Time dispersion in quantum electrodynamics

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Abstract. If we use the path integral approach, we can write quantum electrodynamics (QED) in a way that is manifestly relativistic. However the path integrals are confined to paths that are on mass-shell. What happens if we extend QED by computing the path integrals over all paths in energy momentum space, not only those on mass-shell? We use the requirement of covariance to do this in an unambiguous way. This gives a QED where the time/energy components appear in a way that is manifestly parallel to the space/momentum components: we have dispersion in time, entanglement in time, full equivalence of the Heisenberg uncertainty principle (HUP) in time to the HUP in space, and so on. Entanglement in time has the welcome side effect of eliminating the ultraviolet divergences. We recover standard QED in the long time limit. We predict effects at scales of attoseconds. With recent developments in attosecond physics and in quantum computing, these effects should be detectable. Since the predictions are unambiguous and testable the approach is falsifiable. Falsification would sharpen our understanding of the role of time in QED. Confirmation would have significant implications for attosecond physics, quantum computing and communications, and quantum gravity.

1. Introduction

> "Look, I don't care what your theory of time is. Just give me something I can prove wrong." – Nathan Gisin at the 2009 Feynman Festival in Olomouc

Is quantum electrodynamics fully relativistic? Quantum electrodynamics (QED) can be developed in a large number of ways. Perhaps the most common is the Hamiltonian/canonical momentum approach. Time plays a special role in this formalism (in defining the canonical momenta) so it is not clear that this approach is completely relativistic. However the canonical momentum formalism is equivalent to the Feynman path integral formulation. And in the Feynman path integral formulation, QED is developed in a relativistic way.

However perhaps even the Feynman path integral formulation is not as fully relativistic as it might be. In it the paths are limited to on mass-shell paths. Consider the simplest possible propagator, the propagator for a massive spinless field:

$$\Delta\left(p_\mu\right) = \frac{\imath}{p^\mu p_\mu - m^2 + \imath\varepsilon} \tag{1}$$

Consider the infinitesimal $\imath\epsilon$. We construct the Feynman diagrams by doing integrals in four momentum $\int d^4p$ over the propagators. The $\imath\epsilon$ identifies one of these four integrals not as a normal but as a contour integral. Say we make the contour integral the one over energy. When we look in detail at this – done in the text – we see that for any fixed value of the three

momentum $\vec{p}$ the value of the fourth component E is fixed by the value of the residues at the poles, typically $E \to E_{\vec{p}} \equiv \sqrt{m^2 + \vec{p}^2}$. The effect is to fix the paths to only the on-shell paths.

We can generalize the paths to include off-shell paths as well. By replacing the contour integral with a normal integral we can include paths that vary in four dimensions. For instance we can write E as $E_{\vec{p}} + \delta E$ and include paths which vary over all values of δE. Letting the paths vary in all four dimensions simultaneously is arguably more in keeping with the "spirit" of relativity, more fully relativistic.

But of course the question is not whether this is more fully in keeping with the spirit of relativity but does including off-shell paths in the path integrals give a more accurate description of nature?

Our goal here is to put this question in a way that is falsifiable with current technology.

If dispersion in time/energy is real, why has it not already been seen? QED has been confirmed to extraordinary precision in a wide variety of experiments, to the point where there is a wikipedia page on "Precision Tests of QED". If such dispersion in energy (and therefore time) is present, wouldn't we have already seen indications of this?

The most obvious estimate of the scale at which such effects should be seen is the Bohr radius a_0 divided by c; the time it would take a photon to cross an atom. This is of order attoseconds: $\frac{a_0}{c} \approx .177as$. This is at the edge of current experimental technology so technically within reach. But it is small enough that associated effects are unlikely to be seen if not specifically looked for.

Factors that make it less likely that dispersion in time would be seen by accident include:

(i) Calculations in QED are normally done by taking the limit as time goes to $\pm\infty$. This will naturally tend to obscure effects at attosecond scale.

(ii) Averaging over many interactions – i.e. shining beams against targets – will tend to average out effects in time.

(iii) It is not something which is expected, so therefore less likely to be seen. The effects of dispersion in time might be hiding within the error bars in some existing data sets.

Objective What we are going to do here is to treat include off-shell and on-shell paths on the same basis when computing the Feynman diagrams and see what breaks. Do we encounter an unavoidable contradiction on the one hand? or can we formulate experimental tests of this idea on the other?

Our objective is to force the question; to extend the paths in QED off-shell in a way that is:

(i) Manifestly covariant,

(ii) Consistent with observation and experiment,

(iii) Self-consistent,

(iv) Has no free parameters,

(v) Falsifiable with current technology.

Literature The work here has its starting point in the path integral approach as originated by Stueckelberg and Feynman [1, 2, 3, 4, 5, 6] and as further developed in [7, 8, 9, 10, 11, 12, 13, 14, 15]. This work is specifically part of the Relativistic Dynamics approach as developed by Horwitz, Fanchi, Piron, Land, Collins, and others [16, 17, 18, 19, 20, 21, 22, 23].

We are also much indebted to general reviews of the role of time in quantum mechanics: [24, 25, 26, 27, 28, 29, 30].

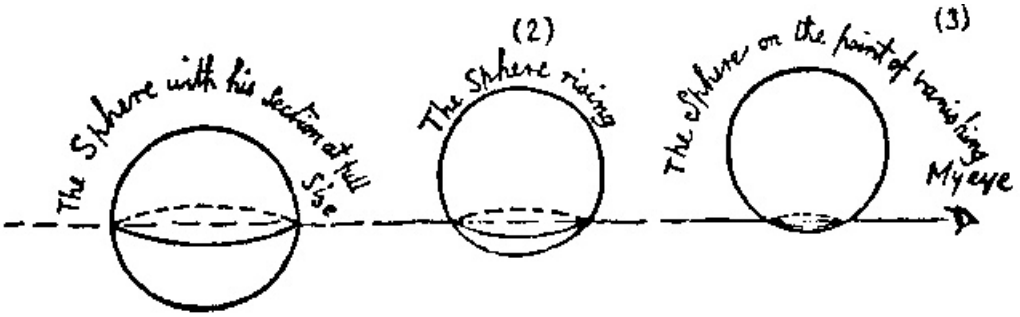

Figure 1. Flatland: A Romance of Many Dimensions – Edwin A. Abbott [52]

And we have taken considerable advantage of the extraordinary literature for QED. References particularly helpful here include [31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 12, 41, 42, 43, 44, 45, 46, 47, 48].

We use the path integral formalism here. Texts on QED typically include a chapter on path integrals. There is also a considerable literature on them in their own right, as [8, 9, 49, 10, 11, 13, 14, 15].

In previous work we have looked at time dispersion in the single particle case [50] (paper A) and at the specific problems created in doing time-of-arrival measurements [51] (paper B). The investigation here extends this work to QED. The extension to QED is necessary to extend the results to high energies/short times, critical for falsification.

Overview In Edwin Abbott's charming 1884 romance Flatland, "A Sphere" – a visitor to Flatland – explains how three dimensions work to "A Square", an inhabitant of Flatland. A Sphere uses rotational symmetry to help take A Square from the idea of a circle or a square to the idea of a sphere or a cube.

We take a similar approach here. We will start with the established rules in standard quantum mechanics (SQM): quantum mechanics applied along the three space dimensions, but time treated classically. We use covariance to extend the rules of SQM to include time as an observable (TQM).

In the early days of quantum mechanics, classical techniques were extended to apply to quantum problems. For instance Feynman used the classical Lagrangian to develop his sum over paths in space. We are extending his approach to include paths in time as well.

Essentially we are completing the square between special relativity and quantum mechanics, adding quantum effects to time on the one side, adding time aspects to quantum mechanics on the other.

Strategy We use path integrals. These are simple, they require only a few basic ingredients: paths, a Lagrangian, a procedure for summing over the paths weighted by the Lagrangian.

In the single particle case, we promote the paths from three space dimensions (3D) to time plus the three space dimensions (4D), while leaving the Lagrangian unchanged.

In QED we again take the paths – now seen as successive values of a field – and promote them from being fields in three space to being fields in four. We are again able to keep the Lagrangian and the rest of the machinery of QED unchanged. In particular, we are able to develop the Feynman rules in TQM in a way that is clearly parallel to the rules in SQM. We get manifest covariance by construction.

As a result, conformity to existing results for SQM can be verified in a straight-forward way. And we can easily pick out experimental tests to look for the differences.

The major sections are:

(i) *Time dispersion in QED.* We use the requirement of covariance to extend the rules of quantum mechanics to include time on the same basis as space.

(ii) *Applications.* We apply these rules to the free case, to the simplest possible scattering case, to a simple mass correction loop, and then to the simplest tree diagrams (Møller, Bhabha, and Compton scattering).

(iii) *Experimental tests.* We propose a specific experiment to test the Heisenberg uncertainty principle in time/energy. The experiment is non-trivial, but appears within reach of current technology. It is, further, only one of many possible tests.

One surprising result is that the simple mass correction loop is convergent without regularization. We might expect that with an additional dimension to integrate over, the usual loop diagrams would become still more divergent, perhaps even unrenormalizable. But instead the combination of dispersion in time and entanglement in time keeps the loop diagrams finite.

In general, any time dependent system monitored by time sensitive detectors should show small but definite effects of dispersion in time. In addition to the Heisenberg uncertainty principle (HUP) in time/energy we can look at forces of "anticipation and regret", diffraction in time, entanglement in time, corrections to existing loop and bound state predictions at short times, and so on.

Summary In general, we expect to see the effects of dispersion in time at scales of attoseconds and less. With recent developments in attosecond physics and in quantum computing, these effects should now be visible. The most dramatic are those involving the HUP in time. The hypothesis is therefore falsifiable in practice.

Since the promotion of time to an operator is done by a straightforward application of agreed and tested principles of quantum mechanics and relativity, falsification will have implications for our understanding of those principles.

Confirmation will have implications for attosecond physics, quantum computing and communications, and quantum gravity.

2. Time dispersion in Quantum Electrodynamics

> "The rules of quantum mechanics and special relativity are so strict and powerful that it's very hard to build theories that obey both." – Frank Wilczek [53]

In this section we work out the rules for extending quantum electrodynamics to include time as an operator. We work up the ladder of complexity till we have all the necessary pieces in place.

(i) Time dispersion and the single particle. We work out the Schrödinger equation for a single particle.

(ii) Spin zero propagator. We work out the Feynman propagator for a spin zero particle with mass greater than zero.

(iii) Photon propagator. We work out the Feynman propagator for photons.

(iv) Dirac propagator. We work out the Feynman propagator for fermions.

(v) Interactions. We note that the usual vertex terms are already TQM-ready.

Because of the need to carefully distinguish between SQM and TQM versions of otherwise familiar objects, some care is required in the notation. We describe our choices in Appendix A.

2.1. Time dispersion and the single particle
We start with the single particle case. The treatment here is largely based on paper A, but reworked to prepare for QED.

We develop the path integral approach for the single particle: we use the standard path integral approach, but with the usual paths generalized from three to four dimensions.

2.1.1. Clock time We start with clock time, defined operationally as what clocks measure: Alice with a stop watch or perhaps the laboratory clock on the wall or a carefully tended Cesium clock. This is also referred to as laboratory time, as in Busch [54] and others. We reserve the letter τ for this.

Note this is *not* the proper time. For a single particle, the use of proper time for the particle would give similar results to those here. However there is no way to extend the proper time approach to QED: the proper time for each massive particle will be in general different, while for photons it will be identically zero.

We give an alternate definition of τ towards the end of this subsection.

2.1.2. Paths Normally in single particle path integrals the paths vary in space but not in time. At each clock tick, the path will be assigned a specific triad of space coordinates. To get the amplitude to go from a starting point A to an endpoint B we will consider the set of all paths from A to B, weighting each by the action. We usually do the sum by breaking up the clock time from A to B into N time steps with each tick of size:

$$\varepsilon \equiv \frac{T}{N} \tag{2}$$

At the end we will take the limit $N \to \infty$. The path is defined by its space coordinates at each clock tick. To sum over the paths, we sum over the associated measure:

$$\mathcal{D}\vec{x} \equiv \prod_{n=0}^{N} d\vec{x}_n \tag{3}$$

Now we extend the paths to include time:

$$\pi\left(\vec{x}_\tau\right) \to \pi\left(t_\tau, \vec{x}_\tau\right) \tag{4}$$

We refer to the time dimension used in this way as coordinate time t, with its properties defined with respect to space by covariance.

The resulting paths are in four dimensions. They curve around in time, so can dart into the future or the past. To be sure, the sum over the paths is in general dominated by the classical paths, whose behavior is more sedate.

We extend the measure to include the sum over 4D paths:

$$\mathcal{D}x \equiv \prod_{n=0}^{N} dt_n d\vec{x}_n \tag{5}$$

2.1.3. Kernel Our primary object is to compute the kernel to go from A to B. This is given by the sum over all paths, weighted by the action, defined as the integral of the Lagrangian along each path:

$$K_\tau\left(x''; x'\right) = \int \mathcal{D}x_\tau \exp\left(\imath \int_0^\tau d\tau' \mathcal{L}\left[x_\tau, \dot{x}_\tau\right]\right) \tag{6}$$

Choice of Lagrangian We need a Lagrangian which is manifestly covariant, which correctly models the behavior of a particle in an electro-magnetic field, and which works equally well in 3D and 4D. We will use the following Lagrangian, which we have from Goldstein [55] and also from Feynman [31]:

$$\mathcal{L}\left[x_\tau, \dot{x}_\tau\right] = -\frac{1}{2} m \dot{x}^\mu \dot{x}_\mu - q \dot{x}^\mu A_\mu\left(x\right) - \frac{m}{2} \tag{7}$$

Convergence of the calculation Path integrals are normally computed by starting at a specific time, then integrating slice-by-slice. For this to make sense, the individual integrals have to converge.

Convergence is normally forced by adding small convergence factors, i.e. rewriting the mass as $m \to m + \imath\epsilon$. Such tricks are not usable here as they generally break covariance. However if we are integrating against a Gaussian test function (GTF), these tricks are not needed in the first place. The GTF itself will keep each step convergent. GTFs are completely general: by using Morlet wavelet analysis we can decompose any normalizable wavelet into sums over GTFs (further discussed in Appendix B).

This give us convergence.

Result In paper A [50] we derive the explicit form of the free kernel:

$$K_\tau\left(x; x'\right) = -\imath \frac{m^2}{4\pi^2 \tau^2} e^{-\frac{\imath m}{2\tau}(x-x')^2 - \imath \frac{m}{2}\tau} \tag{8}$$

or in momentum space:

$$K_\tau\left(p; p'\right) = \exp\left(\imath \frac{E^2 - \vec{p}^2 - m^2}{2m} \tau\right) \delta^4\left(p - p'\right) \tag{9}$$

This matches the non-relativistic kernel found in introductory quantum mechanics textbooks except that it now includes paths in time. From the momentum form, we can see that deviations off-shell will be punished.

2.1.4. Schrödinger equation One normally derives the path integral formula from the Schrödinger equation, see for instance Schulman or Kleinert [8, 14]. However we can also start with the path integral formula and get the Schrödinger equation by taking the short time limit of the path integral expression and running their derivations "in reverse". We get:

$$\imath \frac{\partial \psi_\tau}{\partial \tau}\left(t, \vec{x}\right) = -\frac{1}{2m}\left(\left(\imath\partial_\mu - qA_\mu\left(t, \vec{x}\right)\right)\left(\imath\partial^\mu - qA^\mu\left(t, \vec{x}\right)\right) - m^2\right)\psi_\tau\left(t, \vec{x}\right) \tag{10}$$

or in momentum space:

$$-2m\imath \frac{\partial \psi_\tau}{\partial \tau} = \left(\left(p_\mu - qA_\mu\right)\left(p^\mu - qA^\mu\right) - m^2\right)\psi_\tau \tag{11}$$

This is formally identical to the Feynman-Stueckelberg equation in the Relativistic Dynamics literature [3, 18, 19, 20, 23].

While the equation 10 is formally the same as the Feynman-Stueckelberg equation, the interpretation and use of the equation here is distinct. We will therefore refer to this as the FS/T: the Feynman-Stueckelberg equation in the TQM context.

Note also the resemblance to the non-relativistic Schrödinger equation:

$$\imath \frac{\partial \psi}{\partial \tau} = \frac{1}{2m}\left(\overrightarrow{p} - q\vec{A}\right)^2 \psi \tag{12}$$

The only difference is that we have added a term that represents dispersion in time:

$$-\frac{1}{2m}\left(\left(\imath\partial_t - q\Phi\left(t,\vec{x}\right)\right)\left(\imath\partial^t - q\Phi\left(t,\vec{x}\right)\right)\right)\psi_\tau\left(t,\vec{x}\right) \tag{13}$$

We will modify the FS/T slightly as part of the extension to QED below.

2.1.5. Long, slow approximation If the dependence on clock time is weak, we get the familiar Klein-Gordon equation with minimal substitution:

$$\left(\imath\partial_\mu - qA_\mu\right)\left(\imath\partial^\mu - qA^\mu\right)\psi - m^2\psi = 0 \tag{14}$$

Is it reasonable to assume that the dependence on clock time is weak? That is that:

$$\imath\frac{\partial\psi}{\partial\tau} \approx 0 \tag{15}$$

or more specifically that the expectation value of dependence on clock time is small:

$$\langle\psi|\,\imath\frac{\partial}{\partial\tau}\,|\psi\rangle \approx 0 \tag{16}$$

Effects of dependence on clock time of order picoseconds To see the relevant scale, we estimate the clock frequency ϖ_p:

$$\varpi_p \sim -\frac{E^2 - \vec{p}^2 - m^2}{2m} \tag{17}$$

We are using ϖ rather than ω for the clock frequency to distinguish it clearly from the usual frequency ω. We will modify the definition of ϖ slightly below, again as part of the extension to QED (equation 53).

In the non-relativistic case E is of order mass plus kinetic energy:

$$E \sim m + \frac{\vec{p}^2}{2m} \tag{18}$$

so we have:

$$E^2 - \vec{p}^2 - m^2 \sim \left(m + \frac{\vec{p}^2}{2m}\right)^2 - \vec{p}^2 - m^2 = \left(\frac{\vec{p}^2}{2m}\right)^2 \tag{19}$$

This is just the kinetic energy, squared. In an atom the kinetic energy is of order the binding energy:

$$\frac{\vec{p}^2}{2m} \sim eV \tag{20}$$

So the numerator is of order eV squared. But the denominator is of order MeV. Therefore we can estimate the clock frequency ϖ_p as:

$$\varpi_p \sim \frac{(eV)^2}{MeV} \sim 10^{-6}eV \tag{21}$$

Energies of millionths of an electron volt $10^{-6}eV$ correspond to times of order millions of attoseconds 10^6as or picoseconds, a million times longer than the natural time scale of the effects we are looking at. Therefore the long, slow approximation (LSA) is reasonable.

Over long times, the clock frequency term will tend to reinforce on-shell components of the wave function with respect to the off-shell components. It is not so much that the off-shell components vanish, it is that averaged over nanoseconds, as by a slow detector, the off-shell

components will average out to approximately zero. We then get what may look like a long, slow collapse of the wave function.

Over short times, we will treat the effects of the dependence on clock time as relatively less significant.

2.1.6. Meaning of laboratory time We now have two different kinds of time in play: coordinate time and clock time. We can reduce the ontological overhead of TQM by combining them. To do this, we take the clock time as the average over coordinate time over the rest of the universe $\mathcal{U}$:

$$\tau \equiv \langle \mathcal{U} | t \, | \mathcal{U} \rangle \tag{22}$$

We are effectively dividing the wave function of the universe into two parts, the small part we are focused on and the large part which is us, the laboratory, and the rest of the universe. We now re-define the clock time as the expectation of the coordinate time of the large part.

Therefore the properties of the clock time are those associated with an expectation value over an Avogadro's number of particles. In particular, it does not go backwards, as such fluctuations are wildly unlikely for the usual statistical dynamics reasons. As with a crowd, composed of individuals, but with the dynamics of the crowd very different from the dynamics of the individual. So we have:

(i) Defined clock time in terms of laboratory clocks.

(ii) Defined the extension of paths to coordinate time using clock time and covariance.

(iii) Worked out rules for quantum mechanics with coordinate time. Coordinate time is now an operator in the same way as the three space dimensions are operators.

(iv) Then turned around and defined clock time as the coordinate time operator applied to the laboratory – and the rest of universe, if it comes to that.

With this, laboratory time is not only not an operator, it is not even a parameter, it is merely an expectation value of the fundamental operator t. Therefore Pauli's theorem [56, 25] does not apply to it. We will continue to use clock time as a short hand for equation 22.

This is a significant variation of the work here from the literature in the Relativistic Dynamics program. In that, the parameter we have been calling clock time is an additional parameter which is introduced because various other parts of the problem then become more tractable. Here it is fixed: defined operationally by clocks and defined theoretically as the average over the coordinate time. This eliminates τ as a degree of freedom. The fewer the degrees of freedom the more falsifiable.

In TQM we have only one time, the coordinate time. The clock time is derivative, useful as scaffolding to get the analysis started, but dispensable once the analysis is in place. The clock time applies in full force only to macroscopic ensembles. The coordinate time represents the underlying reality.

2.1.7. Choice of laboratory frame So we understand what is meant by laboratory time in Alice's lab. But what if Bob is working in his laboratory moving at relativistic speeds relative to Alice's? Whose time should we use?

If the speeds are not too great, we can argue the effects will be of second order so may be neglected on a first attack.

However in the interests of achieving a clean and complete treatment we note we can define an invariant reference frame, to whose judgments both Alice and Bob must defer. (This is analogous to the way we can work in the center-of-mass frame, take advantage of the resulting simplicity, and then transform back to a specific laboratory frame at the end.)

In [57] Weinberg shows we can use Einstein's equations of general relativity to define an appropriate energy-momentum tensor of local spacetime. See Appendix D for specifics.

Since this is an energy-momentum tensor, we can use it to define a "local rest frame of spacetime" or $\mathcal{V}$ (for vacuum) frame. We take $\mathcal{V}$ as the required common frame. The defining laboratory time is therefore the clock time in this frame. Alice and Bob can agree on this, then perform the necessary Lorentz transforms from and to their respective frames confident they will make the same physical predictions.

2.2. Spin zero propagator

Having established a foundation in the single particle case, we extend TQM to the case of a massive spin zero particle. This is the core case for managing the transition from single particle quantum mechanics to QED. The photon and fermion cases will turn out to be relatively straightforward extensions of this.

We start with the SQM form. We use as a starting point the careful and detailed treatment in Klauber's text [45], but adapt his notation and techniques to the requirements of TQM. We give only the key "twists and turns".

For SQM and then for TQM, we look at:

(i) the free solutions and their associated Fock space,

(ii) the field operators constructed as sums over the free solutions,

(iii) the propagator constructed as a sum over the field operators.

2.2.1. Spin zero propagator in SQM We start with the Lagrangian for SQM:

$$\mathcal{L}^S \equiv \partial_\tau \phi \partial^\tau \phi - \nabla\phi\nabla\phi - m^2\phi^2 \tag{23}$$

The corresponding Euler-Lagrange equation is the Klein-Gordon equation:

$$\left(\partial_\tau \partial^\tau - \nabla^2 + m^2\right) \phi_\tau(x) = 0 \tag{24}$$

The free solutions of this are:

$$\phi_\tau^{(\vec{k})}(\vec{x}) \sim \exp\left(-\imath\omega_{\vec{k}}\tau + \imath\vec{k}\cdot\vec{x}\right), \omega_{\vec{k}} \equiv \sqrt{m^2 + \vec{k}^2} \tag{25}$$

Fock space The corresponding spin zero Fock space is built up in the usual way as appropriately symmetrized combinations of the free single particle solutions:

$$\phi_{\vec{k}}(\vec{x}) = \frac{1}{\sqrt{V}} \exp\left(\imath\vec{k}\cdot\vec{x}\right) \tag{26}$$

We are using box normalization to a volume V here. This is useful for dimension checking. We will shift back and forth freely between box and continuous normalization.

We use the occupation number representation for Fock space:

$$\left|\{n_{\vec{k}}\}\right\rangle \tag{27}$$

where n is an integer from zero to infinity and the wave functions are fully symmetric. The creation and annihilation operators are defined by their effects on Fock space:

$$a_{\vec{k}}\left|n_{\vec{k}}\right\rangle = \sqrt{n_{\vec{k}}}\left|n_{\vec{k}} - 1\right\rangle, a_{\vec{k}}^\dagger\left|n_{\vec{k}}\right\rangle = \sqrt{n_{\vec{k}} + 1}\left|n_{\vec{k}} + 1\right\rangle \tag{28}$$

with the 3D commutators being:

$$\left[a_{\vec{k}}, a^{\dagger}_{\vec{k}'}\right] = \delta^3\left(\vec{k} - \vec{k}'\right) \tag{29}$$

All other commutators are zero. We make *no* use of the usual interpretation of Fock space in terms of harmonic oscillators. The creation and annihilation operators are defined entirely by their effects on Fock space.

Field Operators Now we build up the spin zero field operators as sums over the free single particle solutions in the interaction picture. We have the sums over the positive frequency components on the left and negative frequency components on the right:

$$\phi^S_\tau(\vec{x}) = \sum_{\vec{k}} \frac{1}{\sqrt{2V\omega_{\vec{k}}}}\left(a_{\vec{k}} \mathrm{e}^{-\imath\omega_{\vec{k}}\tau + \imath\vec{k}\cdot\vec{x}} + a^{\dagger}_{\vec{k}} \mathrm{e}^{\imath\omega_{\vec{k}}\tau - \imath\vec{k}\cdot\vec{x}}\right) \tag{30}$$

We mark SQM parts with a superscript S. The normalization factor $\frac{1}{\sqrt{2\omega_{\vec{k}}}}$ corresponds to the convention of normalizing beams to energy/volume (see for instance Feynman [31]).

Feynman propagator The SQM Feynman propagator is defined as the time-ordered vacuum expectation value of two of these field operators. This definition is key to evaluating the S matrix as a sum over Feynman diagrams:

$$\imath\Delta^S_{\tau_x\tau_y}(\vec{x} - \vec{y}) \equiv \left\langle 0 \left| T\left\{\phi^S_{\tau_x}(\vec{x}), \phi^S_{\tau_y}(\vec{y})\right\} \right| 0 \right\rangle \tag{31}$$

T is the time-ordering operator; if $\tau_y < \tau_x$, then the y operator is on the right and vice versa. We break ϕ up into its positive and negative frequency parts:

$$\phi^S_\tau(\vec{x}) = \phi^{S+}_\tau(\vec{x}) + \phi^{S-}_\tau(\vec{x}) \tag{32}$$

In a vacuum expectation value, the only non-zero terms are those with an annihilation operator a on the left and a creation operator $a^\dagger$ on the right. As a result most of the terms vanish.

When $\tau_x > \tau_y$ the only non-zero term is:

$$\langle 0|\, \phi^{S+}_x(\vec{x})\, \phi^{S-}_y(\vec{y})\, |0\rangle \tag{33}$$

By taking advantage of:

$$0 = -\langle 0|\, \phi^{S-}_y(\vec{y})\, \phi^{S+}_x(\vec{x})\, |0\rangle \tag{34}$$

we can rewrite this in terms of the commutator:

$$\imath\Delta^{S+}_{xy}(\vec{x} - \vec{y}) = \langle 0|\left[\phi^{S+}_x(\vec{x}), \phi^{S-}_y(\vec{y})\right]|0\rangle \tag{35}$$

We expand the operators and use the commutators. To simplify the calculations, we shift from discrete sums to continuous integrals by replacing $\Sigma \to \int, V \to (2\pi)^3$. We use the vacuum product $\langle 0 \mid 0\rangle = 1$. We are left with a pure number:

$$\imath\Delta^{S+}_{xy}(\vec{x} - \vec{y}) = \frac{1}{(2\pi)^3}\int d\vec{k} \frac{e^{-\imath\omega_{\vec{k}}\tau_{xy} + \imath\vec{k}\cdot(\vec{x}-\vec{y})}}{2\omega_{\vec{k}}} \tag{36}$$

The same development on the negative frequency side gives:

$$\imath\Delta^{S-}_{xy}(\vec{x} - \vec{y}) = \frac{1}{(2\pi)^3}\int d\vec{k} \frac{e^{\imath\omega_{\vec{k}}\tau_{xy} - \imath\vec{k}\cdot(\vec{x}-\vec{y})}}{2\omega_{\vec{k}}} \tag{37}$$

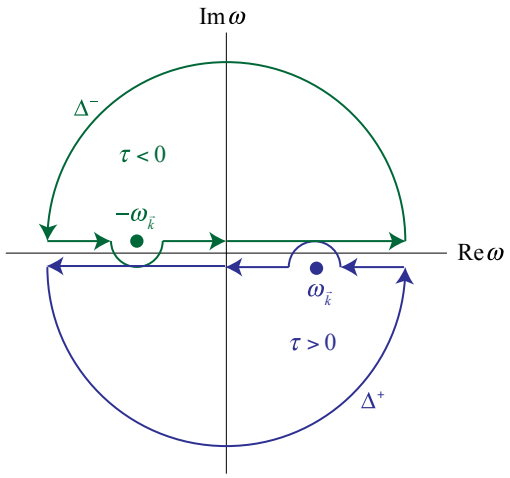

Figure 2. Contour integral for Feynman propagators – after Klauber [45]

We could also show this by interchanging $x \leftrightarrow y$.

We see clearly here that time and space are being treated differently: there is no integral over the energy coordinate ω; the value of $\omega_{\vec{k}}$ is fixed by $\vec{k}$ rather than being allowed to roam. This implies no dispersion in ω – and therefore none in time.

Combining the positive and negative parts we get the full spin zero propagator in SQM:

$$\imath\Delta^{S}_{xy}\left(\vec{x}-\vec{y}\right)=\frac{1}{(2\pi)^3}\int d^3\vec{k}\frac{\mathrm{e}^{-\imath\omega_{\vec{k}}\tau_{xy}-\imath\vec{k}\cdot\left(\vec{x}-\vec{y}\right)}}{2\omega_{\vec{k}}}\theta\left(\tau_{xy}\right)+\frac{\mathrm{e}^{\imath\omega_{\vec{k}}\tau_{xy}+\imath\vec{k}\cdot\left(\vec{x}-\vec{y}\right)}}{2\omega_{\vec{k}}}\theta\left(-\tau_{xy}\right) \tag{38}$$

We refer to this as the "unpacked form". Effectively it carries positive frequency components into the future; negative into the past.

Now we turn this into something that is more covariant in appearance. We start with the positive frequency side. We replace the integrand with a contour integral:

$$\frac{e^{-\imath\omega_{\vec{k}}\tau}}{2\omega_{\vec{k}}}\theta\left(\tau\right)=\frac{\imath}{2\pi}\int\limits_{-\infty}^{\infty}d\omega\frac{e^{-\imath\omega\tau}}{\left(\omega-\omega_{\vec{k}}+\imath\varepsilon\right)\left(\omega+\omega_{\vec{k}}\right)} \tag{39}$$

with a pole at $\omega=\omega_{\vec{k}}-\imath\epsilon$. This gives the positive frequency side as:

$$\imath\Delta^{S+}_{xy}\left(\vec{x}-\vec{y}\right)=\frac{\imath}{(2\pi)^4}\int\limits_{-\infty}^{\infty}d\omega\int d\vec{k}\frac{e^{-\imath\omega\tau_{xy}+\imath\vec{k}\cdot\left(\vec{x}-\vec{y}\right)}}{\left(\omega-\omega_{\vec{k}}+\imath\varepsilon\right)\left(\omega+\omega_{\vec{k}}\right)} \tag{40}$$

The negative frequency part can be replaced by a contour integral in the same way. We take the pole at $\omega=-\omega_{\vec{k}}+\imath\varepsilon$:

$$\frac{e^{\imath\omega_{\vec{k}}\tau}}{2\omega_{\vec{k}}}\theta\left(-\tau\right)=\frac{\imath}{2\pi}\int\limits_{-\infty}^{\infty}d\omega\frac{e^{-\imath\omega\tau}}{\left(\omega-\omega_{\vec{k}}\right)\left(\omega+\omega_{\vec{k}}-\imath\varepsilon\right)} \tag{41}$$

Inserting this back in the previous expression we get:

$$\imath\Delta^{S-}_{xy}\left(\vec{x}-\vec{y}\right)=\frac{\imath}{(2\pi)^4}\int\limits_{-\infty}^{\infty}d\omega\int d\vec{k}\frac{e^{-\imath\omega\tau_{xy}+\imath\vec{k}\cdot\left(\vec{x}-\vec{y}\right)}}{\left(\omega-\omega_{\vec{k}}\right)\left(\omega+\omega_{\vec{k}}-\imath\varepsilon\right)} \tag{42}$$

We could also just flip $x \leftrightarrow y$ again. Combined form spelled out:

$$\imath\Delta_{xy}^{S}\left(\vec{x}-\vec{y}\right)=\frac{\imath}{\left(2\pi\right)^{4}}\int d\omega d\vec{k}\frac{e^{-\imath\omega\tau_{xy}+\imath\vec{k}\cdot\left(\vec{x}-\vec{y}\right)}}{\omega^{2}-\vec{k}^{2}-m^{2}+\imath\varepsilon} \tag{43}$$

The momentum space part is:

$$\imath\Delta_{\omega}^{S}\left(\vec{k}\right)=\frac{\imath}{\omega^{2}-\vec{k}^{2}-m^{2}+\imath\varepsilon} \tag{44}$$

This is the propagator for the original Klein-Gordon equation. If we apply the Klein-Gordon equation (24) in momentum space we have:

$$\left(\omega^{2}-\vec{k}^{2}-m^{2}\right)\Delta_{\omega}^{S}\left(\vec{k}\right)=1 \tag{45}$$

The $+\imath\epsilon$ gives the propagator Feynman boundary conditions.

Energy is only a virtual 4th dimension So we can see that the propagator includes only on-shell paths. For instance, fixing the momentum $\vec{k}$ at any point fixes the energy ω. The integral over ω sees only the poles in the contour integral; it does not see the whole of ω space. Given this, the usual practice of referring to the particles associated with these propagators as virtual is correct. Our goal in building TQM is, in a certain sense, to make them real.

2.2.2. Spin zero propagator in TQM We do the same thing for TQM, replacing all 3D functions with 4D functions. The dependence on clock time and the normalization will require a bit of thought. We start with the TQM plane waves. They have four coordinates rather than three:

$$\phi_{w,\vec{k}}\left(t,\vec{x}\right)=\frac{1}{\sqrt{TV}}\exp\left(-\imath wt+\imath\vec{k}\cdot\vec{x}\right) \tag{46}$$

The T is the length of a box in time, starting well before anything interesting happens, and finishing well after everything interesting is done. It represents box normalization in time. The Fock space is built up of appropriately symmetrized products of these. It is given in the occupation representation by:

$$\left|\{n_k\}\right\rangle \tag{47}$$

where n is an integer from zero to infinity and the wave functions are fully symmetric.

The creation and annihilation operators are defined by their effects in Fock space:

$$a_k\left|n_k\right\rangle=\sqrt{n_k}\left|n_k-1\right\rangle, a_k^{\dagger}\left|n_{\vec{k}}\right\rangle=\sqrt{n_k+1}\left|n_k+1\right\rangle \tag{48}$$

with 4D commutators:

$$\left[a_k,a_{k'}^{\dagger}\right]=\delta^{4}\left(k-k'\right) \tag{49}$$

All other commutators are zero. Again, per discussion above, we make *no* use of the usual interpretation in terms of harmonic oscillators.

Field Operators Now we extend the SQM field operator to TQM. For a first cut we take this as:

$$\phi_\tau\left(t,\vec{x}\right)=\sum_{w,\vec{k}}\frac{1}{\sqrt{TV}}\frac{1}{\sqrt{2\omega_{\vec{k}}}}\left(a_{w,\vec{k}}\mathrm{e}^{-\imath\varpi_k\tau-\imath wt+\imath\vec{k}\cdot\vec{x}}+a^{\dagger}_{w,\vec{k}}\mathrm{e}^{\imath\varpi_k\tau+\imath wt-\imath\vec{k}\cdot\vec{x}}\right) \tag{50}$$

The two questions here are:

(i) What should we use for ϖ_k?

(ii) And what should we use for $\frac{1}{\sqrt{2\omega_{\vec{k}}}}$? This depends to a considerable extent on the answer to the first question, so we will tackle that first.

Dependence on clock time We need a way to define the dependence on clock time – the clock energy – in a way that works equally well for massive spin zero particles, fermions, and photons. This is a non-trivial problem. For instance, the previous formula for the single particle ϖ_p has a $1/m$ factor which makes it unsuitable for use with photons. It is also unclear how best to extend the single particle approach to the Dirac equation: for instance, should antiparticles use the same sign for ϖ_p as particles do? We require an approach which lets us treat all kinds of particles uniformly.

To do this, recall we are using the local rest frame of spacetime as our reference point. What if we argue that the momentum of a particle should be understood not as an absolute but as relative to the average four momentum of the vacuum $\mathcal{P}$?

We do this by replacing the particle's four momentum k with its four momentum relative to $\mathcal{P}$; $k\rightarrow k-\mathcal{P}$. With this ansatz we rewrite the Klein-Gordon equation as:

$$\left(\left(k-\mathcal{P}\right)^2-m^2\right)\psi=0 \tag{51}$$

By working in the rest frame of local spacetime $\mathcal{V}$ (as above) we reduce spacetime's four momentum to just its energy $\mathcal{P}\rightarrow\left(\mathcal{E},\vec{0}\right)$. We replace its energy $\mathcal{E}$ with its complementary time operator $\mathcal{E}\rightarrow\imath\frac{\partial}{\partial\tau_V}$. The laboratory time is defined as $\tau\equiv\langle t\rangle$ so is *also* going opposite to the time of the vacuum so $\imath\frac{\partial}{\partial\tau_V}\rightarrow-\imath\frac{\partial}{\partial\tau}$:

$$\left(k^2-m^2\right)\psi=-2w\imath\frac{\partial}{\partial\tau}\psi \tag{52}$$

We refer to this approach as the Machian hypothesis. We give a more detailed treatment in Appendix D.2. This is the FS/T for the single particle case (equation 11) with the substitution $m\rightarrow w$ or in coordinate space $m\rightarrow\imath\frac{\partial}{\partial t}$. Looking forwards, we will see this gives reasonable results for photons and fermions as well. In energy momentum space we can write the clock time dependence of the wave function as $\psi\sim\exp\left(-\imath\varpi_k\tau\right)$ to get:

$$\imath\frac{\partial}{\partial\tau}\psi=\varpi_k\psi,\varpi_k\equiv-\frac{w^2-\vec{k}^2-m^2}{2w} \tag{53}$$

In general we expect that off-shell components will tend to average out: $\langle w\rangle\approx\omega_{\vec{k}}$. Therefore we can expand the clock frequency in terms of $\delta w\equiv w-\omega_{\vec{k}}$:

$$\varpi_k\approx-\delta w+\frac{\left(\delta w\right)^2}{2\omega_{\vec{k}}} \tag{54}$$

Using ω as the complementary variable to τ, we write in this equation in terms of clock frequency ω and coordinate energy w:

$$\left(w^2 - \vec{k}^2 - m^2 + 2\omega w\right)\psi = 0 \tag{55}$$

From the analysis of the LSA above, we expect that the $2\omega w$ term will have little effect at short times, only coming into its own at longer times, i.e. on the "legs" of the Feynman diagrams.

For the rest of the text, we will treat the Machian hypothesis as a formal hypothesis useful for achieving a consistent treatment. In Appendix D.3 we take a quick look at some of the implications of treating this hypothesis as real.

Normalization There are two requirements for the normalization. The first is that the resulting propagator should be a propagator for (55), that in momentum space it should look like:

$$\imath\Delta_\omega(k) \sim \frac{\imath}{w^2 - \vec{k}^2 - m^2 + 2\omega w} \tag{56}$$

The second is that it should obey Feynman boundary conditions, specifically its dependence on clock time should go as:

$$\exp(-\imath\varpi_k\tau)\,\theta(\tau) + \exp(\imath\varpi_k\tau)\,\theta(-\tau) \tag{57}$$

This is required so that the construction of the S matrix in TQM will go in parallel to the construction in SQM. These two requirements strongly constrain the normalization. We can meet them if we take:

$$\frac{1}{\sqrt{2\omega_{\vec{k}}}} \to \frac{1}{\sqrt{2w}} \tag{58}$$

We therefore take as the TQM operator:

$$\phi_\tau(t,\vec{x}) = \sum_{w,\vec{k}} \frac{1}{\sqrt{TV}}\frac{1}{\sqrt{2w}}\left(a_{w,\vec{k}}\mathrm{e}^{-\imath\varpi_k\tau-\imath wt+\imath\vec{k}\cdot\vec{x}} + a^\dagger_{w,\vec{k}}\mathrm{e}^{\imath\varpi_k\tau+\imath wt-\imath\vec{k}\cdot\vec{x}}\right) \tag{59}$$

The differences from the original guess are the normalization $\frac{1}{\sqrt{2w}}$ and the precise definition of ϖ.

Propagator We can now derive the unpacked form of the propagator in close parallel to the derivation for SQM. The propagator is defined by:

$$\imath\Delta_{xy}(x-y) = \langle 0\,|T\{\phi_x(x),\phi_y(y)\}|\,0\rangle \tag{60}$$

We are using the same conventions and approach as for SQM, but generalizing $\vec{x}\to x, \vec{y}\to y$.

We again break the wave function into its positive and negative frequency parts:

$$\phi_x(x) = \phi^+_x(x) + \phi^-_x(x) \tag{61}$$

As with SQM, for $\tau_x > \tau_y$ most terms are zero. We are left with only:

$$\left\langle 0\left|\phi^+_x(x)\,\phi^-_y(y)\right|0\right\rangle \tag{62}$$

We rewrite this in terms of the commutator:

$$\imath\Delta^+_{xy}(x-y) = \left\langle 0\left|\left[\phi^+_x(x),\phi^-_y(y)\right]\right|0\right\rangle \tag{63}$$

which we write in turn as the integral:

$$\imath\Delta_{xy}^{+}(x-y)=\frac{1}{(2\pi)^4}\int d^4k\frac{e^{-\imath\varpi_k\tau_{xy}-\imath k(x-y)}}{2w} \tag{64}$$

As with SQM, for $\tau_x<\tau_y$ we can get the results for the propagator by interchanging $x\leftrightarrow y$:

$$\imath\Delta_{xy}^{-}(x-y)=\frac{1}{(2\pi)^4}\int d^4k\frac{e^{\imath\varpi_k\tau_{xy}+\imath k(x-y)}}{2w} \tag{65}$$

We flip the sign of k to line this up with the positive frequency side. Since ϖ_k is odd in w, it flips sign as well:

$$\imath\Delta_{xy}^{-}(x-y)=-\frac{1}{(2\pi)^4}\int d^4k\frac{e^{-\imath\varpi_k\tau_{xy}-\imath k(x-y)}}{2w} \tag{66}$$

The result is the full propagator:

$$\imath\Delta_{xy}(x-y)=\frac{1}{(2\pi)^4}\int d^4k\frac{\mathrm{e}^{-\imath\varpi_k\tau_{xy}-\imath k(x-y)}}{2w}\theta(\tau)-\frac{\mathrm{e}^{-\imath\varpi_k\tau_{xy}-\imath k(x-y)}}{2w}\theta(-\tau) \tag{67}$$

The second term differs in overall sign and in the sign of the clock frequency from the second term for SQM.

We now rewrite the integrand in terms of a contour integral over ω. We do this first to match as closely as possible the development in SQM and secondly to let us write the S matrix expansion in a way that gives us conservation not only of coordinate energy but also of clock energy. We use the representations of the Heaviside unit step function:

$$\begin{aligned}\theta(\tau)&=\lim_{\varepsilon\to0^+}-\frac{1}{2\pi\imath}\int_{-\infty}^{\infty}\frac{1}{\omega+\imath\varepsilon}e^{-\imath\omega\tau}d\omega\\ \theta(-\tau)&=\lim_{\varepsilon\to0^+}\frac{1}{2\pi\imath}\int_{-\infty}^{\infty}\frac{1}{\omega-\imath\varepsilon}e^{-\imath\omega\tau}d\omega\end{aligned} \tag{68}$$

We write the propagator in terms of ω:

$$\exp(-\imath\varpi_k\tau)\,\theta(\tau)-\exp(-\imath\varpi_k\tau)\,\theta(-\tau)=\frac{\imath}{2\pi}\int_{-\infty}^{\infty}d\omega\exp(-\imath\omega\tau)\left(\frac{1}{\omega-\varpi_k+\imath\varepsilon}+\frac{1}{\omega-\varpi_k-\imath\varepsilon}\right) \tag{69}$$

which implies:

$$\imath\Delta_{\omega}(k)=\frac{1}{2w}\left(\frac{\imath}{\omega-\varpi_k+\imath\varepsilon}+\frac{\imath}{\omega-\varpi_k-\imath\varepsilon}\right) \tag{70}$$

We multiply out the $2w$ in the denominator to get:

$$\imath\Delta_{\omega}(k)=\frac{\imath}{w^2-\vec{k}^2-m^2+2w\omega+2w\imath\varepsilon}+\frac{\imath}{w^2-\vec{k}^2-m^2+2w\omega-2w\imath\varepsilon} \tag{71}$$

where both sides are inverses of equation 55, with Feynman boundary conditions.

Conservation of clock energy in TQM The unpacked form makes the physical meaning more transparent: the normalization is obvious, the direction in time is obvious, the fact that the expression is a relativistic invariant is obvious.

But the packed form does show the propagator as a function of ω, the Fourier transform of the clock time. The packed form always travels with an implicit:

$$\exp\left(-\imath\omega\tau\right) \tag{72}$$

which will be used in the inverse Fourier transform back to clock space.

This has considerable practical advantages. At a typical vertex, if we have for example an incoming external line, an outgoing line, and a exchanged photon the associated packed propagators will give factors of:

$$\exp\left(\imath\varpi_{out}\tau\right)\exp\left(\mp\imath\omega_{\gamma}\tau\right)\exp\left(-\imath\varpi_{in}\tau\right) \tag{73}$$

where the factors of $\imath\epsilon$ make sure the sign of the photon part is correct. If we have an overall integral over clock time to $\mp\infty$, then we get integrals of the form:

$$\int_{-\infty}^{\infty} d\tau \exp\left(\imath\varpi_{out}\tau\right)\exp\left(\mp\imath\omega_{\gamma}\tau\right)\exp\left(-\imath\varpi_{in}\tau\right) \tag{74}$$

associated with each vertex. And after we have done all these integrals we have conservation of clock energy at each vertex plus an overall conservation of clock energy for the diagram. In SQM:

$$\delta\left(\sum\Omega_{out}-\sum\Omega_{in}\right) \tag{75}$$

in TQM:

$$\delta\left(\sum\varpi_{out}-\sum\varpi_{in}\right) \tag{76}$$

This works for both SQM and TQM; the math is the same. This is helpful in practical calculations.

But it is also a crutch: it depends in a critical way on being able to take the limits of the integral over clock time to infinity, and therefore limits the applicability of the S matrix in SQM to long times. If the limits are for short times, then the conservation of clock energy will be at best approximate. This is troubling in SQM.

But it is not a problem in TQM: in TQM only the *coordinate energy* is real, the clock energy – like its companion clock time – is ultimately a statistical variable. If it fluctuates a bit here and there, well that is to be expected when you are dealing with statistical variables.

We finesse this problem here by looking only at diagrams where the limits of the clock time integrals may be taken to $\pm\infty$ but which still allow a direct comparison of TQM to SQM.

2.2.3. Long and short time scales The TQM propagator is significantly more complex than the SQM equivalent. The problem is that we are treating time at two different levels: the low level quantum realm where coordinate time is fully symmetric with space and the higher level macroscopic realm of clock time, laboratories, and observers. The propagator, like its complementary equation, is a bridge between two disparate realms of analysis. In practice this can be difficult to work with.

From the LSA the clock term:

(i) is small: $|\varpi_k| << \omega_{\vec{k}}$,

(ii) averages to zero: $\langle \varpi_k \rangle \approx 0$, and

(iii) only takes effect over longer terms i.e. picoseconds.

At short times we expect it will not play much of a role, Therefore it is convenient to split the analysis into long (picoseconds) and short (attosecond) times (with femtosecond times left for negotiation).

Long time scales It will normally take quite a few picoseconds for a wave packet to get from the interaction zone to the detector, arguably enough time for the clock frequency to play a significant role in shaping the wave packet. For instance, if there is decoherence en route, the on-shell terms will be preferentially favored over the off-shell. And if the detector itself is not sensitive to sub-picosecond changes, the detector will be unlikely to see off-shell components of the wave function.

We have therefore a natural way to understand how the wave packet evolves from what is initially a fully four dimensional wave packet (as it leaves the interaction zone) to what appears to be on-shell at it registers at a detector.

Short time scales In SQM the combination of clock frequency and clock time give a clear direction in time: $\omega > 0 \Leftrightarrow \theta(\tau), \omega < 0 \Leftrightarrow \theta(-\tau)$. But in TQM, at sub-picosecond times, we have $\exp(-\imath \varpi_k \tau) \approx 1$ and the clock frequency approximately zero, as likely to be negative as positive. The clock time/clock frequency pair no longer provides reliable directionality in time.

Nor should it.

The clock time is defined as the expectation over the coordinate time, only valid at longer times and for statistical assemblies. Backported to extremely short times and small numbers of individual particles, the use of clock time is suspect. Just as it is improper to infer from the macroscopic behavior of a gas the details of the motion of a specific molecule within it.

Recall our fundamental hypothesis, that coordinate time is to be completely defined by covariance and the rules for the three space dimensions. In SQM the expectation of the three momentum gives the direction in space: $\langle p_x \rangle > 0$ implies we are going in the positive x direction, $\langle p_x \rangle < 0$ that we are going in the negative x direction, and so on. Therefore if we have $\langle w \rangle > 0$ we should be going forwards in time; if $\langle w \rangle < 0$ backwards in time.

(Admittedly this latter case is perhaps less often seen in the laboratory. For discussions of what this might look like in practice see Schulman [58] and also Greenberger and Svozil [59]).

So at short times, the clock time/clock frequency should have little or nothing to do with the direction in time. That should be defined by the wave function itself.

Therefore at short times, we approximate the exponential of the clock time as one. Since the left side is forwards in clock time and the right side backwards, we add the two to get the short-time propagator. (We take $\theta(0) \equiv 1/2$ to have exactly $\theta(\tau) + \theta(-\tau) = 1$).

We therefore take as our propagator for attosecond times:

$$\imath \Delta_{\omega}^{\mathrm{A}}(k) \approx \frac{\imath}{w^2 - \vec{k}^2 - m^2} \tag{77}$$

With space time form:

$$\imath \Delta_{\tau}^{\mathrm{A}}(x) = \frac{\imath}{(2\pi)^5} \int d^4k d\omega \frac{\mathrm{e}^{-\imath\omega\tau - \imath kx}}{w^2 - \vec{k}^2 - m^2} \tag{78}$$

We will refer to this as the attosecond propagator, tagging it by a superscript A to make this clear. We expect it will start to fail at picosecond and greater times.

So at attosecond times, we have *no* imposed direction in time. We have no dependence in the propagator on clock time, not even via an $\imath\epsilon$. And we have a natural map to the SQM propagator: $w \to \omega$.

The attosecond propagator directly addresses the question posed in the introduction: what do we get if we apply the replacement $\omega_{\vec{k}} \to w$ to the Feynman propagators? And replace the contour integrals with real ones (by dropping the $\imath\epsilon$'s)?

Quantum energy and quantum time We define the "quantum energy" as the difference between the coordinate energy and the energy value (the classical energy) associated with the parallel SQM calculation. For free particles this is $\delta w \equiv w - \omega_{\vec{k}}$. More generally we can write the quantum energy as the coordinate energy less the value expected from SQM: $\delta E \equiv E - E^S$. This latter definition works within Feynman diagrams as well, where E^S is the "virtual energy" or the energy associated with a virtual particle.

We define in parallel the "quantum time" as the difference between the coordinate time and the clock time: $\delta t \equiv t - \tau$.

We are primarily focused on the quantum energy here, but the quantum time has its uses as well. Both serve as measures of the difference between TQM and SQM. And in that sense summarize the effect we are looking for.

Summary At sub-picosecond times we can use the attosecond time propagator. On the legs, at longer times, we will use the FS/T equation (52) and the associated single particle solutions.

This division makes sense when we are looking at high speed scattering experiments, where the interaction zone is at attosecond scale, but then the products of the interactions take journeys that can be nanoseconds or longer.

In more complex cases we may need to fall back on the full propagator.

Next we develop TQM versions of photon and fermion propagators. The polarization and spin parts will turn out to be relatively minor complications from a TQM point-of-view; we have just navigated the trickier parts of the analysis. Once these are ready, we will turn to applications.

2.3. Photon propagator

We use the same approach here as for spin zero case. The addition of polarization turns out to be an inessential complication from the point of view of TQM.

2.3.1. Photons in SQM

Fock space We have as the basis functions:

$$\frac{1}{\sqrt{2V\omega_{\vec{k}}}}\varepsilon_r^\mu\left(\vec{k}\right)\exp\left(-\imath\omega_{\vec{k}}\tau + \imath\vec{k}\cdot\vec{x}\right) \tag{79}$$

$$\omega_{\vec{k}} \equiv \left|\vec{k}\right| \tag{80}$$

with the polarization vectors:

$$\begin{array}{l}\varepsilon_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix}, \varepsilon_2 = \begin{pmatrix} 0 & 1 & 0 & 0 \end{pmatrix}, \\ \varepsilon_3 = \begin{pmatrix} 0 & 0 & 1 & 0 \end{pmatrix}, \varepsilon_4 = \begin{pmatrix} 0 & 0 & 0 & 1 \end{pmatrix}\end{array} \tag{81}$$

We build up the associated Fock space from these as above; the only difference is that the Fock space labels have a polarization index as well:

$$\left|\left\{m_{r\vec{k}}\right\}\right\rangle \tag{82}$$

We have the usual creation and annihilation operators, indexed by polarization as well as the three space momenta: $a_{r\vec{k}}, a^{\dagger}_{r'\vec{k}'}$. Commutators:

$$\left[a_{r\vec{k}}, a^{\dagger}_{r'\vec{k}'}\right] = \delta_{rr'}\delta^{3}\left(\vec{k}-\vec{k}'\right) \tag{83}$$

All other commutators are zero.

Field Operators The field operators include a sum over the polarization vectors as well as over the three space momenta:

$$A^{(S)\mu}_{\tau}\left(\vec{x}\right) \equiv \sum_{r,\vec{k}} \frac{1}{\sqrt{2V\omega_{\vec{k}}}} \varepsilon^{\mu}_{r}\left(\vec{k}\right) \begin{pmatrix} a_{r}\left(\vec{k}\right)\exp\left(-\imath\omega_{\vec{k}}\tau + \imath\vec{k}\cdot\vec{x}\right) \\ +a^{\dagger}_{r}\left(\vec{k}\right)\exp\left(\imath\omega_{\vec{k}}\tau - \imath\vec{k}\cdot\vec{x}\right) \end{pmatrix} \tag{84}$$

Propagator The propagator is again defined as the vacuum expectation value of the time ordered product of two field operators:

$$\imath D^{(S)\mu\nu}_{21}\left(\vec{x},\vec{y}\right) \equiv \left\langle 0\right| T\left\{A^{\mu}_{x}\left(\vec{x}\right), A^{\nu}_{y}\left(\vec{y}\right)\right\}\left|0\right\rangle \tag{85}$$

And using the same methods as earlier (see also the more detailed treatment in [45]) we get the unpacked SQM photon propagator:

$$\imath D^{(S)\mu\nu}_{\tau}\left(\vec{x}\right) = -\imath g^{\mu\nu}\int \frac{d\vec{k}}{2\omega_{\vec{k}}} \begin{pmatrix} \exp\left(-\imath\omega_{\vec{k}}\tau + \imath\vec{k}\cdot\vec{x}\right)\theta\left(\tau\right) \\ +\exp\left(\imath\omega_{\vec{k}}\tau - \imath\vec{k}\cdot\vec{x}\right)\theta\left(-\tau\right) \end{pmatrix} \tag{86}$$

We use the same approach as with the SQM spin zero case to rewrite the propagator in terms of an integral over clock frequency:

$$\imath D^{(S)\mu\nu}_{\tau}\left(\vec{x}\right) = \frac{-\imath g^{\mu\nu}}{\left(2\pi\right)^{4}}\int d\omega d\vec{k}\frac{\exp\left(-\imath\omega\tau + \imath\vec{k}\cdot\vec{x}\right)}{\omega^{2}-\vec{k}^{2}+\imath\varepsilon} \tag{87}$$

$$\imath D^{(S)\mu\nu}_{\omega}\left(\vec{k}\right) = \frac{-\imath g^{\mu\nu}}{\omega^{2}-\vec{k}^{2}+\imath\varepsilon} \tag{88}$$

The same comments about the virtual character of the SQM particles earlier apply here as well.

2.3.2. Photons in TQM From the perspective of TQM, the SQM photon is a hybrid of 3D and 4D approaches: the time coordinate is clock time, but the vector field is basically a four dimensional object, requiring no adjustment to promote it to TQM.

In Lorenz gauge the individual components of the vector potential obey the Klein-Gordon equation. Therefore in TQM the application of the Machian hypothesis gives:

$$-2w\imath\frac{\partial}{\partial\tau}A^{\nu} = \left(w^{2}-\vec{k}^{2}\right)A^{\nu} \tag{89}$$

with 4D solutions:

$$A^{(k)\mu}_{\tau}\left(x\right) = \frac{\varepsilon^{\mu}_{r}\left(k\right)\exp\left(-\imath\varpi_{k}\tau - \imath kx\right)}{\sqrt{2TVw}} \tag{90}$$

and clock frequency as defined above (with the exception of no mass term):

$$\varpi_k \equiv -\frac{w^2 - \vec{k}^2}{2w} \tag{91}$$

Note that the polarization part is the same in the SQM and TQM free wave functions.

Fock space In TQM the Fock space is built up from the 4D solutions, appropriately symmetrized:

$$|\{m_{rk}\}\rangle \tag{92}$$

The index r to the polarization vectors is unchanged, but the three vector $\vec{k}$ is promoted to a four vector $w, \vec{k}$. We have the 4D commutation relations in complete parallel:

$$\left[a_{rk}, a^{\dagger}_{r'k'}\right] = \delta_{rr'}\delta^4\left(k - k'\right) \tag{93}$$

All other commutators are zero.

Field Operators The field operator is similarly uncomplicated:

$$A^{\mu}_{\tau}\left(t, \vec{x}\right) \equiv \sum_{r,w,\vec{k}} \frac{1}{\sqrt{2TVw}} \varepsilon^{\mu}_{r}\left(w, \vec{k}\right) \begin{pmatrix} a_{rk} \exp\left(-\imath\varpi_k\tau - \imath wt + \imath\vec{k}\cdot\vec{x}\right) \\ +a^{\dagger}_{rk} \exp\left(\imath\varpi_k\tau + \imath wt - \imath\vec{k}\cdot\vec{x}\right) \end{pmatrix} \tag{94}$$

Feynman propagator The Feynman propagator is the vacuum expectation value of the time ordered product of the TQM vector potentials:

$$\imath D^{\mu\nu}_{\tau} \equiv \langle 0 \left| T\left\{A^{\mu}_{\tau}\left(x\right), A^{\nu}_{\tau}\left(y\right)\right\}\right| 0\rangle \tag{95}$$

By the same procedure as for spin zero (equation 67):

$$\imath D^{\mu\nu}_{\tau}\left(x\right) = -\frac{\imath g^{\mu\nu}}{\left(2\pi\right)^4} \int \frac{dwd\vec{k}}{2w} \left(\exp\left(-\imath\varpi_k\tau - \imath kx\right)\theta\left(\tau\right) - \exp\left(-\imath\varpi_k\tau + \imath kx\right)\theta\left(-\tau\right)\right) \tag{96}$$

in momentum space:

$$\imath D^{\mu\nu}_{\tau}\left(k\right) \equiv -\imath\frac{g^{\mu\nu}}{\left(2\pi\right)^4} \left(\frac{\exp\left(-\imath\varpi_k\tau\right)\theta\left(\tau\right)}{2w} - \frac{\exp\left(-\imath\varpi_k\tau\right)\theta\left(-\tau\right)}{2w}\right) \tag{97}$$

We use the same approach as with the TQM spin zero case to rewrite the propagator in terms of an integral over clock frequency:

$$\imath D^{\mu\nu}_{\tau}\left(k\right) \equiv \frac{-\imath g^{\mu\nu}}{\left(2\pi\right)^5} \int_{-\infty}^{\infty} d\omega \frac{\exp\left(-\imath\omega\tau\right)}{2w} \left(\begin{array}{c} \dfrac{\imath}{w^2 - \vec{k}^2 + 2w\omega + 2w\imath\varepsilon} \\ +\dfrac{\imath}{w^2 - \vec{k}^2 + 2w\omega - 2w\imath\varepsilon} \end{array}\right) \tag{98}$$

In momentum space:

$$\imath D^{\mu\nu}_{\omega}\left(k\right) = \frac{-\imath g^{\mu\nu}}{2w} \left(\frac{\imath}{w^2 - \vec{k}^2 + 2w\omega + 2w\imath\varepsilon} + \frac{\imath}{w^2 - \vec{k}^2 + 2w\omega - 2w\imath\varepsilon}\right) \tag{99}$$

or:

$$\imath D_{\omega}^{\mu\nu}(k) = -\imath g^{\mu\nu}\Delta_{\omega}(k) \tag{100}$$

So the photon propagator is a polarization wrapper for the spin zero propagator. This point considerably simplifies the subsequent analysis. From the arguments above, we have also the attosecond form:

$$\imath D_{\omega}^{(A)\mu\nu}(k) = -\imath g^{\mu\nu}\frac{\imath}{w^2 - \vec{k}^2} \tag{101}$$

2.4. Dirac propagator
We now extend TQM to include the Dirac propagator (Dirac [60]).

2.4.1. Dirac propagator in SQM

Dirac equation We start by reviewing the Dirac equation in a way that will prepare for the extension to TQM. Dirac equation in SQM:

$$\left(\imath\gamma_o\partial_\tau + \imath\vec{\gamma}\cdot\nabla - m\right)\psi = 0 \tag{102}$$

We use standard choices for γ^μ:

$$\gamma^0 \equiv \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \gamma^{1,2,3} \equiv \begin{pmatrix} 0 & \sigma^{1,2,3} \\ -\sigma^{1,2,3} & 0 \end{pmatrix} \tag{103}$$

This has positive solutions (particles):

$$\psi_{1,2}^{(S)\vec{p}}(\tau,\vec{x}) = u_{1,2}^{S}(\vec{p})\exp\left(-\imath E_{\vec{p}}\tau + \imath\vec{p}\cdot\vec{x}\right) \tag{104}$$

$$u_1^S = \sqrt{\frac{E_{\vec{p}}+m}{2m}}\begin{pmatrix} 1 \\ 0 \\ \frac{p^3}{E_{\vec{p}}+m} \\ \frac{p^1+\imath p^2}{E_{\vec{p}}+m} \end{pmatrix}, u_2^S = \sqrt{\frac{E_{\vec{p}}+m}{2m}}\begin{pmatrix} 0 \\ 1 \\ \frac{p^1-\imath p^2}{E_{\vec{p}}+m} \\ -\frac{p^3}{E_{\vec{p}}+m} \end{pmatrix} \tag{105}$$

and negative solutions (anti-particles):

$$\psi_{3,4}^{(S)\vec{p}}(\tau,\vec{x}) = v_{2,1}^{S}\exp\left(\imath E_{\vec{p}}\tau - \imath\vec{p}\cdot\vec{x}\right) \tag{106}$$

$$v_2^S = \sqrt{\frac{E_{\vec{p}}+m}{2m}}\begin{pmatrix} \frac{p^3}{E_{\vec{p}}+m} \\ \frac{p^1+\imath p^2}{E_{\vec{p}}+m} \\ 1 \\ 0 \end{pmatrix}, v_1^S = \sqrt{\frac{E_{\vec{p}}+m}{2m}}\begin{pmatrix} \frac{p^1-\imath p^2}{E_{\vec{p}}+m} \\ \frac{-p^3}{E_{\vec{p}}+m} \\ 0 \\ 1 \end{pmatrix} \tag{107}$$

These solutions define a Fock space:

$$\left|\left\{n_{s\vec{p}}\right\}\right\rangle \tag{108}$$

where $n_{r\vec{p}}$ the occupation number is 0 or 1; s indexes u_1, u_2, v_2, v_1, and the product wave functions are anti-symmetric under interchange. As with spin zero, we define creation and annihilation operators to navigate Fock space. The $c^\dagger, c$'s create and annihilate the particles; the $d^\dagger, d$'s the anti-particles. Their anti-commutation relations are:

$$\left[c_{s\vec{p}}, c^{\dagger}_{s'\vec{p}'}\right]_{+} = \left[d_{s\vec{p}}, d^{\dagger}_{s'\vec{p}'}\right]_{+} = \delta_{ss'}\delta^{3}\left(\vec{p}-\vec{p}'\right) \tag{109}$$

with all other anti-commutators zero.

Field Operators We define the field operators in parallel with those in the spin zero case, but with anti-commutators for commutators. The normalization is given by the $\sqrt{\frac{m}{VE_{\vec{p}}}}$. $c, c^{\dagger}$ are the annihilation and creation operators for $u^{S}_{1,2}$; $d, d^{\dagger}$ the annihilation and creation operators for $v^{S}_{1,2}$. The resulting field operator is:

$$\psi^{S}_{\tau}\left(\vec{x}\right) = \sum_{s=1}^{2}\sum_{\vec{p}}\sqrt{\frac{m}{VE_{\vec{p}}}}\begin{pmatrix} c_{s\vec{p}}u^{S}_{s}\left(\vec{p}\right)\exp\left(-\imath E_{\vec{p}}\tau + \imath\vec{p}\cdot\vec{x}\right) \\ +d^{\dagger}_{s\vec{p}}v^{S}_{s}\left(\vec{p}\right)\exp\left(\imath E_{\vec{p}}\tau - \imath\vec{p}\cdot\vec{x}\right)\end{pmatrix} \tag{110}$$

We define adjoints $\bar{u} \equiv u^{\dagger}\gamma_{0}, \bar{v} \equiv v^{\dagger}\gamma_{0}, \bar{\psi} \equiv \psi^{\dagger}\gamma_{0}$:

$$\bar{\psi}^{S}_{\tau}\left(\vec{x}\right) = \sum_{s=1}^{2}\sum_{\vec{p}}\sqrt{\frac{m}{VE_{\vec{p}}}}\begin{pmatrix} d_{s\vec{p}}\bar{v}^{S}_{s}\left(\vec{p}\right)\exp\left(-\imath E_{\vec{p}}\tau + \imath\vec{p}\cdot\vec{x}\right) \\ +c^{\dagger}_{s\vec{p}}\bar{u}^{S}_{s}\left(\vec{p}\right)\exp\left(\imath E_{\vec{p}}\tau - \imath\vec{p}\cdot\vec{x}\right)\end{pmatrix} \tag{111}$$

Dirac propagator The derivation of the Feynman propagator runs in parallel to the derivation for spin zero, except for replacing the simple wave functions with spinors and the commutators with anti-commutators. We define the Feynman propagator as time-ordered vacuum expectation value:

$$\imath S^{S}_{xy}\left(\vec{x}-\vec{y}\right) \equiv \left\langle 0\left|T\left\{\psi^{S}_{x}\left(\vec{x}\right), \bar{\psi}^{S}_{y}\left(\vec{y}\right)\right\}\right|0\right\rangle \tag{112}$$

Inside a vacuum sandwich, only the $cc^{\dagger}, dd^{\dagger}$ terms survive. This lets us rewrite the propagator in terms of the anti-commutators using the same procedure as for the spin zero case. We have the particle case when $\tau_{x} > \tau_{y}$:

$$\imath S^{S+}_{xy}\left(\vec{x}-\vec{y}\right) = \left\langle 0\left|\left[\psi^{S+}_{x}\left(\vec{x}\right), \bar{\psi}^{S-}_{y}\left(\vec{y}\right)\right]_{+}\right|0\right\rangle \tag{113}$$

and the anti-particle case when $\tau_{y} > \tau_{x}$:

$$\imath S^{S-}_{xy}\left(\vec{x}-\vec{y}\right) = -\left\langle 0\left|\left[\bar{\psi}^{S+}_{y}\left(\vec{y}\right), \psi^{S-}_{x}\left(\vec{x}\right)\right]_{+}\right|0\right\rangle \tag{114}$$

Then for the first term we have:

$$\imath S^{S+}_{xy}\left(\vec{x}-\vec{y}\right) = \frac{m}{\left(2\pi\right)^{3}}\int d\vec{p}\sum_{s=1,2}u^{S}_{s}\left(\vec{p}\right)\bar{u^{S}}_{s}\left(\vec{p}\right)\frac{\exp\left(-\imath E_{\vec{p}}\tau_{xy} + \imath\vec{p}\cdot\left(\vec{x}-\vec{y}\right)\right)}{E_{\vec{p}}} \tag{115}$$

We use:

$$\frac{\not{p}^{S}+m}{2m} = \sum_{s=1,2}u^{S}_{s}\left(\vec{p}\right)\bar{u}^{S}_{s}\left(\vec{p}\right) \tag{116}$$

to get the positive frequency part of the propagator as:

$$\imath S^{S+}_{xy}\left(\vec{x}-\vec{y}\right) = \frac{1}{\left(2\pi\right)^{3}}\int d\vec{p}\left(\not{p}^{S}+m\right)\frac{\exp\left(-\imath E_{\vec{p}}\tau_{xy} + \imath\vec{p}\cdot\left(\vec{x}-\vec{y}\right)\right)}{2E_{\vec{p}}} \tag{117}$$

We define $\not{p}^S \equiv \gamma_0 E_{\vec{p}} - \vec{\gamma}\cdot\vec{p}$ to distinguish this construction from $\not{p} \equiv \gamma_0 E - \vec{\gamma}\cdot\vec{p}$, needed for TQM.

For the negative frequency part we use:

$$\frac{\not{p}^S - m}{2m} = \sum_{s=1,2} v_s^S\left(\vec{p}\right)\bar{v}_s^S\left(\vec{p}\right) \tag{118}$$

to get the negative frequency part of the propagator as:

$$\imath S_{xy}^{S-}\left(\vec{x}-\vec{y}\right) = -\frac{1}{\left(2\pi\right)^3}\int d\vec{p}\left(\not{p}^S - m\right)\frac{\exp\left(\imath E_{\vec{p}}\tau_{xy} - \imath\vec{p}\cdot\left(\vec{x}-\vec{y}\right)\right)}{2E_{\vec{p}}} \tag{119}$$

Putting the two pieces back together:

$$\imath S_{xy}^{S}\left(\vec{x}-\vec{y}\right) = \frac{1}{\left(2\pi\right)^3}\int d\vec{p}\begin{pmatrix}\left(\not{p}^S + m\right)\frac{\exp\left(-\imath E_{\vec{p}}\tau_{xy} + \imath\vec{p}\cdot\left(\vec{x}-\vec{y}\right)\right)}{2E_{\vec{p}}}\theta\left(\tau_{xy}\right) \\ -\left(\not{p}^S - m\right)\frac{\exp\left(\imath E_{\vec{p}}\tau_{xy} - \imath\vec{p}\cdot\left(\vec{x}-\vec{y}\right)\right)}{2E_{\vec{p}}}\theta\left(-\tau_{xy}\right)\end{pmatrix} \tag{120}$$

This is again the "unpacked form":

$$\imath S_{\tau}^{S}\left(\vec{x}\right) = \frac{\imath}{\left(2\pi\right)^4}\int d\omega d\vec{p}\frac{\not{p}^S + m}{\omega^2 - \vec{p}^2 - m^2 + \imath\varepsilon}\exp\left(-\imath\omega\tau + \imath\vec{p}\cdot\vec{x}\right) \tag{121}$$

As with the spin zero case, once the three space momenta have been picked, the value of the energy component is forced: there is no dispersion in energy.

The momentum space form is:

$$\imath S_{\tau}^{S}\left(\vec{p}\right) = \imath\frac{\not{p}^S + m}{\omega^2 - \vec{p}^2 - m^2 + \imath\varepsilon} \tag{122}$$

or:

$$\imath S_{\omega}^{S}\left(\vec{p}\right) = \imath\left(\not{p}^S + m\right)\Delta_{\omega}^{S}\left(\vec{p}\right) \tag{123}$$

2.4.2. Dirac propagator in TQM

Dirac equation We now add in the time coordinate. We again apply the Machian hypothesis, this time to get the TQM form of the Dirac equation:

$$\left(\not{p} - \not{\mathcal{P}} - m\right)\psi_\tau = 0 \tag{124}$$

As in the spin zero case:

$$\left(\not{p} - m\right)\psi_\tau = \mathcal{P}\psi_\tau, \mathcal{P}\psi_\tau \to \mathcal{M}\psi_\tau \to \imath\gamma_0\frac{\partial}{\partial\tau_V}\psi_\tau \to -\imath\gamma_0\frac{\partial}{\partial\tau}\psi_\tau \tag{125}$$

The result is the TQM form of the Dirac equation:

$$\left(\not{p} - m\right)\psi_\tau = -\imath\gamma_0\frac{\partial}{\partial\tau}\psi_\tau \tag{126}$$

Dirac propagator To get the corresponding propagator we use the Machian hypothesis to form an ansatz for the TQM propagator then plug this ansatz in the equation to verify it is correct.

First we observe that Dirac's approach had its roots in the Klein-Gordon equation: it was an attempt to get a version of the Klein-Gordon equation which did not have negative energy solutions. We can see this directly from the Dirac equation. Apply $\not{p}+m$ to it using $\not{p}\not{p}=p^2$:

$$\left(\not{p}+m\right)\left(\not{p}-m\right)\psi=\left(p^2-m^2\right)\psi=0 \tag{127}$$

This is still true in TQM:

$$\left(\left(\not{p}-\not{\mathcal{P}}\right)+m\right)\left(\left(\not{p}-\not{\mathcal{P}}\right)-m\right)\psi=\left(\left(p-\mathcal{P}\right)^2-m^2\right)\psi=0 \tag{128}$$

We extend (122) in the obvious way: replacing 3D with 4D objects and replacing p with $p-\mathcal{P}$:

$$\imath S_\tau\left(p\right)=\int d\omega\frac{\left(\not{p}-\not{\mathcal{P}}\right)+m}{\left(\left(p-\mathcal{P}\right)^2-m^2\right)+\imath\varepsilon}\exp\left(-\imath\omega\tau\right) \tag{129}$$

We can see that this propagator is the inverse of the TQM Dirac equation 128.

To see more clearly what this means we again apply the Machian hypothesis, this time to get ω:

$$\mathcal{P}\rightarrow\left(\mathcal{M},\vec{0}\right)\rightarrow\left(\imath\frac{\partial}{\partial\tau_V},\vec{0}\right)\rightarrow\left(-\imath\frac{\partial}{\partial\tau},\vec{0}\right)\rightarrow\left(-\omega,\vec{0}\right) \tag{130}$$

We get:

$$\imath S_\tau\left(p\right)=\int d\omega\frac{\left(\not{p}+\gamma_0\omega\right)+m}{E^2-\vec{p}^2-m^2+2E\omega+\omega^2+\imath\varepsilon}\exp\left(-\imath\omega\tau\right) \tag{131}$$

We get a term proportional to ω in the numerator and terms proportional to ω,ω^2 in the denominator. We drop the ω^2 term as on the grounds that if the $E\omega$ term is small, the ω^2 term is going to be exceptionally small. The term linear in ω in the numerator will be replaced by ϖ when we do the contour integral. This in turn will average to zero to lowest order. We will drop this term for now as well.

Therefore we take as the lowest order TQM propagator for fermions:

$$\imath S_\omega\left(p\right)=\imath\left(\begin{array}{c}\frac{\left(\not{p}+m\right)}{E^2-\vec{p}^2-m^2+2\omega E+2E\imath\varepsilon}\\+\frac{\left(\not{p}+m\right)}{E^2-\vec{p}^2-m^2+2\omega E-2E\imath\varepsilon}\end{array}\right) \tag{132}$$

or in "unpacked" form:

$$\imath S_\tau\left(p\right)=\frac{\imath}{\left(2\pi\right)^4}\left(\frac{\mathrm{e}^{-\imath\varpi_p\tau}}{2E}\theta\left(\tau\right)-\frac{\mathrm{e}^{-\imath\varpi_p\tau}}{2E}\theta\left(-\tau\right)\right) \tag{133}$$

From the arguments above, we have the attosecond form:

$$\imath S_\omega^{\mathrm{A}}\left(p\right)=\imath\frac{\left(\not{p}+m\right)}{E^2-\vec{p}^2-m^2} \tag{134}$$

We have implicitly promoted $E_{\vec{p}}\rightarrow E$ in the various elements of the spinor formalism:

$$u^S\left(\vec{p}\right)\rightarrow u\left(p\right),v^S\left(\vec{p}\right)\rightarrow v\left(p\right),\sqrt{\frac{m}{E_{\vec{p}}}}\rightarrow\sqrt{\frac{m}{E}} \tag{135}$$

2.5. Vertexes

2.5.1. QED interaction in SQM For interactions, we take the Fock space as the product of the photon and fermion Fock spaces. Here it is enough to consider one species of fermion, say electrons. For photon and electrons we have:

$$\left|\left\{n^{e}_{s\vec{p}}\right\}\right\rangle \left|\left\{m^{\gamma}_{r\vec{k}}\right\}\right\rangle \tag{136}$$

The interaction term is:

$$-e\bar{\psi}^{S}_{\tau}\left(\vec{x}\right)A^{(S)\nu}_{\tau}\left(\vec{x}\right)\gamma_{\nu}\psi^{S}_{\tau}\left(\vec{x}\right) \tag{137}$$

The dependence on the coordinates is already absorbed into the definition of the Feynman diagram, so this is reduced to sums over terms of the form:

$$-e\bar{\psi}^{S}\left(\vec{p}'\right)A^{(S)\nu}_{\tau}\left(\vec{k}\right)\gamma_{\nu}\psi^{S}\left(\vec{p}\right) \tag{138}$$

The $\psi^{S}\left(\vec{p}\right)$ are built up over sums of the $u^{S}_{s}\left(\vec{p}\right), v^{S}_{s}\left(\vec{p}\right)$. For an electron $e = -\left|e\right|$.

2.5.2. QED interaction in TQM The full Fock space in TQM is also a product of the fermion and photon Fock spaces:

$$\left|\left\{n^{e}_{sp}\right\}\right\rangle \left|\left\{m^{\gamma}_{sk}\right\}\right\rangle \tag{139}$$

The vertex term is:

$$-e\bar{\psi}_{\tau}\left(t,\vec{x}\right)A^{\nu}_{\tau}\left(t,\vec{x}\right)\gamma_{\nu}\psi_{\tau}\left(t,\vec{x}\right) \tag{140}$$

Again, the dependence on the coordinates is already absorbed into the definition of the Feynman diagram, so this is reduced to sums over terms of the form:

$$-e\bar{\psi}\left(p'\right)A^{\nu}_{\tau}\left(k\right)\gamma_{\nu}\psi\left(p\right) \tag{141}$$

The $\psi\left(p\right)$ are built up over sums of the $u_{s}\left(p\right), v_{s}\left(p\right)$.

2.5.3. Conservation of momentum at a vertex In SQM the integrals over the 3D plane waves give conservation of three momentum at each vertex:

$$\delta^{3}\left(\vec{p}_{out} - \vec{p}_{in}\right) \tag{142}$$

As noted earlier, we get conservation of clock energy at a vertex from the integrals over clock time. For this to work we need to take the limit as $\tau \to \pm\infty$. If we are looking at short time S matrix elements – legitimate if infrequent – we will not get exact conservation of clock energy at each vertex, but only an approximation thereof, corresponding to *not* taking the limit in:

$$\lim_{T\to\infty}\int_{-T}^{T}\frac{d\tau}{2\pi}\exp\left(-\imath\left(\omega_{out}-\omega_{in}\right)\tau\right) = \lim_{T\to\infty}\frac{\sin\left(\left(\omega_{out}-\omega_{in}\right)T\right)}{\pi\left(\omega_{out}-\omega_{in}\right)} = \delta\left(\omega_{out}-\omega_{in}\right) \tag{143}$$

If the time range T is sufficiently small the difference between the *sin* and the δ function could become noticeable. So we see that in SQM the clock energy is not present on exactly the same basis as the three space momenta.

To a reasonable first approximation, TQM may be thought of as SQM with a fourth space dimension. So we get conservation of 4D momentum at each vertex:

$$\delta^4 \left(p_{out} - p_{in} \right) \tag{144}$$

If we able to take the limit as clock time goes to infinity, we get a fifth conservation condition at each vertex and over the entire Feynman diagram:

$$\delta \left(\sum \varpi_{out} - \sum \varpi_{in} \right) \tag{145}$$

but we can easily do without this at shorter times.

To keep the comparisons between SQM and TQM as straightforward as possible, we will finesse this problem by looking primarily at the scattering of GTFs which only intersect for very short times. We can therefore extend the integrals to infinity at will since, except over the short interaction zone, the integrand is zero.

2.5.4. Normalization factors To go from SQM to TQM we changed the normalization on the field operators from $\frac{1}{\sqrt{2\omega_{\vec{k}}}}$ to $\frac{1}{\sqrt{2w}}$ and from $\sqrt{\frac{m}{E_{\vec{p}}}}$ to $\sqrt{\frac{m}{E}}$.

At any vertex, the field operators either connect to a propagator or to an outside line. If they connect to a propagator their normalization factor joins up with the normalization from the other side to give normalizations which are accounted for in the calculation of the propagators:

$$\frac{1}{\sqrt{2\omega_{\vec{k}}}} \frac{1}{\sqrt{2\omega_{\vec{k}}}} \rightarrow \frac{1}{2\omega_{\vec{k}}}, \frac{1}{\sqrt{2w}} \frac{1}{\sqrt{2w}} \rightarrow \frac{1}{2w}, \ldots \tag{146}$$

If on the other hand they are connected to an external line the normalization is passed to the overall definition of the S; we get the familiar exterior factors, as:

$$\prod_{boson}^{exterior} \sqrt{\frac{1}{2\omega_{\vec{k}}}}, \prod_{boson}^{exterior} \sqrt{\frac{1}{2w}}, \ldots \tag{147}$$

In SQM there is a conventional understanding that on the legs the virtual particles will drop off and we will be left with the true 3D on-shell wave function. But in TQM there is no such transition: the wave packets on the legs are fully four dimensional in character.

So ultimately, in TQM if we consider the "Feynman diagram of the universe" we can divide it up into S matrices as we please. Where the legs of two S matrices meet, the two associated exterior factors will join to help form the propagator for the leg. There is no physical transition from the system *under* examination to the system *doing* the examination; it is a matter of convention and names where and how we choose to make the divisions.

3. Applications

> "In other words, we are trying to prove ourselves wrong as quickly as possible, because only in that way can we find progress." – Richard P. Feynman [61]

With the basic tools built, we apply them to a starter set of cases:

(i) Free particles. The creation, propagation, and detection of wave functions which are dispersed in time presents specific questions.

(ii) Spin-zero particle exchange. This serves as the first real test of how TQM plays out in practice.

(iii) Simple mass correction loop. We show that renormalization is not needed in TQM; the combination of dispersion in time and entanglement in time keep the loop integrals finite.

(iv) The three basic second order scattering diagrams in QED:

(a) Møller scattering. Electron-electron scattering by photon exchange.
(b) Bhabha scattering. Electron-positron scattering by photon exchange and by photon creation/annihilation.
(c) Compton scattering. Electron-photon scattering.

This starter set is sufficient to expose a fair number of experimental possibilities; we work out the general Feynman rules in Appendix C.

3.1. Free particles

"nothing can be created out of nothing" – Lucretius [62]

To see the effects associated with dispersion in time we need to look at wave functions which have finite dispersion in time/energy. Plane waves and δ functions are equally unsuitable. Gaussian test functions (Appendix B) are well-suited as models for this: they are easy to work with and they are completely general – any wave function may be represented as a sum over GTFs using Morlet wavelet analysis.

In SQM the use of GTFs is a convenience, but in TQM the use of GTFs is mandatory: the convergence of the path integrals in the single particle case and in loop diagrams depends on the use of GTFs.

To lay a correct foundation for the examination of scattering problems in TQM we will need to look at the initial values, the propagation, and the detection – the birth, life and death – of GTFs.

3.1.1. Initial wave function The first problem is how do we create a GTF of known dispersion in time/energy? This problem is solved in paper A. There we showed that if we are given the norm, average momentum, and average dispersion in momentum we can get a maximum entropy estimate of the corresponding GTF in energy using the method of Lagrange multipliers.

As maximum entropy estimates tend to be robust, we have what we need for falsifiability.

We summarize for use here the results of paper A. We assume we have a description of the wave function in momentum space with the expectation, average relativistic mass, and dispersion in relativistic mass defined:

$$\begin{aligned} &\langle 1 \rangle = 1 \\ &\bar{E} \equiv \langle E \rangle = \sqrt{m^2 + \langle \vec{p} \rangle^2} \\ &\left\langle E^2 \right\rangle = \left\langle m^2 + \vec{p}^2 \right\rangle = m^2 + \left\langle \vec{p}^2 \right\rangle \end{aligned} \tag{148}$$

$\bar{E}$ is the average relativistic mass.

By the method of Lagrange multipliers the corresponding GTF in energy is:

$$\varphi_0(E) = \sqrt[4]{\frac{1}{\pi\sigma_E^2}} e^{\imath\left(E-\bar{E}\right)\tau_0 - \frac{\left(E-\bar{E}\right)^2}{2\sigma_E^2}} \tag{149}$$

$$\sigma_E^2 = 2(\Delta E)^2 = 2\left(\left\langle E^2 \right\rangle - \bar{E}^2\right) \tag{150}$$

With this estimate of the GTF in energy we can get the corresponding GTF in time by taking the Fourier transform. In this case we have:

$$\varphi_0(t) \equiv \sqrt[4]{\frac{1}{\pi\sigma_t^2}} e^{-\imath\bar{E}t - \frac{(t-\tau_0)^2}{2\sigma_t^2}}, \sigma_t = \frac{1}{\sigma_E} \tag{151}$$

The dispersions in energy are likely to be of order eV, if we are starting with atomic wave functions. Assuming we are starting with minimum uncertainty packets (expected) the uncertainty in time will be the inverse of this:

$$\Delta t = \frac{1}{\Delta E} \tag{152}$$

and therefore of order attoseconds.

We will refer to this as the "entropic estimate". It will not pick up any complex structure: to the entropic estimate, everything looks like an s state. But it should get the order-of-magnitude of the time part of the wave function right, which is the critical marker for falsifiability. If no more specific estimate is available, it should do.

3.1.2. Propagation of free wave functions in clock time In general, the three or four dimensional GTFs in momentum space are solutions of the corresponding free equation with appropriate choices of the clock frequency. If we have the wave function as time zero as a function of momentum then we have it at time τ later. In SQM:

$$\varphi_\tau^S\left(\vec{p}\right) = \varphi_0^S\left(\vec{p}\right) \exp\left(-\imath E_{\vec{p}} \tau\right) \tag{153}$$

In TQM:

$$\varphi_\tau\left(E, \vec{p}\right) = \varphi_0\left(E, \vec{p}\right) \exp\left(-\imath \varpi_p \tau\right) \tag{154}$$

These two results are exact.

The square root in the definition of the relativistic mass for SQM and the $1/2w$ factor in the definition of the clock frequency for TQM can make these two expressions difficult to work with analytically. A quadratic approximation offers useful insight. We will use this approach when analyzing the loop corrections, for instance. Note that $\varpi_p = 0$ when we are exactly on-shell, increasing quadratically as we get far enough off-shell:

$$\varpi_p = -\frac{\left(\bar{E} + \delta E\right)^2 - \bar{E}^2}{2\left(\bar{E} + \delta E\right)} \approx -\delta E + \frac{\left(\delta E\right)^2}{2\bar{E}} \tag{155}$$

We expect therefore that if TQM wave packets start on-shell (per the entropic estimate) they will tend to stay approximately on-shell subsequently. This means we can take the initial wave packets in a scattering experiment as on-shell. We will take advantage of this in the next subsection.

3.1.3. Detection of the wave function The issues associated with the detection of a wave function in time were discussed at length in paper B [51]. To keep this work self-contained we give a summary.

Effect of convoluted paths The paths in TQM are significantly more complex than those in SQM. In particular while paths in SQM can go left and right, up and down, forwards and back, paths in TQM can also go into the future and into the past. We expect these excursions will be of order attoseconds and be centered around the classical or SQM values. (This will keep us from attracting the unwelcome attention of the time police.)

When we calculate the paths in TQM we typically calculate the wave function at the detector as a function of clock time:

$$\varphi_\tau\left(t, x\right) \tag{156}$$

This gives the sum not of all paths to t, x but only of those paths with length τ. To get the full amplitude at t, x we will need to sum over paths of all lengths:

$$\varphi(t,x) = \int d\tau \varphi_\tau(t,x) \tag{157}$$

We are, as required, taking coordinate time as fundamental with clock time playing a secondary role, here letting us categorize the paths by length.

The problem of measurement

> "To re-emphasize, from a broader perspective, the main argument that is being articulated in this section is that quantum measurements, in the interferometric and polarization domains, can be described without resorting to the concept of the collapse of the wave function or the collapse of the probability amplitude." – p185 Duarte and Taylor [63]

The fundamental problem here is that ultimately we have to treat the detectors (and even the observers) as also quantum mechanical systems. They are, after all, made of atoms, and atoms are unavoidably quantum systems. There is no such thing as a classical atom – in classical mechanics electrons spiral into the nucleus in a minute fraction of a second due to loss of energy from Larmor radiation [64].

There is of course an extraordinary literature on the "problem of measurement". This problem is not a central focus of this work. In paper B, where this problem was a central focus, we took as a starting point an analysis by Marchewka and Schuss [65, 66, 67, 68] who made a strong argument (from probability conservation) that the probability current correctly gives the detection rate.

In a certain sense this merely postpones the problem: to compute the probability current you must first compute the wave function, which means that you have to first solve the problem of the interaction of the particle's wave function with that of the detector – including absorption, reflection, loss, emission, emission followed by re-absorption, and so on. Rather a lot to consider.

However if you are prepared to posit an ideal detector, with no loss or lag, you can use the probability current at the detector as giving a reasonable estimate of the detection rate as a function of time. The beauty of this approach, in the current context, is that it:

(i) is easy to use,
(ii) should give a reasonable first estimate in general,
(iii) is typical of what workers often do, so clearly not that bad in practice,
(iv) and most importantly here, works the same way for SQM and TQM, so creates no bias.

In SQM We take a detector in the y-z plane placed at $x = L$. For a simple GTF headed, say, left to right with momentum p_0, the detection rate is:

$$D_\tau^S = \frac{p_0}{m} \rho_\tau(x)|_{x=L} \tag{158}$$

$\rho_\tau(x)$ has a dispersion in space given by σ_x. But it is the resulting dispersion of the detection rate D_τ^S in clock time that we are interested in here. This is:

$$\sigma_\tau^S = \frac{1}{m v_0 \sigma_x} \bar{\tau} \tag{159}$$

This is proportional to $\bar{\tau}$ the average time of flight: the greater the average time of flight the greater the uncertainty in clock time at the detector. It is inversely proportional to σ_x because

of diffraction (see Appendix B.3.2). And it is inversely proportional to the speed of the packet: the slower the packet the more spread out it will be by the time it reaches the detector. The packet functions like a train; the slower the train the longer the time from when the engine arrives at the station till the time when the caboose does.

The corresponding uncertainty in clock time, the number we are most interested in, is:

$$\Delta_\tau^S = \frac{1}{\sqrt{2}}\sigma_\tau^S \tag{160}$$

We are seeing uncertainty in clock time, but it is *entirely* the product of dispersion in space. As Busch and other workers have pointed out, in SQM there is a HUP for time/energy, but it is not on the same basis at all as the HUP for momentum/space.

In TQM In TQM we will have dispersion in time in addition to the dispersion in space. Suppose the initial wave function is a product of time and space parts:

$$\varphi_\tau(t,x) = \varphi_\tau^T(t)\,\varphi_\tau^S(x) \tag{161}$$

We may be getting the wave function in time from the entropic estimate. And we are ignoring the more general possibility that time and space are entangled. Using again the probability current, now in TQM, we showed that the detection rate is now the product of the time and space parts:

$$D_\tau(t) = D_\tau^S \rho_\tau^T(t) \tag{162}$$

where the probability distribution in coordinate time as a function of clock time is:

$$\rho_\tau^T(t) \approx \sqrt{\frac{1}{\pi\sigma_\tau^{(T)2}}}e^{-\frac{1}{\sigma_\tau^{(T)2}}(t-\tau)^2}, \sigma_\tau^T \equiv \frac{\bar{\tau}}{m\sigma_t} \tag{163}$$

σ_τ^T is proportional to the average time to the detector. It is inversely proportional to the initial dispersion in coordinate time σ_t. To get the full detection rate at coordinate time t we have to sum over the clock time:

$$D(t) = \int d\tau D_\tau^S \rho_\tau^T(t) \tag{164}$$

This is a convolution of the S and the T distributions. The total dispersion squared is the sum of the squares of the dispersions:

$$\sigma_t^2 = \sigma_\tau^{(S)2} + \sigma_\tau^{(T)2} \tag{165}$$

This is a "nice" result in that if we compute the dispersion in clock time in SQM and the dispersion from the time part of TQM we need only to add them to get the total dispersion. If we monitor the dispersion of arrival times long enough, we should see – to whatever level of statistical certainty is desired – whether the dispersion is better described by SQM or by SQM + TQM.

Slow train problem The main problem here is the $1/v_0$ factor in the dispersion from space. From the entropic estimate we expect that the dispersions in space and time will in general be of the same order, $\sigma_x \sim \sigma_t$. But if the wave packet is going at non-relativistic speeds, the effects of the space part will be far greater than those from time, making it hard to pick out any effects from TQM.

There are (at least) four possible solutions to the slow train problem:

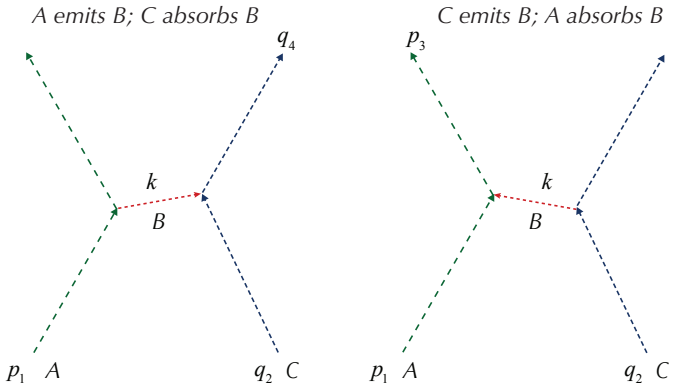

Figure 3. An A and a C particle exchange a B particle

(i) Use a faster train. Get v_0 closer to one. This in fact is part of the motivation for this paper, as we need QED to work with high speeds.

(ii) Wait longer for the train. Collect enough data points that even a small difference becomes statically significant to whatever level is desired.

(iii) Break the train up into its individual cars. If we do this, we can see the dispersion in time on a per car basis, which should make it stand out more. Or in less figurative language, run the wave packet through a strong magnetic field. Discussed further below in subsubsection 3.4.1.

(iv) Diffract the train. (This is a quantum train.) Use the Heisenberg uncertainty principle in time. We use a second wave packet, chosen to be narrow in time, as effectively a single slit in time. The single slit in time acts as a single slit in space would, diffracting the particle with correspondingly increased uncertainty in time-of-arrival.

3.1.4. Summary At this point we have achieved a measure of falsifiability.

(i) We write the TQM prediction of the uncertainty in time at the detector as $(\Delta t)^2_D$,

(ii) We write the SQM prediction of the uncertainty in (clock time) at the detector as $(\Delta\tau)^{(S)2}_D$.

(iii) Our signal is the difference, meaning that part of the uncertainty in time at the detector which is not accounted for by SQM:

$$(\Delta t)^{(T)2}_D \equiv (\Delta t)^2_D - (\Delta\tau)^{(S)2}_D \tag{166}$$

For this to be well-defined, we need a good estimate of the initial wave function in coordinate time. This is why the entropic estimate is critical: it gives a simple but robust estimate of the initial uncertainty in coordinate time.

However this does not yet take us to feasibility. We see that for non-relativistic wave packets, the SQM prediction for uncertainty in time at the detector may be much larger than the additional contribution from TQM; therefore the space contribution may drown out the time contribution.

To get from falsifiability in principle to falsifiability in practice we need a way to increase the relative size of the time contribution.

To do this we look next at the simplest possible scattering problem. This continues our development of TQM and it defines the basic effects we can use to see its effects.

3.2. Spin zero scattering

3.2.1. Objective We now look at the simplest possible scattering problem. We work with a toy model consisting of three spin zero particles: A, B and C. A and C both have mass m. They do not interact directly, but only by exchanging B's. B's have mass μ. A's and C's are stand-ins for fermions, B is a stand-in for photons. We will refer to this as the ABC model.

The very considerable practical advantage of looking at spinless particles is that most of the effects that distinguish TQM from SQM have nothing to do with spin or polarization.

The field operators for A's are for *SQM:*

$$A_\tau^S(\vec{x}) \equiv \sum_{\vec{p}} \frac{1}{\sqrt{2VE_{\vec{p}}}} \begin{pmatrix} a_{\vec{p}} \exp\left(-\imath E_{\vec{p}}\tau + \imath\vec{p}\cdot\vec{x}\right) \\ +a_{\vec{p}}^\dagger \exp\left(\imath E_{\vec{p}}\tau - \imath\vec{p}\cdot\vec{x}\right) \end{pmatrix} \tag{167}$$

and for *TQM:*

$$A_\tau(t,\vec{x}) \equiv \sum_{p} \frac{1}{\sqrt{2TVE}} \begin{pmatrix} a_p \exp\left(-\imath\varpi_p\tau - \imath px\right) \\ +a_p^\dagger \exp\left(+\imath\varpi_p\tau + \imath px\right) \end{pmatrix} \tag{168}$$

B and C's are the same.

The interaction Lagrangian has the form:

$$V = \lambda\frac{A^2B}{2} + \lambda\frac{C^2B}{2} \tag{169}$$

with specific terms of the form:

$$\frac{\lambda\left(a+a^\dagger\right)\left(b+b^\dagger\right)\left(a+a^\dagger\right)}{2} + \frac{\lambda\left(c+c^\dagger\right)\left(b+b^\dagger\right)\left(c+c^\dagger\right)}{2} \tag{170}$$

We will assume λ is small, so that perturbation theory makes sense. We will look at the case where an A and C exchange a B.

To help make the diagrams more readable, we identify the A's by p's, B's by k's, and C's by q's. We use $\bar{E}_{1,2,3,4}, \omega$ for the clock energies for SQM. We use $E_{1,2,3,4}, w$ for the coordinate energies and $\varpi_{1,2,3,4}, \varpi_k$ for the clock energies in TQM. A will start with momentum p_1, finish with p_3; C will start with q_2, finished with q_4; and the exchanged B will have momentum k.

We will work in the center-of-mass frame. For definiteness we will assume the particles are approaching each other along the x-axis and are scattered along the y-axis. Results for the z-axis follow from symmetry. The coordinate system is given by $(x,y,z) = r\left(\sin\theta\cos\varsigma, \sin\theta\sin\varsigma, \cos\theta\right)$. In the case of GTFs, we will work in the center-of-time frame as well: arrange for $\tau = 0$ to correspond to the time of closest approach of the opposing GTFs.

We assume we have detectors well off to the left and right, placed at $x = \pm L$, running in the y, z plane, and time-sensitive. And further we will assume that the detectors are far enough away that the position of the detector divided by time-of-flight from the interaction zone is a good proxy for the velocity and therefore for the momentum. With this arrangement it is easy to map detector time and position to momentum. To first order:

$$p_x = \gamma m v_x, p_y = \gamma m v_y, E = \gamma m \tag{171}$$

For SQM $v_x = \frac{L}{\tau}, v_y = \frac{y}{\tau}$; for TQM $v_x = \frac{L}{t}, v_y = \frac{y}{t}$. We will compute the energy component indirectly, by using the three velocity to compute $\gamma = \frac{1}{\sqrt{1-\vec{v}^2}}$. We will look first at the case where the initial wave functions are defined as plane waves; then as GTFs.

We have conservation of clock energy as well, for both SQM and TQM. For plane waves it is customary to take the limit as the integral over clock time goes to $\pm\infty$, which in turn gives

conservation of clock energy. As noted earlier, for GTFs we argue that if the interaction time is small an integration over the interaction will be zero outside of that time, so that the $\pm$ limits may be extended to $\pm\infty$ without affecting the value of the integral. So we can again take the limit as clock time goes to $\pm\infty$ and therefore have conservation of clock energy.

3.2.2. Plane waves We can see the plane wave case as Møller scattering without the spin or polarization. In terms of the Mandelstam variables [69] we are working in t-channel, with the energy-momentum of the exchanged particle being given by $p_3 - p_1$ or $q_4 - q_2$:

$$t = (p_1 - p_3)^2 = (q_2 - q_4)^2 \tag{172}$$

The S matrix is given in each case as a product of the appropriate δ functions, normalization factors, and the matrix element. We start with the matrix element as the interesting bit. In this simple case, this is λ^2 times the propagator for the exchanged particle.

SQM For SQM the propagator is:

$$\Delta_\omega^S\left(\vec{k}\right) = \frac{1}{\omega^2 - \vec{k}^2 - \mu^2 + \imath\varepsilon} \tag{173}$$

We can compute this from the values of the external momenta and the δ functions at each vertex. If the A particle emits the B, then we have:

$$\left(\omega, \vec{k}\right) = \left(\bar{E}_3 - \bar{E}_1, \vec{p}_3 - \vec{p}_1\right), \left(\bar{E}_4, \vec{q}_4\right) = \left(\bar{E}_2 + \omega, \vec{q}_2 + \vec{k}\right) \tag{174}$$

while if the C particle emits the B we have:

$$\left(\omega, \vec{k}\right) = \left(\bar{E}_4 - \bar{E}_2, \vec{q}_4 - \vec{q}_2\right), \left(\bar{E}_3, \vec{p}_3\right) = \left(\bar{E}_1 + \omega, \vec{p}_1 + \vec{k}\right) \tag{175}$$

In both cases we have overall conservation of momentum:

$$\left(\bar{E}_3 + \bar{E}_4, \vec{p}_3 + \vec{q}_4\right) = \left(\bar{E}_1 + \bar{E}_2, \vec{p}_1 + \vec{q}_2\right) \tag{176}$$

This forces the magnitude of the three vectors for both A and C to be the same before and after the interaction. From conservation of clock energy:

$$2m^2 + \vec{p}_1^2 + \vec{q}_2^2 = 2m^2 + \vec{p}_3^2 + \vec{q}_4^2 \tag{177}$$

In the center-of-mass frame:

$$\vec{p}_1^2 = \vec{q}_2^2, \vec{p}_3^2 = \vec{q}_4^2 \tag{178}$$

Combining these two results we get:

$$2m^2 + 2\vec{p}_1^2 = 2m^2 + 2\vec{p}_3^2 \Rightarrow \vec{p}_1^2 = \vec{p}_3^2 \tag{179}$$

and from this we see there is no clock energy available for the unfortunate B. As the propagator is not being integrated over, we may drop the $\imath\epsilon$. We have:

$$\Delta_\omega^S\left(\vec{k}\right) = -\frac{1}{\vec{k}^2 + \mu^2} \tag{180}$$

We set $\vec{p} \equiv \vec{p}_1 = -\vec{q}_2$ and write:

$$\vec{k} = |\vec{p}|\left(\cos\left(\theta\right), \sin\left(\theta\right), 0\right) \tag{181}$$

so the propagator is:

$$\Delta_{\omega}^{S}\left(\vec{k}\right) = -\frac{1}{2\vec{p}^{2}\left(1-\cos\left(\theta\right)\right)+\mu^{2}} \tag{182}$$

and the matrix element $(-\imath\lambda)^2$ times this:

$$\mathcal{M}^{\mathcal{S}} = \frac{\lambda^{2}}{2\vec{p}^{2}\left(1-\cos\left(\theta\right)\right)+\mu^{2}} \tag{183}$$

The external factors are:

$$N^{S} = \sqrt{\frac{1}{2\bar{E}_{1}}}\sqrt{\frac{1}{2\bar{E}_{2}}}\sqrt{\frac{1}{2\bar{E}_{3}}}\sqrt{\frac{1}{2\bar{E}_{4}}} \tag{184}$$

In the center-of-mass frame they are all equal, so there is just one $\bar{E}$. The δ functions are:

$$D^{S} = (2\pi)^{4}\delta\left(\left(\bar{E}_{3}+\bar{E}_{4}\right)-\left(\bar{E}_{1}+\bar{E}_{2}\right)\right)\delta^{3}\left(\left(\vec{p}_{3}+\vec{q}_{4}\right)-\left(\vec{p}_{1}+\vec{q}_{2}\right)\right) \tag{185}$$

The full S matrix is:

$$S^{S} = D^{S}N^{S}\mathcal{M}^{S} \tag{186}$$

The probability is given by the square of this. The four δ functions make clear that this is a formal object; it will not acquire meaning until we integrate over the GTFs.

TQM The calculation for the plane wave for TQM is the same. We will take the SQM values as the starting point. Recall the quantum energy δw is defined as the difference between the coordinate energy and the SQM energy. Since the SQM energy is zero, the quantum energy is just the coordinate energy w itself.

We have from the entropic estimate that the average energy in the initial value of a TQM wave function will match that in SQM, so we have:

$$E_{1} = \bar{E}_{1}, E_{2} = \bar{E}_{2} \Rightarrow \delta E_{1} = 0, \delta E_{2} = 0 \tag{187}$$

From this we also get $\varpi_1 = \varpi_2 = 0$. Or to put it another way, wave functions in TQM start on-shell. We have four δ functions in momentum plus one in clock energy at each vertex, forcing the four momenta for the exchanged B in the same way as with SQM.

If the A particle emits the B, then we have:

$$E_{3} = \bar{E}-w, \vec{p}_{3} = \vec{p}_{1}-\vec{k}; E_{4} = \bar{E}+w, \vec{q}_{4} = -\vec{p}_{1}+\vec{k} \tag{188}$$

while if the C particle emits the B we have:

$$E_{3} = \bar{E}+w, \vec{p}_{3} = \vec{p}_{1}+\vec{k}; E_{4} = \bar{E}-w, \vec{q}_{4} = -\vec{p}_{1}-\vec{k} \tag{189}$$

In both cases we have overall conservation of momentum:

$$E_{3}+E_{4} = E_{1}+E_{2}, \vec{p}_{3} = -\vec{q}_{4} \tag{190}$$

The only remaining ambiguity is whether we have $\varpi_3 = \varpi_4 = 0$ or not, whether the two outgoing particles are forced to be on-shell or not. To check this we expand in powers of w.

We have for the two external clock energies if A emits the B:

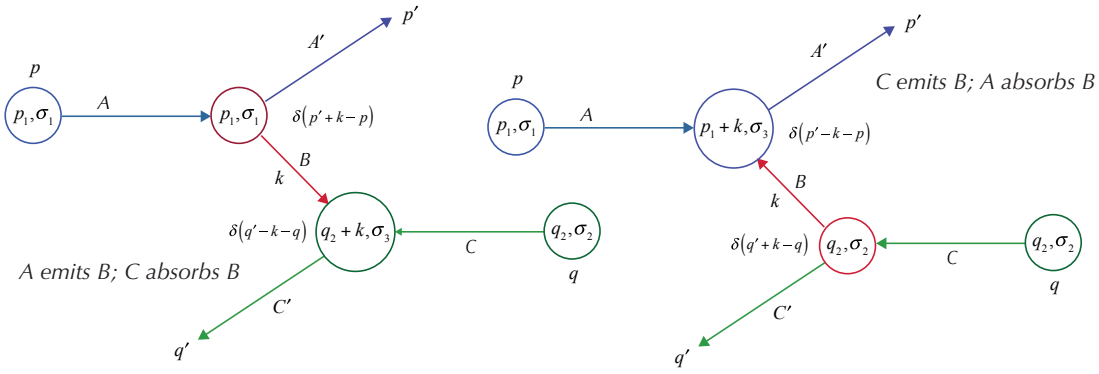

Figure 4. Calculation of dispersion in exchange of B particle

$$\varpi_3 = -\frac{\left(\bar{E}-w\right)^2 - \left(\vec{p}_1 - \vec{k}\right)^2 - \mu^2}{\bar{E}-w}, \varpi_4 = -\frac{\left(\bar{E}+w\right)^2 - \left(\vec{p}_1 - \vec{k}\right)^2 - \mu^2}{\bar{E}+w} \tag{191}$$

and the same thing with $w \to -w$ if C emits the B. Therefore the sum of the two, in either case, is:

$$-\frac{\left(\bar{E}-w\right)^2 - \left(\vec{p}_1 - \vec{k}\right)^2 - \mu^2}{2\left(\bar{E}-w\right)} - \frac{\left(\bar{E}+w\right)^2 - \left(\vec{p}_1 - \vec{k}\right)^2 - \mu^2}{2\left(\bar{E}+w\right)} = 0 \tag{192}$$

We already know, since the incoming lines are on-shell, that:

$$\left(\bar{E}\right)^2 - \left(\vec{p}_1\right)^2 - \mu^2 = 0 \tag{193}$$

Using this and clearing the fractions we get:

$$2\bar{E}w^2 = 0 \Rightarrow w = 0 \tag{194}$$

So here the quantum energy w of the exchanged particle is zero just as the classical energy is. And therefore the clock energies ϖ_3, ϖ_4 are also zero. And therefore both outgoing lines are on-shell. To put it another way, an analysis using on-shell plane waves shows no effects of TQM. We will need to work with GTFs to see the effects of dispersion in time/energy.

3.2.3. Gaussian test functions

Calculation of the propagation of the GTFs through the diagram Our initial wave functions will be direct products of GTFs in x, y, z and (in TQM) t. The value of the t wave function will be inferred from the other three parts using the entropic estimate.

For GTFs the calculation is essentially the same for all three or four components of the momentum, which we will represent as p, q, p', q'.

We will start by assuming we are dealing with fixed input GTFs, functions of p, q centered on p_1, q_2 with dispersions σ_1, σ_2. We wish to compute the amplitude to measure the outgoing momenta with values p', q'.

We can compute this by integrating over the S matrix from the plane wave case. We express this in a generic notation:

$$\psi\left(p',q'\right)=\int dpdqdkS\left(p',q';p,q\right)\varphi_1\left(p\right)\varphi_2\left(q\right) \tag{195}$$

First we assume the incoming GTFs are narrow in energy/momentum, that the dispersions is a small fraction of the corresponding momenta:

$$\frac{\sigma_{|\vec{p}|}}{\vec{p}}\ll 1 \tag{196}$$

We refer to this as the "narrow beam" approximation. Our GTFs are more rounded than simple plane waves but each is still relatively focused on a specific value of the momentum. They inhabit a kind of halfway house between plane wave and GTF.

This lets us make some useful approximations for the normalizations and for the matrix elements. In particular we can approximate each external factor by its average. For SQM we can ignore the variation caused by the GTF:

$$\sqrt{\frac{m}{E_{\vec{p}_{1,2}}}}\approx\sqrt{\frac{m}{\bar{E}_{1,2}}} \tag{197}$$

for TQM we can ignore both this and variation caused by the quantum energy ($\delta E\equiv E-E_{\vec{p}}$):

$$\sqrt{\frac{m}{E_{1,2}}}\approx\sqrt{\frac{m}{E_{\vec{p}_{1,2}}}}\approx\sqrt{\frac{m}{\bar{E}_{1,2}}} \tag{198}$$

By the same argument, for SQM we replace the technically correct $k_i+\delta k_i$ in the denominator of the matrix element by k_i:

$$\frac{\imath}{\left(\vec{k}+\delta\vec{k}\right)^2+\mu^2}\rightarrow\frac{\imath}{\left\langle\vec{k}^2\right\rangle+\mu^2} \tag{199}$$

For TQM we do the same:

$$\frac{\imath}{\left(\delta w\right)^2-\left(\vec{k}+\delta\vec{k}\right)^2+\mu^2}\rightarrow\frac{\imath}{\left\langle\vec{k}^2\right\rangle+\mu^2} \tag{200}$$

This means we can pull out from inside the integral a common factor (the same for SQM and TQM) of:

$$\bar{S}\equiv\prod_{i=1}^{4}\sqrt{\frac{m}{\bar{E}_i}}\frac{\imath}{\vec{k}^2+\mu^2} \tag{201}$$

leaving the integral as purely an integral over the dispersions in the GTFs and the δ functions at the vertexes.

There are three of these integrals for SQM plus one more – over energy – for TQM:

$$I\left(p'q'\right)=\int dpdqdk\begin{pmatrix}\delta\left(p'+k-p\right)\delta\left(q'-k-q\right)\theta\left(\tau\right)\\+\delta\left(p'-k-p\right)\delta\left(q'+k-q\right)\theta\left(-\tau\right)\end{pmatrix}\varphi\left(p\right)\varphi\left(q\right) \tag{202}$$

where the sign of τ depends on whether we are looking at the case where A emits B and C absorbs it or where C emits a B and A absorbs it.

What we have here is a "stick-and-cloud" approximation. The plane wave analysis gives the average result – the stick – then the integrals over the dispersions pull in the quantum effects we are interested in here, the clouds. With the δ functions we have only one real integral to do.

For the first pair, we do the integral over p first. This turns $\varphi_1(p) \to \varphi_1(p'+k)$. Next we do the integral over k and the second δ function. This leaves:

$$\psi^{(1)}(p',q') = \int dq \varphi_1(p'+k)\,\varphi_2(q'-k) \tag{203}$$

This is a convolution of two Gaussians. This results in a single Gaussian of the form:

$$\exp\left(-\frac{((p'+q')-(p_1+q_2))^2}{2\sigma_3^2}\right) \tag{204}$$

with dispersion:

$$\sigma_3^2 = \sigma_1^2 + \sigma_2^3 \tag{205}$$

What this is saying is that the resulting amplitude is a GTF centered on the average value of the total initial momentum, with a total dispersion which is a Pythagorean sum over the two initial dispersions. The second pair of δ functions goes exactly the same way, but in reverse.

At this point we need to check the counting. Each vertex has a factor of $1/2$, for a total of $1/4$. We can have the B emit from the first vertex or the second and from A or from C. This is a counter-balancing factor of four, for a total value of one. This is as expected. Therefore the final distribution is still given by equation 204.

Compare to conservation of energy with plane waves We define:

$$\delta p \equiv p' - \bar{p}, \delta q \equiv q' - \bar{p} \tag{206}$$

giving:

$$\delta p + \delta q = p' + q' - (p+q) \tag{207}$$

and:

$$\psi(\delta p, \delta q) = \exp\left(-\frac{(\delta p+\delta q)^2}{2\sigma_3^2}\right) \tag{208}$$

So the incoming total value of the momentum in question has an average of $\bar{p}$, but is dispersed around that average. The outgoing value is also dispersed around the average, with the dispersion given by the Pythagorean sum.

The full outgoing wave function is the product of three of these functions for SQM; the product of four of these functions – the three from SQM and a fourth on the energy axis – for TQM. Defining:

$$\varphi_{3,4}^{\mu}(\delta p_\mu, \delta q_\mu) \equiv \exp\left(-\frac{(\delta p_\mu + \delta q_\mu)^2}{2\left(\sigma_\mu^{(1)2} + \sigma_\mu^{(2)2}\right)}\right) \tag{209}$$

we get for SQM:

$$\varphi_{3,4}^{S}(\vec{p}',\vec{q}') = \bar{S}(\vec{p}_3,\vec{q}_4;\vec{p}_1,\vec{q}_2)\prod_{i=1}^{3}\varphi_{3,4}^{i}(\delta p_i, \delta q_i) \tag{210}$$

and for TQM we have the product of the energy part with this:

$$\varphi_{3,4}(p',q') = \varphi_{3,4}^{\mu=0}(\delta p_0, \delta q_0)\,\varphi_{3,4}^{S}(\vec{p}',\vec{q}') \tag{211}$$

For our purposes the SQM part is the carrier; the energy GTF is the signal.

Distribution in three space for both SQM and TQM The distribution in the three space dimensions is the same for both SQM and TQM. We pick a specific scattering angle θ.

Suppose we have in the p_θ direction the final dispersion $\hat{\sigma}_\theta$:

$$\hat{\sigma}_\theta^2 = \hat{\sigma}_\theta^{(1)2} + \hat{\sigma}_\theta^{(2)2} \tag{212}$$

then we have σ_θ, the space dispersion in the θ direction, as:

$$\sigma_\theta^2 = \frac{1}{\hat{\sigma}_\theta^2} \tag{213}$$

Then from the previous section the effective dispersion in clock time is:

$$\sigma_\tau^{(S)2} = \frac{1}{\bar{p}^2 \sigma_\theta^2} \bar{\tau}^2 = \frac{\hat{\sigma}_\theta^{(1)2} + \hat{\sigma}_\theta^{(2)2}}{\bar{p}^2} \bar{\tau}^2 \tag{214}$$

Distribution in coordinate time/energy for TQM TQM we have a distribution in energy as well:

$$\rho\left(E\right) \sim \exp\left(-\frac{\left(\delta E_3 + \delta E_4\right)^2}{\left(\sigma_E^{(1)2} + \sigma_E^{(2)2}\right)} \right) \tag{215}$$

This is centered on the average in energy. Therefore if δE for one of the outgoing legs is positive, the mostly likely value of δE for the other is its negative. But what does δE mean in operational terms? The most direct way to determine this is to run the outgoing particles through a detector that responds to energy. But to be consistent with the treatment so far, we will convert this into time-of-arrival measurements.

If $\delta E > 0$ then the particle is traveling "hot" and will arrive a bit early at the detector. For a fixed angle θ, if we have the expected time-of-arrival as $\bar{\tau}$, then we will measure a δt which is "early", so negative. The other particle is most likely to be traveling exactly that amount "cold" and therefore to arrive (to first order) exactly $-\delta t$ "late", so positive.

We don't have an absolute distribution for either particle, what we have is a correlated distribution: if, say, the C particle exactly on time, then the A particle will have a distribution in time centered on $\bar{\tau}$, with dispersion:

$$\rho\left(t\right) = \exp\left(-\frac{\left(\delta t\right)^2}{\sigma_t^2} \right), \sigma_t^2 = \frac{1}{\sigma_E^2}, \sigma_E^2 = \sigma_E^{(1)2} + \sigma_E^{(2)2} \tag{216}$$

If the C particle is a bit off-center, the A particle's own distribution will shift in the opposite direction. So we have the distribution in coordinate time as we leave the interaction zone.

Diffraction with respect to coordinate time in TQM There is one more piece to the puzzle: we have the dispersion in coordinate time as we leave the interaction zone. But this is not the dispersion in coordinate time at the detector, some temporal distance in the particle's future. Referring to the appendix on GTFs , we have the probability density at the detector as:

$$\rho_\tau^T\left(t\right) = \sqrt{\frac{1}{\pi \sigma_t^2 \left(1 + \frac{\bar{\tau}^2}{\bar{E}^2 \sigma_t^4}\right)}} \exp\left(-\frac{\left(t - \bar{\tau}\right)^2}{\sigma_t^2 \left(1 + \frac{\bar{\tau}^2}{\bar{E}^2 \sigma_t^4}\right)} \right) \tag{217}$$

with uncertainty in time as:

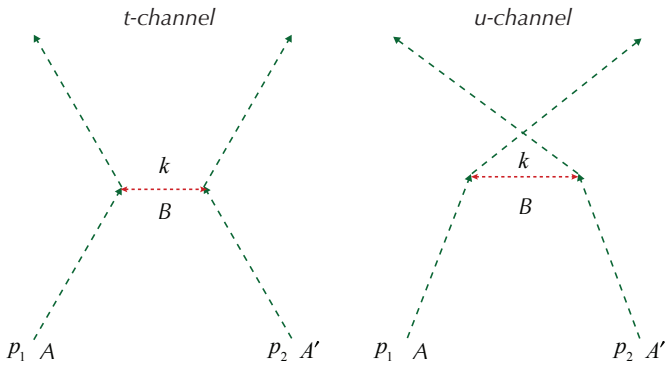

Figure 5. Two A particles exchange a B particle

$$(\Delta t)^2 \equiv \left\langle t^2 \right\rangle - \bar{\tau}^2 = \frac{\sigma_t^2}{2}\left|1 + \frac{\bar{\tau}^2}{\bar{E}^2\sigma_t^4}\right| \to \frac{\bar{\tau}^2}{2\bar{E}^2\sigma_t^2} \tag{218}$$

so both the TQM part and the SQM part scale as $\bar{\tau}$, as one would expect intuitively. But the TQM contribution to the uncertainty in time at the detector is inversely proportional to the initial dispersion in coordinate time. The smaller we can make the post-interaction σ_t, the greater the dispersion in time at the detector. How to make σ_t small?

Single slit in time Suppose we think of the particle C as a gate in time, a single slit in time, with A being the particle passing through the slit. We need C narrow in time, so we make its dispersion in energy large, enough to dominate the sum in (216). We can do this by setting its original dispersion in three momentum large and relying on the entropic estimate to set the dispersion in energy large as well. Now the dispersion in energy in A (post-interaction) is large and its dispersion in coordinate time is therefore small: A will be strongly localized in time. We will have the desired small σ_t.

By judicious manipulation of the uncertainty from SQM – reducing it by increasing the velocity in space and by reducing the dispersion in momentum – while at the same time using the shortest possible wave function for C, we should be able to use C as a time gate, and get a workable ratio of:

$$\frac{\Delta_t^{(T)2}}{\Delta_t^{(T)2} + \Delta_\tau^{(S)2}} \tag{219}$$

enough to make the effects of TQM visible if it is correct. Or *prove* it is false, if it is not correct.

3.2.4. Scattering of indistinguishable particles If we replace the C particle with a second A particle we have two indistinguishable particles to deal with. We therefore cannot tell whether the particle we detect at location 3 is the one that started at location 1 or the one that started at location 2. The diagram on the left is again the t-channel as above. The crossed diagram on the right is associated with the u-channel:

$$u = (p_1 - q_4)^2 = (p_2 - q_3)^2 \tag{220}$$

The sum of the t and u channel diagrams gives the total amplitude. The core matrix element will go as:

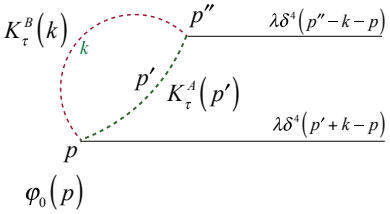

Figure 6. Simple mass correction loop

$$\lambda^2\left(\frac{1}{(\vec{p}_3-\vec{p}_1)^2+\mu^2+\imath\varepsilon}+\frac{1}{(\vec{p}_3-\vec{p}_2)^2+\mu^2+\imath\varepsilon}\right) \tag{221}$$

Since we are dealing with bosons, the relative sign is positive and the two amplitudes add. We see we have symmetry under the interchange of $1 \leftrightarrow 2$ as expected.

By our fundamental hypothesis this rule must apply in TQM for time and the three space coordinates as a whole. To take advantage of this, consider a starting wave function which is symmetric under the interchange of both time and space, but anti-symmetric under each separately. This is not that hard to arrange: we showed in paper A that if the left side has, say, large dispersions in time and space, the right side small – like one of those comedy duos with a tall wide partner and a short thin one – then the total wave function will be the sum of a part which is symmetric in both time and space and a part which is anti-symmetric in time and space separately but symmetric in the combination.

The anti-symmetric part will then give rise to a probability distribution which has a part symmetric in space and time separately and a part anti-symmetric in space and in time, but still symmetric in the combination. For instance it might be symmetric going from $+y, +t \to -y, -t$, but anti-symmetric going $+y \to -y$. Identical yet opposite considerations apply to fermions.

The result is we will see small apparent violations of space (anti-)symmetry in TQM where it is mandated in SQM. As we suspect that this effect is of the second or third order of practicality, we do not pursue it further here. But it is suggestive of the large number of experimental possibilities that TQM generates.

3.2.5. Primary effects Judging by this simple example, the first order of corrections from TQM comes less from the propagators and normalization factors; more from the simple presence of time dispersion in the initial wave functions. There are three kinds of effects we might look for:

Increased dispersion in time/energy These are expected typically small, of scale attoseconds. However they are expected present in all cases.

Diffraction in time In a scattering experiment, we can use one particle, narrowly focused in time, to act as a kind of "single slit in time" experiment with respect to another. This provides a direct test of the HUP in time/energy, and appears to be the most promising of the candidate experiments.

Entanglement in time/energy Anti-symmetry in time is an example of this. These in some ways the most interesting, but may be a bit subtle for use in a first attack on the problem.

3.3. Simple loop diagram

3.3.1. Ultraviolet divergences In the founding days of QED the problem of the loop divergences was one of the most troubling. If you look at essentially any loop diagram in QED, the associated integral diverges. For instance, the loop diagram for an electron to emit and then absorb a photon is divergent.

We will work, as above, with our simple ABC model which has the same problem and for the same reason. Consider the amplitude for an A particle to emit a B particle and then absorb it. We assume a starting momentum for the A particle of p and integrate over all possible values of the B particle's four momentum k:

$$L = \int d\omega d\vec{k} \frac{\imath}{\omega^2 - \vec{k}^2 - \mu^2 + \imath\varepsilon} \frac{\imath}{(E-\omega)^2 - \left(\vec{p} - \vec{k}\right)^2 - m^2 + \imath\varepsilon} \tag{222}$$

This integral does not converge. If we focus on the high momentum part of the integral, the integrand goes as $1/k$. If we impose a high momentum cutoff Λ, the integral diverges logarithmically:

$$\int^{\Lambda} \frac{dk}{k} \sim \ln(\Lambda) \tag{223}$$

This was very troubling to the founders of QED. This particular form of the loop shows up as simple corrections to the mass of a particle, so would be expected small. And these infinities show up in all the basic loop diagrams.

Renormalization Now suppose we were to compare the value of this loop diagram taken at a specific value of p to the value of the same loop diagram taken at a slightly different starting value $p' = p + \delta p$:

$$L' = \int d\omega d\vec{k} \frac{\imath}{k^2 - \mu^2 + \imath\varepsilon} \frac{\imath}{(p + \delta p - k)^2 - m^2 + \imath\varepsilon} \tag{224}$$

The difference between the two loop diagrams goes as:

$$L' - L = \frac{\partial L}{\partial p} \delta p \tag{225}$$

The derivative picks up an extra factor of the A propagator in the denominator:

$$\frac{\partial L}{\partial p} \to -\frac{2\imath(p-k)}{\left((p-k)^2 - m^2 + \imath\varepsilon\right)^2} \tag{226}$$

causing the integrand to acquire an extra factor of $1/k$:

$$\frac{1}{k^4} \to \frac{1}{k^5} \tag{227}$$

and thereby making the integral converge. So that while the original loop is infinite, the differences between nearby loops are finite. Since we do not actually measure any mass directly, but only by comparison – every scale needs a weight on the left side and on the right – we are not strictly required to compute the absolute value of any loop but only the difference between it and another. The calculations are tricky (see for example [33, 38, 45, 47]) but give not only finite results but extraordinarily good agreement with experiment.

The comparison program – referred to as renormalization – has been worked out to a high degree of sophistication. It can be done to all orders, for not only QED but for the entire Standard Model, and in a way that is covariant. So we have a procedure that makes no sense (why are these basic loop diagrams infinite?) yet agrees perfectly and to many places with experiment.

This problem would appear to be even more severe in the case of TQM. TQM has one more dimension to integrate over, so we would expect its divergences to be linear, perhaps even beyond the reach of a renormalization program. This would not keep us from using TQM as a way to develop a program to develop interesting experiments involving time and quantum mechanics, but it would make clear that TQM could not be an entirely satisfactory theory in its own right.

3.3.2. Simple loop calculation We are not going to show that TQM is fully renormalizable. But we will show that simple loop diagrams in TQM – we will focus on the A, B mass correction diagram – are in fact finite and do not require regularization in the first place, provided they are done in a way that is consistent with the spirit of TQM. It is still true that we never measure a mass or any other quantity in isolation but only in comparison with other measurements – but the associated integrals are not only finite but small.

The key is to take advantage of two points:

(i) We have to start with a physically realistic wave function, one which is normalizable itself. Our GTFs will work well for this.

(ii) In computing each step in a loop diagram we have to take advantage of the entanglement in time of that step with the previous. This lets each step take advantage of the finite character of the previous steps to make sure that it stays finite as well. This entanglement in time is not available in SQM, since it is a quantum feature not available to any theory that treats time classically.

We will do the calculation in two stages:

(i) Calculate the value of the loop for a fixed clock time.

(ii) Take the Fourier transform of the fixed clock time result, to get the actual mass correction.

The attosecond form of the propagator is not suitable for this. This is one of the places where that approximation breaks down. But the full, "unpacked" propagator works. We compute the loop from past to future, so need only the positive time branches of the propagators for the A and B particles. Since we are only interested in the questions "is the loop integral finite?" and "does the loop integral make sense?" we ignore the squared coupling constant and an overall factor of $\imath$. We are after the loop, the whole loop, and nothing but the loop.

We start as usual with a GTF. We again use the narrow beam approximation, assuming that for all four momentum components $\frac{\sigma}{|p|} \ll 1$.The propagator for the A particle is:

$$K_\tau^A(p) = \frac{1}{2E}\exp(-\imath\varpi_p\tau), \varpi_p = -\frac{E^2 - \vec{p}^2 - m^2}{2E} \tag{228}$$

The propagator for the B particle is:

$$K_\tau^B(k) = \frac{1}{2w}\exp(-\imath\omega_k\tau), \varpi_k = -\frac{w^2 - \vec{k}^2 - \mu^2}{2w} \tag{229}$$

For fixed clock time the loop integral is:

$$L_\tau(p) = \int d^4k \hat{K}_\tau^A(p-k)\,\hat{K}_\tau^B(k)\,\hat{\varphi}_0(p-p_0) \tag{230}$$

We are using the sample GTF (B.11) from the GTF appendix.

In the narrow beam approximation we may estimate the normalization factor using:

$$E \to E_0 \equiv \sqrt{m^2 + \vec{p}_0^2}, w \to \mu \tag{231}$$

Since these factors are in the denominator, this approximation will make the integral more divergent rather than less. Since they are now constant, we can pull them out of the integral. For the rest of the analysis we will work with a re-scaled loop integral $L \to 4E_0\mu L$.

We will take the same step with respect to the factors of clock time in the exponentials:

$$\varpi_k^B \approx -\frac{w^2 - \vec{k}^2 - \mu^2}{2\mu}, \varpi_p^A \approx -\frac{E^2 - \vec{p}^2 - m^2}{2E_0}, \tag{232}$$

This will tend to make the arguments of the exponentials oscillate less, again making the integrals more divergent rather than less.

The loop integral is a pair of convolutions in momentum space. If we shift to coordinate space, these convolutions will turn into the coordinate space equivalents. The GTF is:

$$\varphi_0(x) = \sqrt[4]{\frac{1}{\pi^4 \det(\Sigma)}} \exp\left(-\imath p_0 x - \frac{(x - x_0)^2}{2\Sigma}\right), \Sigma = \hat{\Sigma}^{-1} \tag{233}$$

The coordinate space forms of the kernels are (compare to Appendix B.3.2):

$$\begin{array}{l} K_\tau^A(x_1; x) = -\imath\frac{E_0^2}{4\pi^2\tau^2} \exp\left(-\frac{\imath E_0}{2\tau}(x_1 - x)^2 + \imath\frac{m^2}{2E_0}\tau\right) \\ K_\tau^B(x_1; x) = -\imath\frac{\mu^2}{4\pi^2\tau^2} \exp\left(-\frac{\imath\mu}{2\tau}(x_1 - x)^2 + \imath\frac{\mu}{2}\tau\right) \end{array} \tag{234}$$

The product equals a single coordinate space kernel:

$$K_\tau^{(M)}(x_1; x) = -\imath\frac{M^2}{4\pi^2\tau^2} \exp\left(-\imath\frac{M}{2\tau}(x_1 - x)^2 + \imath\frac{M}{2}\tau\right) \tag{235}$$

with a modified mass $M \equiv E_0 + \mu$, a prefactor $-\imath\frac{1}{4\pi^2}\frac{E_0^2\mu^2}{\tau^2 M^2}$, and a post-factor $\exp\left(-\imath\frac{M}{2}\tau + \imath\frac{m^2}{2E_0}\tau + \imath\frac{\mu}{2}\tau\right)$. In the limit as $\mu \to 0$ we have $M \to E_0$.

So the loop term in coordinate space is (without prefactor and post-factor):

$$L'_\tau(x_1) = \int d^4x K_\tau^{(M)}(x_1; x)\, \varphi_0(x) \tag{236}$$

This is just the integral to advance a wave function of mass M a distance τ in time, so we have:

$$L'_\tau(x_1) = \varphi_\tau^{(M)}(x_1) \tag{237}$$

We have a correction that shows a spread in time, but at the slightly slower rate associated with the slightly larger mass M. The full correction is greater at shorter clock times, as one would expect. Now we transform back to momentum space:

$$\begin{array}{l} L'_\tau(p_1) = \int d^4p \hat{K}_\tau^{(M)}(p_1; p)\, \hat{\varphi}_0(p) \\ \hat{K}_\tau^{(M)}(p_1; p) = \exp\left(\imath\frac{p^2 - M^2}{2M}\tau\right)\delta^4(p_1 - p) \end{array} \tag{238}$$

So the loop correction for fixed clock time (folding back in the prefactor and post-factor):

$$L_\tau(p_1) = -\imath\frac{1}{4\pi^2}\frac{m^2\mu^2}{M^2\tau^2} \exp\left(\imath\frac{p_1^2}{2M}\tau + \imath\frac{m^2}{2E_0}\tau + \imath\frac{\mu}{2}\tau\right)\hat{\varphi}_0(p_1) \tag{239}$$

At this point the value of the loop correction at a particular value of p is independent of the specific shape of the incoming wave function. We are therefore free to drop the initial wave function from the analysis. We rewrite the loop without the initial wave function, but fold back in the trailing factor:

$$L_\tau(p) = -\imath\frac{1}{4\pi^2}\frac{m^2\mu^2}{M^2\tau^2}\exp\left(-\imath\varpi_p^M\tau\right), \varpi_p^M \equiv -\frac{p^2}{2(E_0+\mu)} - \frac{m^2}{2E_0} - \frac{\mu}{2} \tag{240}$$

The $\varpi_p^M \to \varpi_p$ as $\mu \to 0$. The initial GTF acted in a way parallel to the regularization factors often employed in SQM, but it comes organically out of the calculation, it is not supplied by hand.

3.3.3. Fourier transform The actual mass correction is given by the Fourier transform of the loop integral. Now that we have the loop integral for a specific value of the clock time we can take the Fourier transform with respect to τ. We have the value of the Fourier transform of the core element:

$$\mathcal{FT}\left[\frac{\exp\left(-\imath\varpi_p^M\tau\right)}{\tau^2}\right] = -\sqrt{\frac{\pi}{2}}\left|\omega - \varpi_p^M\right| \tag{241}$$

and therefore of the loop:

$$\hat{L}_\omega(p) = \imath\frac{1}{4\pi^2}\frac{m^2\mu^2}{M^2}\sqrt{\frac{\pi}{2}}\left|\omega - \varpi_p^M\right| \tag{242}$$

The loop correction is zero if $\omega = \varpi$. However in TQM this is a set of measure zero: the 4D wave functions will form a cloud around the average, so the correction will be proportional to the uncertainty in the coordinate energy:

$$\left\langle\left|\omega - \varpi_p^M\right|\right\rangle \sim \left\langle\frac{(\delta E)^2}{2(E_0+\mu)}\right\rangle \approx \frac{\sigma_E^2}{4(E_0+\mu)} \tag{243}$$

So although the initial wave function dropped out of the loop calculation, its influence, like the smile of the Cheshire cat, is still felt.

3.3.4. Implications This is a simple result for a toy model. But it was achieved without artifice. We did not have to add regularizing or convergence factors by hand to the integrals; the initial wave function acted as the convergence factor. And the results make sense: the correction is small, and still smaller when the mass of the B is smaller.

This is perhaps not that surprising. In classical electrodynamics comparable calculations, as of the self-energy of the electron, suffered from linear divergences. The transition from classical electrodynamics to QED softened these divergences from linear to logarithmic.

TQM represents a further step from the classical to the quantum mechanical approaches. We are applying quantization procedures used in space and applying them in time as well. We see a further "softening" of the integrals, to the point where they are finite.

Therefore we have addressed our motivating question: is TQM ruled out as a theory in its own right by not being renormalizable?

And we have provided additional evidence to suggest that TQM is worth exploring experimentally.

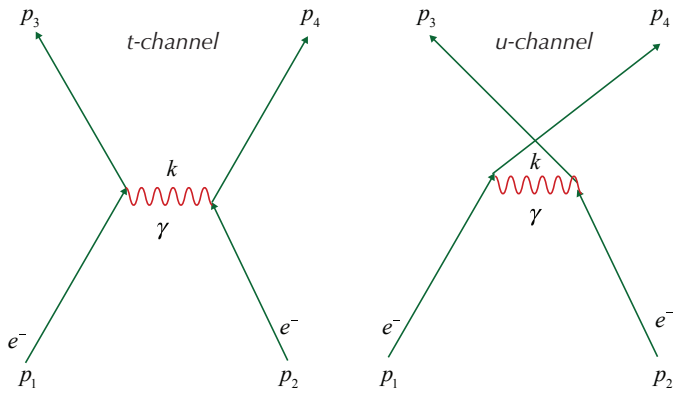

Figure 7. Møller scattering

3.4. Second order-of-magnitude tree diagrams
We discuss here the second order tree diagrams. There are three:

(i) Møller scattering: two electrons exchange a photon.
(ii) Bhabha scattering: an electron and a positron either exchange a photon or else annihilate and then recreate themselves.
(iii) Compton scattering: a photon and an electron scatter.

We see photons and fermions as composed of a spin zero component times a polarization vector or times a spinor. To compute the associated S matrices, we have to look at the input wave functions, the external normalization factors, the polarization vectors, and the spinors. We are primarily looking for additional dispersion in time-of-flight measurements.

If we use the narrow beam approximation, we will be able to ignore the effects of the changes to the external normalization factors, spinors, and polarization vector to lowest order. The SQM part of the diagram will function, as above, largely as a carrier. The principle effects of TQM will be:

(i) The general presence of dispersion in time, leading to additional uncertainty at a detector.
(ii) Diffraction in time, as in the HUP in time/energy.
(iii) Entanglement in time, as in the effects of anti-symmetry in the time part of the wave function.

3.4.1. Møller scattering Møller scattering is the term for electron-electron scattering via the exchange of an electron. From our point of view Møller scattering is a relatively minor generalization of the spin zero scattering problem in subsection 3.2.4. The differences between the SQM and TQM versions of this are in the external factors, the photon propagator, and the spinors at the vertex.

We use Mandelstam variables t, u to characterize the interaction:

$$\begin{aligned} t &= (p_1 - p_3)^2 = (p_2 - p_4)^2 \\ u &= (p_1 - p_4)^2 = (p_2 - p_3)^2 \end{aligned} \tag{244}$$

The heart of the S matrix is the photon propagator. Here the mapping from SQM to SQM is simple, if we use the attosecond propagator:

$$\imath D_\omega^{(S)\mu\nu}\left(\vec{k}\right) = \frac{-\imath g^{\mu\nu}}{\omega^2 - \vec{k}^2 + \imath\varepsilon} \to \imath D_\omega^{(A)\mu\nu}(k) = \frac{-\imath g^{\mu\nu}}{w^2 - \vec{k}^2} \tag{245}$$

We are mapping $\omega \to w$ (coordinate frequency to clock frequency). In SQM, $\omega \to 0$, so the propagator is just:

$$\imath D_{\omega}^{(S)\mu\nu}\left(\vec{k}\right) = \frac{\imath g^{\mu\nu}}{\vec{k}^2} \tag{246}$$

The same analysis as in the spin zero case gives same result for the TQM propagator, so it too is:

$$\imath D_{\omega}^{(A)\mu\nu}\left(k\right) = \frac{\imath g^{\mu\nu}}{\vec{k}^2} \tag{247}$$

And therefore the values of the t and u variables are the same for both SQM and TQM:

$$\frac{1}{t} \approx \frac{1}{t^S}, \frac{1}{u} \approx \frac{1}{u^S} \tag{248}$$

where the SQM context merely means we replace coordinate energy w by its average ω, which is zero.

In the external factors we have to map $E_{\vec{p}} \to E$:

$$\sqrt{\frac{m}{E_{\vec{p}}}} \to \sqrt{\frac{m}{E}} \tag{249}$$

This is where the narrow beam approximation is useful. We are assuming that the variation in energy and the other components of the four momentum are no more than, say, 1% of the average for each component. Therefore if we are looking for order-of-magnitude changes only, we can use the SQM external factor for TQM.

The vertex contribution is more complex. To go from the ABC model to QED in SQM we have:

$$\imath\lambda ABA, \imath\lambda CBC \to -\imath e\bar{\psi}A^{\nu}\gamma_{\nu}\psi \tag{250}$$

To then go from SQM to TQM we replace $E_{\vec{p}} \to E$ in the spinors:

$$-\imath e\bar{\psi}^S\left(\vec{p}'\right)A^{\nu}\left(\vec{k}\right)\gamma_{\nu}\psi^S\left(\vec{p}\right) \to -\imath e\bar{\psi}\left(p'\right)A^{\nu}\left(k\right)\gamma_{\nu}\psi\left(p\right) \tag{251}$$

The ψ's are sums over the u, v's. The u, v's depend on E via factors of $\sqrt{\frac{E+m}{2m}}, \frac{1}{E+m}$. We can expand these in power series of δE:

$$\begin{aligned}
\sqrt{\frac{E+m}{2m}} &= \sqrt{\frac{E_{\vec{p}}+m}{2m}} + \frac{\delta E}{2m\sqrt{E_{\vec{p}}+m}} - \frac{\left(\delta E\right)^2}{16m\sqrt{E_{\vec{p}}+m}^3} + \dots \\
\frac{1}{E+m} &\approx \frac{1}{E_{\vec{p}}+m} - \delta E\frac{1}{\left(E_{\vec{p}}+m\right)^2} + \left(\delta E\right)^2\frac{1}{\left(E_{\vec{p}}+m\right)^3} - \dots
\end{aligned} \tag{252}$$

In the narrow beam approximation we drop the first order and higher terms in δE so that $u, v \approx u^S, v^S$. The implication is that the matrix elements in TQM are the same as in SQM:

$$S \approx S^S \tag{253}$$

All the usual trace-tricks will work the same way: use the SQM procedures replacing $E_{\vec{p}} \to E$ throughout, then drop the quantum energy component $\delta E \equiv E - E_{\vec{p}}$ leaving us right back where we started with the SQM case.

This implies that the first order effects are, as with the spin zero case, a function of the GTFs, rather than the propagator, spinors, or the normalizations.

If we may write the incoming wave functions as the direct product of a spinor part and a GTF, then we have for any SQM calculation the same formula for the TQM signal as with the spin zero case. We have to lowest order reduced the Møller calculation to the ABC one.

Samurai versus pirate Now that we are dealing with charged particles we can employ some additional techniques to help prove TQM effects do not exist. Consider a charged particle going through a magnetic field. It will have a radius of curvature given by:

$$r = \frac{mv}{qB} = \frac{p_x}{eB} \tag{254}$$

so the faster it is going, the greater the radius of curvature. So the larger p_x, the less the path of the particle is bent. This provides us a way to address the "slow train" problem. We can start with the FS/T equation for a single particle, using the minimal substitution to include the magnetic and electric fields. We ignore spin:

$$\imath E \frac{\partial \psi_\tau}{\partial \tau} = -\frac{\left((p_\mu - qA_\mu)\left(p^\mu - qA^\mu\right) - m^2 \right)}{2} \psi_\tau \tag{255}$$

We can use this equation to calculate the motion of a TQM wave packet as function of x, y, t. We can use the Klein-Gordon equation with the minimal substitution to calculate the motion of the particle in the SQM case.

Suppose the particle is going in the x direction and the magnetic field is in the z direction. The force from the magnetic field will be in the y direction. The slower particles will have a smaller radius of curvature, so will be pushed further in the y direction. They will also arrive later, so if the particles are being bent to the right and time is being tracked going up, the main trace will go to the right and up. It will look a bit like a sword trace.

Since the magnetic field has no effect on the time part, the dependence on p_x and on E are separate. If SQM is true, then the dispersion in time at each y position will be small. If TQM is true, then the dispersion in time at each y position will be greater, depending on the specifics significantly greater. For SQM the sword trace will be narrow, looking like the thin precise scar left by a skilled samurai's katana. But if TQM is true, then the sword trace will be broader, looking more like the undisciplined scar left by a pirate's cutlass.

3.4.2. Bhahba scattering In Bhabha scattering we look at an electron scattering from a positron. They can either do this by exchanging a photon, with results very like those for the Møller case, or they can interact by annihilating with emission of a photon, which then decays into an electron-positron pair.

There are two slight differences from the Møller case for our purposes. The exchanges are described by the s and t channels:

$$\begin{aligned} s &= (p_1 + p_2)^2 = (p_3 + p_4)^2 \\ t &= (p_1 - p_3)^2 = (p_2 - p_4)^2 \end{aligned} \tag{256}$$

This is the first point at which the Machian hypothesis has come up as a possible issue. Recall this gave us a well-defined value for the magnitude and sign of the clock frequency. The symmetry between particle and anti-particle means that the magnitude of the clock frequency can't plausibly change. The only reasonable possibility is that the sign changes for anti-matter. However for a normal wave packet the clock frequency will be averaged over values of the coordinate energy centered on the on-shell case. Therefore the sign of the clock frequency will be averaged away.

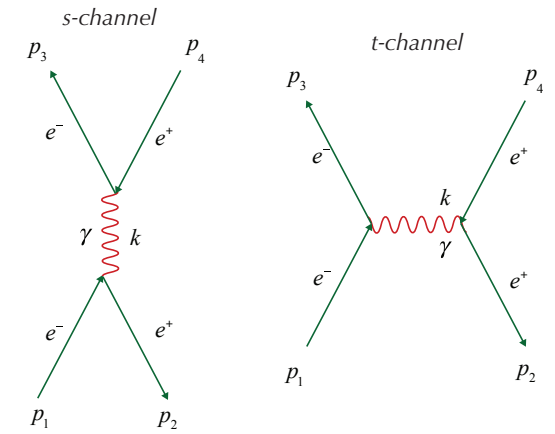

Figure 8. Bhabha scattering

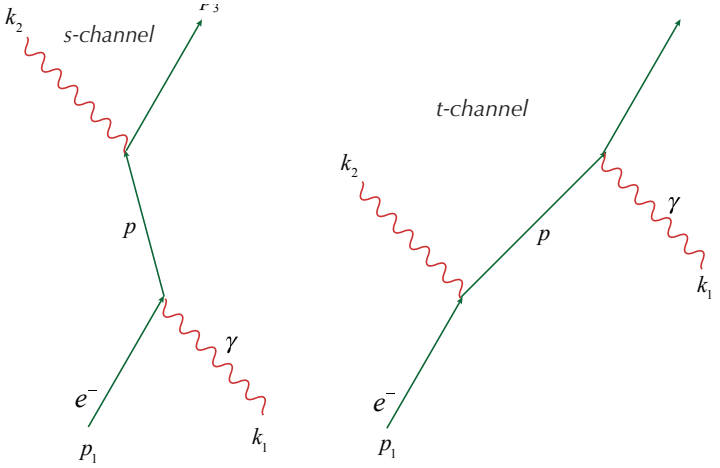

Figure 9. Compton scattering

The other point is that for the first time, the photon propagator will be carrying a non-zero energy component, in the pair annihilation / pair creation branch (s-channel):

$$\imath D_{\omega}^{(S)\mu\nu}\left(\vec{k}\right) = \frac{-\imath g^{\mu\nu}}{\omega^2 - \vec{k}^2 + \imath\varepsilon} \rightarrow \imath D_{\omega}^{(A)\mu\nu}\left(k\right) = \frac{-\imath g^{\mu\nu}}{\left(\omega + \delta w\right)^2 - \vec{k}^2} \approx \frac{-\imath g^{\mu\nu}}{\omega^2 - \vec{k}^2} \tag{257}$$

Here again we invoke the narrow beam approximation to argue that the dependence on δw in the denominator of the propagator will be averaged out in first order and small in second.

And we are left with again the position that all significant dependence on TQM will be in the GTF part of the wave function, there will be no first order dependence on the spinor parts.

The results for the GTFs are essentially the same as with the ABC case: the two final dispersions in energy will be the averages of the initial dispersions in energy. If one of the two incoming particles has a much greater dispersion in energy, it will dominate the result. This particle will then act as a narrow gate in time with respect to the other, with results already discussed.

3.4.3. Compton scattering The exchanges are described by the s and t channels:

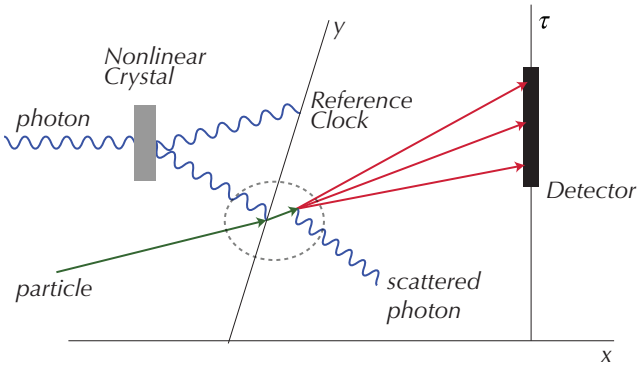

Figure 10. Test of Heisenberg uncertainty principle in time

$$s = (p_1 + k_1)^2 = (p_3 + k_2)^2$$
$$t = (p_1 - k_2)^2 = (k_1 - p_3)^2 \tag{258}$$

This is our first chance to see the fermion propagator in action:

$$\imath S_\omega^S(\vec{p}) = \imath \frac{\left(\not{p}^S + m\right)}{E^{(S)2} - \vec{p}^2 - m^2} \rightarrow \imath S_\omega^{\mathrm{A}}(p) = \imath \frac{(\not{p} + m)}{E^2 - \vec{p}^2 - m^2} \tag{259}$$

We see that the situation is not much different than with the two previous cases. The difference between $\not{p}^S$ and $\not{p}$ drops out in the narrow beam approximation, as does the difference between E^S and E in the denominator. We are left with SQM as carrier and the TQM GTF in time/energy the signal.

The main point of interest here is that as our technology for creating short photon pulses is now extremely sophisticated [70, 71], the changes of using a short pulse of light as a "narrow gate in time" should be good. We discuss this next.

4. Experimental Tests

> "In so far as a scientific statement speaks about reality, it must be falsifiable: and in so far as it is not falsifiable, it does not speak about reality." – Karl Popper [72]

Our goal in this investigation has not been to argue that TQM is a correct extension of QED but rather that it is falsifiable, to give Gisin and his peers "something they can prove wrong".

There is no question that due to the small size of expected effects the associated experiments will be difficult. But at the same time there is a compensating variety of experiments: any time dependent system monitored by time sensitive detectors should show small but definite effects of dispersion in time.

Here we look at one possible experiment, define a figure of merit for such experiments, and then review experimental possibilities in general.

4.1. Heisenberg uncertainty principle in time/energy

> "Ah, but a man's reach should exceed his grasp, Or what's a heaven for?" – Robert Browning

We sketch out a simple candidate solution for using the HUP in time/energy to falsify TQM.

(i) Suppose we start with a photon with a narrow width in time (as in Lindner's classic experiment [73]).
(ii) We send this through a non-linear crystal to split the photon into two equal-but-opposite photons, as in tests of Bell's theorem.
(iii) We send one of these to a reference clock to provide a start point for a time-of-flight measurement.
(iv) We use the other as a narrow gate in time, per above.
(v) We arrange for this photon to scatter an electron via Coulomb scattering.
(vi) We measure the electron's time-of-arrival at a detector.
(vii) We compute the time-of-flight as the difference between the time-of-arrival and the reference time.
(viii) We do this enough times to build up a time-of-flight distribution which will either conform to the predictions of TQM or falsify those.

Dr. Klag was kind enough to point out that this will not, in fact, work. The photons that are split by non-linear crystals in Bell's theorem tests are much longer in time than those Lindner used. As we probably need ultra-short photons, this is likely to be a problem.

Still this does show the essential elements of a realistic experiment:

(i) We need a time-of-flight, so need in general both a start time and a time of arrival.
(ii) We need something that can act as a narrow gate in time.
(iii) We are likely to need many many data points.

We next propose a "figure of merit" to pick among candidate solutions.

4.2. Figure of merit

> "by recording single electron detection events diffracting through a double-slit, a diffraction pattern was built up from individual events." – Bach et al [74]

The primary effect of dispersion in time will be to increase the uncertainty in time from what would otherwise be predicted by standard QED. The fundamental metric is equation 166, repeated here for convenience:

$$(\Delta t)_D^{(T)2} \equiv (\Delta t)_D^2 - (\Delta t)_D^{(S)2} \tag{260}$$

The term on the left is the signal $(\Delta t)_D^{(T)2}$: the uncertainty squared predicted by TQM $(\Delta t)_D^2$ minus the uncertainty squared predicted by SQM $(\Delta t)_D^{(S)2}$.

If we are looking at time-of-flight as the prediction, then the distribution in time-of-flight will build up one event at a time. If we assume, say, Gaussian predictions for both TQM and SQM we will expect in general a wider, flatter Gaussian for TQM.

Assume we pick a degree of confidence, say the traditional five sigmas (giving a one in 3.5 million chance that the distribution was assigned incorrectly). There are many different statistical tests for making this sort of discrimination between two Gaussian distributions; we will assume one appropriate to the specific situation has been chosen.

The chosen degree of confidence will in turn imply a minimal sample size N to achieve it. Now suppose our apparatus can run T tests per second. The number of seconds to achieve the targeted level of confidence is then:

$$S = \frac{N}{T} \tag{261}$$

Our proposed figure of merit is the log to the base ten of the number of seconds required to achieve a five sigma level of confidence that TQM is falsified:

$$M \equiv \log_{10}(S) \tag{262}$$

If we need 100,000 tests and can run one test per second, then our figure of merit is:

$$\log_{10}(100000) = 5 \tag{263}$$

We choose a log scale because we expect there will be considerable variation in the efficiency of various experimental arrangements. In this way, arrangements that generate a stronger signal can be fairly compared to those that generate more tests per second and so on. The smaller S is the better of course. If S is longer than the mean-time-between-failure (MTBF) of the apparatus, then the specific experiment is not practical. And of course if it is longer than the duration of the associated grant, that too will be a problem.

One advantage of having a reasonably well-defined figure of merit is that this makes it easier for an AI system e.g. [75] to compare experimental possibilities. AI systems have the specific advantage in this case that they know nothing about time and are therefore less likely to be distracted by preconceptions.

4.3. TQM as an experiment factory

> "Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality" – Hermann Minkowski [76]

With respect to the falsifiability of TQM, the small size of the basic effect may be compensated for by the large number of experimental possibilities. If quantum mechanics should in fact be extended in the time direction then essentially any time dependent apparatus with time dependent detectors may provide a possible line of attack. By hypothesis, *all* quantum mechanical phenomena seen in space – interference, diffraction, uncertainty, entanglement, tunneling, ... – apply in time as they do in space.

Effects of TQM on the legs We have focused here on the applications within QED. This is needed to treat interactions correctly. However there are many interesting effects at the single particle level.

These may described using the FS/T equation for a spinless particle (52):

$$\left((E - q\Phi)^2 - \left(\vec{p} - q\vec{A}\right)^2 - m^2\right)\psi_\tau = -2E\imath\frac{\partial\psi_\tau}{\partial\tau} \tag{264}$$

or the TQM Dirac equation (126):

$$\left(\not{p} - q\not{A} - m\right)\psi_\tau = -\imath\gamma_0\frac{\partial}{\partial\tau}\psi_\tau \tag{265}$$

We include the vector potential via the minimal substitution. We can also make use of the TQM equation for the free vector potential (89):

$$\left(w^2 - \vec{k}^2\right)A^\nu = -2w\imath\frac{\partial}{\partial\tau}A^\nu \tag{266}$$

All three equations are defined with reference to the rest frame of the vacuum $\mathcal{V}$, so are invariant. As noted, these can often be usefully simplified by rewriting the energy in terms of the quantum energy: $E = E^S + \delta E$ where E^S is the value of the energy in SQM, and then assuming that δE is small (as in the "narrow beam" approximation).

Reuse of existing SQM results We will often start with an existing solution in SQM, perhaps a hard-earned part analytic/part numeric approximation specific to a complex mashup of lab-built and off-the-shelf tech. In many cases we will be able to reuse these pre-existing solutions.

Assume we can use such a pre-existing solution to compute a time-of-arrival distribution. The uncertainty in time-of-arrival is defined as:

$$\Delta_{\tau}^{(S)2} \equiv \int_{-\infty}^{\infty} d\tau \Big(\tau - \langle\tau\rangle^{S}\Big)^{2} \rho_{\tau}^{S}, \langle\tau\rangle^{S} \equiv \int_{-\infty}^{\infty} d\tau\tau\rho_{\tau}^{S} \tag{267}$$

We can also use the existing SQM solution to compute a first order estimate of the corresponding GTF in coordinate time/energy by using the entropic estimate. This will usually be much less sophisticated than our existing SQM solution. But for falsification we are interested primarily in order-of-magnitude numbers. We therefore use the estimated GTF in time to infer the corresponding probability distribution in time $\rho_{\tau}(t)$.

The total uncertainty in coordinate time will be given by:

$$\Delta_{t}^{2} \equiv \int_{-\infty}^{\infty} dt\rho(t)(t - \langle t\rangle)^{2}, \langle t\rangle \equiv \int_{-\infty}^{\infty} dt\rho(t)t, \rho(t) \equiv \int_{-\infty}^{\infty} d\tau\rho_{\tau}(t) \tag{268}$$

The difference between these two metrics is our signal. By this method existing SQM results for time-of-flight and the like may be converted into tests of TQM.

Use of combinations of approaches Further as suggested in the Samurai and Pirate experiment 3.4.1, a combination of approaches may be useful. Post-filtering of the results of an interaction to make the effects of dispersion in time more obvious; pre-filtering to get a cleaner result are possible approaches.

4.3.1. Three classes of effects The primary effects of TQM may be categorized as the common, the dramatic, and the subtle.

Common By the fundamental hypothesis the effects of dispersion in time are omnipresent. In a detector they will show as additional uncertainty in time-of-flight measurements. Within a quantum system, they will act as forces of "anticipation and regret": causing interactions to start sooner and last longer than otherwise would be the case.

But by the initial estimate they are expected small, of order attoseconds or less. There are several ways to address this problem:

(i) Statistical approaches are implicit in the figure of merit.

(ii) Scattering through a crystal is another approach; all of the atoms of the crystal can act in combination to achieve an effect. Therefore we can try sending a beam through a time crystal [77, 78, 79, 80, 81, 82, 83, 84, 85]; look for diffraction effects specifically from TQM.

Dramatic Diffraction effects, especially those associated with the Heisenberg uncertainty principle in time/energy, would appear to present the most dramatic possibilities. In SQM a single slit in time *clips* the wave function in time: the narrower the gate, the less the dispersion in time-of-flight. But in TQM a single slit in time *diffracts* the wave function in time: the narrower the gate, the greater the dispersion in time-of-flight. The effects go in opposite directions, so the contrast may, in principle, be set *arbitrarily* great.

Subtle While it is natural to propose tests using SQM wave functions as carrier, the TQM part as signal; by the fundamental hypothesis a wave function is always to be understood as fully entangled in time and space. Several lines of attack arise out of this including:

(i) Effects of duality:
  (a) The single particle Lagrangian we started with (equation 7) is symmetric under the interchange of $t, \Phi \leftrightarrow x, A_x$. So the effects of sending a charged particle though a time-varying electric field are dual to sending a charged particle through a space-varying magnetic field. For instance we can start with the Aharonov-Bohm effect with respect to magnetic fields [86] and ask if the Aharonov-Bohm effect with respect to the electric field [87] has possibilities.
  (b) In general, Maxwell's equations are symmetric under an interchange of electric and magnetic fields, of the time-space and space-space components of the Maxwell stress tensor $F_{\mu\nu}$. We can look for effects associated with this.
  (c) And of course we can look at any experiment in space and ask if we interchange $t \leftrightarrow x$, do any interesting possibilities present themselves?

(ii) Effects of anti-symmetry in time:
  (a) For bosons: create a wave function which is anti-symmetric in both time and space parts, then look for anti-symmetry under reflection solely in the space part.
  (b) For fermions: create a wave function which is anti-symmetric in the time part but symmetric in the space part, then look for symmetry under reflection solely in the space part.
  (c) And in both cases we can look directly for anti-symmetry in time.

(iii) We can look at EPR effects in time, Bell's theorem tests in time, Greenberger–Horne–Zeilinger experiments, and so on.

(iv) We can look for the effects of tunneling in time, which may have interesting practical applications in steganography (the art of concealing the true message within an apparently innocent one).

4.3.2. Variations on existing fundamental tests of quantum mechanics Another, bottom up, line of attack is to take existing lists of foundational tests of quantum mechanics, to see if there is an "in time" variation for specific tests. Possible starting points are Lamoreaux [88], Ghose [89], and the three hundred or so experiments detailed in Auletta [90]. Examples are the "single slit in time" (as in the text), the "double slit in time" (as Lindner op cit), and so on.

And of course any experiment that explicitly mentions time, i.e. the Delayed Choice Quantum Eraser [91, 92] is a possible starting point.

4.3.3. What if TQM is confirmed but with qualifications? This in our own opinion is the most likely result. We are pushing classical mechanics in a quantum direction, historically a profitable line of attack. And we are making aggressive use of established principles, also often effective.

But we are extrapolating from a 3D theory to a 4D one, from shadow to substance. Any such extrapolation has ambiguities, whether done by a CAT scan or a philosopher.

The figure of merit provides a convenient way to categorize deviations from the simple extrapolation proposed by TQM. The points of maximal deviation provide a natural guide for followup experiments.

4.3.4. What if TQM is falsified? On the other hand, if TQM is falsified, then the falsification itself should suggest further experimental possibilities. The most obvious falsification would be that we do not in fact see the dispersion in time/energy that is the central prediction. There are two main possibilities here:

(i) There is a frame in which the violation is maximal. This would be a preferred frame, anathema to relativity, and of great interest to the contrarian experimenter.

(ii) The absence is uniform across frames. The restriction of the paths to on-shell paths is confirmed. This would perhaps not be entirely in the spirit of relativity. But it would at a minimum help clarify the relationship between the HUP in energy/time and the HUP in space/momentum. For instance it would show how to transform between these two in one frame and in another. As the precise relationship between these two has been the subject of a considerable literature (e.g.[93, 94, 95, 96, 54, 97]) this would be interesting as well.

4.3.5. There are no null experimental results Therefore there are no null experiments. Either we will have a variety of novel phenomena to explore or our understanding of the role of time/energy in QED will be deepened.

5. Discussion

"Anything that is not compulsory is forbidden." – Murray Gell-Mann [98]

The main problem here has been to extend QED to include time while keeping it consistent with all that has gone before. The approach has been to use the path integral formulation but keep everything but the paths themselves the same. We then extended the paths in a way that is manifestly covariant.

While there may be alternative ways to the same end, the requirement of manifest covariance should cause the results to be consistent to the first order of magnitude. Since the relevant scales, of attoseconds, are now accessible by experiment the hypothesis that the wave function should be extended in time is therefore falsifiable.

Evidence for confirmation Obviously no one writes a paper as long as this to define a hypothesis that has no chance of being confirmed. We argue that the odds of it being confirmed are perhaps even better than that given various advantages:

(i) The treatment of the time/energy and space/momentum coordinates is manifestly symmetric. Simply being able to do this is interesting.

(ii) We have a clear explanation of why time normally appears asymmetric at the level of the observer (due to statistical effects at the scale of Avogadro's number) while still at the particle level being completely symmetric.

(iii) We have a treatment which goes smoothly from the single to the multiple particle cases.

(iv) We do not see the ultra-violet divergences. We still have to normalize the loops, but we no longer have to regularize them: that drops out of the formalism.

(v) And we have Gell-Mann's principle: what is not forbidden is compulsory. If there is not a conservation principle or symmetry rule forbidding dispersion in time, it would be surprising *not* to see dispersion in time.

Implications of falsification As noted, if we falsify TQM we would at a minimum get a clearer understanding of the relationship between the HUP in energy/time and the HUP in space/momentum, especially with regard to the way they transform from one frame to another.

Implications of confirmation High speed chemical and biological interactions, i.e. attosecond scale, should show effects of time dispersion. For instance, if molecules can sense into the future, it may affect their ability to find optimal configurations.

There are potential applications for quantum communication and quantum computing. With TQM we have an additional channel to use for calculation/communication but also an additional channel to act as a source of decoherence.

The implications for quantum gravity are particularly interesting: with manifest covariance, elimination of the ultra-violet divergences, some recent work by Horwitz, and earlier work by Verlinde, we appear to have all the pieces needed to construct a complete, covariant, and convergent theory of quantum gravity. Leaving the question of the odds of this being correct to one side, we note that recent advances in technique mean such a theory has a reasonable chance of being falsifiable as well. We explore this in slightly more detail in the Appendix D.3.

Conclusion Any time dependent quantum phenomena viewed at a sufficiently short time scale (attoseconds or less) and with sufficiently time sensitive detectors should either display novel phenomena along the time/energy axis or at a minimum deepen and make more precise our understanding of the role of time in quantum mechanics.

Acknowledgments
I thank my long time friend Jonathan Smith for invaluable encouragement, guidance, and practical assistance.

I thank Ferne Cohen Welch for extraordinary moral and practical support.

I thank Martin Land, L. P. Horwitz, James O'Brien, Tepper Gill, Petr Jizba, Matthew Trump, and the other organizers of the International Association for Relativistic Dynamics (IARD) 2018, 2020, and 2022 Conferences for encouragement, useful discussions, and hosting talks on the papers in this series in the IARD conference series. I also thank my fellow participants in the 2022 conference – especially Pascal Klag, Bruce Mainland, Luca Smaldone, Howard Perko, Philip Mannheim, Alexey Kryukov, Ariel Edery, and others – for many excellent questions and discussions.

I thank Steven Libby for several useful conversations and in particular for insisting on the extension of the original ideas to the high energy limit and therefore to QED.

I thank Larry Sorensen for many helpful references. I thank Ashley Fidler for helpful references to the attosecond physics literature.

I thank Avi Marchewka for an interesting and instructive conversation about various approaches to time-of-arrival measurements.

I thank Hou Yau for an interesting discussion of variations on these themes and for directing my attention to his paper[99].

I thank Asher Yahalom for insisting that a better explanation of the clock time than simply what clocks measure was required

I thank Thomas Cember for useful clarifications of several points in the argument.

I thank the reviewer who drew my attention to Horwitz's work on gravity [100].

I think Danko Georgiev of the journal Quanta for very practical suggestions and advice.

I thank Y. S. Kim for organizing the invaluable Feynman Festivals, for several conversations, and for general encouragement and good advice.

I thank Catherine Asaro, Julian Barbour, Gary Bowson, Howard Brandt, Daniel Brown, Ron Bushyager, John G. Cramer, J. Ferret, Robert Forward, Fred Herz, J. Peřina, Linda Kalb, A. Khrennikov, David Kratz, Andy Love, Walt Mankowski, O. Maroney, John Myers, Paul Nahin, Marilyn Noz, R. Penrose, Stewart Personick, V. Petkov, H. Price, Matt Riesen, Terry Roberts, J. H. Samson, Lee Smolin, L. Skála, Arthur Tansky, R. Tumulka, Joan Vaccaro, L. Vaidman, A. Vourdas, H. Yadsan-Appleby, S. Weinstein, and Anton Zeilinger for helpful conversations over the years.

I thank the organizers of several QUIST, DARPA, Perimeter Institute conferences I've attended and the very much on topic conferences *Quantum Time* in Pittsburgh in 2014 and *Time and Quantum Gravity* in San Diego in 2015.

And none of the above are in any way responsible for any errors of commission or omission in this work.

Appendix A. Notation

We are using natural units throughout.

Since the text usually alternates between SQM and TQM sections, the meaning of an object should often be clear from context. In general SQM objects will have three vectors (i.e. $\vec{p}$) as arguments and TQM objects will have four vectors (i.e. p) as arguments. Where necessary we use a superscript S to mark an SQM object, a superscript T to mark a purely time object, and absence of an explicit mark to indicate a fully relativistic (i.e. TQM object). For example:

$$\begin{aligned} \psi_\tau(t,\vec{x}) &= \psi_\tau^T(t)\,\psi_\tau^S(\vec{x}) \\ \psi_\tau(E,\vec{p}) &= \psi_\tau^T(E)\,\psi_\tau^S(\vec{p}) \end{aligned} \tag{A.1}$$

In paper A we used an over-bar and over-tilde for the same markings, but we had to abandon that usage because it conflicted with the use of an over-bar to mark adjoint spinors in the Dirac equation. We still use the occasional over-bar to indicate average as $\bar{E} \equiv \left\langle \sqrt{m^2+\vec{p}^2} \right\rangle$. We are putting the clock time at the bottom right in the position of a traditional index, in honor of its frequent use as an index in the time-slicing used to compute path integrals. To further streamline the frequent references to clock time we replace τ_1 with just the 1. And if we are looking at differences between two times we just put the two indexes in:

$$K_{\tau_1} \to K_1; K_{\tau_2;\tau_1} \to K_{2;1} \to K_{21} \tag{A.2}$$

The complementary variable for clock time is always ω; the complementary variable for coordinate time may be E, w, k_0, p_0 depending on context. It is usually obvious when a particular object refers to momentum or to coordinate space. When it might not be obvious, we use an over-hat to mark the momentum space form, i.e. $\hat{\phi}(p)$ for a four dimensional plane wave or $\hat{\sigma}_x \equiv \sigma_{p_x}$ for the dispersion in p_x. We use the Greek letter ϖ for the clock energy/clock frequency (energy and frequency are the same in natural units of course):

$$\varpi_k \equiv -\frac{w^2 - \vec{k}^2 - \mu^2}{2w} \tag{A.3}$$

We define $E_{\vec{p}}$ as the relativistic mass $E_{\vec{p}} \equiv \sqrt{m^2+\vec{p}^2}$.

We use a superscript A to tag specific propagators as Attosecond, primarily meant for use at attosecond scales (77). We use ϕ for plane waves, φ for Gaussians, and ψ for general wave functions.

Appendix B. Gaussian Test Functions

> "That's a great deal to make one word mean," Alice said in a thoughtful tone.
> "When I make a word do a lot of work like that," said Humpty Dumpty, "I always pay it extra."
> – Lewis Carroll *Through the Looking Glass* 1871

Appendix B.1. Uses of GTFs
By Gaussian test functions (GTFs) we mean functions of the general form:

$$\varphi_0(x) = \sqrt[4]{\frac{1}{\pi\sigma^2}} e^{\imath p_0 x - \frac{(x-x_0)^2}{2\sigma^2}} \tag{B.1}$$

We generally take them as normalized to one. We refer to the σ as the dispersion. The uncertainty in the associated dimension is given by the dispersion divided by $\sqrt{2}$:

$$\Delta x \equiv \sqrt{\left\langle (x - \langle x \rangle)^2 \right\rangle} = \frac{\sigma}{\sqrt{2}} \tag{B.2}$$

We can get a rough approximation of any normalizable wave function by using the GTF with the same uncertainty:

$$\varphi_0^{(\text{TYPICAL})}(x) \equiv \sqrt[4]{\frac{1}{2\pi(\Delta x)^2}} e^{\imath \langle p_0 \rangle x - \frac{(x-\langle x_0 \rangle)^2}{4(\Delta x)^2}} \tag{B.3}$$

(i) We are making this use of GTFs when we estimate the initial wave functions using the entropic estimate of the uncertainty in subsection 3.1.1. This corresponds loosely to a statistician's use of the mode to summarize a population.

(ii) By using Morlet wavelet analysis, we can represent any normalizable wave function as a sum over Gaussians (see [101]).

(iii) Since the GTFs in momentum space are exact solutions of the various free equations in TQM (and in SQM for that matter) we can use them as starting functions in perturbation expansions.

(iv) A final and perhaps surprising use here is that the use of normalizable functions – whether typical GTFs or sums over GTFs – is critical for ensuring convergence of path integrals in general and loop diagrams in particular.

Appendix B.2. Starting GTFs
Appendix B.2.1. In momentum and space In p_x:

$$\varphi_0(p_x) = \sqrt[4]{\frac{1}{\pi\sigma_{p_x}^2}} e^{-\imath\left(p_x - p_x^0\right)x_0 - \frac{\left(p_x - p_x^0\right)^2}{2\sigma_{p_x}^2}} \tag{B.4}$$

In x:

$$\varphi_0(x) = \sqrt[4]{\frac{1}{\pi\sigma_x^2}} e^{\imath p_0 x - \frac{(x-x_0)^2}{2\sigma_x^2}} \tag{B.5}$$

where $\sigma_{p_x} = \frac{1}{\sigma_x}$. We can simplify slightly by taking $\hat{\sigma}_x \equiv \sigma_{p_x}$. The y, z GTFs are the same, replacing $x \to y, z$ and $p_x \to p_y, p_z$. In momentum space:

$$\varphi_0^S(\vec{p}) = \sqrt[4]{\frac{1}{\pi^3 \det\left(\hat{\Sigma}^S\right)}} e^{-\imath(\vec{p}-\vec{p}_0)\cdot\vec{x}_0 - (\vec{p}-\vec{p}_0)\cdot\frac{1}{2\hat{\Sigma}^S}\cdot(\vec{p}-\vec{p}_0)}, \hat{\Sigma}^S \equiv \begin{pmatrix} \hat{\sigma}_x^2 & 0 & 0 \\ 0 & \hat{\sigma}_y^2 & 0 \\ 0 & 0 & \hat{\sigma}_z^2 \end{pmatrix} \tag{B.6}$$

and in coordinate space:

$$\varphi_0^S(\vec{x}) = \sqrt[4]{\frac{1}{\pi^3 \det(\Sigma^S)}} e^{\imath \vec{p}_0 \cdot \vec{x} - (\vec{x}-\vec{x}_0)\cdot \frac{1}{2\Sigma^S}\cdot(\vec{x}-\vec{x}_0)}, \Sigma^S = \begin{pmatrix} \sigma_x^2 & 0 & 0 \\ 0 & \sigma_y^2 & 0 \\ 0 & 0 & \sigma_z^2 \end{pmatrix} = \frac{1}{\hat{\Sigma}^S} \tag{B.7}$$

Appendix B.2.2. In energy and time We get the time and energy forms taking $p_x \to E, x \to t$ and complex conjugating. If we take the wave function in energy as:

$$\varphi_0(E) \equiv \sqrt[4]{\frac{1}{\pi\sigma_E^2}} e^{\imath(E-E_0)t_0 - \frac{(E-E_0)^2}{2\sigma_E^2}} \tag{B.8}$$

we have the wave function in time as:

$$\varphi_0(t) \equiv \sqrt[4]{\frac{1}{\pi\sigma_t^2}} e^{-\imath E_0 t - \frac{(t-t_0)^2}{2\sigma_t^2}} \tag{B.9}$$

where $\sigma_E = \frac{1}{\sigma_t}$.

Appendix B.2.3. In time/energy and space/momentum We can get four dimensional wave functions by taking the direct product of the wave functions in time/energy and space/momentum. In four momentum space:

$$\varphi_0(E, \vec{p}) = \varphi_0^T(E)\, \varphi_0^S(\vec{p}) \tag{B.10}$$

spelled out:

$$\varphi_0(p) = \sqrt[4]{\frac{1}{\pi^4 \det\left(\hat{\Sigma}\right)}} e^{\imath(p-p_0)_\mu x_0^\mu - (p-p_0)^\mu \frac{1}{2\hat{\Sigma}^{\mu\nu}}(p-p_0)^\nu}, \hat{\Sigma}^S \equiv \begin{pmatrix} \hat{\sigma}_t & 0 & 0 & 0 \\ 0 & \hat{\sigma}_x^2 & 0 & 0 \\ 0 & 0 & \hat{\sigma}_x^2 & 0 \\ 0 & 0 & 0 & \hat{\sigma}_z^2 \end{pmatrix} \tag{B.11}$$

In coordinate space:

$$\varphi_0(t, \vec{x}) = \varphi_0^T(t)\, \varphi_0^S(\vec{x}) \tag{B.12}$$

spelled out:

$$\varphi_0(x) = \sqrt[4]{\frac{1}{\pi^4 \det(\Sigma)}} e^{-\imath p_0^\mu x_\mu - (x-x_0)^\mu \frac{1}{2\Sigma^{\mu\nu}}(x-x_0)^\nu}, \Sigma = \begin{pmatrix} \sigma_t & 0 & 0 & 0 \\ 0 & \sigma_x^2 & 0 & 0 \\ 0 & 0 & \sigma_x^2 & 0 \\ 0 & 0 & 0 & \sigma_z^2 \end{pmatrix} = \frac{1}{\hat{\Sigma}} \tag{B.13}$$

We are treating time and space as disentangled. We can entangle by generalizing the dispersion matrix Σ to be an arbitrary positive definition matrix.

Appendix B.3. GTFs as a function of clock time
We look at the evolution of the GTFs as a function of clock time.

Appendix B.3.1. Momentum space

Non-relativistic and SQM GTFs The behavior of the momentum space GTFs is simple. In the non-relativistic case:

$$\varphi_{\tau}^{NR}\left(\vec{p}\right)=\exp\left(-\imath\frac{\vec{p}^{2}}{2m}\tau\right)\varphi_{0}^{S}\left(\vec{p}\right) \tag{B.14}$$

The GTFs in SQM are similar:

$$\varphi_{\tau}^{S}\left(\vec{p}\right)=e^{-\imath\sqrt{m^{2}+\vec{p}^{2}}\tau}\varphi_{0}^{S}\left(\vec{p}\right) \tag{B.15}$$

If the dispersion in three space momenta is not too great we may write:

$$\sqrt{m^{2}+\vec{p}^{2}}\approx\bar{E}_{0}+\frac{\left(\vec{p}-\vec{p}_{0}\right)^{2}}{2E_{0}},\bar{E}_{0}\equiv\sqrt{m^{2}+\vec{p}_{0}^{2}},\vec{p}_{0}\equiv\left\langle\vec{p}\right\rangle \tag{B.16}$$

Note we are taking the average relativistic mass $\bar{E}_0$ as the reference point, not the bare mass m. This means that the utility of the approximation can survive to much greater velocities; all that is required is that the dispersion of the momentum be small relative to the average momentum. As $\bar{E}_0 \to m$ we get the non-relativistic form, modulo a constant and therefore uninteresting overall factor of $\exp\left(-\imath\bar{E}_0\tau\right)$. Note also that the diagonal form of the clock time dependence is a result of using direct product GTFs in the three space dimensions.

For TQM the behavior is, of course, a bit more complex. We have:

$$\varphi_{\tau}\left(p\right)=\exp\left(\imath\frac{E^{2}-\vec{p}^{2}-m^{2}}{2E}\tau\right)\varphi_{0}\left(p\right)=\exp\left(\imath\frac{E^{2}-\vec{p}^{2}-m^{2}}{2E}\tau\right)\varphi_{0}^{T}\left(E\right)\varphi_{0}^{S}\left(\vec{p}\right) \tag{B.17}$$

So even though the energy and the three momentum parts start disentangled, they become entangled as a result of the $\frac{\vec{p}^2}{2E}$ term. For now, we will deal with this by again assuming that the dispersions in energy/momentum are not that great, so that it makes sense to write $\delta E = E - \bar{E}_0$ and therefore:

$$-\frac{E^{2}-\vec{p}^{2}-m^{2}}{2E}=\varpi_{p}=-\frac{\left(\bar{E}_{0}+\delta E\right)^{2}-\left(\vec{p}_{0}+\delta\vec{p}\right)^{2}-m^{2}}{2\left(\bar{E}_{0}+\delta E\right)}\approx-\delta E+\frac{\left(\delta E\right)^{2}-2\vec{p}\cdot\delta\vec{p}-\left(\delta\vec{p}\right)^{2}}{2\bar{E}_{0}} \tag{B.18}$$

We can then divide the TQM GTF into the energy part:

$$\varphi_{\tau}\left(E\right)=\sqrt[4]{\frac{1}{\pi\sigma_{E}^{2}}}\exp\left(\imath\delta E\left(\tau+t_{0}\right)-\imath\frac{\left(\delta E\right)^{2}}{2\bar{E}_{0}}\tau-\frac{\left(\delta E\right)^{2}}{2\sigma_{E}^{2}}\right) \tag{B.19}$$

and momentum space part:

$$\varphi_{\tau}^{TQM}\left(\vec{p}\right)=\exp\left(\imath\frac{2\vec{p}\cdot\delta\vec{p}+\left(\delta\vec{p}\right)^{2}}{2\bar{E}_{0}}\tau\right)\varphi_{0}^{S}\left(\vec{p}\right) \tag{B.20}$$

The space part in TQM does not evolve in quite the same way as the space part in SQM; it seems better in practice to compare SQM as a whole with TQM as a whole.

Appendix B.3.2. Coordinate space forms

Non-relativistic GTFs We start with the non-relativistic form:

$$\varphi_\tau^{NR}(\vec{x}) = \varphi_\tau^{NR}(x)\,\varphi_\tau^{NR}(y)\,\varphi_\tau^{NR}(z) \tag{B.21}$$

We look at the x direction [102, 8]:

$$\varphi_\tau^{NR}(x) = \sqrt[4]{\frac{1}{\pi\sigma_x^2}}\sqrt{\frac{1}{f_\tau^{(x)}}}e^{\imath p_x^{(0)}x-\frac{1}{2\sigma_x^2 f_\tau^{(x)}}(x-x_\tau)^2-\imath\frac{p_x^{(0)2}}{2m}\tau},\, f_\tau^{(x)} = 1-\imath\frac{\tau}{m\sigma_x^2} \tag{B.22}$$

with average position in x:

$$x_\tau = x_0 + v_x^{(0)}\tau, v_x^{(0)} = \frac{p_x^{(0)}}{m} \tag{B.23}$$

We have y, x the same. The corresponding probability density is:

$$\rho_\tau^{NR}(\vec{x}) = \rho_\tau^{NR}(x)\,\rho_\tau^{NR}(y)\,\rho_\tau^{NR}(z) \tag{B.24}$$

And again focusing on the x direction:

$$\rho_\tau^{NR}(x) = \sqrt{\frac{1}{\pi\sigma_x^2\left(1+\frac{\tau^2}{m^2\sigma_x^4}\right)}}\exp\left(-\frac{(x-x_\tau)^2}{\sigma_x^2\left(1+\frac{\tau^2}{m^2\sigma_x^4}\right)}\right) \tag{B.25}$$

Notice the kink in behavior at the point where $\frac{\tau}{m\sigma_x^2}\approx 1$. At this turning point, the uncertainty goes from being proportional to σ to being proportional to τ/σ.

$$(\Delta x)^2 \equiv \langle x_\tau^2\rangle - \langle x_\tau\rangle^2 = \frac{\sigma_x^2}{2}\left|1+\frac{\tau^2}{m^2\sigma_x^4}\right| \tag{B.26}$$

So if the dispersion starts small it will end large. This may be understood in terms of the HUP in space/momentum. If the initial uncertainty in space is small, the uncertainty in momentum goes as $1/\sigma_x$ so is correspondingly large. Given a bit of time (clock time here) the large dispersion in momentum causes the wave function to spread out in space, creating correspondingly large dispersion in space. We may think of this as diffraction at work. The behavior for the SQM GTFs is essentially the same.

TQM GTFs The time part is in close parallel to the x part:

$$\varphi_\tau^{T}(t) = \sqrt[4]{\frac{1}{\pi\sigma_t^2 f_\tau^t}}e^{\imath E_0 t_0-\frac{(t-t_\tau)^2}{2\sigma_t^2 f_\tau^t}},\, f_\tau^t \equiv 1+\frac{\imath\tau}{E_0\sigma_t^2} \tag{B.27}$$

with:

$$t_\tau = t_0 + v_t\tau, v_t = \gamma = \frac{E_0}{m} \tag{B.28}$$

Probability density:

$$\rho_\tau^{T}(t) = \sqrt{\frac{1}{\pi\sigma_t^2\left(1+\frac{\tau^2}{E_0^2\sigma_t^4}\right)}}\exp\left(-\frac{(t-t_\tau)^2}{\sigma_t^2\left(1+\frac{\tau^2}{E_0^2\sigma_t^4}\right)}\right) \tag{B.29}$$

Again note the shift in uncertainty in time at the point where $\frac{\tau}{E_0\sigma_t^2}\approx 1$.

$$(\Delta t)^2 \equiv \left\langle t^2 \right\rangle - t_\tau^2 = \frac{\sigma_t^2}{2} \left| 1 + \frac{\tau^2}{E_0^2 \sigma_t^4} \right| \tag{B.30}$$

For TQM, the HUP in time/energy is fully equivalent to the HUP for space/momentum.

TQM Kernels In the analysis of the mass loop correction we will work directly with the kernels in the narrow beam approximation. We ignore the normalization factor of $\frac{1}{2\bar{E}_0}$ here. We developed this in some detail in paper A. In momentum space:

$$\hat{K}_\tau \left(p; p' \right) = \delta^{(4)} \left(p - p' \right) \exp \left(-\imath \frac{E^2 - \vec{p}^2 - m^2}{2\bar{E}_0} \tau \right) \tag{B.31}$$

and in coordinate space:

$$K_\tau \left(x; x' \right) = -\imath \frac{\bar{E}_0^2}{4\pi^2 \tau^2} e^{-\imath \bar{E}_0 \frac{\left(t - t' \right)^2}{2\tau} + \imath \bar{E}_0 \frac{\left(\vec{x} - \vec{x}' \right)^2}{2\tau} - \imath \frac{m}{2} \tau} \tag{B.32}$$

Appendix C. Feynman rules

In the text we develop the Feynman rules for a few simple cases; here we show we can extend the Feynman rules to all orders. This is not a given. The problem is that in most treatments of the path integral, the Hamiltonian part acts like a kind of locomotive pulling the sum over paths forward. With TQM there is no natural equivalent to the Hamiltonian, since there is no dependence of the Lagrangian on clock time and therefore no canonical momenta and therefore no non-trivial Hamiltonian.

To see this in the simplest case, consider the Lagrangian for a free spin zero massive particle above (equation 23). The obvious choice for the corresponding TQM Lagrangian is:

$$\mathcal{L} \left[\phi \right] = \frac{1}{2} \partial_t \phi \partial_t \phi - \frac{1}{2} \nabla \phi \nabla \phi - \frac{m^2}{2} \phi^2 \tag{C.1}$$

In fact our requirement of complete covariance does not give us any real alternatives. The derivatives with respect to time must be with respect to coordinate time if they are to form part of a four vector with the ∇ operator. We can write the action as either the integral of d^4x or the integral of $d\tau d^4x$ over this. But we still have no dependence on clock time in the Lagrangian. Therefore this Lagrangian goes not give us an equation of motion. It has no kinetic energy term; it is all potential. Therefore while we can usually develop path integrals with the Hamiltonian or the Lagrangian approaches, with TQM we *must* use the Lagrangian formulation.

Other major differences are that:

(i) Most 3D objects are promoted to 4D.

(ii) The treatment of clock time has a different character; it still orders the Dyson series and the like, but most of the dependence on time is via the dependence on coordinate time.

(iii) We are primarily interested in interactions over short times: the effects we are interested in are of order attoseconds. Over picoseconds – or still more glacial intervals – the effects of time dispersion are likely to be averaged out.

Appendix C.1. Dyson series

Appendix C.1.1. Dyson series in SQM We start with the familiar case of the Dyson expansion for the S matrix in SQM, in the interaction picture:

$$S_{fi} = \langle f | \exp\left(-\imath \int_{\tau_i}^{\tau_f} d\tau \int d\vec{x} \mathcal{H}_I \right) | i \rangle \tag{C.2}$$

with interaction Hamiltonian:

$$H_I \equiv \int d\vec{x} \mathcal{H}_I \tag{C.3}$$

We expand the exponential in a power series:

$$S_{fi} = \sum_{n=0}^{\infty} \frac{(-\imath)^n}{n!} \int_{\tau_i}^{\tau_f} d\tau_1 \int_{\tau_i}^{\tau_f} d\tau_2 \ldots \int_{\tau_i}^{\tau_f} d\tau_n H_I(\tau_1) H_I(\tau_2) \ldots H_I(\tau_n) \tag{C.4}$$

and then time order the individual terms, which disposes of the $1/n!$:

$$S_{fi} = \sum_{n=0}^{\infty} (-\imath)^n \int_{\tau_i}^{\tau_f} d\tau_1 \int_{\tau_i}^{\tau_f} d\tau_2 \ldots \int_{\tau_i}^{\tau_f} d\tau_n T\{H_I(\tau_1) H_I(\tau_2) \ldots H_I(\tau_n)\} \tag{C.5}$$

We then use Wick's theorem to replace the difficult-to-work-with time ordered terms with normal ordered terms plus "contractions" – the Feynman propagators. The sum over all topologically distinct ways of doing this is the sum over all Feynman diagrams.

The handling of clock time requires particular attention in this context. For instance at each vertex, the integrals over three space induce in a natural way a δ function in three momentum. The usual δ function over clock energy is produced by the taking the S matrix to run from $\tau = -\infty \to +\infty$. This guarantees conservation of clock energy, treated as the fourth component of a four vector, at each vertex and also for the S matrix overall.

But this conservation law is purchased at the expense of limiting the domain of applicability in a serious way. If we want to apply the theory to short times (as in this investigation) we have to accept that conservation of laboratory energy may be approximate. So we have an unnatural limitation of the domain of applicability of the theory or else have to take our chances with conservation of laboratory energy. In the text we are able to finesse the problem by focusing on the interaction of individual wave packets. They are set to interact for only short periods of time, so we can let the limits of clock time integrals go to $\pm\infty$ without changing the numeric value of the integral. But the problem remains in the general case.

Appendix C.1.2. Dyson series in TQM Wick's theorem is general, applying equally to SQM and TQM. It can be used in TQM in the same way as in SQM, letting the time ordered elements in the Dyson series be expressed as sums over products of normal ordered operators and of propagators. The topology and symmetry of the two series are the same.

However the individual objects go in general from 3D to 4D. The initial and final wave functions go from 3D plane waves to 4D and the propagators change as described in the text. At each vertex we go from an integral over clock time and three space dimensions to an integral over clock time, coordinate time, and the three space dimensions. The coordinate time and the three space dimensions are part of a four vector, the clock time is treated as a separate object (but really the time coordinate of the rest frame of the vacuum, per discussion).

For the most part the effect is as if we had simply added a 4th space dimension to each object in the series, one that comes in with opposite sign with respect to the three space dimensions. With this it is possible to write out each term in the Dyson series and solve for any specified problem to any required order, with one further change. We will assume, as with the SQM case,

that we are working in the interaction picture. Therefore we need only consider the interaction potential; the rest of the dependence on clock time is contained in the basis states.

The TQM Dyson series for the S matrix is then:

$$S_{fi} = \langle f | \exp\left(\imath \int_{\tau_i}^{\tau_f} d\tau \int dt d\vec{x} \mathcal{L}_I \right) | i \rangle \tag{C.6}$$

The interaction Lagrangian is:

$$\mathcal{L}_I = -V_I \tag{C.7}$$

so:

$$\mathcal{H} = -\mathcal{L} = V_I = \mathcal{H}_I \tag{C.8}$$

The two sign flips of the potential cancel out and we are left with the same arguments for the exponential, albeit with four dimensions to integrate over rather than three:

$$\int d\vec{x} \mathcal{H}_I \to \int dt d\vec{x} \mathcal{H}_I \tag{C.9}$$

Appendix C.2. Feynman rules in TQM

Therefore we can write the Feynman rules for TQM down by inspection, with the substitutions noted. The basic topology is unchanged, the symmetry factors are unchanged. The external factors are adjusted per discussion; the external δ functions go from three in three momentum plus clock energy to four in four momentum plus clock energy.

Appendix C.2.1. S matrix The S matrix in SQM is:

$$S_{fi} = \delta_{fi} + (2\pi)^4 \delta\left(\Omega_f - \Omega_i\right) \delta^3\left(\vec{P}_f - \vec{P}_i\right) \left(\prod^{\text{all external bosons}} \sqrt{\frac{1}{2V\omega_{\vec{k}}}} \right) \left(\prod^{\text{all external fermions}} \sqrt{\frac{m}{VE_{\vec{p}}}} \right) \mathcal{M}^{\mathcal{S}} \tag{C.10}$$

while the S matrix in TQM is the formally similar:

$$S_{fi} = \delta_{fi} + (2\pi)^5 \delta\left(\varpi_f - \varpi_i\right) \delta^4\left(P_f - P_i\right) \left(\prod^{\text{all external bosons}} \sqrt{\frac{1}{2TVw}} \right) \left(\prod^{\text{all external fermions}} \sqrt{\frac{m}{TVE}} \right) \mathcal{M} \tag{C.11}$$

In both cases the amplitude $\mathcal{M}$ is a sum over all topologically distinct terms:

$$\mathcal{M} = \sum_{n=0}^{\infty} \mathcal{M}^{(n)} \tag{C.12}$$

In TQM the raw interaction vertex is unchanged:

$$-e\gamma^{\mu} \tag{C.13}$$

The sign rules are the same as in SQM. For our purposes, writing out the usual Feynman diagrams while adding an additional -1 for each flip of identical fermions suffices.

Appendix C.2.2. Incoming and outgoing wave functions The handling of the incoming and outgoing wave functions is different. The incoming wave functions in SQM are generally taken as plane waves. In TQM we have to use GTFs. In SQM both incoming and outgoing legs are taken as on-shell. In TQM the 4D wave functions oscillate around the on-shell values, now taken as averages rather than absolutes, more guidelines than rules.

To go from SQM to TQM we replace the relativistic mass $E_{\vec{p}}$ in each spinor with the coordinate energy E, but leave them otherwise unchanged. So each spinor becomes a function of all four components of the momentum:

(i) Initial electron: $u_s^S(\vec{p}) \to u_s(p)$
(ii) Each final electron: $\bar{u}_s^S(\vec{p}) \to \bar{u}_s(p)$
(iii) Each initial positron: $\bar{v}_s^S(\vec{p}) \to \bar{v}_s(p)$
(iv) Each final positron: $v_s^S(\vec{p}) \to v_s(p)$

The polarization vectors for the photons are unchanged. We change the argument from a three vector to a four vector to indicate we are changing the context.

(i) Each initial photon: $\varepsilon_{r,\mu}\left(\vec{k}\right) \to \varepsilon_{r,\mu}(k)$
(ii) Each final photon: $\varepsilon_{r,\mu}\left(\vec{k}\right) \to \varepsilon_{r,\mu}(k)$

Appendix C.2.3. Propagators The propagators are different, as noted in some detail in the text. The apparently covariant character of the SQM propagators falls apart under close examination: there is dispersion in the x, y, z directions but not in t. As a result in SQM the intermediate particles are virtual; in TQM they are real.

Photon propagator in SQM (88):

$$\imath D_\omega^{(S)\mu\nu}\left(\vec{k}\right) = \frac{-\imath g^{\mu\nu}}{\omega^2 - \vec{k}^2 + \imath\varepsilon} \tag{C.14}$$

Fermion propagator in SQM (122):

$$\imath S_\omega^S(\vec{x}) = \imath \frac{\omega\gamma_0 - \vec{p}\cdot\vec{\gamma} + m}{\omega^2 - \vec{p}^2 - m^2 + \imath\varepsilon} \tag{C.15}$$

The $\imath\epsilon$ is used to pick out the contours so that positive frequencies are associated with forwards in time; negative with backwards in time. In TQM, at least in this initial analysis, we have found it helpful to break out the forward and backward components and deal with them separately.

Photon propagator in TQM (99):

$$\imath D_\omega^{\mu\nu}(k) = \frac{-\imath g^{\mu\nu}}{2w}\left(\frac{\imath}{w^2 - \vec{k}^2 + 2w\omega + 2w\imath\varepsilon} + \frac{\imath}{w^2 - \vec{k}^2 + 2w\omega - 2w\imath\varepsilon}\right) \tag{C.16}$$

with:

$$\varpi_k \equiv -\frac{w^2 - \vec{k}^2}{2w} \tag{C.17}$$

and short time limit:

$$\imath D_\omega^{(A)\mu\nu}(k) = -\imath g^{\mu\nu}\frac{\imath}{w^2 - \vec{k}^2} \tag{C.18}$$

Feynman propagator for fermions (132):

$$\imath S_{\omega}\left(p\right)=\imath\frac{\left(\not{p}+m\right)}{E^{2}-\vec{p}^{2}-m^{2}+2\omega E+2E\imath\varepsilon}+\imath\frac{\left(\not{p}+m\right)}{E^{2}-\vec{p}^{2}-m^{2}+2\omega E-2E\imath\varepsilon} \tag{C.19}$$

with short time limit:

$$\imath S_{\omega}^{(A)}\left(p\right)=\imath\frac{\not{p}+m}{E^{2}-\vec{p}^{2}-m^{2}} \tag{C.20}$$

Appendix C.2.4. Vertexes

SQM SQM vertex (138):

$$-e\bar{\psi}^{S}\left(\vec{p}'\right)A^{(S)\nu}\left(\vec{k}\right)\gamma_{\nu}\psi^{S}\left(\vec{p}\right) \tag{C.21}$$

The vertex is accompanied by δ functions in the three space momenta:

$$\delta^{3}\left(\vec{p}_{out}-\vec{p}_{in}\right) \tag{C.22}$$

and in clock energy if we are taking the limit as $\tau\pm\infty$:

$$\delta\left(\sum\Omega_{out}-\sum\Omega_{in}\right) \tag{C.23}$$

TQM TQM uses the same vertex except for the obvious replacement of three vector by four vector (141):

$$-e\bar{\psi}\left(p'\right)A^{\nu}\left(k\right)\gamma_{\nu}\psi\left(p\right) \tag{C.24}$$

The vertex is accompanied by δ functions in the four momenta:

$$\delta^{4}\left(p_{out}-p_{in}\right) \tag{C.25}$$

and in clock energy if we are taking the limit as $\tau\pm\infty$:

$$\delta\left(\varpi_{out}-\varpi_{in}\right) \tag{C.26}$$

Appendix D. The rest frame of the vacuum

"If you look long enough into the void, the void begins to look back through you."
– Frederick Nietzsche

Appendix D.1. Energy-momentum of spacetime

In the development in the text we have to pick a specific laboratory frame to define the clock time τ; we have therefore dependence on that choice. For laboratories going at non-relativistic velocities, the corrections to clock time will result in a correction to a correction, therefore not of first order, therefor not essential to falsifiability.

Still we would like to define TQM in a completely frame independent way. We noted in paper A that we can make an invariant choice of frame by taking advantage of an observation from Weinberg [57]. Consider the Einstein field equations:

$$G_{\mu\nu}\equiv R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=-8\pi GT_{\mu\nu} \tag{D.1}$$

Rewrite as:

$$(G^{\mu\nu} + 8\pi G T^{\mu\nu})_{;\nu} = 0 \tag{D.2}$$

We may use this to associate an energy momentum tensor with local spacetime. Define:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{D.3}$$

where $h_{\mu\nu}$ vanishes at infinity but is not assumed small. The part of the Ricci tensor linear in h is:

$$R^{(1)}_{\mu\nu} \equiv \frac{1}{2}\left(\frac{\partial^2 h^\lambda_\lambda}{\partial x^\mu \partial x^\nu} - \frac{\partial^2 h^\lambda_\mu}{\partial x^\lambda \partial x^\nu} - \frac{\partial^2 h^\lambda_\nu}{\partial x^\lambda \partial x^\mu} + \frac{\partial^2 h_{\mu\nu}}{\partial x^\lambda \partial x_\lambda}\right) \tag{D.4}$$

The exact Einstein equations may be written as:

$$R^{(1)}_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} R^{(1)\lambda}_\lambda = -8\pi G\left(T_{\mu\nu} + t_{\mu\nu}\right) \tag{D.5}$$

where $t_{\mu\nu}$ is defined as quadratic in h and higher:

$$t_{\mu\nu} \equiv \frac{1}{8\pi G}\left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R^\lambda_\lambda - R^{(1)}_{\mu\nu} + \frac{1}{2}\eta_{\mu\nu} R^{(1)\lambda}_\lambda\right) \tag{D.6}$$

Weinberg then argues we may interpret $t_{\mu\nu}$ as the energy-momentum of the gravitational field itself.

If we can associate an energy momentum with spacetime, we may define a local rest frame with respect to that energy momentum tensor. We will refer to this as $\mathcal{V}$, the rest frame of the vacuum. If we define clock time with respect to this frame, we have an invariant definition of the clock time. Presumably this invariant frame is in free fall. A laboratory in Near Earth Orbit would do. More practically, we can make the associated calculations to correct for our earth bound laboratory being in an accelerated frame.

Appendix D.2. The four dimensional Schrödinger equation in the rest frame of the vacuum
Weinberg was working in terms of spacetime, an essentially classical concept. Now let us replace spacetime with a quantum vacuum. We will assume that it is full of real (rather than virtual) particles in a statistical ensemble.

With this context we return to the 4D Schrödinger equation:

$$\imath \frac{\partial}{\partial \tau}\psi = -\frac{p^2 - m^2}{2m}\psi \tag{D.7}$$

We define the energy momentum operators in the rest frame of the vacuum as:

$$\mathcal{E} \equiv \imath \frac{\partial}{\partial \tau_{\mathcal{V}}}, \overrightarrow{\mathcal{P}} \equiv -\imath \vec{\nabla}_{\mathcal{V}} \tag{D.8}$$

with four momentum:

$$\mathcal{P} = \left(\mathcal{E}, \vec{\mathcal{P}}\right) \tag{D.9}$$

We are using capital script letters for values and operators associated with the vacuum. We form the invariant $\mathcal{Q}^2$ for the invariant difference between the foreground and the vacuum momentum:

$$\mathcal{Q}^2 \equiv (\mathcal{P} - p)^2 \tag{D.10}$$

Suppose that the Klein-Gordon equation should really not be written as an absolute but as relative to the local spacetime. This is clearly reasonable. Then the Klein-Gordon equation becomes, in an invariant form:

$$\mathcal{Q}^2\psi = m^2\psi \tag{D.11}$$

Any realistic quantum vacuum will be some sort of quantum soup of photons and other bosons, electrons and other fermions. But at core the free equations for all of these are the Klein-Gordon equation with a light flavoring of spin/polarization added. We therefore take the vacuum as obeying an averaged Klein-Gordon equation of its own:

$$\left(\mathcal{P}^2 - \mathcal{M}^2\right)|\mathcal{V}\rangle = 0 \tag{D.12}$$

using $\mathcal{P}, \mathcal{M}$ as conveniently vague local averages over the mass-energy of the vacuum. Write the system of vacuum plus particle as a direct product:

$$|\psi\rangle\,|\mathcal{V}\rangle \tag{D.13}$$

We expand the Klein-Gordon equations:

$$\left(\mathcal{P}^2 - 2p\mathcal{P} + p^2 - m^2 - \mathcal{M}^2\right)|\psi\rangle\,|\mathcal{V}\rangle = 0 \tag{D.14}$$

The purely vacuum part cancels by assumption $\left\langle \mathcal{P}^2 - \mathcal{M}^2 \right\rangle \approx 0$. Further we choose to work in the rest frame of the vacuum so that $\mathcal{P} \to \mathcal{E}, \vec{0}$:

$$\langle 2E\mathcal{E}\rangle = \left\langle p^2 - m^2 \right\rangle \tag{D.15}$$

The energy of the vacuum $\mathcal{E}$ is, in coordinate space, given by the time operator of the vacuum $\mathcal{E} \equiv \imath\frac{\partial}{\partial\tau_{\mathcal{V}}}$:

$$\imath\frac{\partial}{\partial\tau_{\mathcal{V}}} = \frac{p^2 - m^2}{2E} \tag{D.16}$$

Since we are looking at the difference between the energy operator of the particle and the vacuum we therefore also need to look at the difference between the energy operator of the laboratory and the vacuum (the laboratory itself is after all nothing but particles). Therefore the clock time or laboratory time is to be understood as the *negative* of the time operator of the vacuum:

$$\frac{\partial}{\partial\tau_{\mathcal{V}}} = -\frac{\partial}{\partial\tau} \tag{D.17}$$

Therefore the correct 4D Schrödinger equation is (if we are in the $\mathcal{V}$ frame):

$$p^2 - m^2 = -2E\imath\frac{\partial}{\partial\tau} \tag{D.18}$$

So the slow drift of the observed system's wave function with respect to the observer is tracked by the cross-term of observed and observer wave functions. The 4D Schrödinger equation reduces to the non-relativistic form (equation 10) using $E \to m$.

Note use of the Machian hypothesis has recovered the FS/T, which we built starting from the single particle path integral approach. It is striking that these two distinct approaches agree.

Appendix D.3. Implications for quantum gravity

In the text proper we treat the Machian hypothesis as a formal hypothesis, useful for extending QED in time in a self-consistent way. However if we are prepared to accept this hypothesis as physical, at least for purpose of argument, there are some interesting implications for quantum gravity.

In the text proper we show in TQM we have:

(i) a fully covariant treatment of time in QED,

(ii) and the elimination of the ultra-violet divergences.

These are two of the principal barriers in the way of getting to quantum gravity.

Further, as noted in the introduction, TQM is a part of the Relativistic Dynamics program so TQM can draw on the extensive Relativistic Dynamics literature. In particular we can take advantage of Horwitz's extension of Relativistic Dynamics to General Relativity [100, 103, 104].

The Horwitz approach does not itself supply a mechanism. However, consider the conventional practice of dropping disconnected diagrams in Feynman diagrams. What if these terms should be seen not as disconnected but as connected to the vacuum?

Consider the mass terms in particular. They typically have a form like:

$$\frac{m^2}{2}\left(a_k a_{-k} + a_k a_k^\dagger + a_k^\dagger a_k + a_k^\dagger a_{-k}^\dagger\right) \tag{D.19}$$

All four of the terms conserve momentum. The leftmost describes pair annihilation; the rightmost pair creation. These are typically thrown away as part of the process of throwing out "disconnected diagrams". What unconscionable waste!

Perhaps a pair annihilation term is really describing two particles descending to the vacuum, to flit round there for a short time, then return as particles spontaneously appearing from the vacuum via the pair creation term. The discarded terms might represent a kind of quantum friction with the vacuum.

This is consistent with Weinberg's approach, providing a mechanism for the exchange of energy/momentum between foreground and vacuum. And consistent with Verlinde's entropic gravity approach [105, 106], which encourages us to treat spacetime as itself a statistical system.

Of course, these provide only "elements" of a theory of quantum gravity. There may be zero, one, or multiple acceptable ways of combining these elements.

To motivate such an effort consider this:

(i) explains the hierarchy problem: "quantum friction" is a qualitatively different mechanism; it is not surprising it would be much weaker than the electromagnetic, weak, or strong interactions.

(ii) treats the mass in foreground and background the same way: the pair creation term in the foreground is also a pair annihilation term in the vacuum, and vice-versa. This suggests an interesting perspective on the equivalence principle.

(iii) resolves the information paradox [107]: as spacetime is "nothing but" the quantum vacuum, information can transfer to it, hide out for a bit, then escape as during the process of black hole evaporation or the like. Total information in foreground plus background is still expected constant, as per the various no-cloning, no-deleting theorems.

(iv) provides a mechanism by which gravity could act as a source of decoherence, as in the Penrose Interpretation [108, 109, 110].

But the primary advantage that such a theory naturally couples interactions at the quantum scale with effects at the scale of the universe. This opens up interesting experimental and observational possibilities.

For instance the foreground (i.e. the particles we observe) should be at a higher energy and less disordered than the vacuum. So we expect a continuous transfer of energy and information from foreground to vacuum over time. We expect this transfer would be a monotonically increasing function of the energy of the foreground, more rapid within a supernova than in the gaps between the stars.

If we take the zero-zero component of the mass-energy tensor of the vacuum as providing a time scale, this transfer could look like an expansion of the vacuum (as it acquires a greater proportion of the total energy). To lowest order this might therefore look like a general expansion of spacetime (continuously increasing dark energy).

Further if the increased energy in the vacuum is identified as dark matter we can predict that the amount of dark matter in a galaxy will be proportional to the time the galaxy has existed (i.e. small for new borns) and to the mass of the galaxy.

Therefore, for essentially no work, we have two qualitative predictions for the evolution of the universe. And we have an interesting line of attack on the problem of quantum gravity. The associated technical problems are obviously non-trivial (e.g. what does the total Lagrangian look like?), but any well-defined and testable hypothesis has considerable value. At a minimum, this should suggest interesting experiments, especially now that we can see quantum effects of gravity (as in Bothwell et al [111]). For candidate quantum gravity experiments see for instance [112, 113, 114, 115, 116].

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