Is General Relativity a simplified theory?

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Abstract. Gravity is understood as a geometrization of spacetime. But spacetime is also the manifold of the boundary values of the spinless point particle in a variational approach. Since all known matter, baryons, leptons and gauge bosons are spinning objects, it means that the manifold, which we call the kinematical space, where we play the game of the variational formalism of an elementary particle is greater than spacetime. This manifold for any mechanical system is a Finsler metric space such that the variational formalism can always be interpreted as a geodesic problem on this space. This manifold is just the flat Minkowski space for the free spinless particle. Any interaction modifies its flat Finsler metric as gravitation does. The same thing happens for the spinning objects but now the Finsler metric space has more dimensions and its metric is modified by any interaction, so that to reduce gravity to the modification only of the spacetime metric is to make a simpler theory, the gravitational theory of spinless matter. Even the usual assumption that the modification of the metric only involves dependence on the metric coefficients of the spacetime variables is also a restriction because in general these coefficients are dependent on the velocities. In the spirit of unification of all forces, gravity cannot produce, in principle, a different and simpler geometrization than any other interaction.

1. Introduction

Things should be made simple, but not simpler. From this sentence attributed to Albert Einstein is where we take the title of this work to show that if the spin concept of elementary particles had been known to physics before General Relativity was born most probably the geometrization of spacetime proposed by its creator should be changed by the geometrization of a different manifold, larger than spacetime, so that today's General Relativity would be considered as a theory of gravitation of simpler and spinless matter.

The variational approach of classical mechanics can always be interpreted as a geodesic statement on the space X of the boundary variables of the variational formalism [1]. But this metric manifold X, is not a pseudo-Riemannian space but rather a Finsler space [2], [3], where the symmetric metric $g_{ij}(x, \dot{x})$ is not only a function of the point $x \in X$, but also of its velocity $\dot{x}$, where the overdot means derivative with respect to some arbitrary evolution parameter. For the relativistic spinless or point particle this manifold X is just the spacetime $\mathbb{R}^4$ and in the free case the metric is Minkowski's metric $\eta_{\mu\nu}$. But if the particle has spin it would have more degrees of freedom, so that the variational approach will be described as a geodesic problem in a larger manifold than spacetime. Interactions and gravitation would modify the metric of this larger manifold, so that to restrict ourselves to the geometrization of the spacetime submanifold is to simplify the problem, or in physical terms, to reduce the gravitational behaviour of real spinning matter to that of spinless and unexistent matter.

In the next section 2, I will make a summary of the variational approach of classical mechanics, which shows how it can be interpreted as a geodesic statement and the way to obtain the metric from the Lagrangian. If the Lagrangian of a free elementary particle is modified by any interaction, the metric of the boundary X manifold is modified. Any interaction modifies the geometry of the X space.

In section 3 we analyze some examples to show the Finsler metric structure of the spacetime of a charged point particle under some external interactions, which include a uniform magnetic field and a constant gravitational field of a point mass M. In section 4 we also consider the example of the point particle under a static Newtonian gravitational potential. In all cases the modification of the metric coefficients involve dependence on the velocities of the point particle. A Riemannian approximation to the metric can be obtained in the low velocity limit. All these analysis are done in a special relativity framework.

In section 5 after a short introduction to the concept of classical elementary particle I will describe the most general X manifold of a relativistic spinning particle which satisfies Dirac's equation when quantized. Since it seems that there are no spinless elementary particles in nature, it is on this larger manifold that the plausible generalization of Einstein's gravitational formalism has to be worked out.

As a general conclusion, General Relativity is a restricted theory of gravitation and therefore a simplified theory in two aspects. One, that the manifold whose geometry is changed by any interaction is larger than spacetime because real elementary particles are spinning particles. The second is that the modification of the metric coefficients should involve dependence on the velocities, i.e., the metric should be a Finsler metric instead of a Riemannian metric.

2. The geodesic interpretation of the variational formalism

Let us consider any mechanical system of n degrees of freedom q_i, $i = 1, \ldots, n$, described by a Lagrangian, $L(t, q_i, dq_i/dt)$, where t is the time. The variational approach is stated in such a way that the path followed by the system makes stationary the action functional

$$\mathcal{A}[q(t)] = \int_{t_1}^{t_2} L(t, q_i, dq_i/dt)dt,$$

between the initial state $x_1 \equiv (t_1, q_i(t_1))$ and final state $x_2 \equiv (t_2, q_i(t_2))$ on the X manifold, which is the $(n+1)$-th dimensional manifold spanned by the time t and the n degrees of freedom q_i. If instead of describing the evolution in terms of time we express the evolution in parametric form $\{t(\tau),\ q_i(\tau)\}$ in terms of some arbitrary evolution parameter τ, then $dq_i/dt = \dot{q}_i/\dot{t}$, where now the overdot means τ-derivative. The variational approach will be written as

$$\int_{\tau_1}^{\tau_2} L(t, q_i, \dot{q}_i/\dot{t})\ \dot{t}d\tau = \int_{\tau_1}^{\tau_2} \widetilde{L}(x, \dot{x})d\tau, \quad \widetilde{L} = L\dot{t},$$

with the same boundary values on the X manifold as before x_1 and x_2. But now the Lagrangian $\widetilde{L}$ is independent on the evolution parameter τ and it is a homogeneous function of first degree of the derivatives $\dot{x}$ [3]. In fact in L each time derivative dq_i/dt has been replaced by a quotient $\dot{q}_i/\dot{t}$ and therefore it is a homogeneous function of zero-th degree of $\dot{t}, \dot{q}_i$. But it is the last term $dt = \dot{t}d\tau$, which makes $\widetilde{L} = L\dot{t}$, homogeneous of first degree of $\dot{x} \equiv \{\dot{t}, \dot{q}_i\}$.

This means that $\widetilde{L}^2$ is a positive definite homogeneous function of second degree of the derivatives $\dot{x}$, so that Euler's theorem on homogeneous functions allows us to write

$$\widetilde{L}^2 = g_{ij}(x, \dot{x})\dot{x}^i\dot{x}^j, \quad i, j = 0, 1, \ldots n$$

where index 0 corresponds to the time variable and the g_{ij} are computed from $\widetilde{L}^2$ by

$$g_{ij}(x,\dot{x}) = \frac{1}{2}\frac{\partial^2 \widetilde{L}^2}{\partial \dot{x}^i \partial \dot{x}^j} = g_{ji}, \quad i,j = 0,\ldots,n. \tag{1}$$

Between the allowed boundary states x_1 and x_2, since $\widetilde{L}^2 > 0$, the metric $g_{ij}(x,\dot{x})$ represents a definite positive metric which transforms as a second rank covariant tensor under transformations which leave $\widetilde{L}$ invariant [2, 3]. The variational problem can be stated as

$$\int_{\tau_1}^{\tau_2} \widetilde{L}(x,\dot{x})d\tau = \int_{\tau_1}^{\tau_2} \sqrt{\widetilde{L}^2(x,\dot{x})}d\tau = \int_{\tau_1}^{\tau_2} \sqrt{g_{ij}(x,\dot{x})\dot{x}^i\dot{x}^j}d\tau =$$
$$= \int_{x_1}^{x_2} \sqrt{g_{ij}(x,\dot{x})dx^i dx^j} = \int_{x_1}^{x_2} ds,$$

where ds is the arc length on the X manifold with respect to the metric g_{ij}. The variational statement has been transformed into a geodesic problem with a Finsler metric. As shown in [1] this is valid even for Lagrangian systems depending on higher order derivatives $L(t, q_i, q_i^{(1)}, \ldots, q_i^{(k)})$, $q_i^{(k)} = d^k q_i/dt^k$. In this case the manifold of the boundary variables X, which will be called the kinematical space from now on, is spanned by the time t, the n degrees of freedom q_i and their corresponding time derivatives up to order $k-1$.

Since $\widetilde{L}$ is homogeneous of first degree in terms of the derivatives $\dot{x}$ can also be decomposed as a sum of terms with dimensions of action if the arbitrary evolution parameter is taken dimensionless,

$$\widetilde{L} = \frac{\partial \widetilde{L}}{\partial \dot{x}^i}\dot{x}^i = F_i(x,\dot{x})\dot{x}^i,$$

where the $F_i(x,\dot{x})$ are homogeneous functions of zero-th degree of the $\dot{x}^i$, so that they involve time derivatives of the different degrees of freedom. The metric coefficients can be expressed as

$$g_{ij} = F_iF_j + \widetilde{L}\frac{\partial^2 \widetilde{L}}{\partial \dot{x}^i \partial \dot{x}^j} = F_iF_j + \widetilde{L}\frac{\partial F_i}{\partial \dot{x}^j} = g_{ji} \tag{2}$$

and are also homogeneous functions of zero-th degree of the $\dot{x}^i$.

As an example, the relativistic point particle of mass m and spin 0 has a kinematical space spanned by time t and the position of the point $\boldsymbol{r}$, so that the free Lagrangian $\widetilde{L}_0 = -mc\sqrt{c^2\dot{t}^2 - \dot{\boldsymbol{r}}^2}$, is clearly a homogeneous function of first degree of the derivatives $\dot{t}$ and $\dot{\boldsymbol{r}}$. With $x^0 \equiv ct$, the Finsler metric becomes

$$\widetilde{L}_0^2 = m^2c^2(\dot{x}_0^2 - \dot{\boldsymbol{r}}^2), \quad g_{\mu\nu} = \frac{1}{2}\frac{\partial^2 \widetilde{L}_0^2}{\partial \dot{x}^\mu \partial \dot{x}^\nu} = m^2c^2\eta_{\mu\nu},$$

where $\eta_{\mu\nu}$ is $\mathrm{diag}(1,-1,-1,-1)$. The interaction with some external electromagnetic field is described by the new Lagrangian $\widetilde{L} = \widetilde{L}_0 + \widetilde{L}_I$, with $\widetilde{L}_I = -eA_\mu(x)\dot{x}^\mu$, so that the variational problem is transformed into a geodesic problem with a new metric on X space, given by

$$\widetilde{L}^2 = \left(\widetilde{L}_0 + \widetilde{L}_I\right)^2, \quad F_\mu = \frac{\partial \widetilde{L}}{\partial \dot{x}^\mu} = -p_\mu - eA_\mu, \quad p_\mu = \frac{mc\dot{x}_\mu}{\sqrt{\dot{x}_\nu\dot{x}^\nu}}$$

$$g_{\mu\nu}(x,\dot{x}) = m^2c^2\eta_{\mu\nu} + e^2A_\mu A_\nu + e(p_\mu A_\nu + p_\nu A_\mu) + eA_\sigma\dot{x}^\sigma\frac{\partial p_\mu}{\partial \dot{x}^\nu}. \tag{3}$$

The modification of the metric vanishes when $e \to 0$. Because p_μ is not explicitely dependent on the variables x, the dependence of the metric on the spacetime coordinates is coming only from the external fields $A_\mu(x)$. But it depends on the $\dot{x}$ variables through its dependence on the p_μ and its derivatives.

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3. Examples of Finsler spaces

In figure 1 we show three possible motions of a charged point particle in its kinematical space, which reduces in this case to the spacetime. The three trajectories are geodesics of spacetime but with respect to three different metrics. In part (a) the motion is free, the trajectory is a straight line. In (b) the particle is under the action of an external uniform magnetic field, and the trajectory has curvature and torsion. In this case the Finsler metric of spacetime is different than the metric in the free case. The external magnetic field modifies the metric. Finally in (c) it is the same free trajectory but as seen by an accelerated observer. According to the equivalence principle this is equivalent to the analysis under a global and constant gravitational field. Also in this case the metric has been modified.

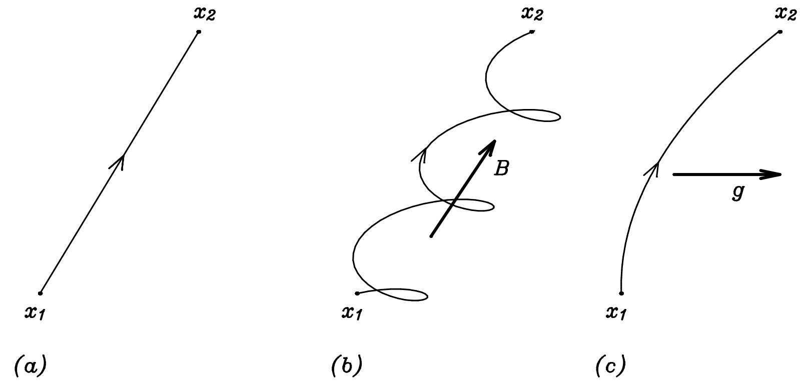

Figure 1. Three motions of a point particle in its kinematical space between the boundary points x_1 and x_2. (a) in the free case, (b) under a uniform magnetic field $\boldsymbol{B}$, and (c) under a uniform gravitational field $\boldsymbol{g}$. In the three cases the kinematical space is the same, the spacetime, the trajectories are geodesics but with respect to three different Finslerian metrics. The spatial part of the trajectories is in the case (a) a straight line with no curvature and no torsion, in (b) with curvature and torsion and in (c) a flat trajectory with curvature.

In case (a) the metric is $g_{\mu\nu} = m^2c^2\eta_{\mu\nu}$, where $\eta_{\mu\nu}$ is diag$(1, -1, -1, -1)$. It is the constant Minkowski metric.

In the case (b), let us assume a uniform magnetic field along OZ axis of intensity B. We can take as the potential vector $\boldsymbol{A} = (0, Bx, 0)$ and scalar potential $A_0 = 0$. The Lagrangian of the point particle under this field is

$$\widetilde{L}_B = -mc\sqrt{\dot{x}_0^2 - \dot{\boldsymbol{r}}^2} + eBx\dot{y}. \tag{4}$$

This Lagrangian leads to the Lorentz force dynamical equation

$$\frac{d\boldsymbol{p}}{dt} = e\,\boldsymbol{u} \times \boldsymbol{B}, \quad \boldsymbol{u} = \frac{d\boldsymbol{r}}{dt}. \tag{5}$$

According to (1) and (3) and calling $K = eBmc$, the Finsler metric coefficients of spacetime are,

$$g_{00} = m^2c^2 + \frac{Kxu^2u_y}{(c^2 - u^2)^{3/2}}, \quad g_{11} = -m^2c^2 + \frac{Kxu_y}{(c^2 - u^2)^{3/2}}\left(c^2 - u_y^2 - u_z^2\right),$$

$$g_{22} = -m^2c^2 + e^2B^2x^2 + \frac{Kxu_y}{(c^2 - u^2)^{3/2}}\left(3c^2 - 3u_x^2 - 2u_y^2 - 3u_z^2\right),$$

$$g_{33} = -m^2c^2 + \frac{Kxu_y}{(c^2-u^2)^{3/2}}\left(c^2 - u_x^2 - u_y^2\right),$$

$$g_{01} = -\frac{Kxcu_xu_y}{(c^2-u^2)^{3/2}}, \quad g_{02} = -\frac{Kxc}{(c^2-u^2)^{3/2}}(c^2 - u_x^2 - u_z^2),$$

$$g_{03} = -\frac{Kxcu_yu_z}{(c^2-u^2)^{3/2}}, \quad g_{12} = \frac{Kxu_x}{(c^2-u^2)^{3/2}}\left(c^2 - u_x^2 - u_z^2\right),$$

$$g_{13} = \frac{Kx}{(c^2-u^2)^{3/2}}u_xu_yu_z, \quad g_{23} = \frac{Kxu_z}{(c^2-u^2)^{3/2}}\left(c^2 - u_x^2 - u_z^2\right),$$

As we see, the metric coefficients are functions of the point, i.e., of the variable x, but they are functions of the three components of the velocity of the particle u_x, u_y, u_z, and therefore the metric depends on both x and $\dot{x}$. The dependence on $\dot{x}$ of the $g_{\mu\nu}$ is a homogeneous function of zero-th degree, and thus it depends on the time derivatives $dr_i/dt = u_i$. If the velocity is negligible with respect to c, the metric coefficients become

$$g_{00} = m^2c^2, \quad g_{02} = -eBmcx, \quad g_{11} = -m^2c^2, \quad g_{22} = -m^2c^2 + e^2B^2x^2, \quad g_{33} = -m^2c^2,$$

vanishing the remaining ones, and since the dependence on the velocity has disappeared the metric has been transformed into a Riemannian metric. Spacetime metric is Riemannian in the low velocity limit.

These metric coefficients give rise to a restricted Lagrangian $\widetilde{L}_R$,

$$\widetilde{L}_R^2 = m^2c^2(c^2\dot{t}^2 - \dot{\boldsymbol{r}}^2) + e^2B^2x^2\dot{y}^2 - 2eBmc^2x\dot{t}\dot{y}, \tag{6}$$

such that when compared with (4) we have an additional term

$$\widetilde{L}_B^2 = \widetilde{L}_R^2 - 2emcBx\dot{y}\left(\sqrt{c^2\dot{t}^2 - \dot{\boldsymbol{r}}^2} - c\dot{t}\right).$$

The Lorentz force dynamical equations (5) are

$$\frac{du_x}{dt} = \frac{eB}{m\gamma(u)}u_y = \frac{1}{\gamma(u)}kcu_y, \quad \frac{du_y}{dt} = -\frac{eB}{m\gamma(u)}u_x = -\frac{1}{\gamma(u)}kcu_x, \quad \frac{du_z}{dt} = 0,$$

with $k = eB/mc$, which gives rise to $u_xdu_x/dt + u_ydu_y/dt + u_zdu_z/dt = \boldsymbol{u}\cdot d\boldsymbol{u}/dt = 0$, so that the motion is at a velocity of constant modulus u, the factor $\gamma(u)$ is constant and the point particle moves at a constant velocity along OZ axis and rotates on the plane XOY with angular velocity $\omega = eB/\gamma(u)m = k\sqrt{c^2-u^2}$. However, the geodesic dynamical equations obtained from the restricted metric (6) are

$$\frac{du_x}{dt} = kcu_y(1 - kxu_y/c), \quad \frac{du_y}{dt} = -kcu_x(1 - kxu_y/c), \quad \frac{du_z}{dt} = 0,$$

which reduce to the above equations when $u/c \to 0$. From the restricted Lagrangian (6) the force acting on the point particle is not longer the Lorentz force.

In the case (c) in a uniform gravitational field $\boldsymbol{g}$, the dynamical equations

$$d\boldsymbol{p}/dt = m\boldsymbol{g}, \tag{7}$$

independent of the mass of the particle, come from the Lagrangian

$$\widetilde{L}_g = \widetilde{L}_0 + m\boldsymbol{g}\cdot\boldsymbol{r}\dot{t}. \tag{8}$$

From the geodesic point of view it corresponds to an evolution in a spacetime with the Finslerian metric given by:

$$g_{00} = m^2c^2 + m^2(\boldsymbol{g}\cdot\boldsymbol{r})^2/c^2 - \frac{m^2c(\boldsymbol{g}\cdot\boldsymbol{r})}{(c^2-u^2)^{3/2}}(2c^2-3u^2),$$

$$g_{11} = -m^2c^2 + \frac{m^2c(\boldsymbol{g}\cdot\boldsymbol{r})}{(c^2-u^2)^{3/2}}(c^2-u_y^2-u_z^2),$$

$$g_{22} = -m^2c^2 + \frac{m^2c(\boldsymbol{g}\cdot\boldsymbol{r})}{(c^2-u^2)^{3/2}}(c^2-u_x^2-u_z^2),$$

$$g_{33} = -m^2c^2 + \frac{m^2c(\boldsymbol{g}\cdot\boldsymbol{r})}{(c^2-u^2)^{3/2}}(c^2-u_x^2-u_y^2),$$

$$g_{01} = -\frac{m^2u^2(\boldsymbol{g}\cdot\boldsymbol{r})}{(c^2-u^2)^{3/2}}u_x, \quad g_{02} = -\frac{m^2u^2(\boldsymbol{g}\cdot\boldsymbol{r})}{(c^2-u^2)^{3/2}}u_y, \quad g_{03} = -\frac{m^2u^2(\boldsymbol{g}\cdot\boldsymbol{r})}{(c^2-u^2)^{3/2}}u_z,$$

$$g_{12} = \frac{m^2c(\boldsymbol{g}\cdot\boldsymbol{r})}{(c^2-u^2)^{3/2}}u_xu_y, \quad g_{23} = \frac{m^2c(\boldsymbol{g}\cdot\boldsymbol{r})}{(c^2-u^2)^{3/2}}u_yu_z, \quad g_{13} = \frac{m^2c(\boldsymbol{g}\cdot\boldsymbol{r})}{(c^2-u^2)^{3/2}}u_xu_z.$$

If again, the velocity is negligible with respect to c, the nonvanishing coefficients are

$$g_{00} = m^2c^2 + m^2(\boldsymbol{g}\cdot\boldsymbol{r})^2/c^2 - 2m^2(\boldsymbol{g}\cdot\boldsymbol{r}), \quad g_{11} = -m^2c^2 + m^2(\boldsymbol{g}\cdot\boldsymbol{r}),$$

$$g_{22} = -m^2c^2 + m^2(\boldsymbol{g}\cdot\boldsymbol{r}), \quad g_{33} = -m^2c^2 + m^2(\boldsymbol{g}\cdot\boldsymbol{r}).$$

i.e.,

$$g_{00} = m^2c^2\left(1-\frac{\boldsymbol{g}\cdot\boldsymbol{r}}{c^2}\right)^2, \quad g_{ii} = -m^2c^2\left(1-\frac{\boldsymbol{g}\cdot\boldsymbol{r}}{c^2}\right), \quad i=1,2,3,$$

where the g_{00} component is the same as the corresponding component of the Rindler metric corresponding to a noninertial accelerated observer or to the presence of a global uniform gravitational field, in General Relativity.

4. Example: Point particle in a Newtonian potential

A final example is the relativistic point particle in the Newtonian potential of a point mass M. The dynamical equations

$$\frac{d\boldsymbol{p}}{dt} = -\frac{GmM}{r^3}\boldsymbol{r}, \tag{9}$$

are independent of the mass of the particle and come from the Lagrangian

$$\widetilde{L}_N = \widetilde{L}_0 + \frac{GmM}{cr}c\dot{t}. \tag{10}$$

If we take into account (1) the metric coefficients are

$$g_{00} = m^2c^2 + \frac{G^2m^2M^2}{c^2r^2} - \frac{Gm^2Mc}{r(c^2-u^2)^{3/2}}(2c^2-3u^2),$$

$$g_{11} = -m^2c^2 + \frac{Gm^2Mc^3}{r(c^2-u^2)^{3/2}} - \frac{Gm^2Mc(u_y^2+u_z^2)}{r(c^2-u^2)^{3/2}},$$

$$g_{22} = -m^2c^2 + \frac{Gm^2Mc^3}{r(c^2-u^2)^{3/2}} - \frac{Gm^2Mc(u_x^2+u_z^2)}{r(c^2-u^2)^{3/2}},$$

$$g_{33} = -m^2c^2 + \frac{Gm^2Mc^3}{r(c^2-u^2)^{3/2}} - \frac{Gm^2Mc(u_x^2+u_y^2)}{r(c^2-u^2)^{3/2}},$$

$$g_{01} = -\frac{Gm^2Mu^2u_x}{r(c^2-u^2)^{3/2}}, \quad g_{02} = -\frac{Gm^2Mu^2u_y}{r(c^2-u^2)^{3/2}}, \quad g_{03} = -\frac{Gm^2Mu^2u_z}{r(c^2-u^2)^{3/2}},$$

$$g_{12} = \frac{Gm^2Mcu_xu_y}{r(c^2-u^2)^{3/2}}, \quad g_{23} = \frac{Gm^2Mcu_yu_z}{r(c^2-u^2)^{3/2}}, \quad g_{31} = \frac{Gm^2Mcu_zu_x}{r(c^2-u^2)^{3/2}},$$

It is a Finsler metric, which in the case of low velocity only the diagonal components survive

$$g_{00} = m^2c^2\left(1 - \frac{2GM}{c^2r} + \frac{G^2M^2}{c^4r^2}\right) = m^2c^2\left(1 - \frac{GM}{c^2r}\right)^2.$$

and the

$$g_{ii} = -m^2c^2\left(1 - \frac{GM}{c^2r}\right), \quad i = 1,2,3.$$

This corresponds to a static and spherically symmetric Riemannian metric which in spherical coordinates and supressing the constant factor m^2c^2, becomes

$$\left(1 - \frac{GM}{c^2r}\right)^2 c^2dt^2 - \left(1 - \frac{GM}{c^2r}\right)(dr^2 + r^2d\Omega^2),$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$.

This metric is not a vacuum solution of Einstein's equations, so that it cannot be transformed into the Schwarzschild metric in isotropic coordinates.

In all the examples, the free Lagrangian $\widetilde{L}_0$ of the spinless particle, has been transformed by the interactions in the way

$$\widetilde{L}_0^2 = m^2c^2\eta_{\mu\nu}\dot{x}^\mu\dot{x}^\nu \quad \Rightarrow \quad \widetilde{L}^2 = g_{\mu\nu}(x,\dot{x})\dot{x}^\mu\dot{x}^\nu, \tag{11}$$

where the new metric $g_{\mu\nu}(x,\dot{x})$ is a Finslerian metric. The low velocity limit of the above metrics produce a Riemannian spacetime approximation which does not give rise to the usual (and expected) dynamical equations. All these examples have been worked out in a special relativity context.

However, General Relativity states that gravity modifies the metric of spacetime producing a new (pseudo-)Riemannian metric $g_{\mu\nu}(x)$, which is related through Einstein's equations to the energy momentum distribution $T^{\mu\nu}$ of all forms of matter and energy. The motion of a test point particle in this gravitational background is a geodesic on spacetime, and therefore can be treated as a Lagrangian dynamical problem with a Lagrangian

$$\widetilde{L}_g^2 = g_{\mu\nu}(x)\dot{x}^\mu\dot{x}^\nu. \tag{12}$$

Because electromagnetism produces a Finsler metric of spacetime, and in the spirit of unification of all interactions, one is tempted to extend the formulation of gravity (12) to the more general *ansatz* (11) by allowing to the metric coefficients produced by gravity to be also a function of the derivatives $\dot{x}$. Otherwise, to assume only a Riemannian metric is to consider that gravity, as far as dynamical equations are concerned, produces a different geometrization than any other interaction. In a region where the gravitational field can be considered uniform, or is given by the Newtonian potential the Lagrangian dynamics of the point particle is equivalent to a geodesic problem in that region where the metric is necessarily a Finsler metric, as seen in the above examples. Any theory of gravitation when restricted to a region of uniform gravitational field or to the sourroundings of a point mass M should reproduce the above dynamical equations (7) and (9), respectively. We expect that, when restricted to inertial observers, general relativity should reproduce the analysis under a special relativity context. But these dynamical equations are derived from Finsler metrics, so that a Riemannian metric $g_{\mu\nu}(x)$ cannot reproduce them. The elimination of the presence of the velocities in the metric coefficients could be interpreted as a low velocity limit of a more general gravitational theory.

5. The spin structure of elementary particles

In the mentioned reference [1] and previous works cited in, an elementary particle is defined as a mechanical system whose kinematical space X (the boundary space of its Lagrangian description) is necessarily a homogeneous space of the Poincaré group $\mathcal{P}$. The idea is that an elementary particle cannot be divided and, if not annihilated with its antiparticle, cannot be deformed so that any state is just a kinematical modification of any one of them [4]. When the initial state x_1 is modified by the dynamics, the subsequent states $x(\tau)$ can always be obtained from it by some change $g \in \mathcal{P}$ of inertial observer $x = gx_1$ and also $x_2 = gx_1$, so that given any two points $x_1, x_2 \in X$ we can always find some $g \in \mathcal{P}$ which links them. It is clear that the point particle manifold, the spacetime, is a homogeneous space of $\mathcal{P}$ and thus fulfills this requirement. Nevertheless, it describes a spinless object and there are no spinless elementary particles in nature. To describe spin we have to enlarge this kinematical space with the above constraint to obtain the largest homogeneous space of $\mathcal{P}$ to describe the elementary particle with the more complex structure. The classical system that when quantized satisfies Dirac's equation corresponds to a kinematical space spanned by the following variables $x \equiv (t, \boldsymbol{r}, \boldsymbol{u}, \boldsymbol{\alpha})$, which are interpreted as the time t, position of the center of charge $\boldsymbol{r}$, velocity of the center of charge $\boldsymbol{u} = d\boldsymbol{r}/dt$ at the speed of light $u = c$, and the orientation $\boldsymbol{\alpha}$ of a cartesian system located at point $\boldsymbol{r}$ [5]. It is a nine-dimensional kinematical space described by four noncompact variables $(t, \boldsymbol{r})$ and five compact ones $(\theta, \phi, \boldsymbol{\alpha})$, being θ, ϕ the orientation of the velocity vector $\boldsymbol{u}$ and the orientation of the particle local frame $\boldsymbol{\alpha}$. The particle has a center of mass $\boldsymbol{q}$ which is expressed in terms of $\boldsymbol{r}$ and its time derivatives. Elementary spinning particles have two distinguished points the center of mass and the center of charge which are different points. The cartesian frame can be taken as the Frenet-Serret triad, so that the angular velocity of the particle can also be expressed in terms of the derivatives of the position of the point $\boldsymbol{r}$.

The free motion of the center of charge $\boldsymbol{r}$ corresponds to a helix of constant curvature and torsion when expressed in terms of the Frenet-Serret triad, and at a velocity of constant absolute value c.

This classical model of elementary particle can be applied for leptons and quarks if, as assumed, they satisfy in the quantum formalism Dirac's equation.

If we want to include gravitation we have to admit arbitrary changes of spacetime coordinates, not only those given by the Poincaré group. This will produce a modification of the metric of the spacetime submanifold, but also the modification of the remaining components of the metric on the whole kinematical X-space. Because all known baryonic and leptonic matter and the gauge bosons are spinning objects we cannot start the geometrization of matter by assuming that it is only the metric of the kinematical space of the point particle which is modified, because there are no spinless objects in nature. We have to geometrize the complete kinematical space of the spinning particle accordingly. We cannot make things simpler.

6. Conclusions

We consider that General Relativity is a constrained, and therefore a simpler formalism for describing gravity for two reasons: One is that the geometrization of spacetime has to be enlarged to consider Finsler metrics instead of pseudo-Riemannian metrics, and another that the manifold which describes the boundary states of spinning matter is larger than spacetime.

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The manifold X of the boundary variables of any Lagrangian dynamical system is always a Finsler metric space, so that any variational approach is equivalent to a geodesic statement on this metric manifold. This metric, which is in general a function of the variables $x \in X$ and their derivatives $\dot{x}$, depends on the interactions, and to assume that gravitation only produces a modification of the metric which is only a function of the point x, is a restriction of a more general formalism which allows for this modification, in the spirit of unification of all interactions.

The second constraint of a gravitational theory is that it has to be applied to a manifold larger than spacetime, because spacetime is the boundary manifold of the spinless point particle and spinless elementary particles seem not to exist in nature.

Without any assumption about general covariance, or any assumption about the dynamical behaviour of spacetime, but in a Lagrangian framework, we have analysed several examples of the Finsler space structure of spacetime under different interactions. In all of them the new metrics are true Finsler metrics which in the case of low velocity limit, and therefore a metric independent on the velocities, resemble the metrics obtained in a general relativity formalism but they are not vacuum solutions of Einstein's equations.

It is possible that the spin structure of matter plays no role in the gravitational analysis of the solar system and in a cosmological background, so that the usual treatment in terms of only spacetime variables is sufficient to describe planetary motions. But in cosmological models, when the velocity of particles is not negligible, redshifts of order 6 and higher have been quoted for several galaxies which correspond to velocities of $0.9c$, a metric dependent on the velocities would produce a different analysis than a Riemannian one.

If we need to take into account the spin content of matter, may be in a neutron star where the magnetic moments of the particles are aligned, or in a gravitational colapse with a huge density of matter where gravitational effects associated to the spin structure are expected, it is unavoidable to enlarge the dynamical formalism to include the spin description of matter, in which the space X is larger than spacetime and the metric should depend also on the velocities. It is on this larger manifold that gravity has to be worked out. The physical restrictions have to applied in the analysis of the particular cases, not at the very begining of the formalism.

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Some other works related to this formalism can be obtained through the webpage <http://tp.lc.ehu.es/martin.htm>