A Test of Newton's Law of Gravity Using the BREN Tower, Nevada

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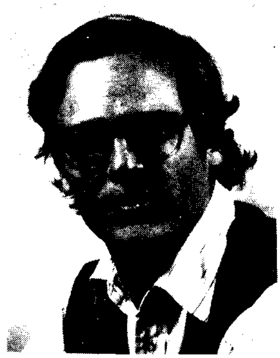

We predicted gravity values on a tower by upward continuing an extensive set of surface data in order to test the $1/r^2$ dependence of Newton's Law of Universal Gravitation. We measured gravity at 12 heights up to 454 m on a tower at the Nevada Test Site, and at 91 locations on the surface of the earth within 2.5 kilometers of the tower. These data have been combined with 60,000 surface gravity measurements within 300 kilometers of the tower and have been used to predict the gravitational field on the tower via a solution of Laplace's equation. A discrepancy between the observed gravity values and the prediction could suggest a breakdown of Newtonian Gravity, but we observe none. Our preliminary results are consistent with the Newtonian hypothesis to within 93±95 μgals at the top of the tower, a result which conflicts with the previously reported 500 μgal non-Newtonian signal seen at 562 meters above the earth.[1,2]

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1. Motivation

Both theoretical[3] and experimental[1,4,5] studies have suggested that the inverse-square law for gravity may be wrong over distances of 10-1000 m. The functional form has been validated on short scales in the lab, and on long scales by observation of orbits. However, it was noted as early as 1971[6] that lab-scale measurements determine the gravitational constant G while orbit studies determine only the product GM_{Earth} and therefore the asymptotic form of the force law. Consequently, many functions are consistent with the observations, and no experiments had been performed to define the law over ranges from 10 m to 10000 m.

Recently, Newton's Law has been tested by measurements over a range of heights both below the ground[1], and above the Earth's surface in air[2] and in ice[3]. These approaches can be understood by writing the Newtonian formula for the magnitude of the gravitational acceleration (defined positive downward) at a point r on a non-rotating Earth as a volume integral over the density distribution, ρ,in the Earth.

$$g(\vec{r}) = \left|\nabla_r U(\vec{r})\right| = G\int \frac{\rho(\vec{r}\,')\,dV'}{\left|\vec{r} - \vec{r}\,'\right|^2} \tag{1}$$

where G is the universal gravitational constant postulated by Newton. Ignoring the ellipticity of the Earth, we can describe its density distribution with two terms: $\bar{\rho}(r)$,which is a model representing our best guess of the average density in each spherical shell of the Earth, and $\Delta\rho(r, \theta, \phi)$, which represents all the deviations from that model, including lateral variations of density within the Earth. The radial component of the Newtonian gravity field is given by integrating over these density distributions

$$g_r(\vec{r}) = \frac{G\bar{m}(r)}{r^2} + G\int \frac{\Delta\rho(r',\theta',\phi')(\vec{r} - \vec{r}\,') \bullet \hat{r}\,dV'}{\left|\vec{r} - \vec{r}\,'\right|^3} = \frac{G\bar{m}(r)}{r^2} + I(\vec{r}) \tag{2}$$

where Gauss' Theorem has been used to simplify the integral over $\bar{\rho}(r)$, $\bar{m}(r)$ is the mass of the model lying beneath the shell of radius r, and the ^ indicates a unit vector. The details of the average density model below the experiment are not important.

All experiments use this Newtonian model to predict the difference between the gravity values measured at two radii. Defining the gravity anomaly to be $\Delta g_r(\vec{r}) = g_r(\vec{r}) - \frac{G\bar{m}(r)}{r^2} = I(\vec{r})$, we have a simple equation for an empirical test of Newton's Law

$$\Delta g(r_1) - \Delta g(r_2) = I(r_1) - I(r_2) \tag{3}$$

where the difference in the observed gravity anomalies at different radii is compared to a volume integral over the density anomalies in the Earth.

The integrals on the r.h.s. contain all the complexities associated with lateral variations of density within the Earth. If these are ignored, equation 3 describes an apparently straightforward test of Newton's Law, first proposed by Airy[7]. In this test, the difference in the average-Earth model at two radii is usually expressed as the sum of two terms: the free-air gradient term, and an integral over vertical variations in density.

Airy method experiments in a mine[4], in boreholes[5,8], and in the air[1] produce deviations from 0 in equation 3 that vary linearly with radius with a derivative on the order of 1-2% of the normal free-air gradient. These results can be interpreted to suggest violations of Newton's Law on the order of 1-4%, but the sign and magnitude of the deviation differ from experiment to experiment. An alternative interpretation suggested by the lateral variation integrals $I(r_1)$ and $I(r_2)$ in equation 3 is that the measurements in the Earth or in ice are detecting the effects of variations in $\Delta\rho$ that cause anomalous free-air gradients.[9,8,5]. Uncertainty about these density variations is the greatest limitation of the Airy experiment.

Anomalous free-air gradients are seen throughout the world, and they normally are removed from the data when inferring density from borehole gravity surveys[10]. Hammer[11] and later Kuo[12] collected gravity measurements in tall buildings in the eastern United States, seeking a calibration method for gravity meters. After accounting for a variety of effects, including the mass of the buildings, they found linear deviations from 0 in equation 3. Believing in Newton's Law, they looked for causes in the Earth, and suggested that the integrals $I(r_1) - I(r_2)$ conspire to produce a correlation between the anomalous free-air gradient and the isostatic gravity anomaly measured on the Earth's surface. The isostatic anomaly is defined as the measured gravity minus the effect of a model of the Earth where the mass in each column of material is the same even though the columns have different heights. In this model, the mass of mountains is "compensated" by low-density roots that are apparently 50 to 150 km deep. A map of isostatic anomalies has features with wavelengths of 10 to 50 km, and indicates the effects of uncompensated mass or mass deficits, including short-wavelength topography, in the crust and upper mantle.

We have examined more recent data to see if it is consistent with Hammer[11] and Kuo's[12] suggestions. Five new determinations of anomalous gradient in the United States from this paper and others[1,8,13], all reported for the purpose of testing the validity of Newton's law, have been added to the seven earlier measurements to produce figure 1. The anomalous gradient, expressed as percent, is plotted as a function of the isostatic anomaly estimated from Woolard's isostatic gravity map of the US[14]. These data suggest that there are variations on the order of 1-2% in the gradient, and that the gradient is low over areas of mass deficit, and high over areas of mass excess. We musct account for the effects of these masses, and that is difficult for a test requiring the determination of a volume integral over un-explored depths in the Earth.

We can write another test of Newton's Law which involves comparing the difference between gravity measurements on a tower directly to observable data, specifically a surface integral of the measurable gravity anomaly on the Earth's surface, rather than to a volume integral over unknown density distributions. Newton's Law (equation 1) implies that the surface integral describes all the contributions of the integral I(r) to the gravity field outside the earth.

2. Derivation of basis of method.

If equation 1 is correct, and the density of air is neglected, then the gravitational potential is a solution to Laplace's equation $\nabla^2 U = 0$ outside the Earth, and so is $r\nabla_r U = rg_r$. Since the solution to Laplace's equation is uniquely determined by values on a closed boundary, we know there is a functional relationship F that operates on the values of rg_r on the entire surface to produce the value at some height above the surface

$$rg_r(\text{air}) = F[r' g_{r'}(\text{surface})] \tag{4}$$

Furthermore, that relationship holds for the Newtonian gravitational field of any proposed mass lying beneath the earth's surface. Using equation 3 to subtract the effect of the whole-earth model $\overline{m}(r)$ from both sides, and dividing through by r gives

$$\Delta g_r(\text{air}) = g_r(\text{air}) - \frac{Gm_{\text{Earth}}(\text{air})}{r^2} = \frac{1}{r}F[r'\Delta g_{r'}(\text{surface})] \tag{5}$$

Equation 5 describes a test of Newton's Law that involves a direct comparison of observable data, with no assumptions about the density within the Earth. This experiment has two limitations: our ability to sample the gravity anomalies on the Earth's surface completely, and our ability to evaluate the function F with sufficient accuracy. Previous experimenters[1,2] used three different approaches to evaluate F, and argued they produced similar results. We have chosen the most simple approximation, to treat our gravity measurements as if they were collected at the same elevation, which turns the functional relationship into a surface integral. The error in this approximation is estimated from numerical studies described later. The advantage of this approximation is that when the surface integral is "discretized" for numerical evaluation, it reduces the surface sampling problem to the estimation of mean values of the gravity anomaly within areas on the earth's surface, and itallows a rather straightforward analysis to estimate the errors due to sampling and truncation of the integral[16].

If the surface data are collected on a sphere of radius a, then equation 4 becomes Poisson's equation:

$$rg_r(r,0,0) = \frac{a^2(r^2 - a^2)}{4\pi}\int g_r(a,\theta,\phi)\frac{d\Omega}{(r^2 + a^2 - 2ar\cos(\theta))^{3/2}} \qquad (6)$$

where $d\Omega$ is an element of solid angle and r the distance of the point of observation from the center of the sphere . If z is the elevation of the point on the tower above the surface (z=r-a) and r' is the distance along the surface of the sphere to a measurement point (r' = aθ) then we can expand Equation 6 in powers of z/a, keep leading terms, and find

$$\Delta g(z,0,0) = \frac{z(1 - 2z/a)}{2\pi}\int_0^\infty\int_0^{2\pi}\Delta g(0,r',\phi)\frac{r'dr'd\phi}{(r^2 + z^2(1 - z/a))^{3/2}} \qquad (7)$$

where the whole-Earth model has been eliminated as in equation 5. The perturbations caused by ignoring the terms $O((z/a)^2)$ are small, about 1 part in 70 million, as is the effect of the ellipsoidal shape of the Earth. Equation 7 describes a relationship between values measured with a gravity meter on the Earth's surface and on a tower, and we use this spherical approximation to test the validity of Newton's Law.

Gravity meters measure the magnitude of the gravity force vector, described here in units of gals, where 1 gal = 1 cm/sec^2. The force vector is dominated by the 980-gal attraction of $\overline{m}(r)$ but also includes the effects of mass anomalies and topography(<50 mgal), the centrifugal effect due to the Earth's rotation (<4 gal), and the tidal forces(<0.3 mgal). The magnitude of the force vector is always nearly the sum of the radial components of these effects. Centrifugal effects are not harmonic, and tidal forces change with time; so these effects do not obey equation 7 and are removed from all gravity data.

Gravity cannot be measured continuously over the surface of the Earth. Thomas[16] has described the steps involved in approximating the integral in equation 7 based on a finite set of samples over a limited portion of the Earth and quantified the size of the resulting errors. The samples are collected in sectors lying in rings around the tower. Several steps are involved in choosing the ring radii and sector sizes. We define the experimental observable to be the difference in Δg at two heights on the tower. This modifies the integral in equation 7 by making the integrand the difference of two terms. Next, we reduce the integral to a sum over sectors where the average gravity value must be measured. We derive the expression for the error in the sum based on the uncertainty in estimating the mean gravity of each sector, using a preliminary dataset to estimate how well a single measurement of gravity represents the average value as a function of sector size.

Based on that preliminary sampling information, we chose sector sizes to minimize the expected sampling error, subject to 2 conditions: a fixed number of nearly square sectors, and a decreasing error with distance so the sum converges. In addition, we estimated the truncation error resulting from limiting the integral to a finite distance.

Thus, we have reduced the test of Newton's law to a problem of estimating the average gravity value in rings around the tower, using a weighted sum of those averages to predict the difference in gravity anomaly values at different heights on the tower.

3. Measurements

All gravity measurements were made with standard LaCoste-Romberg gravity meters[17]. One standard model G meter, and three model D meters, which measures a smaller range of gravity values, and has less systemmatic error, were used. Many measurements were repeated to check reproducibility. One of the model D meters was rebuilt and calibrated by the manufacturer before this experiment. The other meter was borrowed from Los Alamos National Laboratory and had been used routinely in the field.

The sources of error in the LaCoste-Romberg instrument and the procedures needed to keep them small are generally well understood. The largest source of error is thermal drift of 100's of μgal, but standard surveying practice reduces its effect to a few μgal for small-scale studies. Drift is removed from the observations by collecting data in loops with repeated stations and assuming that the drift occurred at a constant rate between those stations. Standard tidal corrections[18] based on the station location, date and time but not local structures or oceanic loading, are also removed from each observation.

From each data point, we removed the effect of a standard "whole-Earth" model WGS84, which places the entire mass of the earth (excluding the average atmosphere above the measurement point) below the geoid. The latitude dependence of that model includes the radial component of the centrifugal effect. This step included the $1/r^2$ factor to produce Δg, as is described in equation 3. Because of the strong dependence of $1/r^2$ on elevation, accurate surveying is required to achieve meaningful values of Δg. We consider the elevation uncertainties in detail in the sections on the tower survey and the surface survey.

The BREN (Bare Reactor Experiment-Nevada) Tower, located on the Nevada Test Site, is an excellent platform for gravity measurements. Built to support a massive reactor, it is stable and free from radio-frequency signals, which might interfere with measurements. The tower rises above Jackass Flats, on gently sloping alluvial deposits from a ring of hills that cover about 50% of the horizon. The slope of 1.5^{o} is nearly constant out to 2 kilometers from the tower; the nearest hills, whose summit is 5 kilometers away, rise to about the same elevation as the tower. Thus, near our tower the topography is more gentle than in the previous tower experiment[1,2]. At a distance of 8-10 tower heights, where it is less important but not negligible, there is more terrain in our experiment.

Measurements on the tower.
Our tower measurements were done with two model D gravimeters in order to check for systemmatic errors of the measurements. The tower measurements were collected at 12 elevations in a series of 11 loops for a total of 42 observations. A typical loop was 1.8 hours long: the drift, never more than 31 μgal, was removed from the data as if it occurred linearly in time. For the 6 stations in the upper half of the tower, each station had 3 or 4 repeated measurements, the average of the sample standard deviations was only 6 μgal, and the maximum excursion from the mean was only 24 μgal. In the lower half of the tower, stations had higher variability: the average sample standard deviation was 22 μgal, and the largest excursion was 52 μgal.

We made our measurements only at sunrise during July through October, when the wind velocities are typically below 3 mph. There were no perceptible tower motions when the wind velocity was that low, and we were able to collect repeatable readings with the more sensitive D meters.

Elevations were measured with a Leitz REDmini 2 EDM system, which was bolted to the railing at each level on the tower. We measured the distance from the railing to five corner reflectors set on the concrete base of the tower, and the distance from the base of the gravimeter to the railing was measured in order to determine the height of the gravimeter. The height measurements were repeated for each gravity measurement, and all repeats at each height fell within ± 15 mm of the mean. There was no detectable correlation of gravity with variations in height at each platform, so we believe that atmospheric changes in the light path are the cause of these distance variations. After the measurements, we had the meter re-certified; its calibration was within specifications of ± 5 mm + 5 ppm, or a maximum calibration error of 12 mm at the top of the tower.

Our results are shown as a solid line labeled "BREN data" in Figure 2. The line through the data is a straight line fit to the individual points, whose scatter is not visible on this plot. We make two observations: the data set on the tower is linear and shows no obvious curvature due to non-Newtonian effects. The second observation is that the data do not agree with the predictions of the globally symmetric model of the earth, differing by 2.8 mgal at the highest station. The gravity gradient on the BREN tower, 0.3030 mg/m, is about 1.8% lower than the model predicts. Very litte can be concluded from these observations alone because topographic effects or geologic variations could conspire to produce an anomalous gradient or mask an exponential gravitational field. In fact, about 50% of the anomalous gradient is predicted by calculating the effect of topography, with an assumed density of 2.2 g/cm^3, out to 300 km. These results illustrate that other measurements are needed to determine the effects of lateral variations before one can study the reliability of the $1/r^2$ force law from a measurement of the vertical variation of gravity

Surface Survey.

We used the method described above to design two surface surveys sampling the gravity field with sufficient density to predict the gravity on the tower within chosen uncertainties. In the first Phase of our experiment, described here, we used a sampling density that limited the uncertainty at the top of the tower to ±100 μgals. For our ultimate survey, to be completed by August of 1989, we will collect nine times as many surface points, which will reduce that uncertainty to about ±30 μgals.

We designed a radially symmetric survey pattern described in Table I

Table I: Parameters for surface survey

	Phase I	Phase II
Our measurements		
inner zone (<170 m)		
rays	9	27
rings	3	9
outer zone (200-2655 m)		
rays	12	36
rings	5	15
Existing Database (2.6-300km)		
rays	12-76	36-76
rings	33	40

To ensure that our sampling within 2.5 km of the tower was not biased by inaccessibility of locations with either high or low elevation, we placed all stations within ± 1 m of locations determined from the regular radial grid pattern. This was easily accomplished, because of the simple terrain and limited vegetation. A square board dug into the alluvial soil was used as a base for both gravity and elevation measurements. All board locations were measured with uncertainties of ± 1 cm near the tower increasing to ± 10 cm at 2.5 km using a theodolite located on a small bunker about 30 m from the tower base. The location accuracy was checked using ten independently surveyed benchmarks lying within 2 km of the tower.

The Phase I gravity survey of 91 stations was completed in a series of 61 loops, with a total of 187 measurements. All stations within 1.5 km of the tower were tied directly to the tower base, and stations beyond that distance tied through a single intermediary station. Loop lengths ranged from 30 to 220 minutes; all loops longer than 125 minutes had stations done on independent loops for consistency checks. Typically, the instrument drifted less than 20 μgals during each loop, although some loops had drifts as high as 50 μgals. One loop was eliminated because its drift was over 100 μgals. Sample standard deviations based on stations with repeated measurements were typically 10 μgals.

Beyond 2.5 km, we used public domain datasets from the USGS[19] and NOAA[20]. Based on our truncation error criteria, we included 60,000 data points out to 300 km from the tower. Multiple samples in a sector were averaged to obtain a sample mean of both gravity and elevation, from which the mean of Δg was calculated. The gravity values have a quoted uncertainty of ± 1 mgal, and the elevation uncertainty can be as large as ± 6 m in the outer zones.

4. Integration of surface data to predict values on tower

The upward continuation results, shown as circles, are compared to the observations in Figure 2. Most of the anomalous gradient is predicted from the upward continuation. Our predictions are within 93 μgals of the observations at all heights on the tower, and the uncertainties at each height are as large as 95 μgals. The error bars in Figure 2 indicate the single standard deviation uncertainty from sampling errors, derived as discussed above, which are the largest source of error in this experiment. At all heights, these errors overlap the observed gravity, providing no evidence for a breakdown of Newtonian gravity.

5. Possible sources of systematic and random error.

Spherical approximation for upward continuation.

The surface field can be uniquely translated into a continuous surface-density distribution, from which the values above the Earth can be readily calculated. For an irregular surface, this transformation requires solution of a Fredholm integral equation that is cumbersome to solve over the distance scales needed for this problem[21]. However, in the approximation that the Earth's surface is a smooth sphere, this integral equation reduces to an integral (equation 7) that can be evaluated easily. This approximation is potentially our most significant source of systemmatic error. Since we cannot correctly predict what we would have measured on the smooth surface, some errors result from our estimates of gravity on that surface. Analysis of the Fredholm integral equation solution for irregular terrain does not produce insights into the size of this effect.

We are using a digital model of the terrain in a 54-km square centered on the tower to estimate the magnitude of the errors from the spherical approximation. Elevation contours (6.1-m interval) were digitized within a rectangle extending 6.1 km north, 5.2 km east, 8.3 km south and 12.0 km west of the tower[22]. For the rest of the square, we used digital terrain sampled on a 15-second grid[23]. The mass of terrain was approximated by a 300x300 grid of density 1.8 gm/cm^3, right-rectangular prisms of identical square cross section, with all bases at an arbitrary depth (-3318 m below sea level) and tops at the height of the smoothed terrain at their centers. At greater distances, the Earth's surface was assumed to be constant to large distances at values determined from the averages of the nearest side of the 54km square. The vertical component of gravity caused by this mass was calculated[24] at all observation points on the tower and at the model elevation in each sector. Using equation 7 to integrate the artificial surface numbers, we found that the spherical approximation underestimates the calculated magnitude of $\Delta g(r_1) - \Delta g(r_2)$ by about 100 μgal for a number of different gridding options. We are continuing this study to include the effects of mass anomalies and to evaluate the effectiveness of including terrain corrections in the definition of Δg.

Random errors and bias due to sampling.

Because gravity is strongly correlated with elevation, the observations must adequately sample the topography, as well as the effects of density variations. Variations in topography and density on a scale large compared to the sectors do not contribute to errors in estimating the sector means. We bound the uncertainty in estimating a sector mean by calculating the sample variance of all the gravity values in a ring of n sectors about the mean gravity for the ring, m_{ring}. Using the assumption that sampling errors in different sectors are independent:

$$\mathrm{Exp}\left(\frac{1}{n-1}\sum\left(\Delta g_i - m_{ring}\right)^2\right) = \frac{1}{n-1}\sum\left(m_{si} - m_{ring}\right)^2 + \sigma_{ring}^{\,2} \qquad (8)$$

The sample variance is a useful bound on the ring variance of the ring mean. In practice we improve this estimate by removing long wavelength effects on the scale of the ring radius. This is done by fitting one or two intersecting planes through $\Delta g(\theta)$ around a ring and removing that "regional" from the data before the ring uncertainty is estimated.

To estimate the uncertainties in the ring means, we assumed that the effects of sampling were uncorrelated from sector to sector. That assumption is valid for our data within 2.6 km of the tower, because the locations were chosen independently of the topography. However, because it may be easier to take measurements in either the valleys or the highlands, there may be a sampling bias in the elevations of measured points beyond 2.6 km. It is important to estimate the magnitude of elevation bias.

We reduce the effect of the sampling by taking averages of many sectors per ring, but the average in each sector can be biased if the sectors are larger than the topographic features influencing the sampling distribution. We used our largest sectors, in the ring from about 270 to 300 km from the tower, to study the magnitude of bias. Here, the sectors are large compared to the data density, and comparable in size to many topographic features in the western US. In many of these sectors, there is a preponderance of data in valleys, but the mountains are sampled on a 4-5 km spacing. We estimated the elevation bias in our outermost ring by subdividing each sector into nearly square subsectors, determining the sector means from the sum of the means in the subsectors, and calculating the ring average as a function of the size of the sub-sectors. The maximum or minimum value as a function of subsector size represents the best averaging we can do with our dataset.

We found that the best subsector size was about 7 km, and that the ring-averaged total gravity and elevation differed from our original values by -7 mgal and 31 m. These numbers indicate that for this ring, Δg is biased by approximately 2.5 mgal. We believe that this represents a reasonable estimate of the elevation sampling bias beyond 80 km, and if it were applied to the data beyond 80 km, it would change the prediction at the top of the tower by only 25 μgal. Inside 80 km, the sector size is below 7km, and more than 85% of the sectors in each ring are filled, except for the ring at 6.3 km, which has only 63% of its sectors filled. Based on the studies of the outermost ring, this data density appears to eliminate much of the bias in sampling topography.

The effects of other sources of error on gravity at the top of the tower have been estimated. These sources include ignoring the ellipsoidal shape of the Earth, uncertainties in gravity meter calibration and measurement error, surveying errors near the tower, changes in geoid height over the range of our surface survey, and the effect of our trucks on the measurements in the tower. Only those discussed in detail above are important.

6. Status and future work

We have used the inverse-square dependence of the force of gravity to successfully predict gravity measurements up to 454 m in the air, within an uncertainty of the order of 100 μgal. To this accuracy level, we see no conflict with Newton's Law. This is in contrast with earlier results from North Carolina[1,2], which are included in figure 2. We are continuing to improve our experiment by collecting more data on the surface and on the tower, by performing numerical studies to better evaluate the uncertainty resulting from our approximations, and to improve our method of upward continuation.

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8. References

1. D. H. Eckhardt, C J. Jekeli, A. R. Lazarewicz, and A. J. Romaides, Phys. Rev. Lett. **60,** 2567, 1988
2 A. J. Romaides, C. Jekeli, A. R. Lazarewicz, D. H. Eckhardt, and R. W. Sands, J. Geophys. Res., **94**, 1563, 1989
3 T. Goldman, R. J. Hughes, and M. M, Nieto, Phys. Lett. B, **171**, 217, 1986
4 F. D. Stacey, and G. J. Tuck, Nature, **292**, 230, 1981
5. M. E. Ander, *et al.* Phys. Rev. Lett. 62, **985**, 1989
6 Y. Fujii, Nature phys. Sci., **234**, 5, 1971
7 Airy, 1854, cited by S. Hammer[10]

8. J. Thomas, P. Vogel, and P. Kasameyer, in Proceedings of the XXIIIrd Rencontre de Moriond, O. Fackler and J. Tran Thanh Van, eds., 585, Editions Frontieres, 1988
9. F. D. Stacey, G. J. Tuck and G. I. Moore, Jour. Geophys. Res. 93, 10575-10587, 1988
10. S. Hammer, Geophysics, 15, 637, 1950
11. S.Hammer, Trans. Amer. Geophys. Union, 19th Annual Meeting, p 72-82, 1938
12 J. T. Kuo, M. Ottaviani, and S. K. Singh, Geophysics, **34**, 235, 1969
13. A. T. Hsui, Science, **237**, 881, 1987
14 G. P. Woolard, in The Earth beneath the Continents, Geophysical Monograph 10, J. Steinhart and T. Smith, eds., 557, American Geophysical Union, Washington D. C., 1966
15. O. D. Kellogg, Foundations of Potential Theory, Ungar (reprinted by Dover), 1929
16 J. Thomas, Phys. Rev. Letters. D, in press, 1989
17 W. P. Telford, L. P. Geldart, R. E. Sheriff and D. A. Keys, Applied Geophysics, 860pp, Cambridge University Press, 1976
18 C. E. Corbato and L. A. Beyer of the US Geological Survey provided the code, which was checked against NEEDS
19 R. W. Saltus, U. S. Geological Survey Open-File report 88-433, 1988
D. A. Ponce, Jour. Geophys. Res., **89**, 9401-9413, 1984.
20 Natl. Oceanic and Atmospheric Admin., User Services, 325 Broadway, Boulder, Colorado
21. B. K. Bhattacharya and K. C. Chan, Geophysics, **42**, 1411, 1977
22 US Geological Survey 1:24,000 topographic maps, for Jackass Flats, Skull Mountain, Spcter Range and Striped Hills 7.5 minute quadrangles, Nye Co., Nevada, revised 1983
23 D. Plouff, described in U.S Geological Survey Open File Report 77-534, 45 p, 1977
24 D. Plouff, Geophysics, **41**, 727, 1976

Figure 1. Measured anomalous vertical gradients in the USA are correlated with isostatic gravity anomalies. Data from buildings[11,12] and towers[1] (circles) support Hammer and Kuo's conjecture that vertical gradient anomalies correlate with isostatic gravity anomalies. Data from boreholes[8,13] (squares) are less well-correlated, probably because of limitations in our knowledge of the sub-surface density.

Figure 2. Results for two tests of Newton's Law. Observed values of the change in Δg as a function of height for experiments on the BREN tower in Nevada, described in this paper, and the WTVD tower in North Carolina[1,2]. The solid lines indicate linear fits to the observations, which are represented by triangles. The dashed lines are linear fits to the predictions based on Newton's Law (equation 7), and those predictions are shown as circles and squares. The measurement errors are negligible on this scale. The stated errors in the predictions are represented by vertical bars. The anomalous gradient we observe on the BREN tower is predicted from the surface survey within our sampling uncertainty.

Figure 1.

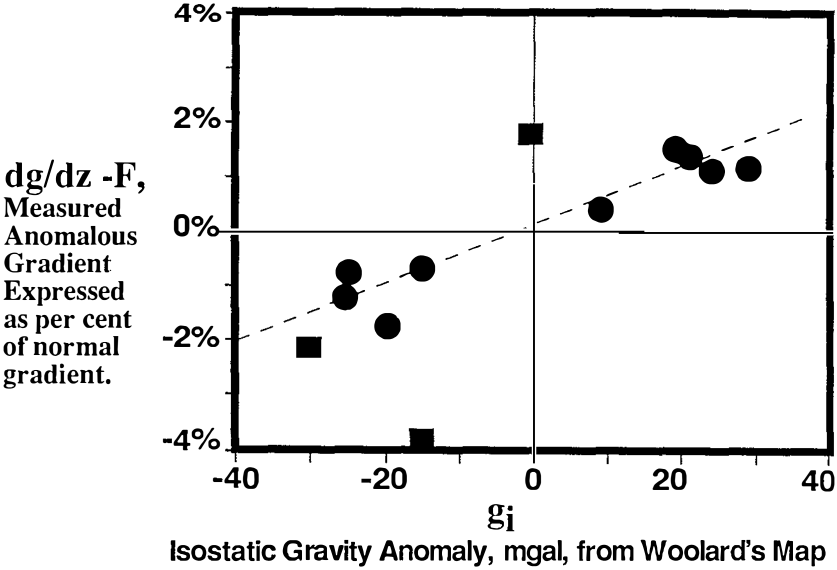

Figure 2.