

The progress of gravitational wave detection in China and its further physical application

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Abstract. Contemporarily, a gravitational wave is one of the most important approaches to gather information from the enormous universe. In short, a gravitational wave is a wave that carries energy, and it is created by the acceleration of massive celestial body propagation with a speed of light. This paper discusses the recent progress of gravitational wave detection in China and clarifies our own opinion on future development. Specifically, a basic description is first presented about the definition and basic knowledge for gravitational wave models and detection methods. Subsequently, this section contains the plan and achievement of the Chinese gravitational wave observatory. Finally, the usages and applications of the gravitational wave to help to detect more phenomena in the universe are demonstrated. These results shed light on a clearer picture of gravitational waves, which may offer a better understanding of the background, principle of detection, and the uses of gravitational waves, i.e., emphasizes its importance in modern astrophysics scientific researches.

Keywords: Gravitational wave (GW), convolutional neural network (CNN), Einstein tensor, Hubble law.

1. Introduction

About a century time ago, when one of the greatest scientists, Einstein, proposed the existence of gravitational waves, people start to have a brief idea of this brand-new concept. However, the essence of the gravitational wave was still vague and remained as an imagination. It was until the late 20th century that people first confirmed this secret code of the universe. In the 1970s, followed by the 80s, Joseph Taylor and his colleagues exhibited the evidence for proving the real existence of gravitational waves in our universe [1]. To infer the existence of gravitational waves, they had observed and studied a pulsar-neutron star binary system. Nearly nine decades after Einstein's assumption in his equation, on the 14th of September in 2015 [2], the first clear gravitational wave signal was detected by the world's largest gravitational wave detector in the US called "LIGO". It gave direct evidence of the existence of the ripple in the space-time structure. Additionally, the first gravitational wave observed by LIGO was created by a binary black hole system [1]. Two massive black holes spun around each other and eventually went into each other and emerged into a single black hole.



Nowadays, scholars can detect gravitational waves much easier than in the previous days by the two super-machines (LIGO in the US and Virgo in the EU). It is worth noticing that China also constructed its own gravitational wave observatory and started its application in 2019. By utilizing these exquisitely accurate machines, researchers can detect the gravitational wave more accurately and precisely and can come up with more details such as the mass, cosmic distance, and energy (by the amplitude of the signal), etc., i.e., gravitational wave is providing an extra approach for people to detect the other celestial body.

In this paper, the next topic, which is a basic description, will give a general idea of gravitational waves first. Then it will discuss in detail about the principle of the detection of gravitational waves. Furthermore, a better data processing solution will be introduced and explained at the end of this topic. Followed by the next part, whose content is describing the plans (including Taiji, Tianqin, and Ali) and preparatory works in China on the gravitational wave observation, including the equation used and data that have been collected. The final main body explains the application of gravitational waves and how it may help people deduce more information about the universe, for instance, the binary system or massive celestial body like black holes, pulsar, and neutron stars, etc.

2. Basic description of gravitational wave

This section will introduce the basic definition of gravitational waves and the principle of the method of detecting accordingly.

2.1. Definition of gravitational wave

A gravitational wave is defined to be the wave that carries energy spreading through the gravitational field; major formations are the acceleration and the perturbation of a massive body. Einstein first hypothesized the concept of a gravitational wave in 1916 according to his equation; additionally, the very first observation of the wave was in September 2015, done by the gravitational wave detector called LIGO located in northern San Diego County, California.

Notice here the most equations are from [3].

Later, people find the solution to prove Einstein's hypothesis under the weak-field limit. The process proceeds to assume that though the field is weak, it can change with time and has no restriction from the test particles' motions. This assumption is crucial as the field varies with time in gravitational radiation. The flat Minkowski space and a small perturbation are the metrics in the inferred equation, respectively.

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \quad |h_{\mu\nu}| \ll 1. \quad (1)$$

Since $h_{\mu\nu}$ is small through the assumption, every term can be dropped higher than first order in the calculations. Obviously, the perturbation $h_{\mu\nu}$ follows the equations of motion by examining Einstein's equations to first order.

The Christoffel symbols are coming up by the equation:

$$\Gamma^{\rho}_{\mu\nu} = \frac{1}{2} g^{\rho\lambda} (\partial_{\mu} g_{\nu\lambda} + \partial_{\nu} g_{\lambda\mu} - \partial_{\lambda} g_{\mu\nu}) \quad (2)$$

$$\Gamma^{\rho}_{\mu\nu} = \frac{1}{2} \eta^{\rho\lambda} (\partial_{\mu} h_{\nu\lambda} + \partial_{\nu} h_{\lambda\mu} - \partial_{\lambda} h_{\mu\nu}) \quad (3)$$

Then calculating the Riemann tensor by computer and by only regarding the derivatives of the Γ 's and not the Γ^2 terms,

$$R_{\mu\nu\rho\sigma} = \eta_{\mu\lambda} \partial_{\rho} \Gamma^{\lambda}_{\nu\sigma} - \eta_{\mu\lambda} \partial_{\sigma} \Gamma^{\lambda}_{\nu\rho} = \frac{1}{2} (\partial_{\rho} \partial_{\nu} h_{\mu\sigma} + \partial_{\sigma} \partial_{\mu} h_{\nu\rho} - \partial_{\sigma} \partial_{\nu} h_{\mu\rho} - \partial_{\rho} \partial_{\mu} h_{\nu\sigma}) \quad (4)$$

The Ricci tensor can be obtained by contracting over μ and ρ .

$$R_{\mu\nu} = \frac{1}{2} (\partial_\sigma \partial_\nu h^\sigma_\mu + \partial_\sigma \partial_\mu h^\sigma_\nu - \partial_\mu \partial_\nu h - \square h_{\mu\nu}) \quad (5)$$

Here the d'Alembertian, $\square = -\partial_t^2 + \partial_x^2 + \partial_y^2 + \partial_z^2$. Contracting again, the Ricci scalar will be inferred,

$$R = \partial_\mu \partial_\nu h^{\mu\nu} - \square h \quad (6)$$

Einstein tensor can be obtained by utilizing the equation (4) and (5),

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} R = \frac{1}{2} (\partial_\sigma \partial_\nu h^\sigma_\mu + \partial_\sigma \partial_\mu h^\sigma_\nu - \partial_\mu \partial_\nu h - \square h_{\mu\nu} - \eta_{\mu\nu} \partial_\rho \partial_\lambda h^{\rho\lambda} + \eta_{\mu\nu} \square h) \quad (7)$$

A gauge transformation is followed by the suggested metric in linearized theory.

$$h^\varepsilon_{\mu\nu} = h_{\mu\nu} + 2\varepsilon \delta(\zeta_\nu) \quad (8)$$

This represents the variation of the metric perturbation under an infinitely small diffeomorphism along the vector field $\mu\zeta_\nu$. This gauge transformation leaves the Riemann tensor; therefore, the curvature remains unchanged. The variables are defined,

$$\Psi = -\frac{1}{6} \delta^{ij} h_{ij} \quad (9)$$

$$s_{ij} = \frac{1}{2} (h_{ij} - \frac{1}{3} \delta^{kl} h_{kl} \delta_{ij}) \quad (10)$$

In the equation above, symbol Ψ indicates that h_{ij} and s_{ij} are traceless strains that contain gravitational radiation. By the consideration of the transverse gauge, the following conditions are imposed,

$$\partial_i s^{ij} = 0 \quad (11)$$

$$\nabla^2 \zeta^j + \frac{1}{3} \partial_j \partial_i \zeta^i = -2\partial_i s^{ij} \quad (12)$$

$$\partial_i w^i = 0 \quad (13)$$

$$\nabla^2 \zeta^0 = \partial_i w^i + \partial_0 \partial_i \zeta^i \quad (14)$$

In the equation, w^i is the vector perturbation. In this gauge, Einstein's equation becomes,

$$G_{00} = \nabla^2 \Psi = 8\pi G T_{00} \quad (15)$$

$$G_{ij} = (\delta_{ij} \nabla^2 - \partial_i \partial_j (\Phi - \Psi)) - \partial_0 \partial_{iwj} + 2\delta_{ij} \partial_0^2 \Psi - \square s_{ij} = 8\pi G T_{ij} \quad (16)$$

Here, T_{ij} is the stress-energy tensor, and Φ is the scalar potential. To settle for gravitational radiation, the freely spreading degrees of liberty of the gravitational field should be learned. No local sources are required for their existence. Hence, people use the transverse gauge with $T_{\mu\nu} = 0$. The 00 equation becomes:

$$\nabla^2 \Psi = 0 \quad (17)$$

With perfect boundary conditions behavior, this implicates that $\Psi = 0$. The 0j equation is,

$$\nabla^2 w_j = 0 \quad (18)$$

this implies $w_j = 0$. The trace of the ij equation coupled with the above results show that $\Phi = 0$

$$\nabla^2 \Phi = 0 \quad (19)$$

This creates the traceless part of the ij equation, which becomes a wave equation for the trace-free strain tensor:

$$\square S_{ij} = 0 \quad (20)$$

The transverse traceless gauge shows,

$$h_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 2S_{ij} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (21)$$

and the equation of motion becomes:

$$\square h_{\mu\nu} = 0 \quad (22)$$

Categorize to the wave equation into the electromagnetism, a variety of equations are found to the equation,

$$h_{\mu\nu} = C_{\mu\nu} e^{ik_{\sigma}x^{\sigma}} \quad (23)$$

It is particularly noticeable that $C_{\mu\nu}$ here is a constant, symmetric (0, 2) tensor which is traceless and purely spatial as $h_{\mu\nu}$ is totally trace-free, transverse and spatial. Putting (21) into (20), find that $k_{\sigma} = 0$, which illustrates that the gravitational wave vector is null. In other words, its speed is equal to the light speed. Given the properties of $h_{\mu\nu}$ and $C_{\mu\nu}$, the graph below emerges:

$$C_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & C_{11} & c_{12} & 0 \\ 0 & C_{12} & -C_{11} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_x & 0 \\ 0 & h_x & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (24)$$

2.2. Principle of detecting gravitational wave

In theory, equation (24)'s model can determine whether the signals of gravitational waves can exist in (s) directly. Nevertheless, gravitational wave signals and noises are proven to have a huge distinction with evidences by considering the condition as being in frequency space, which means that the obtained result (s) can be projected and analyzed into frequency space.

$$y = f(s) \quad (25)$$

$$z = g(s) \quad (26)$$

Before judging if there is any gravitational wave in (s), it is worth noticing that:

$$y = h(z) = h(g(s)) \quad (27)$$

The letter z represents that s is in frequency space. According to this instance, the wavelet packet (WP) decomposition is represented by a letter g . The function h is given by a convolutional neural network (CNN), a machine that will be introduced specifically later in the paper.

In fact, that equations (26) and (27) are unique precepts of equation (25). Two different sequential procedures of equation (25) are created by the precept of equation (26) and (27): pre-process the detail s , which is observed by using the equation “ $g(s)=z$ ” and find if there is any existence of any gravitational wave signal in the detail s by another equation: “ $h(z)=y$ ”. The separate precept can somehow change the sophisticated working process into an easier way. In some professional areas, the feature extraction is the name of the previous equation “ $g(s)=z$ ”. This equation aims to make the inferring process of equation (27) simpler by finding a more reasonable calculation of the observed data s here and eliminating false data. Fig. 3 is an example expressing a flow diagram.

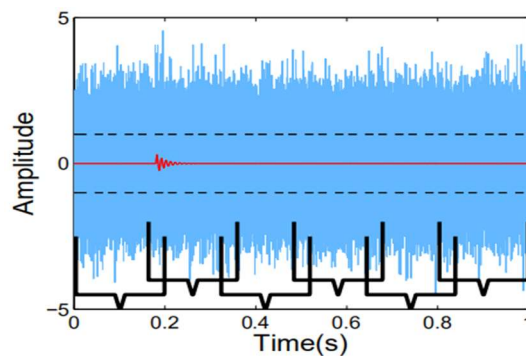


Figure 1. A sample of simulated data analyzes the data of both the universe noise and the gravitational wave signal [4].

As shown in Fig. 1, the object of examination is the blove/peripheral winding, constituted by both gravitational wave signals and noises in the universe. The crimson line can be considered as the hypothetical standard gravitational wave signal. The $A(S/N)$ in this stage gives a small range between 0.15 and 1.05. To be visible, this observed data collection (which is displayed by Fig. 1) demonstrates a sample with the equation: “ $A(S/N)$ ” to be exactly 0.3. The gravitational wave detection system usually changes the position of the detection window on the stream (the place to obtain gravitational wave) to a correct position to gain the fragmentary detail of the gravitational wave for further examination. The indicated brackets on Fig. 1 are the area to locate the windows. The “S” “D”, “noise” (where SD is the standard deviation) are displayed by the dash-line above and below the standard gravitational wave signal.

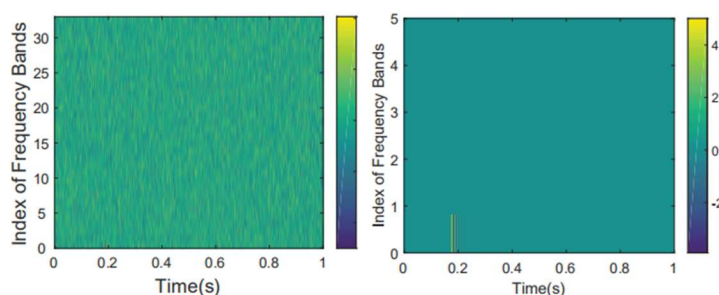


Figure 2. The distinction of the gravitational wave signal compared with the universe noise in a frequency against time diagram [4]

Both graphs (as shown in Fig. 2) analyze the detail presented in the previous picture in the way of frequency against time. Here, the color is indicating the extent of how intense the details are. The gravitational wave signals different frequencies, despite there are some relationships between each of them. The gravitational wave is always giving faint signals which are hard to detect. To make the signal of the gravitational wave more visible, the axis representing frequency is decreased from $[0, 32]$ to $[0, 5]$ in the right graph.



Figure 3. The flow chart of how data are collected, processes, and calculated [4]

However, the extant investigations based on CNN for the gravitational wave signal are mainly lead by inserting a series of times into the network (relating to convolutional neural network) regardless of the influence of the specific category of the process of frequency. The flow chart of a CNN is given in Fig. 3. Hence, the extend of the necessity of temporal frequency examination, and the influences from its approaches are still the controversial trouble waiting to be settled in CNN-based plan of gravitational wave detection. Later there will be the explanation of the decomposition of WP for the gravitational wave signal detection plan based on using the network CNN.

2.3. Applying Convolutional neural network

A very common problem of gravitational wave detection is that the time for gravitational wave signal to occur is random in the window, which detects the signal that may occur in the time-amplitude analysis. Following a WP decomposition, a two-dimensional diagram will result in some deviation or even false results in the time-frequency analysis. CNN is designed to settle the suggested problem. Two important terms should be noticed when using CNN. One is “convolution kernel (CK),” and the other one is “pooling”. They play different roles on their layers respectively. One is for the calculation or computation, and the other is for the process of redundancy.

In the instance of a convolutional layer accepting data of either a matrix or a tensor, the CK is defined using it. In this layer, changing the position of a CK on the data to be analyzed, do calculation between and the data below the CK. On account of the convolution calculation, the occurrences of some distinguishable traits of gravitational wave signals can be collected. The output from the layer of the NN before is the same as the input into a convolutional layer. The prior layer has a possibility to just be the CNN input layer (as exhibited in Fig. 4).

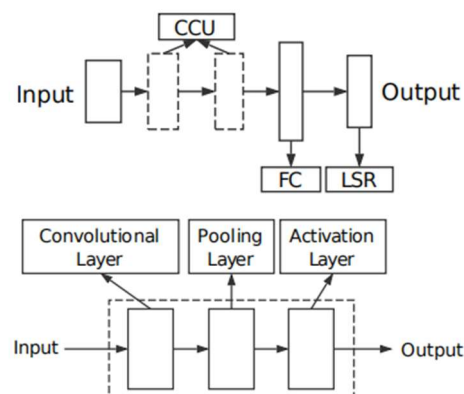


Figure 4. An expanded view of CNN gives the general inner structure of CNN and how data is processed within it [4].

In the outputs of a convolutional layer, there is a huge likelihood of having a number of redundancies. Consequently, it can cause a worse accuracy in data, and a downshift of the calculation efficiency on test data. To settle the problem, a process called pooling is selected. The pooling decreases redundancy by combining the response of the convolution in each of the pooling windows. The combining process is completed by calculating the maxima of the responses.

3. The gravitational wave plans in China

This section will illustrate the development of the gravitational wave plans in China

3.1. The Taiji plan

Observing the gravitational wave (GW) tells gravity's nature and helps further explore the Universe. Specifically, a gravitational wave could be applied to measure the Hubble constant according to a way about the standard siren. Even though there is some problem with degeneracy, the accuracy of the Hubble constant is about 2% after the system of detectors observe for 5 years. With the use of the gravitational wave antenna, like Taiji, and the network of LISA-Taiji, the degree of accuracy about the Hubble constant could be improved to less than 1% and avoid the problem of degeneracy. This method asks Taiji scientific collaboration to set up a three-step plan, which includes sending Taiji-1 in 2019 to prepare the essential technique for Taiji-2, which consists of two artificial satellites that will be used to state the technology during 2023-2025.

According to the observation about the redshift, which belongs to the spectral line of a remote galaxy, the farther a galaxy is from us, the sooner it disappears. This motion fits the Hubble law, which demonstrates that the universe is expanding [5].

$$V = H \times d \quad (27)$$

where H_0 means the average expansion rate about the Cosmos, or Hubble constant. V_H shows the galaxy's velocity, and d expresses the distance between the observers and the galaxy. One way to determine the Hubble constant is a standard siren. In 2017, a gravitational wave signal (event GW170817) was observed from the combination of a binary neutron star system. Considering that the distance to the source is determined straightforwardly from the gravitational wave measurements, GW170817 was seen as a "standard siren" that could help measure the Hubble constant.

When the gravitational wave detector is surrounding the sun, the position and direction of the sun are changing to the satellite. Thus, the motion of the detector adjusts the measured signal depends on the dimensional orientation of the source, which means that the distance and the inclination of a gravitational source will not degenerate.

3.2. The detection scheme of TianQin

TianQin plan started in 2014. Its purpose is to detect the gravitational waves, which have frequencies between 10–4 Hz and 1 Hz. It was special in many ways: it uses geocentric orbits, which is very rare in the gravitational wave detectors, the constellation plane and ecliptic plane are perpendiculars, and its frequency coincides with LISA and DECIGO. TianQin is expected to get the gravitational waves from the Galactic ultra-compact binaries (GCBs), the stellar-mass black hole binaries (SBHBs), the merger of massive black hole binaries (MBHBs). By getting the gravitational waves from these sources, TianQin may provide important information about the history of galaxies, the nature of gravity and black holes, and the expansion history of the Cosmos. To make the technology of the TianQin program more mature, there are four steps to implement [6]

- Step 0: Using lunar laser ranging devices to get accurate orbit information from the satellites
- Step 1: Using one satellite to test the maturity of the inertial reference technology
- Step 2: Using one satellite to test the maturity of the inter-satellite laser interferometry
- Step 3: Launching three satellites to compose the space-based GW observatory

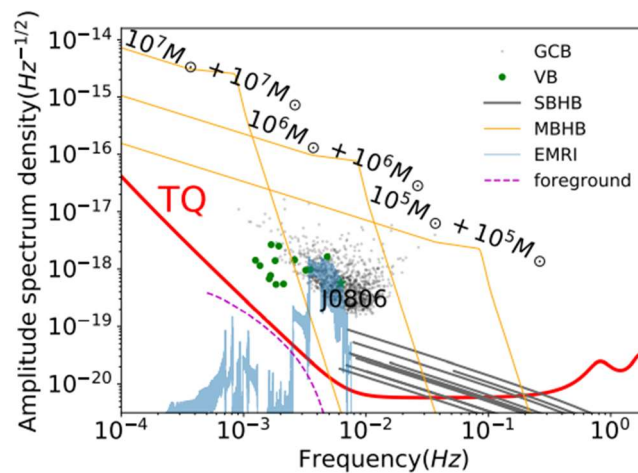


Figure 5. The source that Tianqin may detect

In Figure 5, the information of the detector received is included in the signal strain of each source, and the instrumental noise (red curve) is approximated by dividing $S_n(f)$ by 10/3, which accounts for the geometric configuration of Tianqin and the location and polarization-sensitive response in the low-frequency limit [5].

3.3. Ali Primordial Gravitational Wave Observatory

Ground detection for GW asks very serious observation environment, which needs highly atmospheric transmittance. This is because the water vapor in the air will reduce the transmittance, increase a lot of photons shot noise, and increase optical loading on the devices [7]. Therefore, the Ali GW observatory is built in the Ali Prefecture of Tibet at an altitude of 5250 meters, where has Dry weather and open views. AliCPT could observe the northern and southern sky and the area with the lowest foreground pollution in the north. It is compounded with Ali CMB Polarization telescope -1 (AliCP-1) and AliCP-2, which are testing now. When AliCPT is getting enough data from the GWS, it is expected to study the CMB polarization hemispherical asymmetry, the cross-correlation between AliCPT and DESI [8]. Learning more about the galactic foreground and calculating the dark energy property, neutrino masses.

4. Further application

4.1. GWs application

Since gravitational waves have the trait that the electromagnetic waves fail to obtain (they do not interact strongly with matter), it is expected to bring more undetected information about astronomical phenomena. Compared with photons existing in electromagnetic signals are ordinarily phase-incoherent, gravitons in GWs are phase coherent. Thus, we can utilize the phase coherence (similar to laser) of GWs to strengthen their detectability. The approach of detecting GW bursts, such as those produced by coalescing compact binaries, is to combine well-simulated functional forms with a specific threshold filtering technique. Sources could be revealed by a factor of approximately the square root of the number of cycles in the waveform.

On most occasions, electromagnetic astronomy is limited in the light of the fact that it works on account of the deep imaging of small fields of view. Therefore, observers endeavor to find out a bunch of messages by using numerous facilities observing on a minor part of the sky [9].

4.2. GW detection for accomplishing some properties of Celestial bodies

Categorizing GW sources and the means for discovering their waves by the frequency band in which they radiate is useful. In general, we would separate the GW spectrum into four parts. Generally, in terms of compact sources, GW frequency is determined by source's size R and mass M .

$$f_{GM}(M) < \frac{1}{4\sqrt{2}\pi} \frac{c^3}{GM} \approx 10^4 \text{ Hz} \left(\frac{M_{\odot}}{M}\right) \tag{28}$$

We can roughly know the size of the upper bound, given the mass achieved by observing data [9]. The size of chirp mass could be calculated by simply analyze the data frequency and frequency derivative of the gravitational waves at any time are required. For instance: we can estimate frequency from the time-frequency plot of the observed gravitational wave strain data (Fig. 6)

$$\mu = \frac{c^3}{G} \sqrt{\left(\left(\frac{5}{96}\right)^3 \pi^{-3} f_{GW}^{-11} (f'_{GW})^3\right)} \tag{29}$$

The size of chirp mass could be calculated from simple analysis of the data, frequency, and frequency derivative of the gravitational waves at any time are required. For instance: we can estimate frequency by using the time-frequency plot of the observed gravitational wave strain data (Fig. 6). We can use the formula (2) to make calculations [10]. It can also estimate the frequencies in Fig. 7.

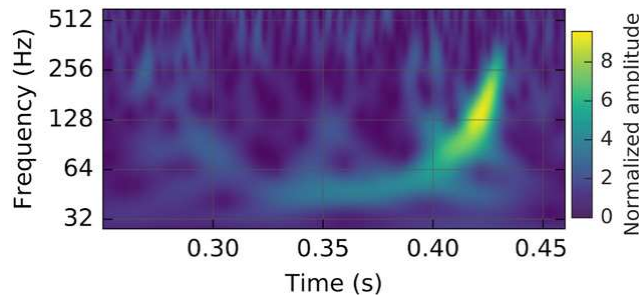


Figure 6. A representation of the strain-data as a time-frequency plot, where the increase in signal frequency (“chirp”) can be traced over time [10]

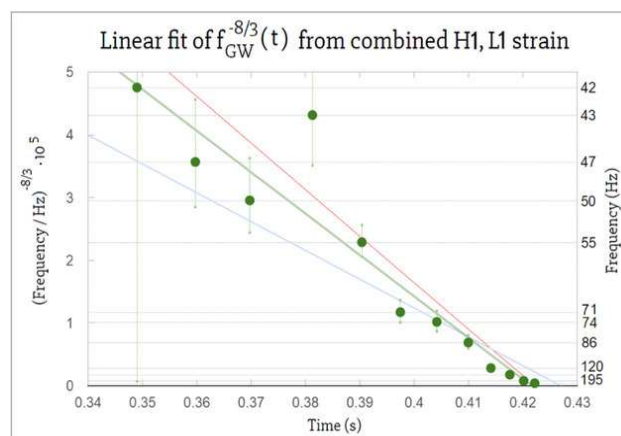


Figure 7. The fitting results of $f_{GW}^{-8/3}(t)$ with a linear model with the combined strain data from H1 and L1 [10]

4.3. Cosmological parameters

The information yielded by GWs can obtain some cosmological parameters that have not been calculated, or accuracy is poor. Independent calculations of the value of the cosmological parameters test the current cosmological paradigm exactly, which is significant for the next generation of the interferometer device and methods of data analysis. Hence, using previous data to confirm cosmological parameters is

meaningful for further GW application. This part will focus on the TianQin parameter ascertain. It is valuable to confirm the post-Newtonian parameter at the level of some scalar-tensor theories of gravity. Shapiro delay may be possible for us to find out the accuracy of the post-Newtonian parameter γ , which is due to the fact that laser signal propagation of TianQin's spacecraft in orbit near the earth is affected by the Earth-Moon system. The recent best value of $\gamma = 1 + (2.1 \pm 2.3) \times 10^{-5}$ was given by the Cassini mission [11]. The uncertainty in the PPN parameter γ is expected with an accuracy of 1.8×10^{-7} measurement accuracy can reach a certain level [12].

5. Conclusion

In the past, scholars explore the universe entirely depending on electromagnetic radiation, e.g., visible light, X-rays, radio waves, microwaves. By detecting the gravitational waves, we obtain information from the objects that we cannot see, proving the existence of the black hole. To detect a gravitational wave, China made three plans, Ali, Taiji, and TianQin, to detect the gravitational waves. When objects move, the curvature of spacetime will be different, and these changes move as the way of gravitational waves, which means that they can be detected by measuring the change in distance between two objects. For the Chinese Ali plan, it is a ground observatory, which was built in the Ngari region, Tibet, China. TianQin plan is working on improving the accuracy of the Hubble constant, and the Taiji plan could receive the specific frequency of gravitational waves. These results offer a guideline for further development in detecting gravitational waves and pave a path to better explore the nature of gravity and the universe.

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