

Emulating Thouless pumping in the interacting Rice–Mele model using superconducting qutrits

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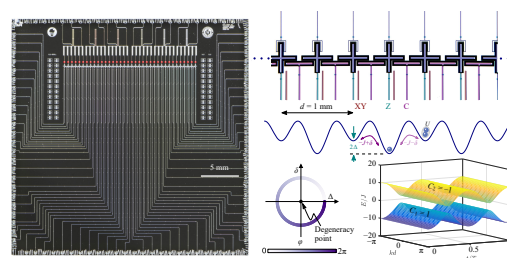
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ABSTRACT

The Rice–Mele model has been a seminal prototypical model for the study of topological phenomena such as Thouless pumping. Here we implement the interacting Rice–Mele model using a superconducting quantum processor comprising a one-dimensional array of 36 qutrits. By adiabatically cycling the qutrit frequencies and hopping strengths in the parametric space, we emulate the Thouless pumping of single and two bounded microwave photons along the qutrit chain. Furthermore, with strong Hubbard interaction inherent in the qutrits we also emulate the intriguing phenomena of resonant tunneling and asymmetric edge-state transport of two interacting photons. Utilizing the interactions and higher energy levels in such fully controlled synthetic quantum simulators, these results demonstrate new opportunities for exploring exotic topological phases and quantum transport phenomena using superconducting quantum circuits.

Keywords quantum simulation, superconducting quantum circuits, superconducting qutrits, Thouless pumping



4] is one of the simplest manifestations of topology in periodically driven quantum systems, which shares the same origin of integer QHE and supports quantized transport of noninteracting particles in one dimension. The topological nature of Thouless pumping makes it robust against modest perturbations such as disorder [5] or interaction [6], and has generated widespread interest for its potential applications, such as current standards [7, 8] and quantum state transfer [9]. While Thouless pumping remains elusive in electron-based condensed matter systems, it has been recently realized in synthetic systems featuring versatility and controllability, including ultracold atoms [10–15], photonic waveguides [16–19], and acoustic waveguides [20, 21], and has also been extended to higher dimensions [22, 23] and momentum space [24]. Recently, Thouless pumping in interacting quantum many-body systems has garnered significant interest [25–33], followed by pioneering experiments with nonlinear photonic waveguides [16, 19] and ultracold atoms with tunable Hubbard interactions [34, 35]. However, the interplay between topology and multi-particle correlations remains hardly explored experimentally, partly because of the challenge in creating and probing site-resolved multi-particle correlations.

In recent years, superconducting quantum circuits have emerged as a promising platform for simulating the phenomena with interaction effects, such as synthetic many-body interactions [36], strongly correlated quantum walks [37, 38], chiral ground-state currents [39], antisymmetric spin exchange [40], and multi-particle bound states [41]. Here we present an experiment of emulating Thouless pumping in the interacting Rice–Mele model using a one-dimensional (1D) array of superconducting qutrits (three-level systems). By adiabatically cycling the qutrit frequencies and hopping strengths, we emulate the characteristic dynamics of Thouless pumping for the single and two bounded microwave photons. Furthermore, with strong Hubbard interaction inherent in the qutrits, we also observe the intriguing phenomena of resonant tunneling and asymmetric edge-state transport of two interacting photons under certain circumstances. These emergent behaviors in the interacting system enrich the physics of the noninteracting counterparts. Utilizing the interactions and higher energy levels in such fully controlled synthetic quantum simulator, these results demonstrate new opportunities for exploring exotic topological phases and quantum transport phenomena using superconducting quantum circuits.

We implement the experiments on a superconducting quantum processor shown in Fig. 1(a), comprising a 1D array of 36 tunably coupled transmon qutrits of the Xmon variety [42] [see Fig. 1(b)]. Each qutrit in the 1D array can be individually addressed and driven into the higher excited state by applying microwave pulse through its dedicated XY drive line. The qutrit frequency and the nearest-neighbour coupling strength can be controlled by applying external flux through the

dedicated Z line or C line of the corresponding qutrit and coupler respectively [43], which correspond to the lattice site potential and nearest-neighbour hopping strength in a lattice model. The physical system in our experiments can be described by the Hamiltonian [44]:

$$H/\hbar = \sum_{j=1}^{N-1} \left(g_{j,j+1} a_j^\dagger a_{j+1} + \text{H.c.} \right) + \sum_{j=1}^N \left(\omega_j a_j^\dagger a_j + \frac{U}{2} a_j^\dagger a_j^\dagger a_j a_j \right), \quad (1)$$

where a_j is the annihilation operator for qutrit on site j , $g_{j,j+1}$ is the nearest-neighbour coupling strength, ω_j is the qutrit angular frequency, $U = -E_C$ is the qutrit anharmonicity, E_C is the charging energy of the qutrit, and H.c. is Hermitian conjugate. Here, we configure the system as a dimerized chain with alternating hopping amplitudes $-J \pm \delta$ and staggered potential energies $\pm\Delta$, as shown in Fig. 1(c), which are realized by tuning the coupling strengths and qutrit frequencies as $g_{j,j+1} = -J + (-1)^j \delta$, $\omega_j = \omega + (-1)^j \Delta$. The physics of this system is captured by the Rice–Mele Hamiltonian [45] with on-site interactions,

$$H_{\text{RM}}/\hbar = \sum_{j=1}^{N-1} \left[[-J + (-1)^j \delta] a_j^\dagger a_{j+1} + \text{H.c.} \right] + \sum_{j=1}^N \left[(-1)^j \Delta n_j + \frac{U}{2} n_j (n_j - 1) \right], \quad (2)$$

where $n_j = a_j^\dagger a_j$ is the photon number operator, $-J \pm \delta$ is the nearest-neighbour hopping strength determined by the tunable couplers, $\pm\Delta$ is the staggered on-site energy determined by the qutrit frequencies, and $U/(2\pi) \approx -190$ MHz is the on-site Hubbard interaction strength determined by the anharmonicity of the qutrits [see Fig. 1(c)]. A pair of neighbouring qutrits constitutes a unit cell with a lattice constant of $d = 1$ mm. When $\Delta = 0$, this Hamiltonian reduces to the Su–Schrieffer–Heeger (SSH) model [46] with on-site interactions. By adiabatically cycling the qutrit frequencies and the coupling strengths as $\Delta = \Delta_0 \cos(\omega t)$ and $\delta = \delta_0 \sin(\omega t)$, we realize the time-varying phase $\varphi = \omega t = 2\pi t/T$, where t is the evolution time and the pump cycle is given as $T = 2\pi/\omega$. The time-evolution can be described as a closed trajectory $\mathcal{C}$ in the Δ – δ parameter plane with the varying phase φ , which allows the emulated photon shift from one sublattice to the other in Thouless pumping, see Fig. 1(d). Because the lattice potential is periodic both in space and time, under periodic boundary condition, one can define the Bloch wavefunction $|\psi_{m,k}(t)\rangle = e^{ikx} |u_{m,k}(t)\rangle$ in the m -th Bloch band with quasimomentum k , and the corresponding topological invariant known as Chern number in a k – t Brillouin zone $C_m = 1/(2\pi) \int_0^T dt \int_{-\pi/d}^{\pi/d} dk \Omega_m(k, t)$, where $\Omega_m(k, t) =$

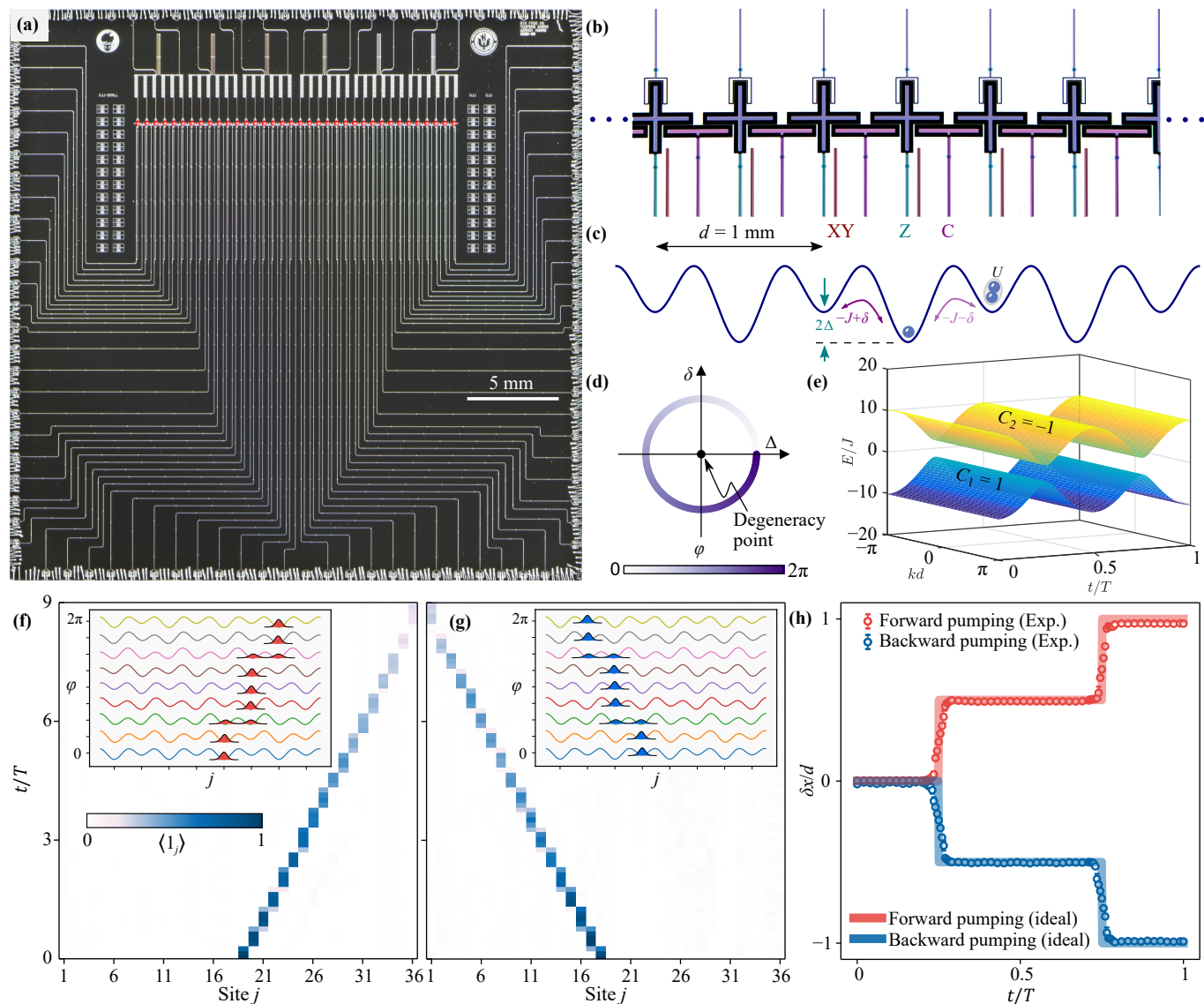


Fig. 1 Emulating Thouless pumping in an array of superconducting qutrits. **(a)** Photograph of the quantum chip (qutrits are marked by red dots). **(b)** Zoomed in micrograph of **(a)**, showing the X-shaped qutrits (light blue) connected with T-shaped tunable couplers (light purple), with their associated control/readout wirings. **(c)** Potential landscape of the interacting Rice-Mele model emulated by the qutrits array. **(d)** Adiabatic cyclic evolution of the Hamiltonian in the Δ - δ parameter space around the degeneracy point at the origin with a varying phase φ . **(e)** Two energy bands in the k - t Brillouin zone with Chern numbers $C_1 = 1$ and $C_2 = -1$ respectively. **(f-g)** Forward (backward) pumping where the system is initialized in the lower (upper) band localized at site $j = 19$ ($j = 18$). Inset: Evolution of the lattice potential and single-particle state during one cycle. **(h)** δx extracted from **(f)** and **(g)**, with $\delta x/d = 0.971(5)$ and $-0.990(3)$ respectively in a pumping cycle, consistent with the Chern number.

$i(\langle \partial_t u_{m,k} | \partial_k u_{m,k} \rangle - \langle \partial_k u_{m,k} | \partial_t u_{m,k} \rangle)$ is the Berry curvature (see the Supplementary Information) and T is the pumping period. Figure 1(e) shows a representative instantaneous spectrum E/J of the Hamiltonian in the k - t Brillouin zone, with a lower band and an upper band. As long as the bandgap never closes, ideally the microwave photons will stay within the same band during the adiabatic pumping process. Since the time-reversal symmetry is broken by the phase sweep, the lower and upper bands have nontrivial Chern numbers

of $C_1 = 1$ and $C_2 = -1$ respectively. The center-of-mass shift of the microwave photons in such topologically nontrivial bands after one pumping cycle is simply given by $\delta x \equiv x(t) - x(0) = C_m d$, where the position operator $x = d/2(\sum j n_j / \sum n_j)$. In Figs. 1(f) and (g), given parameters $\Delta_0/(2\pi) = 80$ MHz, $\delta_0/(2\pi) = J/(2\pi) = 8$ MHz, $T = 0.4$ μ s, we emulate the forward (backward) pumping, where the population $\langle n_j \rangle = |\langle \psi(t) | n_j \rangle|^2$ for $n = 1$ is measured, and the system is initially prepared as $|1_{j=18,19}\rangle$. The initial state localized at site j is prepared

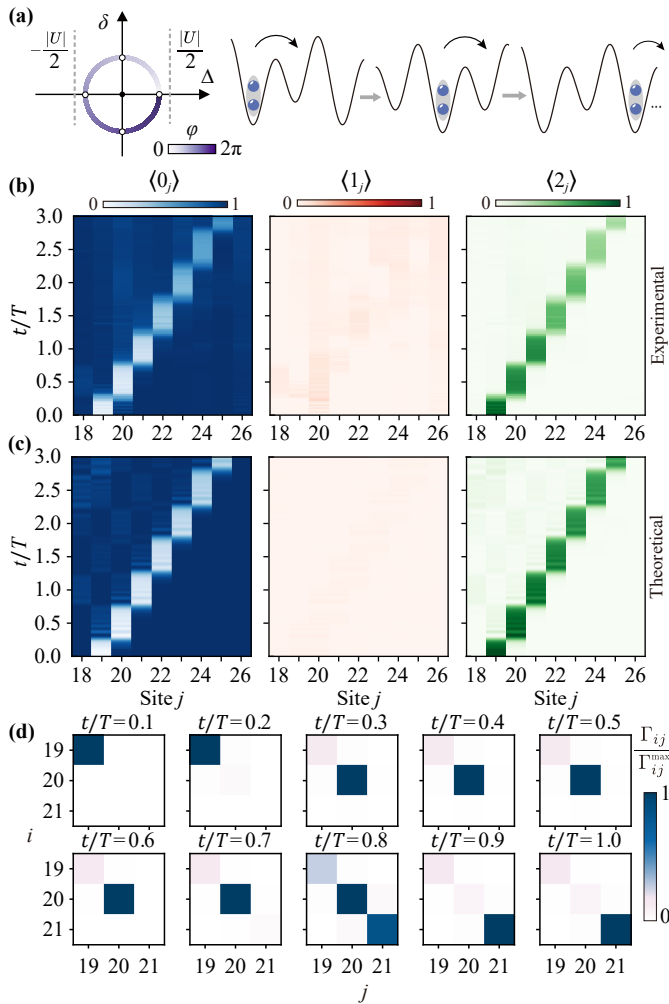


Fig. 2 Emulation of the Thouless pumping for two bounded microwave photons. **(a)** Schematic showing two bounded microwave photons unidirectionally transported through the barriers. **(b, c)** Experimental data (b) and numerical simulation (c) of the populations of the $|n_j\rangle$ states for $n = 0, 1$ and 2 of each site j during the pumping process, where the $|1_j\rangle$ state is unpopulated. **(d)** The normalized two-site correlations $\Gamma_{ij}/\Gamma_{ij}^{\max}$ of site i and j during the pumping at different evolution times $t = 0.1T, 0.2T, \dots, T$.

by applying a π pulse to excite the qutrit from the ground state to the first excited state $|1_j\rangle = a_j^\dagger|0^{\otimes N}\rangle$. Figure 1(h) shows the center-of-mass displacement δx extracted from Figs. 1(f) and (g), yielding $\delta x/d = 0.971(5)$ and $-0.990(3)$ in a pumping cycle respectively, consistent with the Chern numbers of the lower and upper bands. We note that the pumping of microwave photon is only valid as long as the potential is varied adiabatically, as this phenomenon is not generically robust to non-adiabatic effects despite its topological nature [47]. The pumping period of $T = 0.4 \mu\text{s}$ chosen here is a balance between adiabaticity and qutrit coherence (see details in the Supplementary Information). Leakage into the couplers is also a dissipation source during the pumping process (see details in the Supplementary

Information).

When the on-site interaction is involved, we need to generalize Thouless pumping from single-particle to multi-particle cases, where the Bloch wave-function is replaced with multi-particle Bloch wave-function $|\psi_{m,K}(t)\rangle$ with K being center-of-mass quasimomentum [26]. The initial two-particle Wannier state approximates to two particles at site j whose preparation requires the second excited state $|2_j\rangle = (a_j^\dagger)^2|0^{\otimes N}\rangle$ of the qutrit at site j . To experimentally prepare the state $|2_j\rangle$, we apply a microwave drive pulse through the XY line of the qutrit with the driving frequency $\omega_d = \omega_{20}/2$ to perform two-photon excitation $|0_j\rangle \rightarrow |2_j\rangle$ [48], where $\omega_{20}/2 = \omega_{10} + U/2$, $\hbar\omega_{mn}$ is the energy difference between states $|m\rangle$ and $|n\rangle$. See Supplementary Information for the experimental calibration of the two-photon excitation process. We focus on the strong interaction regime where $|U| \gg |-J \pm \delta|$, and vary the double-well bias to explore the transport behavior in different regimes. If $2|\Delta_0| < |U|$, the interaction dominates and protects two bounded microwave photons in the same site. In this case, the on-site interaction U induces an effective hopping strength of $2[-J + (-1)^j\delta]^2/U$ between the bounded microwave photons at the j -th and $(j+1)$ -th sites [26], and the two photons in the same site are shifted unidirectionally as a whole [26, 29], see the schematic in Fig. 2(a). To emulate the Thouless pumping of two bounded microwave photons, we perform the quantum simulation of the interacting Rice–Mele model with $\Delta_0/(2\pi) = 8 \text{ MHz}$, $\delta_0/(2\pi) = J/(2\pi) = 12 \text{ MHz}$, and a pumping period of $T = 0.4 \mu\text{s}$. Limited by the qutrit decoherence and non-adiabatic effects, we are not able to emulate the Thouless pumping of two bounded microwave photons on the full chain. Instead we choose a subset of the qutrits with index 18–26 and prepare a second excited state $|2_{j=19}\rangle$ that largely overlap with two-particle Wannier state to emulate this pumping. The population of the Fock states $|n_j\rangle$ for $n_j = 0, 1$ and 2 of each site j are simultaneously measured and shown in Fig. 2(b), where the $|1_j\rangle$ state of each site is unpopulated. The corresponding theoretical results are shown in Fig. 2(c), see the Supplementary Information for details. To manifest the nature of bounded photons more clearly, we measure the density–density correlations $\Gamma_{ij} = \langle a_i^\dagger a_j^\dagger a_i a_j \rangle$ of site i and j [37, 49] at different evolution times t within a pumping period, as shown in Fig. 2(d). For the Fock states $|n_j\rangle$ for $n_j = 0, 1$ and 2 , the correlation $\Gamma_{ij} = \langle 2_i 0_j \rangle$ ($\langle 1_i 1_j \rangle$) when $i = j$ ($i \neq j$). The probability of finding microwave photons at sites i and j are concentrated along the diagonal of the normalized correlation, $\Gamma_{ij}/\Gamma_{ij}^{\max}$, indicating that the two photons always appear at the same site.

If $2|\Delta_0| > |U|$, the double-well bias 2Δ balances the interaction U four times in each pumping cycle, where single-photon resonant tunneling happens and the bounded photons are destroyed [26]. In this case, the

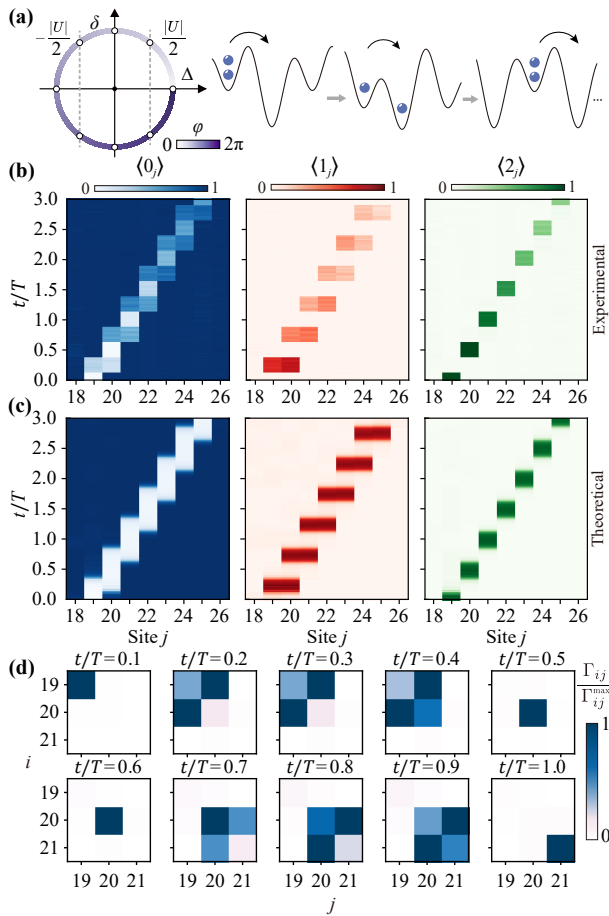


Fig. 3 Emulation of the resonant tunneling for interacting photons. (a) Schematic showing interacting photons transported unidirectionally through the barriers one by the other. (b, c) Experimental data (b) and numerical simulation (c) of the populations during the pumping process, where the $|1_j\rangle|1_{j+1}\rangle$ state is populated during the transition between $|2_j\rangle|0_{j+1}\rangle$ and $|0_j\rangle|2_{j+1}\rangle$. (d) The normalized correlations $\Gamma_{ij}/\Gamma_{ij}^{\max}$ during the pumping at different evolution times $t = 0.1T, 0.2T, \dots, T$.

two microwave photons in one site are unidirectionally transported through the barriers one by the other, see Fig. 3(a), which is a different kind of pumping process for the microwave photons in the interacting Rice–Mele model. In Fig. 3(b), we emulate the resonant tunneling with $\Delta_0/(2\pi) = 150$ MHz, an order of magnitude larger than that in Fig. 2(b), while $\delta_0/(2\pi) = J/(2\pi) = 12$ MHz and $T = 0.4$ μ s remain unchanged. The corresponding theoretical results are shown in Fig. 3(c). In this case, the $2|\Delta_0|/(2\pi) = 300$ MHz is clearly larger than $|U|/(2\pi) = 190$ MHz. Similarly, the population of the Fock states $|n_j\rangle$ of each site are simultaneously measured. Different from that in Fig. 2(b), the $|1_j\rangle|1_{j+1}\rangle$ state is populated during the transition between $|2_j\rangle|0_{j+1}\rangle$ and $|0_j\rangle|2_{j+1}\rangle$. We measure the density–density correlations Γ_{ij} at different evolution times t within a period, see Fig. 3(d), where the strong correlations observed in diagonal elements suggest the two photons appear at the same site. Furthermore, strong correlations are also observed in off-diagonal elements at specific times, suggesting the phenomena that two bounded microwave photons are destroyed and split into single photon in two sites.

Finally, we explore the interaction effects to the transport of edge states [50, 51]. The edge-state pumping can be realized by adiabatically modulating the hopping amplitudes and on-site energy in the Rice–Mele model [Eq. (2)] as $\Delta = \Delta_0 \sin(2\pi t/T_e)$, $\delta = -\delta_0 \cos(2\pi t/T_e)$, where T_e is the pumping period for the edge states. In the dimerized chain with even sites configured as noninteracting Rice–Mele model, the system has a symmetric energy spectrum respective to zero energy, benefiting from the generalized chiral symmetry [52, 53], which provides reversible channels of edge-state transport from left to right edge or vice versa (see the Supplementary Information for details). When the on-site interaction is involved, the energy spectrum becomes asymmetric as shown in Fig. 4(a), which gives a flat (steep) energy

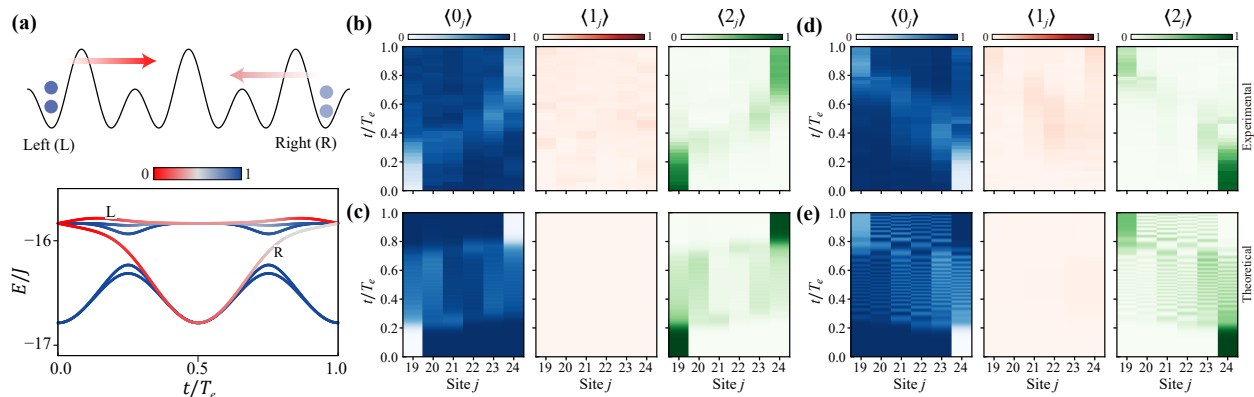


Fig. 4 Emulation of asymmetric edge-state transport. (a) The evolution of the left and right edge states in the potential well and the corresponding energy band. (b, c) Experimental data (b) and numerical simulation (c) of the left edge state transport. (d, e) Experimental data (d) and numerical simulation (e) of the right edge state transport. The colors in (b)–(e) denote the Fock state populations, whereas the colors in the energy band in (a) denote the probability of instantaneous states occupying the eigenstates.

band for the pumping starting from the left (right) edge, and thus the non-adiabatic effects of right-to-left pumping are stronger than left-to-right pumping for a given period T_e . We perform the quantum simulation of this edge-state transport in a subset of qutrits with index 19–24 isolated from other qutrits, with $\Delta_0/(2\pi) = 0.5$ MHz, $\delta_0/(2\pi) = J/(2\pi) = 12$ MHz and $T_e = 4$ μ s, an order of magnitude larger than T . In Figs. 4(b) and (d), the population of the Fock states $|n_j\rangle$ for $n_j = 0, 1$ and 2 of each site j are simultaneously measured, where the state $|2_{j=19}\rangle$ ($|2_{j=24}\rangle$) is prepared at $t = 0$. The initial left (right) edge state is transported into the right (left) edge of site $j = 24$ ($j = 19$) at $t = T_e$, with a population of $\langle 2_{j=24} \rangle = 0.51(8)$ ($\langle 2_{j=19} \rangle = 0.27(6)$). The corresponding theoretical results are shown in Figs. 4(c) and (e), where the transported population into the right (left) edge $\langle 2_{j=24} \rangle = 0.97$ ($\langle 2_{j=19} \rangle = 0.51$) at $t = T_e$, mainly limited by the non-adiabatic effect for a finite evolution time.

In conclusion, we have simulated the phenomena of Thouless pumping in the interacting Rice–Mele model using a 1D array of superconducting qutrits. Utilizing the interactions and higher energy levels in such fully controlled synthetic quantum simulator, these results demonstrate new opportunities for exploring exotic topological phases and quantum transport phenomena. Forthcoming efforts could be made to extend to two-dimension using flip-chip packaged superconducting processors [54–56] and the realization of an analogy of the 4D integer quantum Hall effect [22, 23].

Declarations The authors declare that they have no competing interests and there are no conflicts.

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