

Inflation and dark energy—theory and observational signatures

Shinji Tsujikawa¹

Department of Physics, Faculty of Science, Tokyo University of Science, 1-3, Kagurazaka, Shinjuku-ku, Tokyo 162-8601, Japan

Abstract

The theories of inflation and dark energy are reviewed paying particular attention to their observational signatures. We discuss the classification of inflationary models, primordial density perturbations including non-Gaussianities, and modified gravity models of inflation. We also review theoretical attempts for finding out the origin of dark energy—such as the cosmological constant, modified matter models, and modified gravity models.

1 Introduction

The inflationary paradigm has been the backbone of high-energy cosmology over the past 30 years. Inflation was first introduced [1, 2] as a way of addressing a number of cosmological problems such as horizon and flatness problems. The striking feature of the inflationary cosmology is that it predicts nearly scale-invariant, gaussian, adiabatic density perturbations in its simplest form [3]. This prediction shows an excellent agreement with all existing and accumulated data within observational errors. In particular the temperature anisotropies of Cosmic Microwave Background (CMB) measured by the Wilkinson Microwave Anisotropy Probe (WMAP) [4, 5] have provided the high-precision dataset from which inflationary models can be seriously constrained.

The first model of inflation proposed by Starobinsky [1] is based on a conformal anomaly in quantum gravity. The model in which the Lagrangian density is given by $f(R) = R + \alpha R^2$, where R is a Ricci scalar, can lead to the sufficient cosmic acceleration with a successful reheating [6]. Moreover this model is still allowed from the recent observations of the CMB temperature anisotropies [7]. The idea of “old inflation” [2], which is based on the theory of supercooling during the cosmological phase transition, turned out to be unviable, because the Universe becomes inhomogeneous by the bubble collision after inflation. The revised version dubbed “new inflation” [8, 9], where the second-order transition to true vacuum is responsible for cosmic acceleration, is plagued by a fine-tuning problem for spending enough time in false vacuum. However these pioneering works opened up a new paradigm for the construction of workable inflation models based on particle physics such as superstring theory and supergravity (see e.g., Refs. [10, 11]).

Most of the inflation models, including the Linde’s chaotic inflation [12], have been constructed by using a slow-rolling scalar field with a sufficiently flat potential. One can discriminate between a host of slow-roll inflation models by comparing the theoretical prediction of the spectral index of scalar metric perturbations as well as the ratio between scalar and tensor perturbations with the temperature anisotropies in CMB (see e.g., [13–15]). There are other classes of models called k-inflation [16] in which the field kinetic energy plays an important role to drive cosmic acceleration. Since in k-inflation the scalar propagation speed is different from the speed of light [17], this can give rise to large non-Gaussianities of primordial perturbations [18, 19]. The models based on the modification of gravity (including the Starobinsky’s model [1]) can lead to some peculiar theoretical predictions for inflationary observables. We shall discuss how a host of models can be distinguished from observations.

The observations of supernovae type Ia [20, 21] have shown that the Universe entered the phase of cosmic acceleration after the matter-dominated epoch. The discovery of this late-time accelerated expansion has opened up a new research field called dark energy (DE), see e.g., [22]. If we try to explain the origin of DE based on particle physics, we encounter a problem associated with a very small energy scale. For example, the vacuum energy appearing in particle physics is usually significantly larger than

¹Email address: shinji@rs.kagu.tus.ac.jp

the observed energy density of dark energy ($\rho_{\text{DE}}^{(0)} \simeq 10^{-47} \text{ GeV}^4$) [23]. In this case we need to find out a mechanism to obtain a tiny value of the cosmological constant Λ consistent with observations.

The first step toward understanding the nature of DE is to clarify whether it is a simple cosmological constant or it originates from other sources that dynamically change in time. The dynamical DE models can be distinguished from the cosmological constant by considering the evolution of the equation of state of DE ($= w_{\text{DE}}$). The scalar field models of DE such as quintessence [24, 25] and k-essence [26, 27] predict a wide variety of variations of w_{DE} , but still the current observational data are not sufficient to provide some evidence for the preference of such models over the Λ CDM model. Moreover we require that the field potentials are sufficiently flat, such that the field evolves slowly enough to drive the cosmic acceleration today. This demands that the field is extremely light ($m_\phi \simeq 10^{-33} \text{ eV}$) relative to typical mass scales appearing in particle physics. However it is not entirely hopeless to construct viable scalar-field dark energy models in the framework of particle physics.

There exists another class of dynamical DE models that modify Einstein gravity. The models that belong to this class are $f(R)$ gravity [28, 29] (f is a function of the Ricci scalar R), scalar-tensor theories [30], Dvali, Gabadadze and Porrati (DGP) braneworld model [31], Galileon gravity [32], and so on. The attractive feature of these models is that the cosmic acceleration can be realized without recourse to a dark energy component. If we modify gravity from General Relativity, however, there are in general stringent constraints coming from local gravity tests as well as a number of observational constraints. Hence the restriction on modified gravity models is quite tight compared to modified matter models such as quintessence and k-essence.

We shall review the above mentioned dark energy models and also discuss the current status of observational and experimental constraints on those models.

2 Inflation

2.1 Dynamics and models of inflation

Most of inflationary models are based on a minimally coupled scalar field ϕ (“inflaton”) with a potential $V(\phi)$. In the flat Friedmann-Lemaître-Robertson-Walker (FLRW) background with a scale factor $a(t)$, the energy density and the pressure of the inflaton are given, respectively, by

$$\rho_\phi = \dot{\phi}^2/2 + V(\phi), \quad P_\phi = \dot{\phi}^2/2 - V(\phi), \quad (1)$$

where a dot represents a derivative with respect to cosmic time t . From the Friedmann equation $3H^2 = 8\pi G\rho_\phi$ ($H \equiv \dot{a}/a$ is the Hubble parameter and G is gravitational constant) and the continuity equation $\dot{\rho}_\phi + 3H(\rho_\phi + P_\phi) = 0$, it follows that

$$H^2 = \frac{8\pi}{3m_{\text{pl}}^2} \left[\frac{1}{2}\dot{\phi}^2 + V(\phi) \right], \quad \ddot{\phi} + 3H\dot{\phi} + V_{,\phi}(\phi) = 0, \quad (2)$$

where $m_{\text{pl}} = G^{-1/2}$ is the Planck mass, and $V_{,\phi} \equiv dV/d\phi$. Combination of these equations gives $\ddot{a}/a = 8\pi(V - \dot{\phi}^2)/(3m_{\text{pl}}^2)$, which means that cosmic acceleration ($\ddot{a} > 0$) occurs for $\dot{\phi}^2 < V$. Inflation can be realized by a sufficiently flat potential along which the field evolves slowly. Under the slow-roll conditions $\dot{\phi}^2 \ll V(\phi)$ and $|\ddot{\phi}| \ll |3H\dot{\phi}|$, we have $H^2 \simeq 8\pi V(\phi)/(3m_{\text{pl}}^2)$ and $3H\dot{\phi} \simeq -V_{,\phi}(\phi)$ from (2).

We define the so-called slow-roll parameters [13]

$$\epsilon_V = \frac{m_{\text{pl}}^2}{16\pi} \left(\frac{V_{,\phi}}{V} \right)^2, \quad \eta_V = \frac{m_{\text{pl}}^2 V_{,\phi\phi}}{8\pi V}, \quad \xi_V^2 = \frac{m_{\text{pl}}^4 V_{,\phi} V_{,\phi\phi\phi}}{64\pi^2 V^2}. \quad (3)$$

The sufficient amount of inflation is realized provided that $\{\epsilon_V, |\eta_V|, |\xi_V^2|\} \ll 1$. At leading order in the slow-roll expansion the parameters (3) reduce to $\epsilon_V \simeq \epsilon \equiv -\dot{H}/H^2$, $\eta_V \simeq 2\epsilon - \dot{\epsilon}/(2H\epsilon)$, and $\xi_V^2 \simeq [2\epsilon - \dot{\eta}/(H\eta)]\eta$ [33]. The cosmic acceleration ends when ϵ_V and η_V grow to of order unity. A useful quantity to describe the amount of inflation is the number of e-foldings, defined by

$$N \equiv \ln \frac{a_f}{a} = \int_t^{t_f} H dt \simeq \frac{8\pi}{m_{\text{pl}}^2} \int_{\phi_f}^{\phi} \frac{V}{V_{,\phi}} d\tilde{\phi} = \frac{2\sqrt{\pi}}{m_{\text{pl}}} \int_{\phi_f}^{\phi} \frac{d\tilde{\phi}}{\sqrt{\epsilon_V}}, \quad (4)$$

where the subscript f denotes the evaluation of the quantity at the end inflation. In order to solve the horizon and flatness problems we require that N is larger than 60 [14].

The inflationary models based on a scalar field can be roughly classified in the following way [34]. The first class (type I) consists of the “large field” models, in which the field evolves over a super-Planckian range during inflation, $\Delta\phi > m_{\text{pl}}$. Chaotic inflation [12] is one of the representative models of this class. The second class (type II) consists of the “small field” models, in which the field moves over a small (sub-Planckian) distance: $\Delta\phi < m_{\text{pl}}$. New inflation [8, 9] and natural inflation [35] are the examples of this type (although in these models there are some cases in which the field evolves over a super-Planckian range). In the first class one usually has $V_{,\phi\phi} > 0$, whereas $V_{,\phi\phi}$ can change the sign in the second class. The third class (type III) consists of the hybrid inflation models [36, 37], in which inflation typically ends by a phase transition triggered by the presence of a second scalar field. The fourth class (type IV) consists of the double inflation models in which there exist two dynamical scalar fields leading to the two stages of inflation. A simple example is two light massive scalar fields [38] (see also Refs. [39]).

We note that several models of inflation cannot be classified in the above four classes. For example, there are some models in which the potential does not have a minimum—such as inflation with an exponential potential [40], quintessential inflation [41], and tachyon inflation [42]. Typically these scenarios suffer from a reheating problem unless some modifications to the potential are taken into account. There exist other models of inflation in which an accelerated expansion is realized without using the potential of the inflaton. For example, k-inflation [16] and ghost inflation [43] belong to this class. In this case inflation occurs in the presence of non-linear kinetic terms of the scalar field. Inflation can also be realized by higher-order curvature terms [1, 44].

2.2 Linear density perturbations and observational constraints

It is possible to distinguish between a host of inflationary models from primordial density perturbations generated during inflation. Consider the general theories [16] described by the action

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\text{pl}}^2}{2} R + P(\phi, X) \right], \quad (5)$$

where $M_{\text{pl}} = (8\pi G)^{-1/2}$ is the reduced Planck mass, R is a Ricci scalar, and $P(\phi, X)$ is the Lagrangian dependent on the field ϕ and the kinetic energy $X = -(1/2)g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$. In addition to the standard canonical field with a potential, the action (5) includes a wide variety of theories such as low energy effective string theory with derivative terms [45], ghost condensate model [46], tachyon field [47], and DBI theories [48]. In the following we shall use the unit $M_{\text{pl}}^2 = 1$, but we sometimes restore M_{pl} when the dimension matters.

The line-element describing scalar perturbations Ψ, B, Φ, E and tensor perturbations h_{ij} about the flat FLRW background is given by [50]

$$ds^2 = -(1 + 2\Psi)dt^2 + 2a(t)B_{,i}dx^i dt + a^2(t)[(1 + 2\Phi)\delta_{ij} + 2E_{,ij} + h_{ij}]dx^i dx^j, \quad (6)$$

One can derive the action for the scalar functions Ψ, B, Φ, E together with the inflaton fluctuation $\delta\phi$. Integrating the action (5) by parts and using the background equations of motion, the second-order action for these perturbations can be written in terms of the gauge-invariant comoving curvature perturbation [51]

$$\mathcal{R} = \Phi - H\delta\phi/\dot{\phi}. \quad (7)$$

Choosing the comoving gauge $\delta\phi = 0$, for example, the second-order action for the curvature perturbation takes the following form [17]

$$S_2 = \int dt d^3x a^3 \frac{\epsilon}{c_s^2} \left[\dot{\mathcal{R}}^2 - \frac{c_s^2}{a^2} \partial^i \mathcal{R} \partial_i \mathcal{R} \right], \quad (8)$$

where $\epsilon = -\dot{H}/H^2$, and c_s^2 is the scalar propagation speed squared, given by [17]

$$c_s^2 = \frac{P_{,X}}{P_{,X} + 2XP_{,XX}}. \quad (9)$$

We write \mathcal{R} in Fourier space with the comoving wave number \mathbf{k} , as

$$\mathcal{R}(\tau, \mathbf{x}) = \frac{1}{(2\pi)^3} \int d^3\mathbf{k} \mathcal{R}(\tau, \mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}}, \quad \mathcal{R}(\tau, \mathbf{k}) = u(\tau, \mathbf{k})a(\mathbf{k}) + u^*(\tau, -\mathbf{k})a^\dagger(-\mathbf{k}), \quad (10)$$

where $a(\mathbf{k})$ and $a^\dagger(\mathbf{k})$ are the annihilation and creation operators, respectively, satisfying the commutation relations $[a(\mathbf{k}_1), a^\dagger(\mathbf{k}_2)] = (2\pi)^3 \delta^{(3)}(\mathbf{k}_1 - \mathbf{k}_2)$, $[a(\mathbf{k}_1), a(\mathbf{k}_2)] = [a^\dagger(\mathbf{k}_1), a^\dagger(\mathbf{k}_2)] = 0$. Note that $\tau = \int a^{-1} dt$ is a conformal time, which can be expressed as $\tau = -1/(aH)$ in the de Sitter background.

The equation for the Fourier mode u follows from the action (8). Introducing new variables $v = zu$ and $z = (a\sqrt{2\epsilon}/c_s)u$, it follows that

$$v'' + (c_s^2 k^2 - z''/z) v = 0, \quad (11)$$

where a prime represents a derivative with respect to τ . In the quasi de Sitter background with a slow variation of c_s^2 we can approximate $z''/z \simeq 2/\tau^2$. The solution to Eq. (11), which recovers the Bunch-Davis vacuum state $v = e^{-ic_s k \tau} / \sqrt{2c_s k}$ in the asymptotic past, ($k\tau \rightarrow -\infty$) is given by

$$u = i \frac{e^{-ic_s k \tau}}{2k^{3/2} \sqrt{\epsilon c_s}} H(1 + ic_s k \tau). \quad (12)$$

After the perturbations leave the Hubble radius ($c_s k \ll aH$), the asymptotic solution for $k\tau \rightarrow 0$ is described by $u \simeq H/[2k^{3/2} \sqrt{\epsilon c_s}]$. The power spectrum $\mathcal{P}_{\mathcal{R}}$ of the curvature perturbation is defined by $\langle \mathcal{R}(\mathbf{k}_1) \mathcal{R}(\mathbf{k}_2) \rangle = (2\pi^2/k_1^3) \mathcal{P}_{\mathcal{R}}(k_1) \cdot (2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2)$. We then obtain [17]

$$\mathcal{P}_{\mathcal{R}} = \frac{1}{8\pi^2 M_{\text{pl}}^2} \frac{H^2}{c_s \epsilon}, \quad (13)$$

which is evaluated at $c_s k = aH$. The spectral index is

$$n_{\mathcal{R}} - 1 \equiv \left. \frac{d \ln \mathcal{P}_{\mathcal{R}}}{d \ln k} \right|_{c_s k = aH} = -2\epsilon - \eta - s, \quad (14)$$

where

$$\epsilon = -\dot{H}/H^2, \quad \eta = \dot{\epsilon}/(H\epsilon), \quad s = \dot{c}_s/(Hc_s). \quad (15)$$

The tensor perturbation h_{ij} satisfies the same equation as that for a massless scalar field. The spectrum of tensor perturbations and its spectral index are given by

$$\mathcal{P}_T = \frac{2H^2}{\pi^2 M_{\text{pl}}^2}, \quad n_T \equiv \left. \frac{d \ln \mathcal{P}_T}{d \ln k} \right|_{c_s k = aH} = -2\epsilon. \quad (16)$$

The tensor-to-scalar ratio is

$$r \equiv \frac{\mathcal{P}_T}{\mathcal{P}_{\mathcal{R}}} = 16c_s \epsilon = -8c_s n_T. \quad (17)$$

In inflation models with a standard kinetic term (i.e. $P = X - V(\phi)$) the propagation speed is $c_s = 1$. In this case it follows that

$$n_{\mathcal{R}} - 1 = -6\epsilon_V + 2\eta_V, \quad n_T = -2\epsilon_V, \quad r = 16\epsilon_V, \quad (18)$$

where we used the slow-roll parameters defined in Eq. (3). The runnings of the spectral indices are given by

$$\alpha_{\mathcal{R}} \equiv \left. \frac{dn_{\mathcal{R}}}{d \ln k} \right|_{k=aH} = 16\epsilon_V \eta_V - 24\epsilon_V^2 - 2\xi_V^2, \quad \alpha_T \equiv \left. \frac{dn_T}{d \ln k} \right|_{k=aH} = -4\epsilon_V (2\epsilon_V - \eta_V). \quad (19)$$

We can evaluate the above observables for given inflaton potentials. Let us consider chaotic inflation [12] with the potential $V(\phi) = V_0 \phi^n$. In this case the number of e-foldings is given by $N = 4\pi/(nm_{\text{pl}})(\phi^2 - \phi_f^2)$,

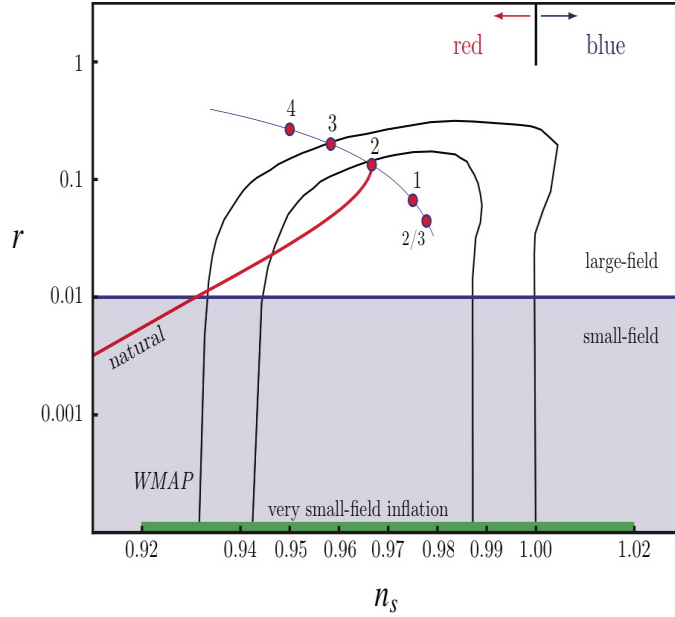


Figure 1: Observational constraints (1σ and 2σ contours) on single-field inflation models in the $(n_{\mathcal{R}}, r)$ plane ($n_{\mathcal{R}}$ is denoted as n_s in the figure). We also show the theoretical prediction of chaotic inflation with $V(\phi) = V_0\phi^n$ (the power $n = 4, 3, 2, 1, 2/3$) and natural inflation with $V(\phi) = V_0[1 - \cos(\phi/\mu)]$. The border between large-field and small-field models is characterized by $r = 0.01$. From Ref. [53].

where $\phi_f = nm_{\text{pl}}/4\sqrt{\pi}$ is value of inflaton at the end of inflation (at which $\epsilon_V = 1$). The scalar spectral index $n_{\mathcal{R}}$ and the tensor-to-scalar ratio r are given by

$$n_{\mathcal{R}} = 1 - \frac{2(n+2)}{4N+1}, \quad r = \frac{16n}{4N+1}. \quad (20)$$

The number of e-foldings relevant to the CMB anisotropies corresponds to $N_{\text{CMB}} = 50-60$. Figure 1 shows theoretical predictions for the models $n = 4, 3, 2, 1, 2/3$ ($N_{\text{CMB}} = 60$), with the bounds constrained by the WMAP 5-year data [52]. The models with $n \leq 3$ are within the 2σ contour, but the self-coupling model ($n = 4$) is not allowed observationally. Hybrid inflation [36] gives rise to the scalar spectral index $n_{\mathcal{R}}$ larger than 1 with a suppressed tensor-to-scalar ratio ($r \ll 1$), which is also in tension with observations.

From Eqs. (4) and (17) we find that r is related to the variation of the field ϕ , as $r = (8/M_{\text{pl}}^2)(d\phi/dN)^2$. The total variation of the field between the time when the perturbations exited the Hubble radius (at $N = N_{\text{CMB}}$) and the end of inflation (at $N = N_f$) is then given by $\Delta\phi/M_{\text{pl}} = \int_{N_f}^{N_{\text{CMB}}} dN \sqrt{r/8}$. Provided that the variation of r is not much, we obtain the Lyth's bound [54]

$$\Delta\phi/M_{\text{pl}} = \mathcal{O}(1) \times (r/0.01)^{1/2}. \quad (21)$$

For large-field inflation with $\Delta\phi > M_{\text{pl}}$ we have $r > 0.01$ (as in chaotic inflation). The small-field models ($\Delta\phi < M_{\text{pl}}$) gives rise to a suppressed tensor-to-scalar ratio: $r < 0.01$. The natural inflation model with the potential $V(\phi) = V_0[1 - \cos(\phi/\mu)]$ belongs to either large-field or small-field, depending on the initial conditions of ϕ . The models motivated by string theory, such as D-brane inflation [55], racetrack inflation [56], and Kähler inflation [57] usually predict a very small tensor-to-scalar ratio ($r < 10^{-3}$). It may be difficult to see the signatures of those models even with the Planck satellite.

In k-inflation the propagation speed c_s is different from 1, so we need to use the results (14) and (17) to confront the models with observations. In such models the non-Gaussianity of curvature perturbations can be large, as we will see in what follows.

2.3 Non-Gaussian perturbations

The standard inflation with a canonical kinetic term predicts a nearly Gaussian distribution of primordial perturbations [58, 59], but in k-inflation it is possible to give rise to large non-Gaussianities. The first non-trivial statistics describing the non-Gaussianities of the curvature perturbation \mathcal{R} is the bispectrum defined by

$$\langle \mathcal{R}(\mathbf{k}_1) \mathcal{R}(\mathbf{k}_2) \mathcal{R}(\mathbf{k}_3) \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) (\mathcal{P}_{\mathcal{R}})^2 B(k_1, k_2, k_3), \quad (22)$$

where B depends on inflation models.

Let us consider k-inflation models described by the action (5). The vacuum expectation value of the three-point correlation function of \mathcal{R} can be obtained by using the interaction picture in quantum field theory, as [60]

$$\langle \mathcal{R}(t, \mathbf{k}_1) \mathcal{R}(t, \mathbf{k}_2) \mathcal{R}(t, \mathbf{k}_3) \rangle = -i \int_{t_0}^t d\tilde{t} \langle 0 | [\mathcal{R}(t, \mathbf{k}_1) \mathcal{R}(t, \mathbf{k}_2) \mathcal{R}(t, \mathbf{k}_3), H_I(\tilde{t})] | 0 \rangle, \quad (23)$$

where t_0 is the initial time during inflation (at which the perturbations are deep inside the Hubble radius) and t is some time after the Hubble radius crossing. The interaction Hamiltonian H_I can be derived by expanding the action (5) at the third-order in perturbations, as $H_I = -L_3$, where L_3 is the third-order Lagrangian (i.e. $S_3 = \int dt L_3$).

In order to evaluate the third-order Lagrangian \mathcal{L}_3 it is convenient to use the ADM metric $ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$ with $h_{ij} = a^2 e^{2\mathcal{R}} \delta_{ij}$. In this case we only need to consider the perturbations of N and N^i at first-order in \mathcal{R} , with the comoving gauge $\delta\phi = 0$ [59]. Using the first-order solution (12) and commutation relations for the creation and annihilation operators, we obtain the three-point correlation long after the Hubble radius crossing [19]:

$$\langle \mathcal{R}(\mathbf{k}_1) \mathcal{R}(\mathbf{k}_2) \mathcal{R}(\mathbf{k}_3) \rangle = (2\pi)^7 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) (\mathcal{P}_{\mathcal{R}})^2 \mathcal{A} / (k_1^3 k_2^3 k_3^3), \quad (24)$$

with

$$\begin{aligned} \mathcal{A} = & \left(\frac{1}{c_s^2} - 1 - \frac{2\lambda}{\Sigma} \right) \frac{3k_1^2 k_2^2 k_3^2}{2K^3} + \left(\frac{1}{c_s^2} - 1 \right) \left(-\frac{1}{K} \sum_{i>j} k_i^2 k_j^2 + \frac{1}{2K^2} \sum_{i \neq j} k_i^2 k_j^3 + \frac{1}{8} \sum_i k_i^3 \right) \\ & + \frac{\epsilon}{c_s^2} \left(-\frac{1}{8} \sum_i k_i^3 + \frac{1}{8} \sum_{i \neq j} k_i k_j^2 + \frac{1}{K} \sum_{i>j} k_i^2 k_j^2 \right) + \frac{\eta}{c_s^2} \left(\frac{1}{8} \sum_i k_i^3 \right) \\ & + \frac{s}{c_s^2} \left(-\frac{1}{4} \sum_i k_i^3 - \frac{1}{K} \sum_{i>j} k_i^2 k_j^2 + \frac{1}{2K^2} \sum_{i \neq j} k_i^2 k_j^3 \right), \end{aligned} \quad (25)$$

where $K = k_1 + k_2 + k_3$, $\Sigma = X P_{,X} + 2X^2 P_{,XX}$, and $\lambda = X^2 P_{,XX} + 2X^3 P_{,XXX}/3$. We take a factorizable shape function B in the form $B = (2\pi)^4 (9f_{nl}/10) [-1/(k_1^3 k_2^3) - 1/(k_1^3 k_3^3) - 1/(k_2^3 k_3^3) - 2/(k_1^2 k_2^2 k_3^2) + 1/(k_1 k_2^2 k_3^3) + (5 \text{ perm.})]$, where the permutations act on the last term in parenthesis. For the equilateral triangles where $k_1 = k_2 = k_3$ it follows that [18, 19]

$$f_{nl}^{\text{equil}} = -\frac{85}{324} \left(\frac{1}{c_s^2} - 1 + \frac{8\lambda}{17\Sigma} \right) + \frac{55}{36} \frac{\epsilon}{c_s^2} + \frac{5}{12} \frac{\eta}{c_s^2} - \frac{85}{54} \frac{s}{c_s^2}. \quad (26)$$

In standard inflation with a canonical kinetic term one has $c_s^2 = 1$, $\lambda = 0$, and $s = 0$, so that f_{nl}^{equil} is of the order of the slow-roll parameters ϵ and η . In k-inflation models such as DBI inflation [48] one has $|f_{nl}^{\text{equil}}| \gg 1$ for $c_s^2 \ll 1$, which can be testable in future observations. Non-Gaussianities in multi-field inflation has been also studied by a number of authors [61]. For the k-inflation models described by the action $P = Xg(Y)$, where g is an arbitrary function in terms of $Y = X e^{\mu\phi/M_{\text{Pl}}}$ (μ is a constant), assisted inflation [62] occurs in the presence of multiple scalar fields [63]. It is of interest to see whether such assisted k-inflation models can give rise to large non-Gaussianities.

2.4 Modified gravitational models of inflation

There are a number of inflation models based on the modification of gravity—such as $f(R)$ gravity and scalar-tensor theories. Let us consider the action in $f(R)$ gravity

$$S = \frac{M_{\text{pl}}^2}{2} \int d^4x \sqrt{-g} f(R), \quad (27)$$

where f is an arbitrary function in terms of the Ricci scalar R . In the Starobinsky's model described by the Lagrangian $f(R) = R + R^2/(6M^2)$ [1] the slow-roll parameter $\epsilon = -\dot{H}/H^2$ is approximately given by $\epsilon \simeq M^2/(6H^2)$, so that inflation occurs for $H \gtrsim M \simeq 10^{13}$ GeV. The mass scale M is determined by the WMAP normalization of the CMB temperature anisotropies [7]. The dynamical analysis of this model was carried out in Refs. [65].

The spectra of scalar and tensor perturbations can be evaluated directly for the Jordan frame action (27). Another way is to transform the action (27) to that in the Einstein frame under the conformal transformation $\tilde{g}_{\mu\nu} = F g_{\mu\nu}$, where $F \equiv \partial f / \partial R$. The action in the Einstein frame is given by [66]

$$S_E = \int d^4x \sqrt{-\tilde{g}} \left[\frac{M_{\text{pl}}^2}{2} \tilde{R} - \frac{1}{2} \tilde{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right], \quad (28)$$

where $\phi = \sqrt{3/2} M_{\text{pl}} \ln F$ and $V(\phi) = (FR - f)M_{\text{pl}}^2/(2F^2)$. Under the conformal transformation the perturbed metric (6) is transformed as

$$d\tilde{s}^2 = F ds^2 = -(1 + 2\tilde{\Psi})d\tilde{t}^2 + 2\tilde{a}(\tilde{t})\tilde{B}_{,i}d\tilde{x}^i d\tilde{t} + \tilde{a}^2(\tilde{t}) \left[(1 + 2\tilde{\Phi})\delta_{ij} + 2\tilde{E}_{,ij} + \tilde{h}_{ij} \right] d\tilde{x}^i d\tilde{x}^j, \quad (29)$$

where $d\tilde{t} = \sqrt{F}dt$ and $\tilde{a} = \sqrt{F}a$. We decompose the conformal factor $F(t, \mathbf{x})$ into the background and perturbed parts: $F(t, \mathbf{x}) = \bar{F}(t) [1 + \delta F(t, \mathbf{x})/\bar{F}(t)]$. In what follows we omit a bar from F . The transformation of scalar metric perturbations is given by

$$\tilde{\Psi} = \Psi + \delta F/(2F), \quad \tilde{B} = B, \quad \tilde{\Phi} = \Phi + \delta F/(2F), \quad \tilde{E} = E, \quad (30)$$

whereas $\tilde{h}_{ij} = h_{ij}$ for tensor perturbations. Under the above transformation one can easily show that the curvature perturbation $\mathcal{R} = \Phi - H\delta F/\dot{F}$ is invariant, i.e. $\tilde{\mathcal{R}} = \mathcal{R}$. Since the tensor perturbation is also invariant, the tensor-to-scalar ratio \tilde{r} in the Einstein frame is identical to that in the Jordan frame.

For example, let us consider the model $f(R) = R + R^2/(6M^2)$. For this model the potential $V(\phi)$ in the Einstein frame is given by

$$V(\phi) = \frac{3}{4} M^2 M_{\text{pl}}^2 \left(1 - e^{-\sqrt{2/3}\phi/M_{\text{pl}}} \right), \quad \phi = \sqrt{3/2} M_{\text{pl}} \ln[1 + R/(3M^2)]. \quad (31)$$

In the regime $\phi \gg M_{\text{pl}}$ this potential is nearly constant ($V(\phi) \simeq 3M^2 M_{\text{pl}}^2/4$), which leads to slow-roll inflation. Meanwhile, in the regime $\phi \ll M_{\text{pl}}$, one has $V(\phi) \simeq (1/2)M^2\phi^2$, so that the field oscillates around $\phi = 0$ with a Hubble damping. During inflation the slow-roll parameters defined in Eq. (15) are approximately given by $\epsilon \simeq 3/(4N^2)$ and $\eta \simeq 3/(4N^2) - 1/N$, where $N \simeq (3/4)e^{\sqrt{2/3}\phi/M_{\text{pl}}}$ is the number of e-foldings from the end of inflation to the epoch of the first horizon crossing during inflation. From Eq. (18) the spectral index of curvature perturbations and the tensor-to-scalar ratio are given, respectively, by

$$n_{\mathcal{R}} - 1 \simeq -2/N, \quad r \simeq 12/N^2, \quad (32)$$

where we have ignored the term of the order of $1/N^2$ in the expression of $n_{\mathcal{R}}$. For the typical value $N = 55$ relevant to the CMB anisotropies we have $n_{\mathcal{R}} \simeq 0.964$ and $r \simeq 4.0 \times 10^{-3}$. These values are allowed by the WMAP5 year constraints: $n_{\mathcal{R}} = 0.960 \pm 0.013$ and $r < 0.22$ (for the negligible running) [52]. The tensor-to-scalar ratio is suppressed compared to chaotic inflation with the potential $V(\phi) = V_0\phi^n$ ($n = 2, 4$), because the potential (31) is almost flat in the regime $\phi \gtrsim M_{\text{pl}}$.

For the model $f(R) = R + R^2/(6M^2)$, reheating proceeds gravitationally through the oscillation of R in the regime $H \lesssim M$ [6]. If a field χ is non-minimally coupled to R through the coupling $\xi R\chi^2/2$,

non-perturbative particle creation called preheating can occur through parametric resonance [67]. In Ref. [68] it was shown that preheating occurs for $|\xi|$ larger than 1.

The equivalence of the curvature perturbation between the Jordan and Einstein frames also holds for scalar-tensor theory with the Lagrangian $\mathcal{L} = F(\phi)R/(2\kappa^2) - (1/2)\omega(\phi)g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - U(\phi)$ [69]. For a non-minimally coupled scalar field with $F(\phi) = 1 - \xi\kappa^2\phi^2$ [70, 71] the spectral indices of scalar and tensor perturbations have been derived by using such equivalence [72]. In particular the self-coupling potential $U(\phi) = \lambda\phi^4/4$ with a largely negative non-minimal coupling ($|\xi| \gg 1$) gives rise to the similar potential to Eq. (31) in the Einstein frame. In this case $n_{\mathcal{R}}$ and r are the same as those given in Eq. (32) [72], so the model can be allowed observationally. Preheating in this model was studied in detail in Ref. [73]. We note that the self-coupling potential with a non-minimal coupling was recently revived as a “Higgs inflation” as a way of realizing inflation with a Higgs field [74].

3 Dark energy

In this section we discuss a number of approaches that have been adopted to try and explain the origin of dark energy. The difference between inflation and dark energy is their associated energy scales—the former is about 10^{13} GeV and the latter is about 10^{-42} GeV. From the view point of particle physics dark energy is more difficult to identify its origin, but still it is not entirely hopeless to do so. In inflationary cosmology the energy density of a scalar degree of freedom should vary in time to end inflation, but dark energy can be either the cosmological constant (constant energy density) or some other dynamical source.

3.1 Cosmological constant

The cosmological constant Λ is the simplest candidate of dark energy and it is favored by a number of observations [5, 20, 21, 75]. However it suffers from a serious problem of energy scale if it originates from a vacuum energy appearing in particle physics. The zero-point energy of some field of mass m with momentum k and frequency ω is given by $E = \omega/2 = \sqrt{k^2 + m^2}/2$ (in the units of $\hbar = c = 1$). Summing over the zero-point energies of this field up to a cut-off scale k_{\max} ($\gg m$), we obtain the vacuum energy density

$$\rho_{\text{vac}} = \int_0^{k_{\max}} \frac{4\pi k^2 dk}{(2\pi)^3} \frac{1}{2} \sqrt{k^2 + m^2} \approx \frac{k_{\max}^4}{16\pi^2}, \quad (33)$$

where we have used the fact that the integral is dominated by the mode with k larger than m . Taking the cut-off scale k_{\max} to be the Planck scale m_{pl} , the vacuum energy density can be estimated as $\rho_{\text{vac}} \simeq 10^{74} \text{ GeV}^4$. This is about 10^{121} times larger than the observed value $\rho_{\text{DE}}^{(0)} \simeq 10^{-47} \text{ GeV}^4$.

Before the observational discovery of dark energy in 1998 [20, 21], most people believed that the cosmological constant was exactly zero and tried to explain why it is so. The vanishing of a constant may imply the existence of some symmetry. In supersymmetric theories the bosonic degree of freedom has its Fermi counter part that contributes to the zero point energy with an opposite sign. If supersymmetry is unbroken, an equal number of bosonic and fermionic degrees of freedom is present such that the total vacuum energy vanishes. However it is known that supersymmetry is broken at sufficient high energies (for the typical scale $M_{\text{SUSY}} = 10^3 \text{ GeV}$). Hence the vacuum energy is generally non-zero in the world of broken supersymmetry.

Even if supersymmetry is broken there is a hope to obtain a vanishing Λ or a tiny amount of Λ . In supergravity theory an (effective) cosmological constant is given by an expectation value of the potential V for chiral scalar fields φ^i :

$$V(\varphi, \varphi^*) = e^{\kappa^2 K} \left[D_i W (K^{ij*}) (D_j W)^* - 3\kappa^2 |W|^2 \right], \quad (34)$$

where $\kappa^2 = 8\pi G = 1/M_{\text{pl}}^2$, K and W are the so-called Kähler potential and the superpotential, respectively (which are functions of φ^i and φ^{i*}). The quantity K^{ij*} is the inverse of the derivative $K_{ij*} \equiv \partial^2 K / \partial \varphi^i \partial \varphi^{j*}$, whereas the derivative $D_i W$ is defined by $D_i W \equiv \partial W / \partial \varphi^i + \kappa^2 W (\partial K / \partial \varphi^i)$.

The breaking of the supersymmetry corresponds to the condition $D_i W \neq 0$. In this case it is possible to find scalar field values giving the vanishing potential ($V = 0$), but this is not in general an equilibrium

point of the potential V . Nevertheless there is a class of Kähler potentials and superpotentials giving a stationary scalar-field configuration at $V = 0$. Consider, for example, the gluino condensation in $E_8 \times E_8$ superstring theory [76]. The reduction of the 10-dimensional action to 4-dimensions gives rise to a so-called modulus field T . This field characterizes the scale of the compactified 6-dimensional manifold. There exists another complex scalar field S of 4-dimensional dilaton/axion fields. The fields T and S are governed by the Kähler potential

$$K(T, S) = -(3/\kappa^2) \ln(T + T^*) - (1/\kappa^2) \ln(S + S^*), \quad (35)$$

where $(T + T^*)$ and $(S + S^*)$ are positive definite. The field S couples to the gauge fields, while T does not. An effective superpotential for S can be obtained by integrating out the gauge fields under the use of the R -invariance [77]:

$$W(S) = M_{\text{pl}}^3 [c_1 + c_2 \exp(-3S/2b)], \quad (36)$$

where c_1, c_2 , and b are constants.

Substituting Eqs. (35) and (36) into Eqs. (34), we obtain the field potential

$$V = \frac{M_{\text{pl}}^4}{(T + T^*)^3 (S + S^*)} \left| c_1 + c_2 \exp(-3S/2b) \left\{ 1 + \frac{3}{2b} (S + S^*) \right\} \right|^2. \quad (37)$$

This potential is positive because of the cancellation of the last term in Eq. (34). The stationary field configuration with $V = 0$ is realized under the condition $D_S W = \partial W / \partial S - W / (S + S^*) = 0$. Note that the derivative, $D_T W = \kappa^2 W \partial K / \partial T = -3W / (T + T^*)$, does not necessarily vanish. When $D_T W \neq 0$ the supersymmetry is broken with a vanishing potential energy. Hence it is possible to obtain a stationary field configuration with $V = 0$ even if supersymmetry is broken.

The discussion above is based on the lowest-order perturbation theory. This picture is not necessarily valid to all finite orders of perturbation theory because the non-supersymmetric field configuration is not protected by any symmetry. Moreover some non-perturbative effect can provide a large contribution to the effective cosmological constant. The so-called flux compactification in type IIB string theory allows us to realize a metastable de Sitter (dS) vacuum by taking into account a non-perturbative correction to the superpotential (coming from brane instantons) as well as a number of anti D3-branes in a warped geometry [78]. Hence it is not hopeless to obtain a small value of Λ or a vanishing Λ even in the presence of some non-perturbative corrections.

Kachru, Kallosh, Linde and Trivedi (KKLT) [78] constructed dS solutions in the type II string theory compactified on a Calabi-Yau manifold in the presence of flux. The construction of the dS vacua in the KKLT scenario consists of two steps. The first step is to freeze all moduli fields in the flux compactification at a supersymmetric anti de Sitter (AdS) vacuum. Then a small number of the anti D3-brane is added in a warped geometry with a throat, so that the AdS minimum is uplifted to yield a dS vacuum with broken supersymmetry. If we want to use the KKLT dS minimum derived above for the present cosmic acceleration, we require that the potential energy V_{dS} at the minimum is of the order of $V_{\text{dS}} \simeq 10^{-47} \text{ GeV}^4$. Depending on the number of fluxes there are a vast of dS vacua, which opened up a notion called string landscape [79]. The question why the vacuum we live in has a very small energy density among many possible vacua has been sometimes answered with anthropic arguments. Some people have studied landscape statistics by considering the relative abundance of long-lived low-energy vacua [80]. These statistical approaches are still under study, but it will be interesting to pursue the possibility to obtain high probabilities for the appearance of low-energy vacua.

In the following we shall consider alternative models of dark energy to the cosmological constant, under the assumption that the cosmological constant problem is solved in such a way that it vanishes completely.

3.2 Modified matter models

In this section we discuss “modified matter models” in which the energy-momentum tensor $T_{\mu\nu}$ on the r.h.s. of the Einstein equations contains an exotic matter source with a negative pressure. The models that belong to this class are quintessence [24, 25], k-essence [26, 27], and perfect fluid models. In what follows we shall discuss quintessence and k-essence. Note that there is a perfect fluid model described by

the equation of state $P = -A/\rho$ ($A > 0$) (Chaplygin gas) [81], but this model was already excluded by the observations of large-scale structure [82].

3.2.1 Quintessence

Many scalar fields are present in particles physics—including string theory and supergravity. We use “quintessence” [25] to denote a canonical scalar field ϕ with a potential $V(\phi)$. The action of quintessence is described by [24]

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\text{pl}}^2}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right] + S_M, \quad (38)$$

where M_{pl} is the reduced Planck mass, and R is the Ricci scalar. As a matter action S_M we consider a perfect fluid with the energy density ρ_M , the pressure P_M , and the equation of state $w_M = P_M/\rho_M$.

In a flat FLRW background the perfect fluid satisfies the continuity equation $\dot{\rho}_M + 3H(\rho_M + P_M) = 0$ and the field ϕ obeys the second of Eq. (2). The field equation of state is given by

$$w_\phi \equiv \frac{P_\phi}{\rho_\phi} = \frac{\dot{\phi}^2 - 2V(\phi)}{\dot{\phi}^2 + 2V(\phi)}, \quad (39)$$

where ρ_ϕ and P_ϕ are given in Eq. (1). We also have the following Friedmann equation

$$H^2 = \frac{1}{3M_{\text{pl}}^2} \left[\frac{1}{2} \dot{\phi}^2 + V(\phi) + \rho_M \right]. \quad (40)$$

During radiation and matter dominated epochs, the energy density ρ_M of the fluid dominates over that of quintessence, i.e. $\rho_M \gg \rho_\phi$. We require that ρ_ϕ tracks ρ_M so that the dark energy density dominates at late times. Whether this tracking behavior occurs or not depends on the form of the potential $V(\phi)$. If the potential is steep so that the condition $\dot{\phi}^2/2 \gg V(\phi)$ is always satisfied, we have $w_\phi \simeq 1$ from Eq. (39). In this case the energy density of the field evolves as $\rho_\phi \propto a^{-6}$, which decreases much faster than the background fluid density.

We require the condition $w_\phi < -1/3$ to realize the late-time cosmic acceleration, which translates into the condition $\dot{\phi}^2 < V(\phi)$. Hence the scalar potential needs to be shallow enough for the field to evolve slowly along the potential. This situation is similar to that in inflationary cosmology and it is convenient to introduce the following slow-roll parameters given in Eq. (3). If the conditions $\epsilon_V \ll 1$ and $|\eta_V| \ll 1$ are satisfied, the evolution of the field is sufficiently slow so that $|\ddot{\phi}| \ll |3H\dot{\phi}|$ and $\dot{\phi}^2 \ll V(\phi)$.

The deviation of w_ϕ from -1 is given by

$$1 + w_\phi = \frac{V_{,\phi}^2}{9H^2(\mu + 1)^2\rho_\phi}, \quad (41)$$

where $\mu \equiv \ddot{\phi}/(3H\dot{\phi})$. This shows that w_ϕ is always larger than -1 for the positive potential. In the slow-roll limit, $|\xi_s| \ll 1$ and $\dot{\phi}^2/2 \ll V(\phi)$, we obtain $1 + w_\phi \simeq 2\epsilon_V/3$ by neglecting the matter fluid in Eq. (40), i.e. $3H^2 \simeq V(\phi)/M_{\text{pl}}^2$. The deviation of w_ϕ from -1 is characterized by the slow-roll parameter ϵ_V .

So far many quintessence potentials have been proposed. Crudely speaking they have been classified into (i) “freezing models” and (ii) “thawing” models [83]. In the class (i) the field was rolling along the potential in the past, but the movement gradually slows down after the system enters the phase of cosmic acceleration. In this case the field equation of state w_ϕ decreases toward -1 . In the class (ii) the field (with mass m_ϕ) has been frozen by Hubble friction (i.e. the term $H\dot{\phi}$) until recently and then it begins to evolve once H drops below m_ϕ . In this case the equation of state of dark energy is $w_\phi \simeq -1$ at early times, which is followed by the growth of w_ϕ . The representative potentials that belong to these two classes are

- Freezing models:

$$(A) \ V(\phi) = M^{4+n} \phi^{-n} \quad (n > 0), \quad (B) \ V(\phi) = M^{4+n} \phi^{-n} \exp(\alpha \phi^2/m_{\text{pl}}^2).$$

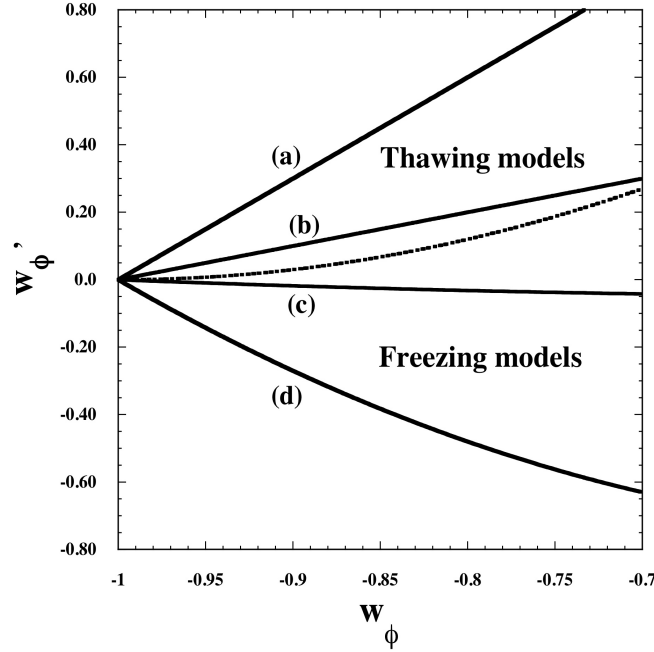


Figure 2: The allowed region in the (w_ϕ, w'_ϕ) plane for thawing and freezing models of quintessence (here primes denote the derivative with respect to $N = \ln a$). The thawing models correspond to the region between two curves: (a) $w'_\phi = 3(1 + w_\phi)$ and (b) $w'_\phi = 1 + w_\phi$, whereas the freezing models are characterized by the region between two curves: (c) $w'_\phi = 0.2w_\phi(1 + w_\phi)$ and (d) $w'_\phi = 3w_\phi(1 + w_\phi)$. The dotted line shows the border between the acceleration and deceleration of the field ($\ddot{\phi} = 0$), which corresponds to $w'_\phi = 3(1 + w_\phi)^2$.

- Thawing models:

$$(C) V(\phi) = V_0 + M^{4-n}\phi^n \quad (n > 0), \quad (D) V(\phi) = M^4 \cos^2(\phi/f).$$

The potential (A) does not possess a minimum and hence the field rolls down the potential toward infinity [24, 25]. This appears, for example, in the fermion condensate model as a dynamical supersymmetry breaking [84]. The potential (B) has a minimum at which the field is eventually trapped (corresponding to $w_\phi = -1$). This potential can be constructed in the framework of supergravity [85].

The potential (C) is similar to the one of chaotic inflation ($n = 2, 4$) used in the early Universe (with $V_0 = 0$), while the mass scale M is very different. Note that the model with $n = 1$ was originally proposed by Linde [86] to replace the cosmological constant by a slowly varying field and then it was revised [87] in connection with the possibility to allow for negative values of $V(\phi)$. The Universe will collapse in the future if the system enters the region with $V(\phi) < 0$. The potential (D) appears as that of the Pseudo-Nambu-Goldstone Boson (PNGB). This was introduced by Frieman *et al.* [88] in response to the first tentative suggestions that the universe may be dominated by the cosmological constant. The small mass of the PNGB model required for the late-time cosmic acceleration is protected against radiative corrections, so this model is favored theoretically. In this model the field is nearly frozen at the potential maximum during the period in which the field mass m_ϕ is smaller than H , but it begins to roll down around the present ($m_\phi \simeq H_0$).

The freezing models and the thawing models are characterized by the conditions $w'_\phi \equiv dw_\phi/d\ln(a) < 0$ and $w'_\phi > 0$, respectively. More precisely the allowed regions for the freezing and thawing models are given by $3w_\phi(1 + w_\phi) < w'_\phi \lesssim 0.2w_\phi(1 + w_\phi)$ and $1 + w_\phi \lesssim w'_\phi \lesssim 3(1 + w_\phi)$, respectively [83]. In Fig. 2 we illustrate these borders in the (w_ϕ, w'_ϕ) plane. While the observational data up to now are not sufficient to distinguish freezing and thawing models by the variation of w_ϕ , we may be able to do so with the next decade high-precision observations.

In supernovae Ia observations the Hubble parameter $H(z)$ is estimated by measuring a luminosity distance d_L . From the observational data it is possible to reconstruct quintessence potentials [89]. The reconstruction process is however subject to two general problems. The first is that finding a model containing a trajectory with a given expansion rate does not guarantee that the trajectory is stable. The second is that the actual observational data such as d_L are known at discrete values of redshifts, so we require some smoothing process for the reconstruction. In spite of these potential problems it will be of interest how the future high-precision observations can restrict the forms of quintessence potentials.

3.2.2 k-essence

Let us consider dark energy models in which the late-time cosmic acceleration is driven by a kinetic energy of a scalar field. The action for such models is described by (5) in the presence of matter fluids:

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\text{pl}}^2}{2} R + P(\phi, X) \right] + S_M. \quad (42)$$

The application of these theories to dark energy was first carried out by Chiba *et al.* [26]. Later this was extended to more general cases and the models based on the action (42) were named “k-essence” [27].

The energy density ρ_ϕ and the pressure P_ϕ of the field are given by $\rho_\phi = 2XP_{,\phi} - P$ and $P_\phi = P$, respectively. The equation of state of k-essence is

$$w_\phi = \frac{P_\phi}{\rho_\phi} = \frac{P}{2XP_{,\phi} - P}. \quad (43)$$

As long as the condition $|2XP_{,\phi}| \ll |P|$ is satisfied, w_ϕ can be close to -1 . For example, in the ghost condensate model [46] given by $P = -X + X^2/M^4$, we have

$$w_\phi = \frac{1 - X/M^4}{1 - 3X/M^4}, \quad (44)$$

which gives $-1 < w_\phi < -1/3$ for $1/2 < X/M^4 < 2/3$. In particular a de Sitter solution ($w_\phi = -1$) is realized at $X/M^4 = 1/2$. Since the field energy density is $\rho_\phi = M^4/4$ at the de Sitter point, it is possible to explain the present cosmic acceleration for $M \sim 10^{-3}$ eV. There is also a modified version of the above model, $P = -X + e^{\lambda\phi/M_{\text{pl}}} X^2/M^4$, which is called dilatonic ghost condensate model [49]. The correction of the type $e^{\lambda\phi/M_{\text{pl}}} X^2/M^4$ can arise as a dilatonic higher-order correction to the tree-level string action.

In k-essence it can happen that the linear kinetic energy in X has a negative sign. Such a field, called a phantom or ghost scalar field [90], suffers from a quantum instability problem unless higher-order terms in X or ϕ are taken into account in the Lagrangian density. In the (dilatonic) ghost condensate scenario it is possible to avoid this quantum instability by the presence of the term X^2 . The quantum stability conditions for k-essence which come from the positive definiteness of the Hamiltonian are given by [49]

$$P_{,\phi} + 2XP_{,\phi\phi} \geq 0, \quad P_{,\phi} \geq 0, \quad (45)$$

$$P_{,\phi\phi} \leq 0. \quad (46)$$

The instability prevented by the condition (46) is of the tachyonic type and generally much less dramatic than the conditions (45). The scalar propagation speed c_s is defined in Eq. (9), which is positive under the conditions (45).

In the dilatonic ghost condensate model ($P = -X + e^{\lambda\phi/M_{\text{pl}}} X^2/M^4$), for example, the conditions (45) are ensured for $e^{\lambda\phi/M_{\text{pl}}} X/M^4 \geq 1/2$ with the sound speed smaller than 1 (speed of light). Some k-essence models have been proposed to solve the coincidence problem of dark energy by the existence of tracker solutions [27]. In such cases, however, it was shown that the sound speed becomes superluminal ($c_s > 1$) before reaching the accelerated attractor [91].

3.3 Modified gravity models

There is another class of dark energy models in which gravity is modified from General Relativity (GR). We discuss a number of cosmological and gravitational aspects of $f(R)$ gravity and DGP braneworld.

3.3.1 $f(R)$ gravity

Let us consider the action (27) in $f(R)$ gravity in the presence of matter fluids:

$$S = \frac{M_{\text{pl}}^2}{2} \int d^4x \sqrt{-g} f(R) + S_M, \quad (47)$$

where f is a function of the Ricci scalar R , and S_M is a matter action for perfect fluids. The field equation can be derived by varying the action (47) with respect to $g_{\mu\nu}$:

$$F(R)R_{\mu\nu}(g) - \frac{1}{2}f(R)g_{\mu\nu} - \nabla_\mu \nabla_\nu F(R) + g_{\mu\nu} \square F(R) = T_{\mu\nu}/M_{\text{pl}}^2, \quad (48)$$

where $F(R) \equiv \partial f / \partial R$, and $T_{\mu\nu}$ is an energy-momentum tensor of matter². The trace of Eq. (48) is given by

$$3\square F(R) + F(R)R - 2f(R) = T/M_{\text{pl}}^2, \quad (49)$$

where $T = g^{\mu\nu}T_{\mu\nu} = -\rho_M + 3P_M$. Here ρ_M and P_M are the energy density and the pressure of matter, respectively.

The de Sitter fixed point corresponds to a vacuum solution at which the Ricci scalar is constant. Since $\square F(R) = 0$ at this point, we obtain

$$F(R)R - 2f(R) = 0. \quad (50)$$

The model $f(R) = \alpha R^2$ satisfies this condition and hence it gives rise to an exact de Sitter solution. It is possible to construct viable dark energy models based on $f(R)$ theories having the late-time de Sitter solution satisfying the condition (50).

The possibility of the late-time cosmic acceleration in $f(R)$ gravity was first suggested by Capozziello [28] in 2002. An $f(R)$ model of the form $f(R) = R - \mu^{2(n+1)}/R^n$ ($n > 0$) was proposed to be responsible for dark energy [29], but this model suffers from a number of problems such as the matter instability [92], absence of the matter era [93], and inability to satisfy local gravity constraints [94]. There are a number of conditions under which $f(R)$ dark energy models are viable. Below we summarize those conditions.

- (i) $f_{,R} > 0$ for $R \geq R_0$, where R_0 is the Ricci scalar today. This is required to avoid the appearance of a ghost.
- (ii) $f_{,RR} > 0$ for $R \geq R_0$. This is required to avoid the negative mass squared of a scalar-field degree of freedom (tachyon) [95].
- (iii) $f(R) \rightarrow R - 2\Lambda$ for $R \geq R_0$. This is required to for the presence of the matter era [93] and for consistency with local gravity constraints [96, 97].
- (iv) $0 < \frac{Rf_{,RR}}{f_{,R}}(r = -2) < 1$ at $r = -\frac{Rf_{,R}}{f} = -2$ [98, 99]. This is required to for the stability and the presence of a late-time de Sitter solution. Note that there is another fixed point that can be responsible for the acceleration [99] (with an effective equation of state $w_{\text{eff}} > -1$).

The examples of viable models that satisfy all these requirements are [96, 97, 100]

$$(A) \quad f(R) = R - \mu R_c \frac{(R/R_c)^{2n}}{(R/R_c)^{2n} + 1} \quad \text{with } n, \mu, R_c > 0, \quad (51)$$

$$(B) \quad f(R) = R - \mu R_c \left[1 - (1 + R^2/R_c^2)^{-n} \right] \quad \text{with } n, \mu, R_c > 0, \quad (52)$$

$$(C) \quad f(R) = R - \mu R_c \tanh(R/R_c) \quad \text{with } \mu, R_c > 0, \quad (53)$$

where μ , R_c , and n are constants. R_c is roughly of the order of the present cosmological Ricci scalar R_0 . A similar model to (C) was also proposed by Appleby and Battye [101]. If $R \gg R_c$ the models are close to the Λ CDM model ($f(R) \simeq R - \mu R_c$), so that GR is recovered in the region of high density. Meanwhile

²There is another way for the variation of the action (47) called the Palatini formalism.

the deviation from GR becomes important when R decreases to the order of R_c . The equation of state w_{DE} of dark energy to confront with supernovae Ia observations can be smaller than -1 for viable $f(R)$ models without the appearance of ghosts [96, 100, 102]. We note, however, that the effective equation of state $w_{\text{eff}} = -1 - 2\dot{H}/(3H^2)$ remains to be larger than -1 as long as the conditions (i)-(iv) listed above are satisfied (apart from small oscillations around the de Sitter attractor if it is a stable spiral). Since the deviation of w_{DE} from that in the Λ CDM model ($w_{\text{DE}} = -1$) is not so significant [96, 103], the viable models such as (51)-(53) can be consistent with the supernovae Ia data fairly easily.

The modification of gravity manifests itself in the effective gravitational coupling that appears in the equation of cosmological perturbations. The matter density perturbation δ_m satisfies the following equation under a quasi-static approximation on sub-horizon scales [104]

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G_{\text{eff}}\rho_m\delta_m = 0, \quad (54)$$

where ρ_m is the energy density of non-relativistic matter, and

$$G_{\text{eff}} = \frac{G}{f_{,R}} \frac{1 + 4mk^2/(a^2R)}{1 + 3mk^2/(a^2R)}. \quad (55)$$

Here $m \equiv Rf_{,RR}/f_{,R}$ is the deviation parameter from the Λ CDM model [99]. In the regime where the deviation from the Λ CDM model is small such that $mk^2/(a^2R) \ll 1$, the effective gravitational coupling G_{eff} is very close to the gravitational constant G . Then the matter perturbation evolves as $\delta_m \propto t^{2/3}$ during the matter dominance. Meanwhile in the regime $mk^2/(a^2R) \gg 1$ one has $G_{\text{eff}} = 4G/(3f_{,R})$, so that the evolution of δ_m during the matter era is given by $\delta_m \propto t^{(\sqrt{33}-1)/6}$.

This unusual evolution of δ_m leaves a number of interesting signatures such as the modification to the matter power spectrum and the effect on weak lensing. For the models (A) and (B), for example, the difference between the spectral indices between the matter power spectrum and the CMB spectrum can be estimates as [97]

$$\Delta n_s = \frac{\sqrt{33} - 5}{6n + 4}. \quad (56)$$

Observationally we do not find any strong signature for the difference of slopes of the two spectra. If we take the mild bound $\Delta n_s < 0.05$, we obtain the constraint $n > 2$. Local gravity constraints on solar system scales can be satisfied for $n > 0.9$ [105] through the chameleon mechanism [106]. Hence, as long as $n > 2$, the models (A) and (B) can be consistent with both cosmological and local gravity constraints. The model (53) can easily satisfy local gravity constraints because of the rapid approach to the Λ CDM model in the regime $R \gg R_c$.

In the strong gravitational background (such as neutron stars), Kobayashi and Maeda [107, 108] pointed out that for the $f(R)$ models such as (51) and (52) it is difficult to obtain thin-shell solutions inside a spherically symmetric body with constant density. For chameleon models with general couplings Q , a thin-shell field profile was analytically derived in Ref. [109] by employing a linear expansion in terms of the gravitational potential Φ_c at the surface of a compact object with constant density. Using the boundary condition set by analytic solutions, Ref. [109] also numerically confirmed the existence of thin-shell solutions for $\Phi_c \lesssim 0.3$ in the case of inverse power-law potentials $V(\phi) = M^{4+n}\phi^{-n}$. Ref. [110] also showed that static relativistic stars with constant density exists for the model (52). The effect of the relativistic pressure is important around the center of the body, so that the field tends to roll down the potential quickly unless the boundary condition is carefully chosen. Realistic stars have densities $\rho_A(r)$ that globally decrease as a function of r . Numerical simulations of Refs. [111] demonstrate that thin-shell solutions are present for the $f(R)$ model (52) by considering a polytropic equation of state even in the strong gravitational background.

3.3.2 DGP model

There is another class of modified gravity models of dark energy based on braneworlds. In braneworlds standard model particles are confined on a 3-dimensional (3D) brane embedded in 5-dimensional bulk spacetime with large extra dimensions. Dvali, Gabadadze, and Porrati (DGP) [31] proposed a braneworld model in which the 3-brane is embedded in a Minkowski bulk spacetime with infinitely large extra

dimensions. Newton's law can be recovered by adding a 4D Einstein-Hilbert action sourced by the brane curvature to the 5D action. The presence of such a 4D term may be induced by quantum corrections coming from the bulk gravity and its coupling with matter on the brane. In the DGP model the standard 4D gravity is recovered for small distances, whereas the effect from the 5D gravity manifests itself for large distances. Interestingly it is possible to realize the late-time cosmic acceleration without introducing a dark energy component [112].

The action for the DGP model is given by

$$S = \frac{1}{2\kappa_{(5)}^2} \int d^5 X \sqrt{-\tilde{g}} \tilde{R} + \frac{1}{2\kappa_{(4)}^2} \int d^4 X \sqrt{-g} R - \int d^5 X \sqrt{-\tilde{g}} \mathcal{L}_M, \quad (57)$$

where \tilde{g}_{AB} is the metric in the 5D bulk and $g_{\mu\nu} = \partial_\mu X^A \partial_\nu X^B \tilde{g}_{AB}$ is the induced metric on the brane with $X^A(x^c)$ being the coordinates of an event on the brane labelled by x^c . The 5D and 4D gravitational constants, $\kappa_{(5)}^2$ and $\kappa_{(4)}^2$, are related with the 5D and 4D Planck masses, $M_{(5)}$ and $M_{(4)}$, via $\kappa_{(5)}^2 = 1/M_{(5)}^3$ and $\kappa_{(4)}^2 = 1/M_{(4)}^2$.

The first and second terms in Eq. (57) correspond to Einstein-Hilbert actions in the 5D bulk and on the brane, respectively. The matter action consists of a brane-localized matter whose action is given by $\int d^4 x \sqrt{-g} (\sigma + \mathcal{L}_M^{\text{brane}})$, where σ is the 3-brane tension and $\mathcal{L}_M^{\text{brane}}$ is the Lagrangian density on the brane. Since the tension is not related to the Ricci scalar R , it can be adjusted to be zero.

The Friedmann equation on the flat FLRW brane is given by [112]

$$H^2 - \frac{\epsilon}{r_c} H = \frac{\kappa_{(4)}^2}{3} \rho_M, \quad (58)$$

where $\epsilon = \pm 1$, $r_c \equiv \kappa_{(5)}^2/(2\kappa_{(4)}^2)$ is a crossover scale, and ρ_M is the matter energy density on the brane satisfying the continuity equation

$$\dot{\rho}_M + 3H(\rho_M + P_M) = 0. \quad (59)$$

If the crossover scale r_c is much larger than the Hubble radius H^{-1} , the first term in Eq. (58) dominates over the second one. In this case the standard Friedmann equation, $H^2 = \kappa_{(4)}^2 \rho_M/3$, is recovered. Meanwhile, in the regime $r_c < H^{-1}$, the presence of the second term in Eq. (58) leads to a modification to the standard Friedmann equation. In the Universe dominated by non-relativistic matter ($\rho_M \propto a^{-3}$), the Universe approaches a dS solution for $\epsilon = +1$:

$$H \rightarrow H_{\text{dS}} = 1/r_c. \quad (60)$$

Hence it is possible to realize the present cosmic acceleration provided that r_c is of the order of the present Hubble radius H_0^{-1} .

Although the DGP braneworld is an attractive model allowing a self acceleration, the joint constraints from data of supernovae Ia, baryon acoustic oscillations, and the CMB shift parameter shows that this model is under strong observational pressure [113]. Moreover, this model contains a ghost mode [114] with the effective Brans-Dicke parameter ω_{BD} smaller than $-3/2$. Hence the original DGP model is effectively ruled out from observational constraints as well as from the ghost problem. It is however possible to construct a generalized DGP model free from the ghost problem by embedding our visible 3-brane with a 4-brane in a flat 6D bulk [115].

In the DGP model a brane-bending mode ϕ (i.e. longitudinal graviton) gives rise to a field self-interaction of the form $\Box\phi(\partial^\mu\phi\partial_\mu\phi)$ through a mixing with the transverse graviton [116]. This can lead to the decoupling of the field ϕ from gravitational dynamics in the local region by the so-called Vainshtein mechanism [117]. It is possible to generalize the field self-interaction $\Box\phi(\partial^\mu\phi\partial_\mu\phi)$ to more general forms in the 4D gravity, such that the Lagrangian is invariant under the Galilean shift $\partial_\mu\phi \rightarrow \partial_\mu\phi + b_\mu$ [32]. In such ‘‘Galileon gravity’’ the late-time cosmic acceleration can be realized without the appearance of ghosts [118].

4 Conclusions

We have discussed theoretical attempts for finding the origin of inflation and dark energy, paying particular attention to their observational signatures. The WMAP observations already ruled out some models

of inflation (such as $V(\phi) = \lambda\phi^4/4$). The future observations such as the Planck will be able to constrain inflationary models further. In particular the detection of gravitational waves and non-Gaussianities will allow us to discriminate between a host of inflation models.

The important step for approaching the origin of dark energy is to clarify whether it is a cosmological constant or it originates from some dynamical source. In doing so, it is important to find some observational signatures for the deviation of the dark energy equation of state from -1 . Modified gravity models of dark energy can be distinguished from other models at the level of perturbations because of the modification of the effective gravitational coupling. The upcoming high-precision observations of large-scale structure, weak lensing, and cosmic microwave background may allow us to discriminate those models from the Λ CDM.

Acknowledgements

S. T. would like to give warm regards to Profs. Kei-ichi Maeda and Takashi Nakamura for the celebration of their 60-th birthdays. S. T. also thanks organizers of the JGRG20 workshop for inviting him to have a talk.

References

- [1] A. A. Starobinsky, Phys. Lett. B **91** (1980) 99.
- [2] D. Kazanas, Astrophys. J. **241** L59 (1980); K. Sato, Mon. Not. R. Astron. Soc. **195**, 467 (1981); Phys. Lett. **99B**, 66 (1981); A. H. Guth, Phys. Rev. D **23**, 347 (1981).
- [3] V. F. Mukhanov and G. V. Chibisov, JETP Lett. **33**, 532 (1981); A. H. Guth and S. Y. Pi, Phys. Rev. Lett. **49** (1982) 1110; S. W. Hawking, Phys. Lett. B **115**, 295 (1982); A. A. Starobinsky, Phys. Lett. B **117** (1982) 175; J. M. Bardeen, P. J. Steinhardt and M. S. Turner, Phys. Rev. D **28**, 679 (1983).
- [4] D. N. Spergel *et al.* [WMAP Collaboration], Astrophys. J. Suppl. **148**, 175 (2003).
- [5] E. Komatsu *et al.*, arXiv:1001.4538 [astro-ph.CO].
- [6] A. A. Starobinsky, in: Proc. of the 2nd Seminar, “Quantum Gravity” (Moscow, 13-15 Oct. 1981), INR Press, Moscow, 1982, pp. 58-72; reprinted in: Quantum Gravity, eds. M. A. Markov and P. C. West, Plenum Publ. Co., N. Y., 1984, pp. 103-128; A. Vilenkin, Phys. Rev. D **32**, 2511 (1985); M. B. Mijic, M. S. Morris and W. M. Suen, Phys. Rev. D **34**, 2934 (1986).
- [7] J. c. Hwang and H. Noh, Phys. Lett. B **506**, 13 (2001).
- [8] A. D. Linde, Phys. Lett. B **108**, 389 (1982).
- [9] A. Albrecht and P. Steinhardt, Phys. Rev. Lett. **48**, 1220 (1982).
- [10] D. H. Lyth and A. Riotto, Phys. Rept. **314**, 1 (1999).
- [11] A. D. Linde, arXiv:hep-th/0503203.
- [12] A. D. Linde, Phys. Lett. B **129** 177 (1983).
- [13] J. E. Lidsey, A. R. Liddle, E. W. Kolb, E. J. Copeland, Rev. Mod. Phys. **69**, 373 (1997).
- [14] A. R. Liddle and D. H. Lyth, *Cosmological inflation and large-scale structure*, Cambridge University Press (2000).
- [15] B. A. Bassett, S. Tsujikawa and D. Wands, Rev. Mod. Phys. **78**, 537 (2006).
- [16] C. Armendariz-Picon, T. Damour and V. F. Mukhanov, Phys. Lett. B **458**, 209 (1999).

- [17] J. Garriga and V. F. Mukhanov, Phys. Lett. B **458**, 219 (1999).
- [18] D. Seery and J. E. Lidsey, JCAP **0506**, 003 (2005).
- [19] X. Chen, M. x. Huang, S. Kachru and G. Shiu, JCAP **0701**, 002 (2007).
- [20] A. G. Riess *et al.* [Supernova Search Team Collaboration], Astron. J. **116**, 1009 (1998).
- [21] S. Perlmutter *et al.* [Supernova Cosmology Project Collaboration], Astrophys. J. **517**, 565 (1999).
- [22] V. Sahni and A. A. Starobinsky, Int. J. Mod. Phys. D **9**, 373 (2000); P. J. E. Peebles and B. Ratra, Rev. Mod. Phys. **75**, 559 (2003); T. Padmanabhan, Phys. Rept. **380**, 235 (2003); E. J. Copeland, M. Sami and S. Tsujikawa, Int. J. Mod. Phys. D **15**, 1753 (2006).
- [23] S. Weinberg, Rev. Mod. Phys. **61**, 1 (1989).
- [24] Y. Fujii, Phys. Rev. D **26**, 2580 (1982); C. Wetterich, Nucl. Phys. B. **302**, 668 (1988); B. Ratra and J. Peebles, Phys. Rev. D **37**, 321 (1988); T. Chiba, N. Sugiyama and T. Nakamura, Mon. Not. Roy. Astron. Soc. **289**, L5 (1997).
- [25] R. R. Caldwell, R. Dave and P. J. Steinhardt, Phys. Rev. Lett. **80**, 1582 (1998).
- [26] T. Chiba, T. Okabe and M. Yamaguchi, Phys. Rev. D **62**, 023511 (2000).
- [27] C. Armendariz-Picon, V. F. Mukhanov and P. J. Steinhardt, Phys. Rev. Lett. **85**, 4438 (2000); C. Armendariz-Picon, V. F. Mukhanov and P. J. Steinhardt, Phys. Rev. D **63**, 103510 (2001).
- [28] S. Capozziello, Int. J. Mod. Phys. D **11**, 483, (2002).
- [29] S. Capozziello, V. F. Cardone, S. Carloni and A. Troisi, Int. J. Mod. Phys. D, **12**, 1969 (2003); S. M. Carroll, V. Duvvuri, M. Trodden and M. S. Turner, Phys. Rev. D **70**, 043528 (2004).
- [30] L. Amendola, Phys. Rev. D **60**, 043501 (1999); J. P. Uzan, Phys. Rev. D **59**, 123510 (1999); T. Chiba, Phys. Rev. D **60**, 083508 (1999); N. Bartolo and M. Pietroni, Phys. Rev. D **61**, 023518 (2000).
- [31] G. R. Dvali, G. Gabadadze and M. Porrati, Phys. Lett. B **485**, 208 (2000).
- [32] A. Nicolis, R. Rattazzi and E. Trincherini, Phys. Rev. D **79**, 064036 (2009).
- [33] A. R. Liddle, P. Parsons and J. D. Barrow, Phys. Rev. D **50**, 7222 (1994).
- [34] E. W. Kolb, arXiv:hep-ph/9910311 (1999).
- [35] K. Freese, J. A. Frieman and A. V. Olinto, Phys. Rev. Lett. **65**, 3233 (1990).
- [36] A. D. Linde, Phys. Rev. D **49**, 748 (1994).
- [37] E. J. Copeland, A. R. Liddle, D. H. Lyth, E. D. Stewart and D. Wands, Phys. Rev. D **49**, 6410 (1994).
- [38] D. Polarski and A. A. Starobinsky, Nucl. Phys. B **385**, 623 (1992); D. Langlois, Phys. Rev. D **59**, 123512 (1999).
- [39] A. A. Starobinsky and J. Yokoyama, arXiv:gr-qc/9502002; A. A. Starobinsky, S. Tsujikawa and J. Yokoyama, Nucl. Phys. B **610**, 383 (2001).
- [40] F. Lucchin and S. Matarrese, Phys. Rev. D **32**, 1316 (1985); J. Yokoyama and K. i. Maeda, Phys. Lett. B **207**, 31 (1988).
- [41] P. J. E. Peebles and A. Vilenkin, Phys. Rev. D **59**, 063505 (1999).
- [42] M. Fairbairn and M. H. G. Tytgat, Phys. Lett. B **546**, 1 (2002); A. Feinstein, Phys. Rev. D **66**, 063511 (2002).

- [43] N. Arkani-Hamed, P. Creminelli, S. Mukohyama and M. Zaldarriaga, JCAP **0404**, 001 (2004).
- [44] J. R. Ellis, N. Kaloper, K. A. Olive and J. Yokoyama, Phys. Rev. D **59**, 103503 (1999); S. W. Hawking, T. Hertog and H. S. Reall, Phys. Rev. D **63**, 083504 (2001); K. i. Maeda and N. Ohta, Phys. Lett. B **597**, 400 (2004).
- [45] M. Gasperini and G. Veneziano, Astropart. Phys. **1**, 317 (1993).
- [46] N. Arkani-Hamed, H. C. Cheng, M. A. Luty and S. Mukohyama, JHEP **0405**, 074 (2004).
- [47] A. Sen, JHEP **0204**, 048 (2002).
- [48] E. Silverstein and D. Tong, Phys. Rev. D **70**, 103505 (2004); M. Alishahiha, E. Silverstein and D. Tong, Phys. Rev. D **70**, 123505 (2004).
- [49] F. Piazza and S. Tsujikawa, JCAP **0407**, 004 (2004).
- [50] J. M. Bardeen, Phys. Rev. D **22**, 1882 (1980); H. Kodama and M. Sasaki, Prog. Theor. Phys. Suppl. **78**, 1 (1984); V. F. Mukhanov, H. A. Feldman and R. H. Brandenberger, Phys. Rept. **215**, 203 (1992).
- [51] V. N. Lukash, Sov. Phys. JETP **52**, 807 (1980); D. H. Lyth, Phys. Rev. D **31**, 1792 (1985); M. Sasaki, Prog. Theor. Phys. **76**, 1036 (1986).
- [52] E. Komatsu *et al.* [WMAP Collaboration], Astrophys. J. Suppl. **180**, 330 (2009).
- [53] D. Baumann, arXiv:0907.5424 [hep-th].
- [54] D. H. Lyth, Phys. Rev. Lett. **78**, 1861 (1997).
- [55] G. R. Dvali and S. H. H. Tye, Phys. Lett. B **450**, 72 (1999); S. Kachru *et al.*, JCAP **0310**, 013 (2003); D. Baumann, A. Dymarsky, I. R. Klebanov and L. McAllister, JCAP **0801**, 024 (2008); S. Panda, M. Sami and S. Tsujikawa, Phys. Rev. D **76**, 103512 (2007); D. Baumann *et al.*, JHEP **03**, 093 (2009).
- [56] J. J. Blanco-Pillado *et al.*, JHEP **09**, 002 (2006).
- [57] J. P. Conlon and F. Quevedo, JHEP **01**, 146 (2006).
- [58] N. Bartolo, S. Matarrese and A. Riotto, Phys. Rev. D **65**, 103505 (2002).
- [59] J. M. Maldacena, JHEP **0305**, 013 (2003).
- [60] K. Koyama, Class. Quant. Grav. **27**, 124001 (2010).
- [61] F. Bernardeau and J. P. Uzan, Phys. Rev. D **66**, 103506 (2002); D. Seery and J. E. Lidsey, JCAP **0509**, 011 (2005); C. T. Byrnes, M. Sasaki and D. Wands, Phys. Rev. D **74**, 123519 (2006); D. Langlois, S. Renaux-Petel, D. A. Steer and T. Tanaka, Phys. Rev. D **78**, 063523 (2008); F. Arroja, S. Mizuno and K. Koyama, JCAP **0808**, 015 (2008).
- [62] A. R. Liddle, A. Mazumdar and F. E. Schunck, Phys. Rev. D **58**, 061301 (1998).
- [63] S. Tsujikawa, Phys. Rev. D **73**, 103504 (2006).
- [64] A. De Felice and S. Tsujikawa, Living Rev. Rel. **13**, 3 (2010).
- [65] K. i. Maeda, Phys. Rev. D **37**, 858 (1988); L. Amendola, S. Capozziello, M. Litterio and F. Occhionero, Phys. Rev. D **45**, 417 (1992).
- [66] K. i. Maeda, Phys. Rev. D **39**, 3159 (1989).
- [67] L. Kofman, A. D. Linde and A. A. Starobinsky, Phys. Rev. Lett. **73**, 3195 (1994); Phys. Rev. D **56**, 3258 (1997).

- [68] S. Tsujikawa, K. i. Maeda and T. Torii, Phys. Rev. D **60**, 123505 (1999).
- [69] N. Makino and M. Sasaki, Prog. Theor. Phys. **86**, 103 (1991); R. Fakir, S. Habib and W. Unruh, Astrophys. J. **394**, 396 (1992).
- [70] T. Futamase and K. i. Maeda, Phys. Rev. D **39**, 399 (1989).
- [71] R. Fakir and W. G. Unruh, Phys. Rev. D **41**, 1783 (1990).
- [72] E. Komatsu and T. Futamase, Phys. Rev. D **59**, 064029 (1999); S. Tsujikawa and B. Gumjudpai, Phys. Rev. D **69**, 123523 (2004).
- [73] S. Tsujikawa, K. i. Maeda and T. Torii, Phys. Rev. D **61**, 103501 (2000).
- [74] F. L. Bezrukov and M. Shaposhnikov, Phys. Lett. B **659**, 703 (2008).
- [75] D. J. Eisenstein *et al.* [SDSS Collaboration], Astrophys. J. **633**, 560 (2005).
- [76] M. Dine, R. Rohm, N. Seiberg and E. Witten, Phys. Lett. B **156**, 55 (1985).
- [77] I. Affleck, M. Dine and N. Seiberg, Nucl. Phys. B **256**, 557 (1985).
- [78] S. Kachru, R. Kallosh, A. D. Linde and S. P. Trivedi, Phys. Rev. D **68**, 046005 (2003).
- [79] L. Susskind, arXiv:hep-th/0302219.
- [80] F. Denef and M. R. Douglas, JHEP **0405**, 072 (2004); R. Blumenhagen *et al.*, Nucl. Phys. B **713**, 83 (2005).
- [81] A. Y. Kamenshchik, U. Moschella and V. Pasquier, Phys. Lett. B **511**, 265 (2001); M. C. Bento, O. Bertolami and A. A. Sen, Phys. Rev. D **66**, 043507 (2002).
- [82] H. Sandvik, M. Tegmark, M. Zaldarriaga and I. Waga, Phys. Rev. D **69**, 123524 (2004).
- [83] R. R. Caldwell and E. V. Linder, Phys. Rev. Lett. **95**, 141301 (2005).
- [84] P. Binetruy, Phys. Rev. D **60**, 063502 (1999).
- [85] P. Brax and J. Martin, Phys. Lett. B **468**, 40 (1999).
- [86] A. D. Linde, “Inflation and quantum cosmology”, in: Three hundred years of gravitation, Eds.: S. W. Hawking and W. Israel, Cambridge University Press (1987), 604.
- [87] R. Kallosh *et al.*, JCAP **0310**, 015 (2003).
- [88] J. A. Frieman, C. T. Hill, A. Stebbins and I. Waga, Phys. Rev. Lett. **75**, 2077 (1995).
- [89] A. A. Starobinsky, JETP Lett. **68**, 757 (1998); T. Nakamura and T. Chiba, Mon. Not. Roy. Astron. Soc. **306**, 696 (1999); T. Chiba and T. Nakamura, Phys. Rev. D **62**, 121301 (2000); T. D. Saini, S. Raychaudhury, V. Sahni and A. A. Starobinsky, Phys. Rev. Lett. **85**, 1162 (2000).
- [90] R. R. Caldwell, Phys. Lett. B **545**, 23 (2002).
- [91] C. Bonvin, C. Caprini and R. Durrer, Phys. Rev. Lett. **97**, 081303 (2006).
- [92] A. D. Dolgov and M. Kawasaki, Phys. Lett. B **573**, 1 (2003).
- [93] L. Amendola, D. Polarski and S. Tsujikawa, Phys. Rev. Lett. **98**, 131302 (2007).
- [94] T. Chiba, Phys. Lett. B **575**, 1 (2003); G. J. Olmo, Phys. Rev. D **72**, 083505 (2005); I. Navarro and K. Van Acoleyen, JCAP **0702**, 022 (2007).
- [95] Y. S. Song, W. Hu and I. Sawicki, Phys. Rev. D **75**, 044004 (2007).

- [96] W. Hu and I. Sawicki, Phys. Rev. D **76**, 064004 (2007).
- [97] A. A. Starobinsky, JETP Lett. **86**, 157 (2007).
- [98] V. Muller, H. J. Schmidt and A. A. Starobinsky, Phys. Lett. B **202**, 198 (1988); V. Faraoni, Phys. Rev. D **72**, 061501 (2005).
- [99] L. Amendola, R. Gannouji, D. Polarski and S. Tsujikawa, Phys. Rev. D **75**, 083504 (2007).
- [100] S. Tsujikawa, Phys. Rev. D **77**, 023507 (2008).
- [101] S. A. Appleby and R. A. Battye, Phys. Lett. B **654**, 7 (2007).
- [102] L. Amendola and S. Tsujikawa, Phys. Lett. B **660**, 125 (2008).
- [103] H. Motohashi, A. A. Starobinsky and J. Yokoyama, Prog. Theor. Phys. **123**, 887 (2010).
- [104] S. Tsujikawa, Phys. Rev. D **76**, 023514 (2007).
- [105] S. Capozziello and S. Tsujikawa, Phys. Rev. D **77**, 107501 (2008).
- [106] J. Khoury and A. Weltman, Phys. Rev. Lett. **93**, 171104 (2004).
- [107] T. Kobayashi and K. i. Maeda, Phys. Rev. D **78**, 064019 (2008).
- [108] T. Kobayashi and K. i. Maeda, Phys. Rev. D **79**, 024009 (2009).
- [109] S. Tsujikawa, T. Tamaki and R. Tavakol, JCAP **0905**, 020 (2009).
- [110] A. Upadhye and W. Hu, Phys. Rev. D **80**, 064002 (2009).
- [111] E. Babichev and D. Langlois, Phys. Rev. D **80**, 121501 (2009); Phys. Rev. D **81**, 124051 (2010).
- [112] C. Deffayet, Phys. Lett. B **502** (2001), 199; C. Deffayet, G. R. Dvali and G. Gabadadze, Phys. Rev. D **65** (2002), 044023.
- [113] M. Fairbairn and A. Goobar, Phys. Lett. B **642**, 432 (2006); R. Maartens and E. Majerotto, Phys. Rev. D **74**, 023004 (2006); Y. S. Song, I. Sawicki and W. Hu, Phys. Rev. D **75**, 064003 (2007).
- [114] A. Lue, R. Scoccimarro and G. D. Starkman, Phys. Rev. D **69**, 124015 (2004); K. Koyama and R. Maartens, JCAP **0601**, 016 (2006).
- [115] C. de Rham *et al.*, Phys. Rev. Lett. **100**, 251603 (2008).
- [116] C. Deffayet, G. R. Dvali, G. Gabadadze and A. I. Vainshtein, Phys. Rev. D **65**, 044026 (2002); M. Porrati, Phys. Lett. B **534**, 209 (2002).
- [117] A. I. Vainshtein, Phys. Lett. B **39**, 393 (1972).
- [118] A. De Felice and S. Tsujikawa, Phys. Rev. Lett. **105**, 111301 (2010).