SUDAKOV LOGARITHMS IN ELECTROWEAK PROCESSES

A. WERTHENBACH and W. BEENAKKER
Department of Physics, University of Durham,
South Road, Durham DH1 3LE, England

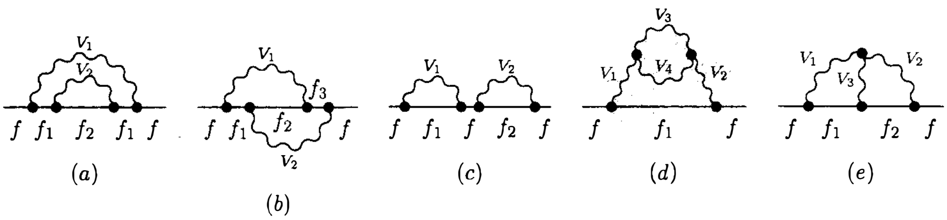
To match the expected experimental precision at future linear colliders, improved theoretical predictions beyond next-to-leading order are required. At the anticipated energy scale of $\sqrt{s} = 1$ TeV the electroweak virtual corrections are strongly enhanced by collinear-soft Sudakov logarithms of the form $\log^2(s/M^2)$, with M being the generic mass scale of the W and Z bosons. By choosing an appropriate gauge, we have developed a formalism to calculate such corrections for arbitrary electroweak processes. As an example we consider here the process $e^+e^- \to f\bar{f}$. In unbroken theories like QED and QCD the Sudakov form factor, resummed to all orders in perturbation theory, simply amounts to exponentiation of the one-loop corrections. However, based on an explicit two-loop calculation we find non-exponentiating terms, originating from the mass gap between the photon and the Z boson in the neutral sector of the Standard Model.

1 Introduction

At the next generation of colliders center-of-mass energies will be reached that largely exceed the electroweak scale. At such energies effects arising from weak corrections are expected to be of the order of 10% or more [1,2,3], and higher order corrections become relevant.
The dominant source of radiative corrections at TeV-scale energies is given by the so-called Sudakov logarithms $\propto \alpha^n \log^{2n}(M^2/s)$, involving particle masses M well below the collider energy $\sqrt{s}$. These corrections have a unique origin, being related to collinear-soft singularities [4]. Recent studies have focused on these Sudakov effects in the process $e^+e^- \to f\bar{f}$ [5,6,7]. Unfortunately the three independent studies are in mutual disagreement.

2 Electroweak Sudakov logarithms in $e^+e^- \to f\bar{f}$

In order to facilitate the calculation of the one- and two-loop Sudakov logarithms, we work in the Coulomb gauge for both massless and massive gauge bosons (the subtleties associated with the Coulomb gauge for massive gauge bosons will be discussed in Ref. [8]). Working in this special gauge has the advantage that all virtual Sudakov logarithms are contained exclusively in the self-energies of the external on-shell particles [8,9] or the self-energies of any intermediate particle that happens to be effectively on-shell. The elegance of this method lies in its universal nature, with the Sudakov logarithms originating from vertex, box etc. corrections being suppressed. The relevant self-energies for the calculation of the Sudakov logarithms involve the exchange of collinear-soft gauge bosons. The collinear-soft exchange of fermions, scalars and ghosts leads to suppressed contributions, since the propagators of these particles do not have the required pole structure.
Our one-loop results [10] are in agreement with the calculations in the literature. At two-loop accuracy one has to take the following five generic sets of diagrams into account:



The fermions f_i are fixed by the exchanged gauge bosons V_i.
Adding up all possible contributions, we find for the full two-loop Sudakov correction factor for right- and left-handed fermions/antifermions [10]

$$\delta^{(2)}_{f_R} = \delta^{(2)}_{\bar{f}_L} = \frac{1}{2}\left(\delta^{(1)}_{f_R}\right)^2 + Q_f^2\,\mathrm{L}(M,M)\left[\frac{4}{3}\mathrm{L}(M,m_f) - \mathrm{L}(M,M)\right] \tag{1}$$

$$\delta^{(2)}_{f_L} = \delta^{(2)}_{\bar{f}_R} = \frac{1}{2}\left(\delta^{(1)}_{f_L}\right)^2 + \left(Q_f^2 - \frac{|Q_f|}{2\sin^2\theta_{\mathrm{w}}}\right)\mathrm{L}(M,M)\left[\frac{4}{3}\mathrm{L}(M,m_f) - \mathrm{L}(M,M)\right]. \tag{2}$$

Here θ_{w} is the weak mixing angle, m_f the mass of the external fermion,

$$\delta^{(1)}_{f_R} = \delta^{(1)}_{\bar{f}_L} = \left(\frac{Y_f^R}{2\cos\theta_{\mathrm{w}}}\right)^2 \mathrm{L}(M,M) + Q_f^2\left[\mathrm{L}_\gamma(\lambda,m_f) - \mathrm{L}(M,M)\right], \tag{3}$$

$$\delta^{(1)}_{f_L} = \delta^{(1)}_{\bar{f}_R} = \left[\frac{C_F}{\sin^2\theta_{\mathrm{w}}} + \left(\frac{Y_f^L}{2\cos\theta_{\mathrm{w}}}\right)^2\right]\mathrm{L}(M,M) + Q_f^2\left[\mathrm{L}_\gamma(\lambda,m_f) - \mathrm{L}(M,M)\right], \tag{4}$$

and

$$\mathrm{L}(M_1, M_2) = -\frac{\alpha}{4\pi} \log\left(\frac{M_1^2}{s}\right) \log\left(\frac{M_2^2}{s}\right) , \tag{5}$$

$$\mathrm{L}_\gamma(\lambda, M_1) = -\frac{\alpha}{4\pi} \left[\log^2\left(\frac{\lambda^2}{s}\right) - \log^2\left(\frac{\lambda^2}{M_1^2}\right)\right] . \tag{6}$$

$Y_f^{R,L}$ denotes the right- and left-handed hypercharge of the external fermion, which is connected to the third component of the weak isospin I_f^3 and the electromagnetic charge $e\,Q_f$ through the Gell-Mann – Nishijima relation $Q_f = I_f^3 + Y_f^{R,L}/2$. The coefficient $C_F = 3/4$ is the Casimir operator in the fundamental representation of $SU(2)$ and λ is the fictitious (infinitesimally small) mass of the photon needed for regularizing the infrared singularity. For the sake of calculating the leading Sudakov logarithms, the masses of the W and Z bosons can be represented by one generic mass scale M. From Eqs. (1) and (2) we deduce our main statement, namely that the virtual electroweak two-loop Sudakov correction factor is not obtained by a mere exponentiation of the one-loop Sudakov correction factor. Based on the explicit two-loop calculation we find non-exponentiating terms, originating from the mass gap between the *massless* photon and the *massive* Z boson in the neutral sector of the SM. The cancellation that takes place in unbroken gauge theories, leading to exponentiation, is upset by the fact that the on-shell poles for photons and Z bosons do not coincide. We have checked that these extra terms vanish in leading-logarithmic approximation if *all* gauge bosons have the same (or roughly the same) mass. From Eqs. (1) and (2) it is clear that the non-exponentiating terms are genuine quadratic large-logarithmic effects. They will not vanish if the fermion mass is of the order of the masses of the weak bosons or if the energy is taken to infinity, since in those cases $\Delta_f \to \mathrm{L}^2(M,M)/3$. Therefore, as far as the virtual Sudakov logarithms are concerned, the SM will never completely behave like an unbroken theory. This is a consequence of the fact that the photon is massless, i.e. $m_f/\lambda \gg \sqrt{s}/M$.

We also note that, in adding up all the contributions, we find that the 'rainbow' diagrams of set (a) yield the usual exponentiating terms plus an extra term similar to the one found in Ref. [6], originating from the charged-current interactions. Whereas in Ref. [6] this extra term was interpreted as the source of non-exponentiation, we observe that it in fact cancels against a specific term originating from the triple gauge-boson diagrams of set (e). In contrast to Ref. [7] we do not observe the exponentiation of the virtual one-loop Sudakov logarithms.

Acknowledgments

A. W. acknowledges support in form of a DAAD Doktorandenstipendium (HSP III).
W. B. acknowledges support in form of a PPARC Research Fellowship.

References

1. P. Ciafaloni and D. Comelli, *Phys. Lett.* **B446**, 278 (1999).
2. W. Beenakker, A. Denner, S. Dittmaier, R. Mertig, and T. Sack, *Nucl. Phys.* **B410**, 245 (1993).
3. W. Beenakker, A. Denner, S. Dittmaier, and R. Mertig, *Phys. Lett.* **B317**, 622 (1993).
4. V. V. Sudakov, *Sov. Phys. JETP* **3**, 65 (1956).
5. J. H. Kuhn and A. A. Penin, *hep-ph/9906545.*
6. P. Ciafaloni and D. Comelli. *Phys. Lett.* **B476**, 49 (2000).
7. V. S. Fadin, L. N. Lipatov, A. D. Martin, and M. Melles, *Phys. Rev.* **D61**, 094002 (2000).
8. W. Beenakker and A. Werthenbach, in preparation.
9. J. Frenkel and J. C. Taylor, *Nucl. Phys.* **B116**, 185 (1976).
10. W. Beenakker and A. Werthenbach, *hep-ph/0005316.*