



# Thermodynamics of FRW Universe: Heat engine

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## ABSTRACT

We assume the non-flat Friedmann-Robertson-Walker (FRW) Universe as a thermodynamical system. We assume the cosmological horizon as an inner trapping horizon which is treated as dynamical apparent horizon of FRW Universe. We write the dynamical apparent horizon radius and temperature on the apparent horizon. We assume that the fluid pressure as thermodynamical pressure of the system. Using Hayward's unified first law, Clausius relation and Friedmann equations with cosmological constant, we obtain the entropy on the apparent horizon. We assume that the cosmological constant provides the thermodynamic pressure of the system. We obtain the entropy, surface area, volume, temperature, Gibb's Helmholtz's free energies, specific heat capacity of the FRW Universe due to the thermodynamic system. We study the Joule-Thomson expansion of the FRW Universe and by evaluating the positive sign of Joule-Thomson coefficient, we determine that the FRW Universe obeys the cooling nature. We also find the inversion temperature and inversion pressure. Next we demonstrate the thermodynamical FRW Universe as heat engine. For Carnot cycle, we obtain the work done and the maximum efficiency. Also for new engine, we study the work done and its efficiency.

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## 1. Introduction

Since the discovery of Hawking's radiation [1,2], the black hole thermodynamics has become an intensive research topic in Astrophysics. From the early discoveries of black hole thermodynamics, it was speculated that the black hole area behaves as thermodynamic entropy [3,4] and surface gravity behaves as temperature [1]. Further, Hawking et al. [5] have analyzed the thermodynamic properties of Schwarzschild-AdS black hole. In the study of black hole chemistry, the negative cosmological constant (i.e.,  $\Lambda < 0$ ) has been assumed as thermodynamic pressure  $P = -\frac{\Lambda}{8\pi} = \frac{3}{8\pi\ell^2}$  where  $\ell$  is the length of AdS black hole [7–12]. The geometry of AdS black hole thermodynamics has been studied by several authors [13–19]. It is well known result that in general relativity, the entropy to black hole horizon [1] and cosmological horizon [6] have the same entropy-area relation  $S = A/4G$  where  $A$  is the proper area of the horizon surface. In the black hole thermodynamics, there are some relations between black hole thermodynamics and Einstein's field equations. Jacobson [20] has observed that Einstein's field equations can be derived from the relation  $S = A/4G$  on any local Rindler causal horizons.

Most discussions of black hole thermodynamics have been concentrated on the stationary black holes. For non-stationary (i.e., dynamical) black holes, Hayward [21–23] has proposed a mecha-

nism of dynamical black hole thermodynamics associated with its trapping horizon. Using this mechanism, for spherical symmetric space-times, the Einstein's field equations can be written in a form of Hayward's "unified first law". From this unified first law along trapping horizon, one can obtain the first law of thermodynamics for dynamical black hole. In Hayward's proposal, outer trapping horizon of dynamical black hole can be used to the apparent horizon. Since the Friedmann-Robertson-Walker (FRW) Universe is treated as one kind of non-stationary spherically symmetric space-times, so similar to the study of non-stationary black hole, we can discuss its thermodynamic properties on the trapping horizon. In the FRW Universe, the outer trapping horizon cannot exist, instead there exists a kind of cosmological horizon like inner trapping horizon. So in the FRW Cosmology, this horizon coincides with the apparent horizon. Cai and Kim [24] have obtained the Friedmann equations in Einstein's gravity, Gauss-Bonnet gravity and Lovelock gravity using the unified first law with the help of entropy on the apparent horizon. Cai and Cao [25] have studied the unified first law and thermodynamics of apparent horizon in FRW Universe in the framework of Einstein theory, Lovelock theory and scalar-tensor theory. Akbar and Cai [26–28] have studied the thermodynamic phenomena of FRW Universe in Einstein's gravity, scalar-tensor gravity,  $f(R)$  gravity, Gauss-Bonnet gravity and Lovelock gravity. From modified entropy area relations, the modified Friedmann equations in FRW Universe have been obtained by several authors [29–34]. From these point of view we'll study

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the thermodynamics in FRW Universe and its associated thermodynamic quantities.

In the black hole thermodynamics, Johnson [35] has proposed the concept of holographic heat engine for AdS black hole, where cosmological constant has been treated as a thermodynamic variable. Subsequently, Johnson [36–39] has studied the Gauss-Bonnet black hole heat engine and Born-Infeld AdS black hole heat engine. Based on the Johnson’s holographic heat engine proposal, several authors have studied the heat engine phenomena and its efficiency for various types of AdS black holes [40–51]. Recently, we have studied the thermodynamics,  $P$ - $V$  criticality, stability, Joule-Thomson expansion and heat engine efficiency due to Carnot cycle and Rankine cycle for AdS black holes [52–54]. In the framework of the Universe, the thermodynamics heat engine has been studied by Pilot [55]. For polytropic gas model of the Universe, the Carnot cycle heat engine phenomena has been studied by Askin et al. [56]. Motivated by these works, here we’ll study the unified first law for FRW Universe and found the form of entropy using Einstein’s field equations and vice versa. Also we study the thermodynamic quantities as well as Joule-Thomson expansion and analyzed the heat engine phenomena for FRW Universe. In section 2, we study the Hayward’s unified first law of thermodynamics for FRW Universe. Using the Clausius relation, we derive the form of entropy using Friedmann equations with cosmological constant. Then in section 3, we obtain the thermodynamic quantities and Joule-Thomson expansion. In section 4, we study the phenomena of heat engine for FRW Universe and study the Carnot engine and new engine with work done and efficiencies. Finally, we present the results of the whole work in section 5.

## 2. Unified first law and entropy of FRW Universe

The line element for homogenous, isotropic and non-flat Friedmann-Robertson-Walker (FRW) Universe is given by

$$ds^2 = -dt^2 + a^2(t) \left[ \frac{dr^2}{1-kr^2} + r^2 (d\theta^2 + \sin^2\theta d\phi^2) \right] \quad (1)$$

where  $a(t)$  is the scale factor and  $k$  ( $= 0, -1, +1$ ) represent the flat, open and closed model of the Universe. We consider the Einstein-Hilbert action as in the form

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[ \frac{R - 2\Lambda}{16\pi G} + \mathcal{L}_m \right] \quad (2)$$

where  $R$  is the Ricci scalar,  $\Lambda$  is cosmological constant,  $\mathcal{L}_m$  is the matter Lagrangian and  $g = \det(g_{ij})$  (choosing  $c = 1$ ). The Einstein’s field equation is  $G_{ij} = 8\pi G T_{ij}$ . Here  $T_{ij}$  is the energy momentum tensor for perfect fluid, given by

$$T_{ij} = (\rho + p)u_i u_j + p g_{ij} \quad (3)$$

where  $\rho$  and  $p$  are respectively the energy density and pressure of perfect fluid. The four velocity  $u_i$  satisfies the relations  $u_i u^i = -1$  and  $u^i \nabla_j u_i = 0$ . The FRW metric (1) can be expressed in the following form [57]

$$ds^2 = h_{ij} dx^i dx^j + \tilde{r}^2 (d\theta^2 + \sin^2\theta d\phi^2) \quad (4)$$

where  $x^0 = t$ ,  $x^1 = r$ ,  $\tilde{r} = a(t)r$  and  $h_{ij} = \text{diag}(-1, \frac{a^2}{1-kr^2})$ . We consider the FRW universe as a thermodynamical system and so the dynamical apparent horizon, a marginally trapped surface with vanishing expansion can be described by the relation  $h^{ij} \frac{\partial \tilde{r}}{\partial x^i} \frac{\partial \tilde{r}}{\partial x^j} = 0$ . From this relation, we obtain the apparent horizon radius  $\tilde{r}_h$  for FRW Universe as in the form

$$\tilde{r}_h = \frac{1}{\sqrt{H^2 + \frac{k}{a^2}}} \quad (5)$$

where  $H = \frac{\dot{a}}{a}$  is the Hubble parameter. Taking derivative with respect to time  $t$ , we obtain

$$\dot{\tilde{r}}_h = -\tilde{r}_h^3 H \left( \dot{H} - \frac{k}{a^2} \right) \quad (6)$$

Since the dynamical apparent horizon is the causal horizon [23,57,58], so it is associated with the gravitational entropy and surface gravity. The surface gravity on the apparent horizon is given by [24]

$$\kappa = \frac{1}{2\sqrt{-h}} \frac{\partial}{\partial x^i} \left( \sqrt{-h} h^{ij} \frac{\partial \tilde{r}}{\partial x^j} \right) \quad (7)$$

where  $h = \det(h_{ij})$ . For FRW Universe, we obtain the surface gravity on the apparent horizon as

$$\kappa = -\frac{1}{\tilde{r}_h} \left( 1 - \frac{\dot{\tilde{r}}_h}{2\tilde{r}_h H} \right) \quad (8)$$

So the temperature on the apparent horizon is obtained by

$$T = \frac{|\kappa|}{2\pi} = \frac{1}{2\pi \tilde{r}_h} \left( 1 - \frac{\dot{\tilde{r}}_h}{2\tilde{r}_h H} \right) \quad (9)$$

Within an infinitesimal time interval  $dt$ , if we assume that the apparent horizon radius  $\tilde{r}_h$  is kept fixed, then  $\dot{\tilde{r}}_h \ll 2\tilde{r}_h H$ . In that approximate case, there is no volume change in it and so we get [24,59]

$$T = \frac{1}{2\pi \tilde{r}_h} \quad (10)$$

which is identical with the Hawking temperature on the black hole horizon.

The Hayward’s unified first law is defined by [23,25,57,58]

$$dE = A\Psi + WdV \quad (11)$$

where  $dE$  is the change of energy inside the apparent horizon. The surface area  $A$  of the apparent horizon is given by

$$A = 4\pi \tilde{r}_h^2 \quad (12)$$

and  $V$  is the volume inside the apparent horizon surface, given by

$$V = \frac{4\pi}{3} \tilde{r}_h^3 \quad (13)$$

The work done by change of the apparent horizon describes the work density and the work density function  $W$  is given by [23,25,57,58]

$$W = -\frac{1}{2} T^{ij} h_{ij} = \frac{1}{2} (\rho - p) \quad (14)$$

The total energy flow through the apparent horizon denotes energy-supply vector and is given by

$$\Psi_i = h^{jk} T_{ik} \frac{\partial \tilde{r}}{\partial x^j} + W \frac{\partial \tilde{r}}{\partial x^i} \quad (15)$$

From this, we obtain the components of energy-supply vector,

$$\Psi_t = -\frac{1}{2} (\rho + p) H \tilde{r}, \quad \Psi_r = \frac{1}{2} (\rho + p) a. \quad (16)$$

So we obtain the energy flux as in the form

$$\Psi = \Psi_i dx^i = -\frac{1}{2} (\rho + p) H \tilde{r} dt + \frac{1}{2} (\rho + p) a dr \quad (17)$$

Therefore the equation (11) yields

$$A\Psi + WdV = -A(\rho + p)H\tilde{r}dt + A\rho d\tilde{r} \quad (18)$$

Since heat is one of the forms of energy, so according to the study of Cai and Kim [24], the heat flow  $\delta Q$  through the apparent horizon is just the amount of energy crossing the apparent horizon during the time interval  $dt$ , i.e.,  $\delta Q = -dE$ . So using (11) and (18), on the apparent horizon, we obtain

$$\delta Q = -dE = A(\rho + p)H\tilde{r}_h dt \quad (19)$$

The first law of thermodynamics (Clausius relation) on the apparent horizon is

$$\delta Q = TdS \quad (20)$$

From equations (19) and (20), we obtain

$$TdS = A(\rho + p)H\tilde{r}_h dt \quad (21)$$

For the FRW metric, the Friedmann equations in presence of cosmological constant in Einstein's gravity are given by

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3} \rho + \frac{\Lambda}{3} \quad (22)$$

and

$$\dot{H} - \frac{k}{a^2} = -4\pi G(\rho + p) \quad (23)$$

We consider the Universe is filled with the fluid content whose energy density and pressure are  $\rho$  and  $p$ , satisfy the energy conservation equation

$$\dot{\rho} + 3H(\rho + p) = 0 \quad (24)$$

which can be obtained from the Einstein's field equations (22) and (23). From equations (5) and (22), we obtain

$$\rho = \frac{3}{8\pi G\tilde{r}_h^2} - \frac{\Lambda}{8\pi G} \quad (25)$$

Also from equations (22) and (23), we obtain

$$p = -\frac{1}{8\pi G\tilde{r}_h^2} + \frac{\Lambda}{8\pi G} \quad (26)$$

Using the form of temperature (10), we now obtain the entropy  $S$  from the Friedmann equations (22) and (23). Putting the expressions (10), (12), (25) and (26) in the equation (21) and then integrating, we obtain the form of entropy as

$$S = \frac{A}{4G} + S_0 \quad (27)$$

where  $S_0$  is an integration constant. If  $S_0 = 0$ , the above entropy is identical form of entropy of the black hole horizon. This is known result for FRW Universe in Einstein's gravity.

For FRW Universe, if we assume that the entropy has the form  $S = \frac{A}{4G} + S_0$  and temperature has the form (10), then using conservation equation (24) with the help of (5), (6) and (21), we obtain the Friedmann equations (22) and (23) which have also been found in [24].

### 3. Thermodynamic quantities in FRW Universe

For thermodynamic system, the entropy on the apparent horizon in the FRW Universe is

$$S = \frac{A}{4G} + S_0 = \frac{\pi\tilde{r}_h^2}{G} + S_0 \Rightarrow \tilde{r}_h = \sqrt{\frac{G}{\pi}}\sqrt{S - S_0} \quad (28)$$

The volume inside apparent horizon can be written in terms of entropy as

$$V = \frac{4\pi}{3} \tilde{r}_h^3 = \frac{4}{3\sqrt{\pi}} G^{3/2}(S - S_0)^{3/2} \quad (29)$$

The temperature (10) can be written in terms of entropy as in the following form

$$T = \frac{1}{2\sqrt{\pi G}\sqrt{S - S_0}} \quad (30)$$

As well as in black hole thermodynamics, the cosmological constant  $\Lambda$  is treated as thermodynamic pressure  $P$  i.e.,  $P = \frac{\Lambda}{8\pi G}$  and allows to variable. So from equations (25) and (26) we obtain

$$\rho = \frac{3}{8G^2(S - S_0)} - P \quad (31)$$

and

$$p = P - \frac{1}{8G^2(S - S_0)} \quad (32)$$

We have taken positive  $\Lambda$ , otherwise  $p$  will always negative which cannot happen in general. Since  $\rho > 0$  so we have  $P < \frac{3}{8G^2(S - S_0)}$  and hence  $p < \frac{2P}{3}$ . Also we can write  $\rho + 3p = 2P$ . The enthalpy function is defined as  $\mathcal{H} = U + PV$  where  $U$  is the energy. Hence, using the first law of thermodynamics, we get

$$d\mathcal{H} = TdS + VdP \quad (33)$$

Now integrating, we obtain the enthalpy function  $\mathcal{H}$  as in the following form,

$$\mathcal{H} = \frac{2\sqrt{S - S_0}}{3\sqrt{\pi G}} + \frac{4}{3\sqrt{\pi}} G^{3/2} \int (S - S_0)^{3/2} dp + \mathcal{H}_0 \quad (34)$$

where  $\mathcal{H}_0$  is integration constant. So the Gibb's free energy can be obtained as [60],

$$\begin{aligned} \mathcal{G} &= \mathcal{H} - TS \\ &= \frac{(S - 4S_0)}{6\sqrt{\pi G}\sqrt{S - S_0}} + \frac{4}{3\sqrt{\pi}} G^{3/2} \int (S - S_0)^{3/2} dp + \mathcal{H}_0 \end{aligned} \quad (35)$$

Also the Helmholtz's free energy can be obtained as [60]

$$\begin{aligned} F &= \mathcal{G} - PV \\ &= -\frac{S_0}{2\sqrt{\pi G}\sqrt{S - S_0}} - \frac{4}{3\sqrt{\pi}} G^{3/2}(S - S_0)^{3/2} p \\ &\quad + \frac{4}{3\sqrt{\pi}} G^{3/2} \int (S - S_0)^{3/2} dp + \mathcal{H}_0 \end{aligned} \quad (36)$$

The specific heat capacity of the FRW Universe in thermodynamical system can be written as [9]

$$C_p = T \left( \frac{\partial S}{\partial T} \right)_p = 2(S_0 - S) \quad (37)$$

Coefficient of thermal expansion is given by

$$\alpha = \frac{1}{V} \left( \frac{\partial V}{\partial T} \right)_p = -12\sqrt{\pi G}\sqrt{S - S_0} \quad (38)$$

The isothermal compressibility is given by [61]

$$\kappa_T = -\frac{1}{V} \left( \frac{\partial V}{\partial P} \right)_T = \frac{3}{2(S - S_0)} \left( \frac{1}{8G^2(S - S_0)^2} - \frac{\partial p}{\partial S} \right)^{-1} \quad (39)$$

The minus sign accounts for the fact that an increase in pressure generally induces a reduction in volume.

Joule-Thomson expansion [62,63] describes that the change of temperature from high pressure regime to low pressure regime, while the enthalpy remains constant. It is an irreversible process, also known as the throttling process. Here, we now examine the Joule-Thomson expansion for FRW Universe. The Joule-Thomson coefficient  $\mu$  is the slope of the isenthalpic curve, defined by [64]

$$\mu = \left( \frac{\partial T}{\partial P} \right)_{\mathcal{H}} \quad (40)$$

which can be written in the following forms

$$\mu = \frac{1}{C_p} \left[ T \left( \frac{\partial V}{\partial T} \right)_p - V \right] \quad (41)$$

or

$$\mu = \frac{1}{S} \left[ P \left( \frac{\partial V}{\partial P} \right)_{\mathcal{H}} + 2V \right] \quad (42)$$

The cooling or heating nature of the Universe can be determined by the sign of  $\mu$ . If  $\mu > 0$ , the cooling process occurs while  $\mu < 0$  describes the heating nature. Now for FRW Universe, we obtain

$$\mu = \frac{8}{3\sqrt{\pi}} G^{3/2} \sqrt{S - S_0} \quad (43)$$

We see that  $\mu > 0$  always, so cooling process occurs in FRW Universe.

Putting  $\mu = 0$  in (41), we can obtain the expansion process of inversion curve and so the inversion temperature is obtained as

$$T_{inv} = V \left( \frac{\partial T}{\partial V} \right)_p = - \frac{1}{12\sqrt{\pi G} \sqrt{S - S_0}} \quad (44)$$

Also by putting  $\mu = 0$  in (42), the inversion pressure can be obtained as

$$P_{inv} = -2V \left( \frac{\partial P}{\partial V} \right)_{\mathcal{H}} = \frac{4(S - S_0)}{3} \left( \frac{1}{8G^2(S - S_0)^2} - \frac{\partial p}{\partial S} \right) \quad (45)$$

The inversion pressure depends on the fluid pressure  $p$  and entropy  $S$  of the system.

#### 4. Heat engine for FRW Universe

Thermodynamically, heat engine is a physical system which converts heat/thermal energy into mechanical energy for doing mechanical work. So heat engine transfers heat from hot region, where part of the heat transforms into physical works while remaining part of the heat is moved to cold region. So the heat engine works in a cyclic manner where the heat/thermal energy produces in one part of the cycle, which can do work in another part of the cycle. In this section, we'll study the Carnot cycle of the heat engine for FRW Universe.

In 1824, Carnot introduced a theoretical thermodynamic cycle, known as Carnot cycle and corresponding classical heat engine is known as Carnot heat engine. Now we assume,  $T_H$  and  $T_C$  are respectively the temperatures of hot and cold regions, which consist of upper and lower isothermal processes. For Carnot heat engine, Johnson [35] has shown the  $P$ - $V$  diagram in a closed path in order to calculate the work done by the heat engine. In the diagram, the heat flows are produced from level 1 to level 2 along the upper isotherm process, given as  $Q_H = T_H \Delta S_{1 \rightarrow 2} = T_H(S_2 - S_1)$  and also the exhausted heat formed from level 3 to level 4 along lower isothermal process, given as  $Q_C = T_C \Delta S_{3 \rightarrow 4} = T_C(S_3 - S_4)$ . In FRW Universe, using relation (29), the entropies  $S_i$ 's are related to the volumes  $V_i$ 's as in the following

$$S_i = \frac{1}{G} \left( \frac{3\sqrt{\pi}}{4} \right)^{2/3} V_i^{2/3} + S_0, \quad i = 1, 2, 3, 4. \quad (46)$$

The total work done by the Carnot heat engine is  $W = Q_H - Q_C$ . The efficiency of the Carnot heat engine is defined by the ratio of total work done and so the amount of heat energy along the upper isotherm process is given as  $\eta_{car} = \frac{W}{Q_H} = 1 - \frac{Q_C}{Q_H}$ . In Carnot cycle, we have the relations  $V_4 = V_1$  and  $V_3 = V_2$ . So the maximum efficiency for Carnot cycle is obtained as  $(\eta_{car})_{max} = 1 - \frac{T_C}{T_H} = 1 - \sqrt{\frac{S_H}{S_C}}$ . Since  $T_H > T_C$ , so maximum efficiency for Carnot cycle always satisfies  $0 < (\eta_{car})_{max} < 1$ .

Johnson [35] has also described a new engine which includes two isobars and two isochores/adiabats, where the heat flows show along top and bottom lines. The total work done along the isobars is given in the form

$$W = \Delta P_{4 \rightarrow 1} \Delta V_{1 \rightarrow 2} = (P_1 - P_4)(V_2 - V_1) \quad (47)$$

The net inflow of heat along the upper isobar is given by

$$\begin{aligned} Q_H &= \int_{T_1}^{T_2} C_p(P_1, T) dT \\ &= \frac{1}{2\sqrt{\pi G}} \left( \sqrt{S_2 - S_0} - \sqrt{S_1 - S_0} \right) \end{aligned} \quad (48)$$

The net exhaust of heat from the lower isobar is given by

$$\begin{aligned} Q_C &= \int_{T_3}^{T_4} C_p(P_4, T) dT \\ &= \frac{1}{2\sqrt{\pi G}} \left( \sqrt{S_4 - S_0} - \sqrt{S_3 - S_0} \right) \end{aligned} \quad (49)$$

So the thermal efficiency for the new heat engine for FRW Universe is given in the form

$$\begin{aligned} \eta_{New} &= \frac{W}{Q_H} = \frac{(P_1 - P_4)(V_2 - V_1)}{Q_H} \\ &= \left[ \frac{8G^2}{3} (p_1 - p_4) + \frac{(S_4 - S_1)}{3(S_1 - S_0)(S_4 - S_0)} \right] \\ &\quad \times \left( S_1 + S_2 - 2S_0 + \sqrt{(S_1 - S_0)(S_2 - S_0)} \right) \end{aligned} \quad (50)$$

where  $p_1$  and  $p_4$  denote the fluid pressures at stages 1 and 4 respectively. We see that the efficiency of the new engine  $\eta_{New} > 0$  always if  $p_1 > p_4$  and  $S_4 > S_1$ .

#### 5. Discussions and concluding remarks

We have assumed the non-flat Friedmann-Robertson-Walker (FRW) Universe as a thermodynamical system. We have also considered the cosmological horizon as a inner trapping horizon which is treated as dynamical apparent horizon of FRW Universe. We have found the dynamical apparent horizon radius and temperature on the apparent horizon. We have assumed that the cosmological constant  $\Lambda$  for the Universe as thermodynamical pressure  $P$  of the system. Using the Hayward's 'unified first law', we have obtained the change of the energy  $dE$  inside the apparent horizon. The Friedmann equations in presence of cosmological constant with the help of conservation equation, we have obtained the entropy on the apparent horizon as in the form  $S = \frac{A}{4G} + S_0$ . If  $S_0 = 0$  then the entropy is identical form on horizon entropy of black hole. Conversely, from entropy-area relation, the Friedmann

equations with cosmological constant can be recovered [24]. Due to the thermodynamic system, we have obtained the surface area, entropy, volume, temperature, Gibb's free energy, Helmholtz's free energy and specific heat capacity of the FRW Universe. We have examined the Joule-Thomson expansion of FRW Universe and evaluated the Joule-Thomson coefficient  $\mu$ . The sign of  $\mu$  presents the key role for heating or cooling nature of the Universe. We have found that  $\mu$  is positive, so we may conclude that in the thermodynamic system, the FRW Universe produces the cooling nature. Putting  $\mu = 0$ , we have obtained the inversion temperature and inversion pressure. The inversion pressure always depends on the nature of the fluid pressure as well as entropy. Next we have demonstrated the thermodynamical FRW Universe as heat engine. For Carnot cycle, we have obtained the work done and its maximum efficiency which satisfies  $0 < (\eta_{Car})_{max} < 1$ . Also we have found the work done and its efficiency for a new engine, which satisfies  $\eta_{New} > 0$  always if  $p_1 > p_4$  and  $S_4 > S_1$ .

### Declaration of competing interest

The author declares that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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