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Quantum Multi-Path Communication Protocol Based on Maximum Flow Theory

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***Abstract*—Quantum networks are an actively researched and promising field, aiming to achieve efficient quantum information transmission by interconnecting quantum nodes. In large-scale quantum networks, end-to-end throughput is a critical factor that affects the overall performance of the network. The maximum flow problem, extensively studied in classical network theory, identifies a set of paths between the source and destination nodes that maximizes the total flow. This study extends the maximum flow problem to quantum networks, focusing on coordinating multiple paths for multi-path quantum communication. We propose a Quantum Multi-Path Communication Protocol (QMCP) that employs maximum flow theory to allocate transmission resources across multiple nodes efficiently, thus maximizing the total transmission capacity from the source to the destination. Our evaluation demonstrates that QMCP significantly enhances end-to-end throughput in quantum networks.**

***Index Terms*—Entanglement Routing, Quantum Communication, Quantum Networks**

I. INTRODUCTION

In recent years, quantum networks have become a focal point in research, leading to successful applications in fields such as quantum key distribution [1], distributed quantum computing [2], and quantum teleportation [3], enabling functionalities that classical networks cannot achieve. Similarly to classical networks, quantum networks transmit information using qubits. However, due to the no-cloning theorem [4], qubits cannot be transmitted by hop-by-hop forwarding. The prevailing approach is to establish end-to-end entanglement connections through entanglement swapping [5], [6], which are then used for end-to-end communication.

The construction of long-distance, large-scale quantum networks requires advancements in both hardware and software. On the hardware side, high-performance quantum repeaters and entanglement sources are required to overcome signal attenuation during long-distance transmission [7], [8]. On the software side, efficient quantum error correction algorithms [9], [10] and entanglement routing protocols [11]–[13] are crucial for building robust quantum networks. Quantum network routing protocols must consider unique entanglement properties, aiming to maximize distribution rates and maintain high fidelity, beyond merely selecting paths. Previous studies have primarily focused on the design and theoretical analysis of long-distance quantum entanglement routing protocols [14]–[16]. Recently, several studies have used fidelity [17] to quantify the quality of entanglement links when designing entanglement routing protocols and have applied this metric in routing decisions [18]–[21]. However, it remains critical to explicitly verify the capacity of the entanglement paths before selecting the appropriate entanglement links.

In this paper, we propose the Quantum Multi-Path Communication Protocol (QMCP), which functions as a quantum counterpart to classical flow control protocols, enabling multi-path entanglement routing. Based on classical maximum flow theory [22]–[24], QMCP calculates and allocates appropriate flows among multiple quantum nodes. To evaluate our protocol and algorithm, we implement a packet-based, event-driven quantum network simulator that concurrently simulates packet switching in classical channels and qubit transmission in quantum channels. Our evaluation results demonstrate that QMCP can flexibly and efficiently establish end-to-end entanglement links across quantum networks of varying scales, thereby enhancing end-to-end throughput.

The contributions of this paper are summarized as follows:

- We propose QMCP, which utilizes the maximum flow theory derived from classical networks to identify optimal path combinations and resource allocation strategies that satisfy the requirements for establishing end-to-end entanglement connections. This approach fully exploits quantum storage resources at the nodes, thereby improving network throughput.
- We conducted extensive simulations, demonstrating that our method significantly improves end-to-end throughput in quantum networks.

II. BACKGROUND

A. Entanglement Connection

Establishing entanglement connections between distant quantum nodes is essential to realize distributed quantum applications. However, as the physical distance of the quantum channel increases, the success probability of direct entanglement distribution between two quantum nodes decreases significantly. To overcome this challenge, a method inspired by quantum teleportation has been proposed. In this method, an entangled qubit is separated from an entangled qubit pair [25]. This separated qubit can then be transferred to another quantum node. As a result, entanglement is established between

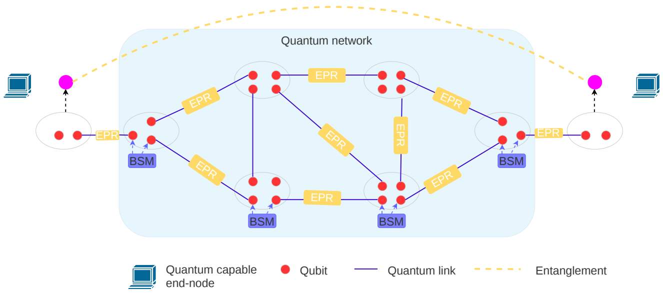

Fig. 1. An example of a quantum network.

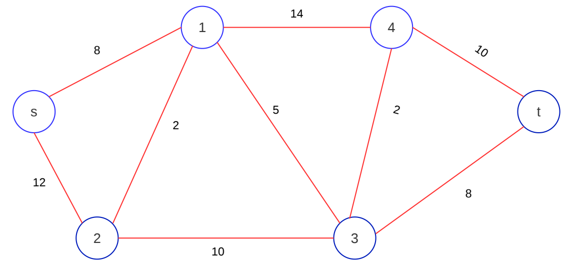

Fig. 2. A motivating example (Solid lines for entanglement links.)

distant quantum nodes. This technique, termed entanglement swapping [5], [6], has been suggested as an effective solution to generate long-distance entanglement between distant quantum nodes.

B. Quantum Networks

A quantum network consists of multiple quantum nodes that are interconnected through quantum links. Since the success rate of qubit transmission decreases exponentially with link length, entanglement-based networks have been proposed to improve the transmission of quantum information by leveraging quantum entanglement [26], [27]. When there is no direct physical link between a source node S and a destination node D, repeater nodes are arranged along the physical path connecting S and D. These repeater nodes are responsible for generating entanglement (Fig. 1) with adjacent nodes and performing entanglement swapping operations, facilitating an end-to-end entanglement link between S and D. Once the entanglement link is successfully established, S can teleport an information qubit to D by consuming the established entanglement.

C. Maximum flow problem

The network flow problem focuses on determining the optimal way to transmit flow through a network and is widely applied in fields such as computer science, communication networks, and operations research. Among these, the maximum flow problem [28] is one of the classic issues in network flow theory, aiming to find the maximum flow from the source node to the sink node. The general formulation can be described as: Given the source node, sink node, and edge capacities, determine a flow allocation that maximizes the total flow from the source to the sink. When solving this problem, the following two constraints must be satisfied: (i) the flow on any edge must not exceed its capacity; (ii) except for the source and sink nodes, the inflow to any node must equal its outflow.

III. DESIGN

A. Motivation

Resources in quantum networks, such as entangled pairs and qubits, are limited and valuable. To optimize network throughput, it is crucial to determine the maximum flow between the source and target nodes. This involves distributing the flow across multiple potential paths to maximize the total transmission capacity. For example, the capacity on each quantum link, ranging from 2 to 14, is shown in Figure 2. In the quantum network topology illustrated, suppose that we consider the entanglement path $s \to 2 \to 3 \to t$. If we allocate a flow of 12 units along this path, the path from node 3 to node t will become congested due to the link capacity limitation. To avoid this bottleneck, we can distribute the flow across multiple paths to optimize the overall network throughput. Specifically, the flow can be allocated as follows:

- $s \to 1 \to 4 \to t$: allocate 6 units of flow;
- $s \to 2 \to 1 \to 4 \to t$: allocate 2 units of flow;
- $s \to 2 \to 3 \to t$: allocate 8 units of flow;
- $s \to 2 \to 3 \to 4 \to t$: allocate 2 units of flow.

By distributing the flow in this manner, we can achieve a maximum total flow of 18 units from the source node s to the target node t. This example demonstrates the importance of proper flow allocation and motivates us to explore methods for addressing the challenges of path selection and flow allocation in quantum networks.

B. Models

We model our quantum network by mimicking the structure of the classical Internet and adopting the following components and assumptions.

1) **Quantum Nodes**: These nodes serve as the fundamental units of the quantum network. Despite potential variations in processing capability and storage capacity across nodes, this model assumes uniform functionality among all nodes.
2) **Quantum Memory and Quantum Link Capacity**: Quantum memory comprises slots, referred to as memory positions, for storing qubits. Since establishing entanglement between quantum nodes necessitates each node to hold one qubit of an entangled pair, we define quantum link capacity in terms of quantum memory capacity. Specifically, the quantum link capacity between nodes is determined by the number of slots available in quantum memory for storing entangled qubits.

IV. QMCP DETAILS

Our protocol consists of two components: entanglement routing and maximum flow entanglement path selection. The path selection algorithm identifies entanglement connection paths and calculates the amount of entanglement distribution

Algorithm 1: Entanglement Routing

```
Input: Current node: node, start expression: message
Function HandleRoutingMessage(node, message):
    next_hop ← GetNextHop(message);
    if isinstance(message, RoutingRequest) then
        if node.qmemory_positions[next_hop] is not full then
            Generate entanglement between node and
             next_hop;
            message.path.append(node.ID);
            ForwardRequest(next_hop, message);
        else
            return Failure(message);
        end
    if isinstance(message,qubit) then
        if node.qmemory_positions[next_hop] is not full then
            node.qmemory[message].append(qubit);
            if len(message.path) == 2 then
                return Success(message);
            else if len(node.qmemory[message]) == 2 then
                m ←
                 BellMeasure(node.qmemory[message]);
                node.qmemory[message].clear();
                ForwardMeasurement(next_hop, m);
            end
        else
            return Failure(message);
        end
    if isinstance(message, Measurement) then
        if node is the destination of message then
            node.meas_result.append(m);
            if len(node.m_result) == len(message.path) - 1
             then
                qubit ← node.qmemory[message].pop();
                Correction(qubit, node.meas_result);
            else
                ForwardMeasurement(next_hop, m);
            end
end
```

for each path, enabling the source node to reorganize its routing table. For the same destination, the entanglement routing protocol can distribute entanglement across different paths, thus ensuring optimal utilization of network resources.

A. Quantum Entanglement Routing

We have adapted classical routing to be suitable for quantum networks and designed the Entanglement Routing algorithm. The main process of this algorithm is illustrated in Algorithm 1. Upon receiving a request, the routing protocol selects the next hop from the routing table and checks the quantum memory for available space to establish entanglement with the next hop. Concurrently, the request is forwarded through a classical channel. Each intermediate node appends its identifier to the request, marking the number of hops traversed. When two corresponding qubits are received, the node performs a Bell measurement and forwards the results to the next hop to complete the entanglement swapping. If the received message is a measurement result, it is either forwarded to the next hop, or, if all intermediate measurements have been collected, it is utilized to perform the correction procedure. This algorithm ensures accurate routing and efficient transmission of quantum messages within the network, while taking memory constraints into account.

Algorithm 2: Maximum Flow Entanglement Path Selection

```
Input: flow_dict, source, sink
Function find_paths(current_node, path, flow):
    if current_node == sink then
        flow ← min(flow_dict[u][v] for (u, v) in
         zip(path[:-1], path[1:]));
        if flow > 0 then
            paths.append((path, flow));
            // Reduce the flow along the path
            for i ← 0 to len(path) - 2 do
                u ← path[i];
                v ← path[i + 1];
                flow_dict_copy[u][v] -= flow;
            end
        return;
    for neighbor in flow_dict[current_node].keys() do
        if flow_dict[current_node][neighbor] > 0 and
         neighbor not in path then
            minflow ← min(flow,
             flow_dict[current_node][neighbor]);
            find_paths(neighbor, path + [neighbor],
             minflow);
    end
end
Output: paths
```

B. Max-Flow Entanglement Path Selection

Here, we assume that the quantum memory capacity of each node in the quantum network is known. The classical maximum flow algorithm [23] is used to calculate a flow dictionary that records the flow from the source node to other nodes and this dictionary is stored in the routing table of the source node. Using this flow dictionary, Algorithm 2 extracts multiple paths to establish entanglement connections through Depth-First Search (DFS). Specifically, the algorithm conducts a recursive search for all possible paths from the source node to the sink node. For each complete path found, the minimum flow is determined by comparing the remaining flow on each edge of the path, and this value is recorded in the path list. Subsequently, the corresponding flow along the path is reduced to avoid repeated use. Finally, the algorithm returns all possible maximum flow paths along with their corresponding flow values.

V. PERFORMANCE EVALUATION

A. Test Case Generation

We conduct extensive simulations using the quantum network simulator NetSquid [29] to evaluate the performance of our algorithm. To construct a network topology with quantum nodes N, we use the Waxman graph model [30] as the foundational structure. This model typically generates network topologies with diverse paths and reasonable density, making it suitable for evaluating network throughput and maximum flow characteristics. In our simulation, the default network consists

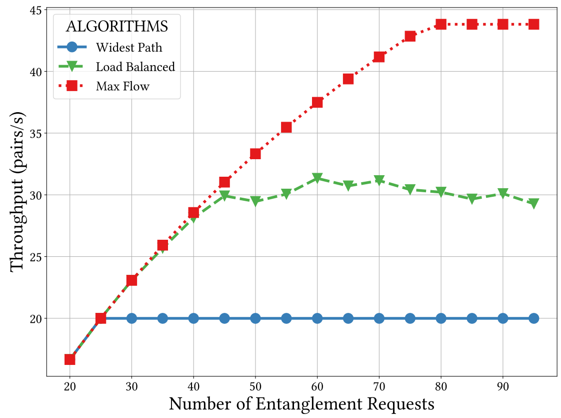

Fig. 3. Throughput v.s. number of entanglement requests.

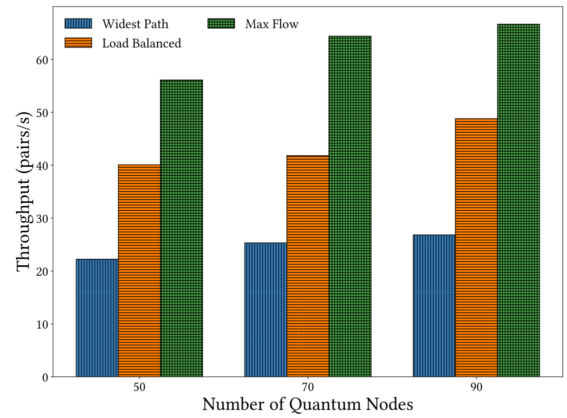

Fig. 4. Throughput v.s. network scale.

of 50 quantum nodes, each node connected via quantum links. On average, each quantum link contains 20 quantum memory units. We simulate network traffic by randomly selecting source-destination pairs, where both the source and destination nodes can be any nodes within the network. The source nodes generate routing requests following a Poisson distribution with fixed parameters, thus simulating the randomness of traffic in a real network.

B. Comparative schemes And Performance Metrics

We compare our algorithm with two baselines: (1) the Widest Path algorithm [31] and (2) the load balance algorithm [32]. The widest path algorithm identifies a path with the maximum minimum transmission capacity and establishes end-to-end entanglement along this fixed path. The load balancing algorithm distributes the entanglement requests evenly across different paths to achieve end-to-end entanglement. To quantify network performance, we use throughput as our metric, defined as the number of successfully generated source-target entanglement connections divided by the time elapsed.

C. Evaluation Results

1) Effect of number of Requests: We simulate varying number of requests while keeping other parameters constant to evaluate algorithm performance under different traffic conditions. Figure 3 shows the relationship between the number of requests and throughput. Since the widest path algorithm selects a fixed path for each request, the storage capacity of nodes on that path is quickly exhausted, leading to network congestion. In contrast, both the maximum flow path selection algorithm and the load balancing algorithm distribute a large number of requests across multiple paths, alleviating node overload on specific paths. However, our path selection algorithm benefits from a more rigorous combination of paths, enabling more efficient utilization of network resources. The numerical results indicate that our maximum flow path selection algorithm outperforms the other two algorithms in terms of network throughput.

2) Effect of number of Quantum Nodes: Finally, we show the performance of different algorithms as the network scale changes. In our experiment, we vary the number of quantum nodes in the network while keeping the number of requests fixed at 100. We randomly select a source-destination pair from the network and apply the three algorithms to choose paths for establishing entanglement connections from the routing table of the source node. Figure 4 plots the relationship between throughput and the number of network nodes, with the results being the average of 20 trials. As expected, the maximum flow selection algorithm consistently maintained higher throughput. As the network scale increased, more quantum resources became available for establishing entanglement connections. The maximum flow algorithm can efficiently select the entanglement paths and distribute traffic, making better use of the available quantum resources. In summary, our path selection algorithm consistently delivers superior throughput across different network scales and under higher traffic conditions.

VI. Conclusion

In this paper, we consider the problem of path combination in quantum networks. We propose a novel Quantum Multi-Path Communication Protocol (QMCP) based on classical maximum flow theory, with the aim of maximizing end-to-end throughput in quantum networks under constrained quantum resources. To evaluate the performance of QMCP, we simulated a quantum network where quantum nodes are interconnected through multiple entanglement links. The simulation results demonstrate that our protocol efficiently establishes entanglement links across quantum networks of varying scales, and our path selection algorithm outperforms other methods in terms of network throughput.

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References

[1] C. H. Bennett and G. Brassard, "Quantum cryptography: Public key distribution and coin tossing," *Theoretical computer science*, vol. 560, pp. 7–11, 2014.

[2] A. S. Cacciapuoti, M. Caleffi, F. Tafuri, F. S. Cataliotti, S. Gherardini, and G. Bianchi, "Quantum internet: Networking challenges in distributed quantum computing," *IEEE Netw.*, vol. 34, no. 1, pp. 137–143, 2020.

[3] D. Bouwmeester, J.-W. Pan, K. Mattle, M. Eibl, H. Weinfurter, and A. Zeilinger, "Experimental quantum teleportation," *Nature*, vol. 390, no. 6660, pp. 575–579, 1997.

[4] W. K. Wootters and W. H. Zurek, "A single quantum cannot be cloned," *Nature*, vol. 299, no. 5886, pp. 802–803, 1982.

[5] C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, and W. K. Wootters, "Teleporting an unknown quantum state via dual classical and einstein-podolsky-rosen channels," *Physical review letters*, vol. 70, no. 13, p. 1895, 1993.

[6] S. Shi and C. Qian, "Concurrent entanglement routing for quantum networks: Model and designs," in *Proceedings of the Annual conference of the ACM Special Interest Group on Data Communication on the applications, technologies, architectures, and protocols for computer communication*, 2020, pp. 62–75.

[7] H.-J. Briegel, W. Dür, J. I. Cirac, and P. Zoller, "Quantum repeaters: the role of imperfect local operations in quantum communication," *Physical Review Letters*, vol. 81, no. 26, p. 5932, 1998.

[8] M. Lucamarini, Z. L. Yuan, J. F. Dynes, and A. J. Shields, "Overcoming the rate–distance limit of quantum key distribution without quantum repeaters," *Nature*, vol. 557, no. 7705, pp. 400–403, 2018.

[9] A. R. Calderbank and P. W. Shor, "Good quantum error-correcting codes exist," *Physical Review A*, vol. 54, no. 2, p. 1098, 1996.

[10] A. Ekert and C. Macchiavello, "Quantum error correction for communication," *Physical Review Letters*, vol. 77, no. 12, p. 2585, 1996.

[11] M. Pant, H. Krovi, D. Towsley, L. Tassiulas, L. Jiang, P. Basu, D. Englund, and S. Guha, "Routing entanglement in the quantum internet," *npj Quantum Information*, vol. 5, no. 1, p. 25, 2019.

[12] K. Chakraborty, F. Rozpedek, A. Dahlberg, and S. Wehner, "Distributed routing in a quantum internet," *arXiv preprint arXiv:1907.11630*, 2019.

[13] C. Li, T. Li, Y.-X. Liu, and P. Cappellaro, "Effective routing design for remote entanglement generation on quantum networks," *npj Quantum Information*, vol. 7, no. 1, p. 10, 2021.

[14] G. Vardoyan, S. Guha, P. Nain, and D. Towsley, "On the stochastic analysis of a quantum entanglement switch," *ACM SIGMETRICS Performance Evaluation Review*, vol. 47, no. 2, pp. 27–29, 2019.

[15] M. Caleffi, "Optimal routing for quantum networks," *Ieee Access*, vol. 5, pp. 22 299–22 312, 2017.

[16] S. Pirandola, "End-to-end capacities of a quantum communication network," *Communications Physics*, vol. 2, no. 1, p. 51, 2019.

[17] M. A. Nielsen and I. L. Chuang, *Quantum computation and quantum information*. Cambridge university press, 2010.

[18] Y. Zhao, G. Zhao, and C. Qiao, "E2e fidelity aware routing and purification for throughput maximization in quantum networks," in *IEEE INFOCOM 2022-IEEE Conference on Computer Communications*. IEEE, 2022, pp. 480–489.

[19] M. Liu, Z. Li, K. Cai, J. Allcock, S. Zhang, and J. C. Lui, "Quantum bgp with online path selection via network benchmarking," in *IEEE INFOCOM 2024-IEEE Conference on Computer Communications*. IEEE, 2024, pp. 1401–1410.

[20] A. Farahbakhsh and C. Feng, "Opportunistic routing in quantum networks," in *IEEE INFOCOM 2022-IEEE Conference on Computer Communications*. IEEE, 2022, pp. 490–499.

[21] L. Ruan, "Minimization of the estimation error for entanglement distribution networks with arbitrary noise," *Physical Review A*, vol. 108, no. 2, p. 022418, 2023.

[22] L. R. Ford and D. R. Fulkerson, "Maximal flow through a network," *Canadian journal of Mathematics*, vol. 8, pp. 399–404, 1956.

[23] G. R. Waissi, "Network flows: Theory, algorithms, and applications," 1994.

[24] T. H. Cormen, C. E. Leiserson, R. L. Rivest, and C. Stein, *Introduction to algorithms*. MIT press, 2022.

[25] A. Einstein, B. Podolsky, and N. Rosen, "Can quantum-mechanical description of physical reality be considered complete?" *Physical review*, vol. 47, no. 10, p. 777, 1935.

[26] N. Sandilya and A. K. Sharma, "Quantum internet: An approach towards global communication," in *2021 9th International Conference on Reliability, Infocom Technologies and Optimization (Trends and Future Directions)(ICRITO)*. IEEE, 2021, pp. 1–5.

[27] Z. Li, K. Xue, J. Li, L. Chen, R. Li, Z. Wang, N. Yu, D. S. Wei, Q. Sun, and J. Lu, "Entanglement-assisted quantum networks: Mechanics, enabling technologies, challenges, and research directions," *IEEE Communications Surveys & Tutorials*, 2023.

[28] A. Schrijver, "On the history of the transportation and maximum flow problems," *Mathematical programming*, vol. 91, pp. 437–445, 2002.

[29] T. Coopmans, R. Knegjens, A. Dahlberg, D. Maier, L. Nijsten, J. de Oliveira Filho, M. Papendrecht, J. Rabbie, F. Rozpędek, M. Skrzypczyk *et al.*, "Netsquid, a network simulator for quantum information using discrete events," *Communications Physics*, vol. 4, no. 1, p. 164, 2021.

[30] B. M. Waxman, "Routing of multipoint connections," *IEEE journal on selected areas in communications*, vol. 6, no. 9, pp. 1617–1622, 1988.

[31] D. Medhi and K. Ramasamy, *Network routing: algorithms, protocols, and architectures*. Morgan kaufmann, 2017.

[32] F. Jiang, Y. Liu, B. Wang, and X. Wang, "A relay-aided device-to-device-based load balancing scheme for multitier heterogeneous networks," *IEEE Internet of Things Journal*, vol. 4, no. 5, pp. 1537–1551, 2017.