THE TRANSVERSE MOTION OF THE PARTICLES IN LINEAR ELECTRON ACCELERATOR CAUSED BY THE ACTION OF NONSYMMETRIC WAVE

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(Presented by O. A. Valdner)

1) It was proved in a number of works (1-3) that injection current in linear electron accelerator is limited. By exceeding the threshold of injection current (because of growth of beam charge density or increase of injection pulse width) electron beam is destroyed.

Quantitative theoretical and experimental analysis of this phenomenon was carried out in work (3).

Calculation in (3) is based on consideration of self-consistent problem of interaction of weakly modulated beam with non-symmetric backward mode. It is similar to linear theory of r.f. backward-wave oscillator (4).

However in linear electron accelerator the process of particles bunching by external field of accelerating (symmetrical) wave ends rather quickly, after which the electrons gathered in separate bunches perform limiting motion with practically unchanged phase.

Therefore it is interesting to research the opposite limiting case of strong modulation of beam. Bearing in mind that velocity of electrons is rapidly increasing up to relativistic values we will consider the process of beam " expansion " on the bases of the research of electromagnetic fields excited by beam and analysis of the deflecting action of the field in assuming predeterminated longitudinal motion of particles. The charged bursts moving in a slow-waves accelerator structure excited spectrum of various frequencies modes (5). The lowest frequency has the symmetric mode E_{01}. It's wave length and fields spatial distribution coincide with the corresponding external accelerating microwave fields characteristics. All the main energetic characteristics of the high current electron accelerator for various operating conditions (6) may be obtained if take into consideration the similar type mode excitation. If system axial symmetry is disturbed (due to asymmetrical distribution of charge in the beam or axial deflection of the beam or distriction of the slow-wave structure ets) the beam also generates nonsymmetric electromagnetic waves frequency of which is high than that of the accelerating field. Below we apply the method describing the processes in the accelerator similar to (6), to account for the influence of the nonsymmetric waveguide harmonics of the field excited by electron beam.

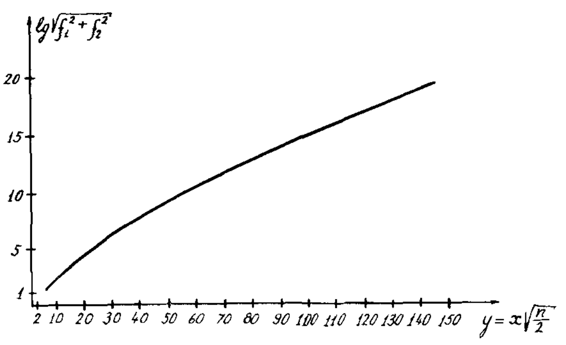

Fig. 1 - Radial displacement amplitude of the bursts as a function of dimensionless variable $y = x\sqrt{n/2}$.

2) Nonsymmetric slow waveguide modes existing in iris-loaded circular waveguide have complicate hybrid character (in contrast to purely " electric " or " magnetic " waves with azimuth symmetry).

As usual in considering the periodic structures, the mode field on each of the excited frequencies is a superposition of space harmonics. All the space harmonics corresponding to a given mode are characterized by identical azimuth symmetry and identical group velocity but differ in phase velocity values. Without considering in detail the properties of nonsymmetric waves (see, for example, (7)) we list expressions for electromagnetic field more significant components with one azimuth variations in the most interesting case, when phase velocity of resonance space harmonic (on which the field interacts with a relativistic beam) is close to velocity of light (constant of this harmonic propagation $h_n = \omega/V_{\Phi n} = h(\omega) + (2\pi/D)n \approx k$:

$$E_z = \frac{e_1}{D} f_n \left(\frac{r}{a}\right)^m \sin(m\varphi + \varphi_o)\, e^{ikz - i\omega t},$$

$$E_r = \left\{ - i \frac{e_1}{D} f_n \frac{h_n a}{2(m+1)} \left(\frac{r}{a}\right)^{m-1} \left(1 + \frac{r^2}{a^2}\right) + \frac{e_2}{D} \tilde{f}_n \cdot \left(\frac{r}{a}\right)^{m-1} \right\} \sin(m\varphi + \varphi_o)\, e^{ikz - i\omega t} \quad [1]$$

$$H\varphi = \left\{ - i \frac{e_1}{D} f_n \frac{h_n a}{2(m+1)} \left(\frac{r}{a}\right)^{m-1} \left(1 + \frac{r^2}{a^2}\right) + i \frac{e_1}{D} f_n \frac{m}{ka} \left(\frac{r}{a}\right)^{m-1} + \frac{e_2}{D} \tilde{f}_n \left(\frac{r}{a}\right)^{m-1} \right\} \sin(m\varphi + \varphi_o)\, e^{ikz - i\omega t}$$

Here e_1 and e_2 — amplitudes of longitudinal E_z and lateral $E\varphi$ electric fields on cylindrical surface $r = a$; f_n and $\tilde{f}_n$ — coefficients of Fourier — expansion for these fields distributions by harmonics $e^{ih_n z}$; D — period of slow-wave structure, a — axial channel structure radius.

Using above cited expressions it is easy to compute lateral components of Lorentz force which acts on charged particle uniformly moving parallel to the structure axis and synchronous to nonsymmetrical wave field [1]:

$$F \left\{ \begin{matrix} r_m \\ \varphi_m \end{matrix} \right. = - e \frac{ie_1}{D} f_n \left(\frac{m}{ka}\right) \left(\frac{r_o}{a}\right)^{m-1} \left\{ \begin{matrix} \sin(m\varphi + \varphi_o) \\ \cos(m\varphi + \varphi_o) \end{matrix} \right\} e^{i\psi} \quad [2]$$

where ψ — phase constant, determining particle position relative to wave maximum, r_o and φ_o — correspond to radial and azimuth particle position. As it follows from expressions [2] the lateral force acting on relativistic particle in wave field differs from zero for nonsymmetrical waves only /$m > 0$/. In this case force radial and azimuth components have the same value. They are shifted by the azimuth at an angle of $\pi/2$. The dependence of lateral deflecting force radial displacement of particle will be $(r_o/a)^{m-1}$ i.e. mainly determined by power m of wave field azimuth asymmetry. Thus, with $m = 1$ the deflecting force is the same throughout the entire cross section ($r \leqslant a$) of accelerator channel. With $m > 1$ the force rapidly reduces as it approaches the axis. Therefore, if consideration is limited by well focused paraxial beams it is necessary in the first place to take into account the effect of the force connected with the waves which have single azimuth variation. We should note also that the considered lateral deflecting force does not depend on dispersion characteristic sigh of eigen waveguide modes.

3) We determine amplitudes of eigen waveguide modes excited by the charge moving parallel to the slowing structure axis with the help of power method which is similar to that considered in (5).

Taking into account the connection of energy density with the energy flux value averaged by time, we have amplitude value of resonant (i.e. moving synchronous with the particle) space harmonic:

$$e_1 \frac{f_n}{D} = \frac{q}{2} \left(\frac{r_o}{a}\right)^m \frac{V_g}{p_1(1 - V_g/V)} \equiv \frac{g}{2} \left(\frac{r_o}{a}\right)^m \frac{1}{p(1 - V_g/V)} \quad [3]$$

Here coefficient p_1, characterizing the distribution of field through the cross section is determined by the formula

$$p_1 \equiv p\, V_g = \frac{N_m}{\left| \dfrac{e_1 f_n}{D} \right|^2} \quad [4]$$

where N_m — power flux through a cross-section of the structure. The coefficient p has the same meaning as in work (5).

It is necessary to note that in deduction the expression [3] for resonant harmonic amplitude approximate field formulas [1] were used wich are true for particle relativistic velocities.

4) Now, let us consider the particle radial displacement under the action of nonsymmetric harmonic of radiated field. The electron beam is treated as consisting of equivalent bursts travelling with constant velocity and following each other by equal distances λ_o (coincide with accelerating microwave field wavelength) begins to inject at the moment of $t = 0$; we consider that azimuth position of all bursts is the same. The radial motion equation of burst number n travelling in radiation field of n of previous bursts ($n = 0$ corresponds to the forward burst) will be:

$$\frac{d}{dt}(m\dot{r}_n) = F_m \quad [5]$$

Disregarding for simplicity the change of particle energy along section we may rewrite the equation as follows:

$$\frac{d^2 \eta_n}{d x^2} = -\sum_{k-1}^{n} \eta_{n-k} \sin k\psi \qquad [6]$$

where dimensionless values are used: $\eta = \frac{r_n(z)}{r_0}$ — burst relative radial displacement ($r_0 = r_n(0)$ — initial deflection, which we assume constant for all the bursts) and $x = z\sqrt{C} = t\,v\,\sqrt{C}$ — dimensionless distance along section. The constant C with dimension of backward area is determined by the expression:

$$C = \frac{eq}{2ka^2 mv^2} \; \frac{V_g}{p_1 (1 - V_g/V)} \qquad [7]$$

Value $k\psi$ in the equation [6] determines phase of the field emitted by the burst with number (n - k) at the point of the considered burst position, minus sign in the right part [6] corresponds to phase reading in the direction opposite to that of propagation.

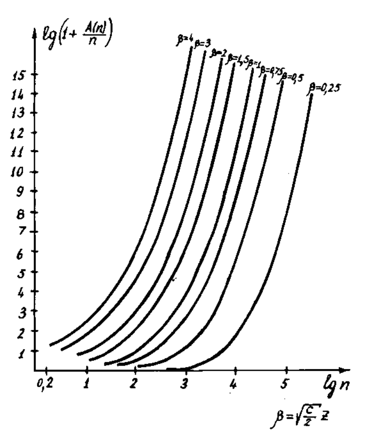

Fig. 2 - Radial displacement amplitude of the bursts as a function of the injection pulse length (the attenuation is neglected).

Solution of the equation [6] with initial conditions: $\eta_n(0) = 1$, $\eta_n(0) = 0$, (i.e. with absence of beam radial divergence at the entrance to the accelerator) can be represented as follows:

$$\eta_n(x) = \sum_{m=0}^{n} a_{mn} \frac{x^{2m}}{(2_m)!} (-1)^m , \qquad [8]$$

where the constant coefficients a_{mn} are determined by the recursion formulas

$$a_{on} = 1, \qquad a_{mm} = \sum_{k-1}^{h-m+1} \sin k\psi \; a_{m-1,\, n-k}. \qquad [9]$$

Values a_{mn} with $m \geqslant 1$ can be represented as polynoms by n power $(m - 1)$. Combinations of trigonometric functions from ψ limited in value serve as their coefficients. At large values of n it is more convenient to use instead of exact formulas [9] the approximate expressions for a_{mn} (corresponding to retention of polynom senior powers by n):

$$a_{mm} \simeq (-1)^m \frac{n^{m-1}}{2^m (m-1)!} \operatorname{cosec}\frac{\psi}{2} \sin\left[\frac{\pi}{2} m + \left(n + \frac{1}{2}\right)\psi\right] = \qquad [10]$$

$$= (-1)^{m+n} \frac{n^{m-1}}{2^m (m-1)!} \sec\frac{\delta}{2} \cos\left(m\frac{\pi}{2} + n\delta\right).$$

In the last expression substitution $\psi = 3\pi + \delta$ ($|\delta| < \pi$) is used accounting that for slowing structures used in accelerators relation of frequencies of nonsymmetric and accelerating symmetric modes is close to 3/2.

Solution of radial motion equation [6] in cases of large n according to [8] and [10] will be:

$$\eta_n = 1 + \frac{(-1)^n}{\cos \delta/2} \left[\sum_{m=1}^{n} \frac{n^{m-1} \cdot x^{2m}}{2^m (2m)!\,(m-1)!} \cos\left(m\frac{\pi}{2} + n\delta\right)\right] =$$

$$= 1 + \frac{(-1)^n}{n \cdot \cos \delta/2} \left[f_1(y) \cos n\delta + f_2(y) \sin n\delta\right] = \qquad [11]$$

$$= 1 + \frac{(-1)^n}{n \cos \delta/2} A(y) \cos(n\delta - \varphi),$$

where:

$$A(y) = \sqrt{f_1^2(y) + f_2^2(y)}, \qquad \varphi = \tan^{-1}\frac{f_2(y)}{f_1(y)}. \qquad [12]$$

In expression [11], [12] new variable is introduced $y = x\,\sqrt{n/2}$), whereas f_1 and f_2 are determined by rapidly converging sums

$$f^1(y) = \sum_{k=1}^{[n/2]} (-1)^k \frac{y^{4k}}{(4k)!\,(2k-1)},$$

$$f_2(y) = \sum_{k=1}^{[(n+1)/2]} (-1)^k \frac{y^{4k-2}}{(4k-2)!\,(2k-2)!}. \qquad [13]$$

Formula [11] represents relative radial displacement of burst as oscillating function of burst number n with amplitude depending on n value, parameters of accelerator C and distance along the section from the point of injection z.

Figure 1 represents in logarithmic scale relation $A = A(y)$ which permits to determine the relative displacement from the axis of n-th burst with the given values of δ, C and z. It is more convenient to use the family of curves $F = \lg(1+A/n) = F(n)$ plotted for various values of parameter $\beta = x/\sqrt{2} = z\sqrt{C/2}$ which is determined by the geometry of accelerator section (see Fig. 2).

Curves $F = F(n, \beta = \text{const})$ determine directly burst maximum deflection from the axis in function of injection pulse length t ($t = h/\nu_0$, where ν_0 — frequency of accelerating field). As it follows from cited diagrams nonsymmetrical harmonic radiation field action leads to the increase of particle deflection from the axis in injection time function according to the law which is somewhat slower than the exponential law.

5) Losses in slow-wave structure of the accelerator result in radiated field amplitude exponential attenuation as the distance from the source with attenuation coefficient $\alpha_1 = \alpha(V_g/V\text{-}V_g)$ increase (α — attenuation constant of appropriate waveguide mode). As before we search solution in form (8). As it was done in the lossless problem we may receive at large values of n the following expression of coefficient a_{mn}:

$$a_{mn} \approx \frac{1}{2^m}\left(\frac{\sin\psi}{\text{ch}\,\gamma - \cos\psi}\right)^m + \frac{n^{m-1}}{2^m (m-1)!} e^{-\gamma n} \cdot \frac{\cos\left(\frac{\pi}{2} m + n\psi\right)\left[\cos\psi - e^{-\gamma}\right] - \sin\left(\frac{\pi}{2} m + n\psi\right)\sin\psi}{\text{ch}\,\gamma - \cos\psi}$$

[14]

where $\gamma = \alpha_1 \lambda_0$ — the nonsymmetric wave field attenuation at the distance between the adjacent bursts.

The second term turns into the expression [12] in the limit case of $\gamma \to 0$; account of first item which is independent of n (and small in comparison with the value [10]) is necessary because the second term decrease exponentially with the increase of n. The resulting displacemente of the burst will be:

$$\eta_n = B(x, \delta) + \frac{(-1)^n}{n} A_1(y)\cos(n\delta + \varphi_1), \qquad [15]$$

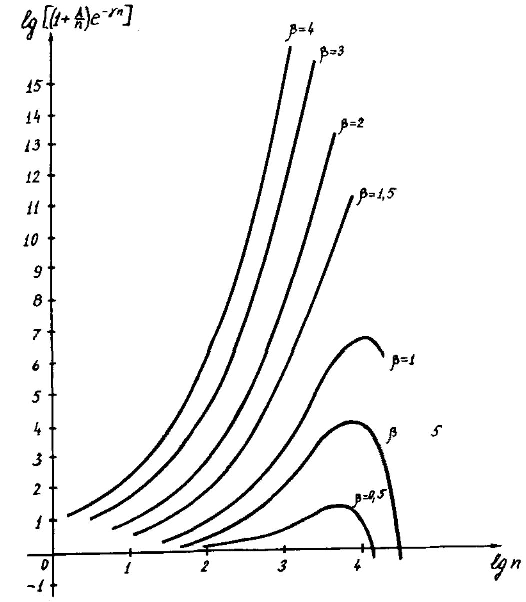

Fig. 3 - Radial displacement amplitude of the bursts as a function of the injection pulse length (the attenuation is taken into account).

where

$$B(x, \delta) = \text{ch}\left[x\sqrt{\frac{\sin\delta}{2(\text{ch}\,\gamma + \cos\delta)}}\right] \approx$$

$$\approx \text{ch}\left(x\sqrt{\frac{1}{2}\tan\frac{\delta}{2}}\right),$$

$$A_1(y) = \sqrt{\frac{2}{\text{ch}\,\gamma + \cos\delta}}\, A(y)\, e^{-\gamma n} \approx \frac{e^{-\gamma n}}{\cos\delta/2} A(y).$$

[16]

Value $B(x, \delta)$ determines the limit value of $n \to \infty$ the relative displacement of n-th burst. As follows from [16] it depends considerably on sign δ(with $\delta > 0$ $B(x, \delta)$ increases exponentially with the increase of argument). The second item in [15] differs from radial deflection amplitude without account of damping $A(y)$ by exponential factor $e^{-\gamma n}$. This factor limited considerably the increase of beam lateral dimensions with time. To illustrate the character of dependence of beam lateral dimension from time the diagrams of the variable part of lateral displacement in the function of burst number n for various values of parameter β with $\gamma = 0{,}001$, $\alpha = 0.35$ np/m,, $\beta_g = 0{,}028$, $\lambda_0 = 0{,}1$ m, are shown in Fig. 3.

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DISCUSSION

CROWLEY-MILLING: Have the calculations been carried out for waveguides with varying parameters, or only for waveguides with uniform dimensions?

VALDNER: The calculations have been carried out for uniform structure. However this theory inable us to calculate any case, because the formula has been obtained without serious limitations.

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INTERACTION OF A BUNCHED BEAM WITH TRANSVERSE MODES IN r.f. CAVITIES *

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High current proton linear accelerators are being designed for use both as injectors for high-energy synchrotrons and as facilities for direct experimentation with mesons and nucleons. It is well known that traveling wave electron linacs exhibit a beam-cavity interaction which leads to transverse beam blow-up (1). We have studied the interaction of high-current proton beams with transverse modes in a standing wave linear accelerator in order to determine the seriousness of this phenomenon for present designs.

The theory of the interaction of a bunched beam with transverse modes in a cavity has been developed (2, 3) in analogy with Wilson's treatment for the traveling wave electron machine. The modes of the first transverse band are assumed to be oscillating with a (normalized) magnetic field amplitude H_j at frequency $\omega_j/2\pi$, with z dependence given by the wave number k_j. As a narrow beam pulse traverses the cavity it contributes to the transverse field because of its transverse displacement, but it also responds dynamically to the existing transverse field. This change in transverse motion generates further changes in the field amplitudes which are of course proportional to those already present.

The amplitude after the traversal of the m^{th} beam pulse can then be written as

$$H_j^{(m+1)} e^{-i\omega_j t} = H_j^{(m)} (1 - \varepsilon_j) + \Sigma_k S_k \left[W_{jk} H_k^{(m)} - \bar{W}_{jk} H_k^{(m)*} \right] - i \left(x_m - i K_j x'_m \right) e^{i \alpha_j/2} \quad [1]$$

Here

$$\varepsilon_j = \frac{\pi \omega_j}{\omega_o Q_j}, \quad S_k = \text{const} \frac{I_o L^2}{p} \frac{R_k}{Q_k} \quad [2]$$

where I_o is the average beam current, $\Delta t = 2\pi/\omega_o$ is the beam bunch separation, L is the cavity

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