum (C) in the range $0 < x < 1$. On the left hand side of the minimum point and the right hand side of the maximum point, $T_{eff}$ decreases with the increasing of $x$. While, in the range between the two extrema $T_{eff}$ increases with increasing $x$. Generally, heat capacity can be defined as $C = \frac{\partial M}{\partial T} = T\frac{\partial S}{\partial T}$. If we also require only positive $T_{eff}$ is meaningful, we find that only in the section (BC) the heat capacity can be positive. This is an unexpected result. This means that when the black hole horizon and the cosmological horizon are too far away (small $x$) or too close (large $x$), RNdS black hole cannot be thermodynamically stable. While RNdS black hole with an intermediate separation between the two horizons may be thermodynamically preferred. For the other two cases ($Q = 0.05$ and $Q = 0.75$), the RNdS black hole is also thermodynamically stable only in a small portion of $x$.

We can further analyze the globally thermodynamic stability of the RNdS black hole according to the Gibbs free energy $G = M - T_{eff}S$. We also take the $Q = 0.5$ case as an example. The section (BC) in Fig. 3 corresponds to that in Fig. 2(d). Thus, in the section (BC) the higher $T_{eff}$ state of the RNdS black hole is more stable. As is shown in the $G - T$ plot, on the surface, for lower effective temperature there are three different states at a temperature and when the effective temperature is high, there is only one state at a temperature. Because the heat capacity is positive only in the red-curve part, globally speaking, the RNdS black hole will evolve along the red curve from the state (B) to the state (C).

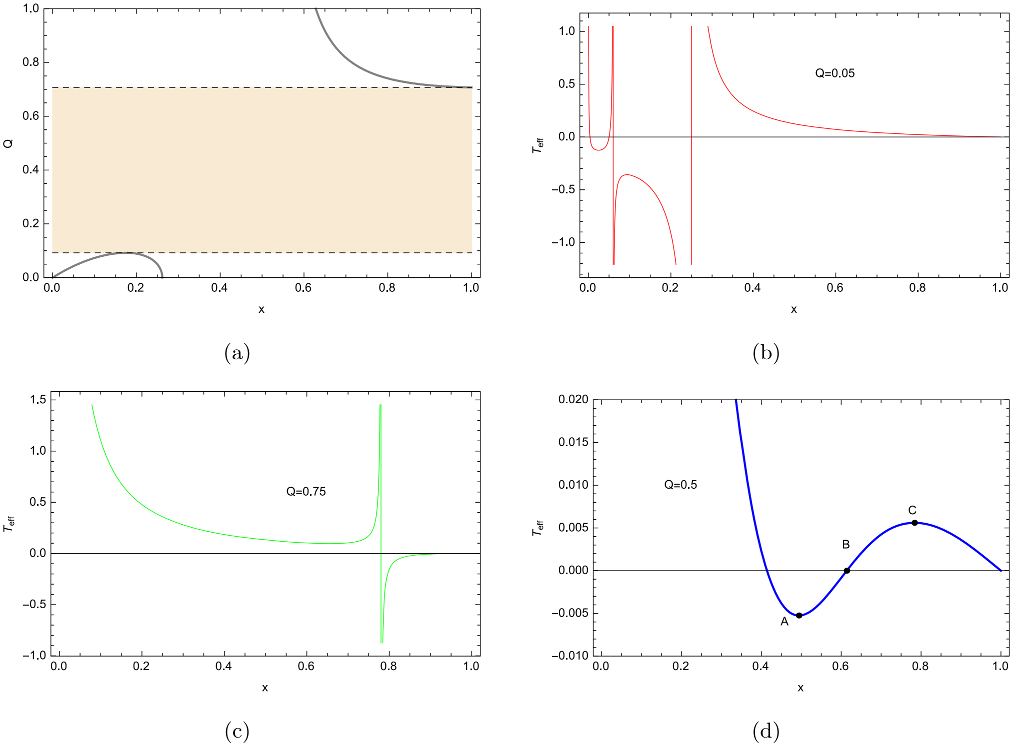


Fig. 2. The panel (a): for $0 < x < 1$ $T_{eff}$ is finite only when $Q$ takes values in the shadow region $0.092 < Q < 0.707$. Other subfigures exhibit the $T_{eff} - x$ curves for RNdS black hole with different $Q$ are also shown. Here we set $r_c = 1$.

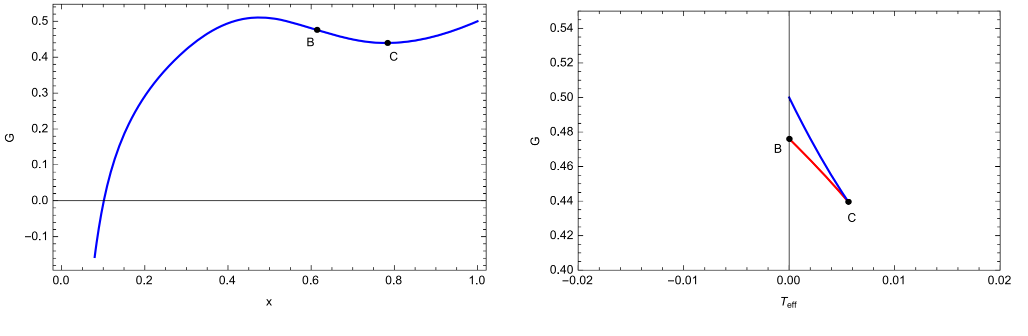


Fig. 3. The Gibbs free energy as function of $x$ and $T_{eff}$ for $r_c = 1$, $Q = 0.5$, respectively. In the right panel, the red/blue curve corresponds to the positive/negative heat capacity.

### 2.2. *SdS black hole*

When $Q = 0$, RNdS black hole turns into SdS black hole. For SdS black hole, no lukewarm case exists. Thus, we cannot derive the corrected entropy directly in the similar way to that of RNdS black hole. However, considering that SdS black hole is just a special case of RNdS black hole and the $f(x)$ derived for RNdS black hole is only implicitly dependent on the charge $Q$, we

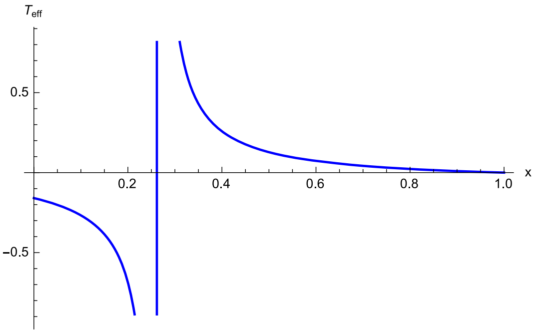


Fig. 4. The effective temperature $T_{eff}$ as function of $x$ for SdS black hole. We have set $r_c = 1$.

conjecture that the function $f(x)$ should has the same form for SdS black hole. Thus, the entropy of SdS black hole also has the form shown in Eq. (14).

The effective temperature $T_{eff}$ for SdS black hole can be derived from two ways. One can start from the metric of SdS black hole and express the quantities $M$, $\Lambda$ as functions of $r_+$, $r_c$. Then, through the effective first law, one can obtain $T_{eff} = \frac{\partial M}{\partial S}\big|_\Lambda$. The other way is more straightforward. One can set $Q = 0$ in Eq. (7) to obtain the effective temperature of SdS black hole. One can verify that the two ways give the same result. Anyway, we can obtain

$$T_{eff} = \frac{(2x+1)(2+x)(1-x)}{4\pi\left(x^2+x+1\right)r_c\left[\left(x^2+x+1\right)f'(x)-(2x+1)f(x)+x^2-1\right]}. \tag{15}$$

Substituting $f(x)$ into the above equation, we can then depict the effective temperature of SdS black hole.

In Fig. 4, it can be seen that the effective temperature diverges at a point. On the left side of the divergent point, $T_{eff}$ is negative. On the right hand side of the divergent point, $T_{eff}$ decrease with the increasing of $x$. This means that SdS black hole is always thermodynamically unstable, either with negative temperature or with negative heat capacity.

At last, we discuss the Nariai limit, namely $x \to 1$. In this case, the black hole horizon and the cosmological horizon apparently coincide, but the volume between them is not zero. The nonzero volume indicates that in this limit the region between the two horizons does not shrink into zero. The two horizons are not really coincident, despite $r_+ = r_c = r_0 = \sqrt{1/\Lambda}$ in this case [1]. There are two kinds of temperatures for de Sitter black holes based on standard normalization and Bousso–Hawking normalization [45]. The standard normalization provide the conventional Hawking temperature, which gives $T_+ = T_c = 0$ in the Nariai limit. Bousso–Hawking normalization indicates that the normalization constant $\gamma_t$ of timelike Killing vector field $K = \gamma_t \frac{\partial}{\partial t}$ cannot set to be $\gamma_t = 1$ as usual, but should be taken as $\gamma_t = \frac{1}{\sqrt{h(r_g)}}$ with $r_g$ a reference point. In this way, the Bousso–Hawking temperature can be calculated along with the new normalization as $T_+^{BH} = \frac{h'(r_+)}{4\pi\sqrt{h(r_g)}}$ and $T_c^{BH} = \frac{h'(r_c)}{4\pi\sqrt{h(r_g)}}$. One can easily find that the Bousso–Hawking temperature is not zero in the Nariai limit, but $T_+^{BH} = T_c^{BH} = \frac{\sqrt{\Lambda}}{2\pi}$.

In either case, the temperatures on the both horizons are the same. In our opinion, in this time the effective temperature $T_{eff}$ should also take the same value. In Fig. 2 and Fig. 4, it is shown that $T_{eff}$ always tends to zero when $x \to 1$. It seems that our effective temperature favors the standard normalization and does not favor the Bousso–Hawking normalization. This conclusion is similar to that in [7].

## 3. Entropy of black holes with quintessence

For static spherically symmetric charged black hole surrounded by quintessence (BHQ), the metric function $h(r)$ in Eq. (1) takes the form [14]

$$h(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\alpha}{r^{3\omega_q+1}}, \tag{16}$$

and $M$ is the mass parameter, $Q$ is the electric charge, $\alpha$ is a normalization factor and $\omega_q$ is the state parameter of quintessence which is confined in the range $-1 < \omega_q < -1/3$. Besides, the parameter $\alpha$ is related to the energy density, $\rho_q = -\frac{\alpha}{2}\frac{3\omega_q}{r^{3(1+\omega_q)}}$. Because $\rho_q$ is positive, $\alpha$ must take *positive* values.

In the following, we will choose $\omega_q = -2/3$ for simplicity. In this case, Eq. (16) becomes

$$h(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \alpha r. \tag{17}$$

It is found that there are three positive real roots for $h(r) = 0$ only if $6\alpha M < 1$ [46]. Among the three roots, the smallest one, $r_-$, corresponds to the black hole Cauchy/inner horizon, the intermediate one, $r_+$, corresponds to the black hole event horizon, and the largest one, $r_q$, corresponds to the "quintessence horizon".

Although this metric is different from the RNdS metric in the last term, one can treat the parameter $\alpha$ in the similar way to the cosmological constant $\Lambda$. When the black hole horizon and the quintessence horizon are viewed as independent each other, there are also respective first laws of thermodynamics:

$$dM = T_+ dS_+ + \Phi_+ dQ + \Theta_+ d\alpha, \quad dM = -T_q dS_q + \Phi_q dQ + \Theta_q d\alpha, \tag{18}$$

where $\Phi_+$ and $\Phi_q$ are electric potentials corresponding to the two horizons. $\Theta_+$ and $\Theta_q$ are the conjugate quantities of $\alpha$ on the two horizons.

Similarly, one can derive the effective first law for the charged black hole surrounded by quintessence:

$$dM = T_{eff} dS + \Phi_{eff} dQ + \Theta_{eff} d\alpha. \tag{19}$$

According to $r_+$, $r_q$ and $Q$, we can express $M$ and $\alpha$ as

$$M = \frac{x^2 r_q^2 + Q^2 x^2 + Q^2 x + Q^2}{2x(x+1)r_q}, \qquad \alpha = \frac{x r_q^2 - Q^2}{x(x+1)r_q^3}. \tag{20}$$

The entropy can also be taken as the form in Eq. (4). Thus, the effective temperature can be derived

$$\begin{aligned} T_{eff} &= \frac{\partial M}{\partial S}\bigg|_{Q,\alpha} = \frac{\partial(M,\alpha)}{\partial(S,\alpha)}\bigg|_{Q} \\ &= \frac{(x-1)\left[Q^2(x+2) - x r_q^2\right]\left[x^2 r_q^2 - Q^2(2x+1)\right]}{B(x)}, \end{aligned} \tag{21}$$

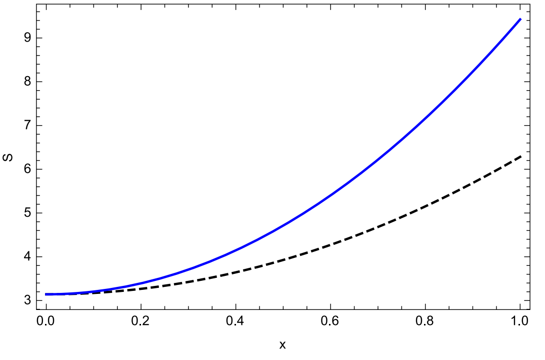


Fig. 5. The solid (blue) curve corresponds to the entropy in Eq. (27) and the dashed (black) curve corresponds to the sum of the two horizon entropy. We have set $r_q = 1$.

where

$$B(x) = 2\pi x(x+1)r_q^3\left[x^2(x+1)r_q^2 - 3Q^2x(x+1)\right]f'(x) + 4\pi x(x+1)r_q^3\left[Q^2(2x+1) - x^2r_q^2\right]f(x) + 2\pi x(x+1)r_q^3\left[2(x-1)x^2r_q^2 - 2Q^2\left(x^3+2x^2-2x-1\right)\right]. \tag{22}$$

In the lukewarm case, $M$ and $Q$ should satisfy [46,47]

$$M = \frac{xr_q^2\left(xr_q + r_q\right)}{x^2r_q^2 + 3xr_q^2 + r_q^2}, \quad Q^2 = \frac{x^2r_q^4}{x^2r_q^2 + 3xr_q^2 + r_q^2}. \tag{23}$$

And the temperatures at black hole horizon and the quintessence horizon are

$$T_+ = T_q = \frac{1-x}{4\pi\left(x^2+3x+1\right)r_q}. \tag{24}$$

Substituting $Q$ in Eq. (23) into Eq. (21), we can get

$$T_{eff} = \frac{(1-x)x}{2\pi\left(x^2+3x+1\right)r_q\left[\left(x^2+1\right)f'(x) - 2xf(x)\right]}. \tag{25}$$

Requiring the temperatures in Eq. (24) and Eq. (25) to be the same, we can obtain

$$f'(x) - \frac{2x}{x^2+1}f(x) = \frac{2x}{x^2+1}. \tag{26}$$

This differential equation has a simple solution: $f(x) = x^2$, if taking the boundary condition $f(0) = 0$. So, the entropy takes the form

$$S = \pi r_c^2\left(1+2x^2\right). \tag{27}$$

In this case, the entropy has been depicted in Fig. 5.

In Fig. 6, we depict the effective temperature for the uncharged and charged black holes surrounded by quintessence. Clearly, although the metrics of de Sitter black holes and the BHQ are different, their effective temperatures have the similar behaviors. Thus, the thermodynamic stability of BHQ is also similar to that of de Sitter black holes studied in the above section.

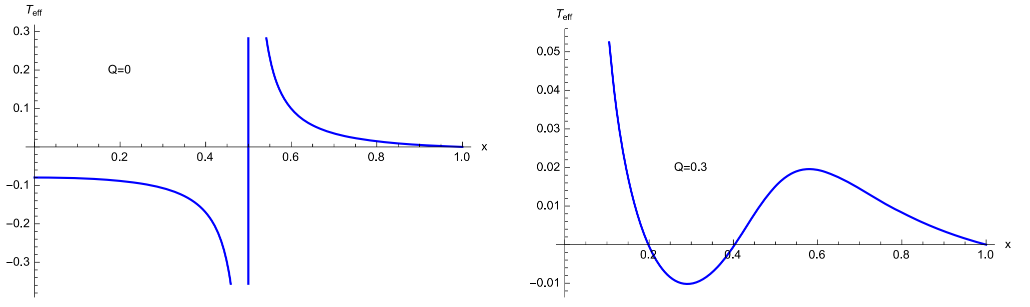


Fig. 6. $T_{eff}$ of BHQ as functions of $x$ in the uncharged case ($Q = 0$) and the charged case ($Q = 0.3$). We also set $r_q = 1$.

## 4. Concluding remarks

In the present paper, we mainly studied the entropy of de Sitter black holes and black holes surrounded by quintessence. Incidently, we analyzed the thermodynamic stability of these black holes. The principle results are as follows.

Firstly, we simply repeat our ideas to derive the entropy. For these black holes with multi-horizons, the black hole event horizon and the cosmological or quintessence horizon are not independent. Thus, we conjecture that the total entropy should take the form of Eq. (4). To derive the function $f(x)$, we use an effective method to obtain an effective temperature $T_{eff}$. In the lukewarm case, the two horizons have the same temperature. We conjecture that $T_{eff}$ also takes that value. In this way, we can obtain an differential equation about $f(x)$. It can be solved exactly.

Secondly, it is found that $f(x)$ is always positive and monotonically increases with the increasing of $x$. This can be understandable because the correlations between the two horizons should be more stronger as they get closer and closer. According to the general definition of heat capacity, we find that the SdS black hole and uncharged BHQ are always thermodynamically unstable due to negative heat capacity. While RNdS black hole and the charged BHQ can be thermodynamically stable only in an intermediate region of $x$ with positive heat capacity.

Finally, we considered the temperature of black holes with multiple horizons. Taking the SdS black hole as an example, in the Nariai case the black hole horizon and the cosmological horizon have the same temperature. According to the standard normalization, it should be zero, and nonzero according to Bousso–Hawking normalization. In the Nariai limit ($x \to 1$), $T_{eff} = 0$. Because we require the effective temperature should equal to the temperatures of the both horizons in the lukewarm and Nariai cases, the standard normalization for the temperatures of multi-horizons black holes is more preferred according to our results.

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