3 Electron Positron and Muon Circular Colliders

3.1 Ground-Up Circular e+e- Higgs Factory Design and Cell Length Optimization

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3.1.1 Introduction

A ``ground-up'' CEPC Higgs Factory design methodology is described. The goals are to find: (i) optimal parameters, (ii) improved understanding (iii) a tentative lattice design. As illustration of the method, six chromaticity-corrected lattices, with cell lengths ranging from 45 m to 280 m, all with identical βy=2 mm or βy=10 mm intersection region optics, are designed and their properties compared. For simplicity only a single ``toy ring'', circumference (76 km), with one interaction point, and a single beam energy (120 GeV) is considered. For the cell-length optimization a figure of merit FOM (essentially integrated luminosity) is maximized consistent with a dimensionless fine tuning penalty function'' or figure of demerit FOD, not being allowed to exceed a conservatively chosen upper limit. The tentative recommendation from this investigation is that the optimal route is (except for obvious changes) to simply copy LEP: 80 m cell length and two-in-one single-ring operation.

A new circular e+e- Higgs factory can have significant luminosity advantages relative to LEP. One LEP parameter that CEPC must not copy is the luminosity of 1032/cm2/s. Some guaranteed improvements (with their improvement factors) are: increased ring-radius x RF power product (3x5≈15); non-interleaved sextupoles (2); full-energy, top-off injection (5); more bunches (110/6≈15); improved intersection region optics (2). It would be double counting to simply multiply these factors. But, barring unforced errors, more than two orders of magnitude improvement is conservatively available. So, with these changes, luminosity in excess of 1034/cm2/s is assured, with little uncertainty or risk.

Possible unforced errors'' that could jeopardize these luminosity improvement factors include too-short cell length, which causes excessively large fine tuning penalty, and local chromatic compensation, which requires strong bends adjacent to the intersection regions (IR). At the high Higgs factory energy the synchrotron radiation from these bends contains hundreds of kilowatts of hard x-rays.

3.1.1.1 *Two results from my 2015 IAS Higgs factory white paper*

For my 2015 IAS Higgs Factory white paper [1] I determined a universal scaling relation for radiation dominated colliding storage rings shown in Figure 1. This graph was introduced primarily in reference to the choice of ring circumference. As such it is not very important for the present paper, which concentrates on optimizing the cell length for constant circumference. In fact, the present paper investigates moving away from this nominal (constant dispersion) behavior (primarily by reducing cell length) to optimize the luminosity.

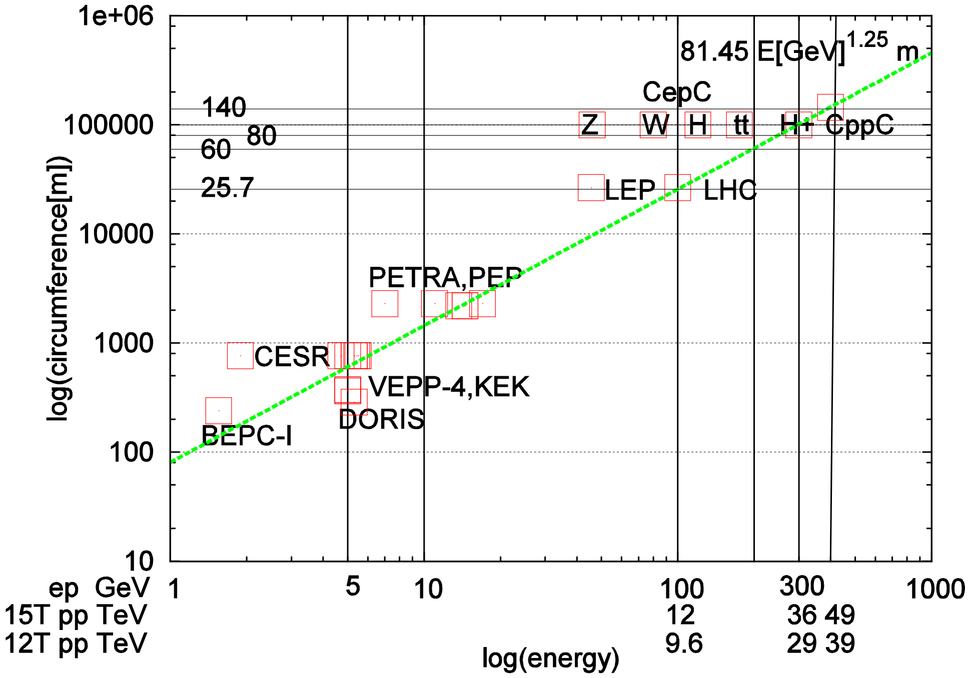

Figure 1. Dependence of circumference on beam energy for radiation-dominated colliders. i.e. GeV-scale electron colliders, and TeV-scale proton colliders of magnetic field 12 T or 15 T.

Of much greater importance for the present paper, also copied from my 2015 white paper, is Table 1, which compares past and future colliding beam rings on the basis of FOD=βy[max]/(l_c<D>) a ``figure of demerit" introduced in that paper; here βy[max] is the maximum vertical beta function anywhere in the ring, l_c is the arc cell length, and <D> is the average dispersion. This formula is justified more fully later in this paper. Though having physical dimension 1/m, this FOD becomes dimensionless after multiplication by an (unknown) positioning length uncertainty, that reflects state-of-the-art construction, positioning, and stabilization precision. The FOD figure of demerit is based on the assumption that construction, positioning, and stabilization uncertainties are comparable in all rings---though possibly improving due to improved technology over time. To the extent this is valid, the degree of conservatism of diverse storage rings can be compared just on the basis of dimensional analysis. The vindication for applying dimensional analysis comes from the degree of constancy exhibited by the entries in the last column of Table 1. The actually-measured values in the upper six rows vary from 5.1 to 49, which can hardly be said to represent constancy. But both electron and proton rings are represented, and the particle energies range over a much greater three orders of magnitude range.

In preparing the present paper I came to realize that an appropriate name for this measure of ring sensitivity is ``fine tuning penalty". Having heard theorists emphasizing their disapproval of theories that required ``fine tuning" for many years, it came to me that accelerator physicists have been facing up to fine tuning difficulties during the same era. Surely there are few instruments more finely tuned than a colliding beam. My ``fine tuning" epiphany reminded me of a line in the Moliere play, ``Le Bourgeois Gentilhomme". Monsieur Jourdain, during a discussion of poetry and prose announces, ``Good heavens, you mean that for more than forty years I have been speaking prose without knowing it." So, as already stated, for parameter optimization, the fine tuning penalty provides a quantitative constraint on the storage ring sensitivity. Tentatively, based on measured values in the table, I have adopted FOD<50 as the maximum allowable fine tuning penalty. It will be easy, later, to haggle about the validity of the fine tuning penalty, for example replacing it by some other ring sensitivity measure.

Table 1. Sampling of collider FOD's ("Fine Tuning Penalties") for previous and planned colliding rings, both p,p and e+e-, low and high energy.

β_y^* m	Ring		$\langle D\rangle$ m	l_c m	β_y^{max} m	$\frac{\beta_y^{max}}{\langle D\rangle l_c}$ FOD 1/m
0.015	CESR	measured	1.1	17	95	5.1
0.08	PETRA	measured	0.32	14.4	225	49
	HERA	measured	1.5	48	2025	28
0.05	LEP	measured	0.8	79	441	7.0
0.007	KEKB	measured	0.5	20	290	29
	LHC	measured	1.6	79	4500	36
0.001	CEPC	design	0.31	47	6000	410
0.001	FCC-ee	design	0.10	50	9025	1805

In describing the ``ground up" optimization methodology, the ``fine tuning penalty" will also be referred to as a ``figure of demerit" (FOD) where, numerically, FOD is given by βy[max]/(l_c<D>). Hands-on experience with any particular ring suggests that increased βy[max] correlates well with increased tuning sensitivity. (A positional uncertainty at a βy=βy[max] quadrupole location produces a positional uncertainty proportional √(βy[max]) elsewhere in the ring, and proportional to βy[max] at all high βy locations.) The previously introduced transverse position uncertainty introduces another length.

With the unknown position uncertainty being a length, the FOD itself has to have inverse length dimensionality. To cancel length-squared, a natural further factor, with dimensions of inverse-length-squared, is the typical sextupole strength S needed to cancel the ring chromaticities. (This is why sextupoles are present in the ring, with their undesirable nonlinear aperture-limiting effects). Since sextupole strengths are not routinely available, it is convenient to replace S by the 1/(l_c<D>) factor, which scales proportionally. (This is because quadrupole q induced chromaticity q δ is cancelled by sextupole induced chromatic compensation -S<D>δ, where δ is fractional momentum offset, and q scales as 1/l_c. The ground-up methodology I recommend includes the following design principles for CEPC and FCC-ee:

- Luminosity is a dependent variable, not an input parameter.
- The ``ground-up" methodology is incompatible with ``defined parameter" colliding beam ring design. For example, luminosity is treated as output, not input.
    - Circular colliders and linear colliders are not the same. This is not inconsequential; currently, by adopting linear-collider-like intersection region optics, neither CEPC nor FCC-ee intersection region designs have adequately appreciated this.
- Transverse sensitivity, upper limit on fine tuning penalty; FOD< 50/m. (This may be too conservative. If so, it can be relaxed later.)
- The Higgs Factory design problem is *not* chromatic mismatch of IR and arcs; it is the loss of off-momentum particles, for example due to the Telnov[2] effect.

Though not exactly a ``design principle" my preliminary ground-up design calculations suggest that local chromatic correction (with its strong bends, large dispersion, and hard x-rays aimed toward the detectors) are unnecessary. (Another quotation from a different Moliere play, ``Nearly all men die of their remedies, and not of their maladies.") To understand this analogy it is necessary to think of chromaticity as the malady, and sextupoles as remedy. In this case the potentially lethal side effects of the sextupole medicine include both reduced dynamic aperture and hard x-rays incident on the IP detector.

3.1.1.2 *Optimization variables*

It is important to distinguish between independent and dependent variables. The main independent variables are:

- l^* = (1/2) free length for detector [m]
- l_c = lattice cell length [m]
- $^*_y$ = vertical beta function at IP [m]
- δ = fractional momentum offset [%]

- The main dependent variables are:
- L = actual luminosity [in units of $10^{34}/cm^2/s$]
- L'_0 = luminosity per momentum acceptance
- $\Delta\delta[max] = \delta[max] - \delta[min]$
- figure of merit,

  $FOM = l^*$ x $\Delta\delta[max]/\sqrt{(\varepsilon_x \varepsilon_y)}$

Rationale for this figure of merit: FOM encapsulates the most important lattice-dependent ``useful" (i.e. including l^* factor) luminosity factors (other than β^*_y).

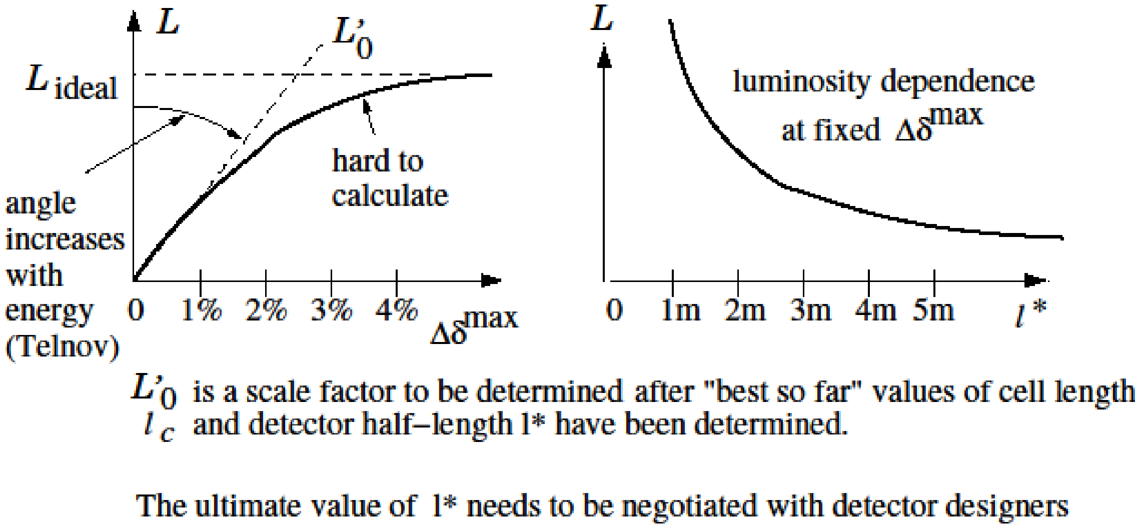

Figure 2. Qualitative luminosity dependencies. Luminosity vs momentum acceptance of the left, luminosity vs IR half-length l^* on the right.

Qualitative luminosity dependencies are sketched in Table 2. Standard luminosity formulas (which ignore momentum acceptance) yield the luminosity labeled L_ideal in the figure on the left. It has always been known that actual luminosity also depends on momentum acceptance. As the figure indicates, the actual luminosity initially increases linearly with momentum acceptance, with slope L'_0. As noted in the figure, it was Telnov[2] who first emphasized that the increasing importance of beamstrahlung with increasing beam energy places increasing demands on the momentum acceptance.

The target for the optimization is to maximize FOM, consistent with limiting FOD. The strategy is to perform multiple scans varying one input parameters while holding constant the other input parameters, including β^*_y. Successive scans establish ``best so far" values of detector half-length l^* and cell length l_c, without exceeding FOD=50/m.

Major variables held constant for this preliminary study have been:

- Ring circumference C ≈ 75 km (midway between CEPC and FCC-ee).
- Beam energy E_0=120 GeV.
- All lattices investigated are ``toy" lattices consisting of *just one intersection region*, inert straight-section opposite, and two dispersion-suppressed arcs.
- *Just two* sextupole families, tuned to cancel both horizontal and vertical chromaticities. Since there are no other nonlinear elements there are no sextupole strengths to be optimized.
- Any benefit from more sophisticated optics, such as more sextupole families or local chromaticity compensation, will necessarily increase the luminosity. Parameter scan policies include:
- When scanning input variables, hold β^*_y constant, but not necessarily small (to avoid lattice tune-up difficulties). β^*_y can be optimized later. This is opposite to the ``defined parameter" approach, which obstinately fixes β^*_y to a very small value, such as 1 mm, thought to be necessary to produce a specified luminosity.
- When scanning cell length *l_c*, the intersection region optics are held constant. The number of arc cells is adjusted to hold circumference *C* (more or less) constant.
- When scanning free length l^* the arcs are held constant, except for tweaking phase advance per cell to adjust β^*_y and sextupole strengths to cancel chromaticities.

One aspect of ground-up design is probing to find favorable and unfavorable dependencies. Inferences gleaned so far include:

- One may as well have *the game as the name*; high beta points in every arc cell can be exploited without doing more harm than one, or a few, points with the same high beta values; e.g. in local chromaticity-correction sectors.
- It is not necessary to ``match" the arc beta functions. Systematic $\beta_y$ ``beats" are found to be harmless. This is the only *radical* deviation from orthodoxy suggested in this paper.
- Also suggested, though not proved in general, is the observation, with best-so-far parameters, that β^*_y can be changed over a substantial range without much change in momentum acceptance.

3.1.2 Six storage ring designs with varied cell length

3.1.2.1 *Chromatic correction in the arcs.*

Non-interleaved sextupole, arc-only chromatic compensation has been based entirely on arcs consisting of repetition of enough identical five-cell sectors having the following fivecell pattern to make two arcs of the proper length**:**

```
fivecell:line=(
  .0    quadhf,sext1,bend,quadvf,
  .125  quadvf,      bend,quadhf,
  ------------------------------------
  .25   quadhf,      bend,quadvf,
  .375  quadvf,      bend,quadhf,
  ------------------------------------
  .50   quadhf,sext1,bend,quadvf,
  .625  quadvf,sext2,bend,quadhf,
  ------------------------------------
  .75   quadhf,      bend,quadvf,
  .875  quadvf,      bend,quadhf,
  ------------------------------------
  1.00  quadhf,      bend,quadvf,
  1.125 quadvf,sext2,bend,quadhf ) 1.25
```

The numbers listed in the margins are tune advances to that location, from the beginning of fivecell. All phase advances per cell are very close to π/2$, but tweaked to control beta functions at the IP. There are just two sextupole families, with strengths sext1 and sext2. Phase advances between matched sextupoles are very close to π, as required to cancel on-momentum sextupole kicks.

Zooming of ring sectors for tuning the six ``toy lattices'' for this study has been possible using the following simple ring design.

```
arc : line = ( dsin, 35*fivecell, dsout )
```

irtoarc : line = (dr01p,qir1p,dr12p,qir2p,dr23p,qir3p,

qir1, dr12, qir2, dr23, qir3)

arctoir : line = (-irtoarc)

ring : line = (irtoarc, arc, farstraight, arc, arctoir)

The ``35" entry in arc is appropriate for lattice CEPC4.0. The corresponding entries for other lattices are given in a later table. Here dsin and dsout are dispersion suppression, while other elements starting with ``d", such as dr01p, dr12p etc. are drifts. Elements qir1p, qir2p, etc. are quadrupoles. To change cell-length (for this investigation) all arc element lengths (including dispersion suppression and far straight) are scaled proportionally, with all quads varying inversely (to hold phase advance per cell almost constant). Optically this resembles zooming a telephoto lens. But (also like the final stage of a zooming telephoto lens) the intersection region optics are held fixed.

3.1.2.2 *Scans leading to best performance (so far)*

My preliminary parameter scans have been organized as illustrated in

Figure 3, and described in the following list. Qualitative observations made during these runs are indicated by circled numbers.

- Scan I, is made ``easy to tune" in spite of the low value β^*_y =2 mm, by the small free length l^*=0.8 m (circled 1). Even so, with lattice tune-up routines not yet developed, the tunes were not carefully controlled.
- Scan II becomes ``hard to tune" for large l^*. To relieve this β^*_y is increased to 10 mm. (circled 2). A surprise during this scan was that momentum acceptance increased (or, at least, did not decrease) with increasing l^*.
- Scan III is to find best case so far; l_c=85 m, l^*=2.0 m. (circled 3). To make tuning easier β^*_y was increased to 10 mm.
- Scan IV is to adjust β^*_y (circled 4) surprisingly, momentum acceptance is nearly independent of β^*_y. But (obviously) FOD increases strongly above its maximum allowable value, as β^*_y is reduced towards 1 mm.

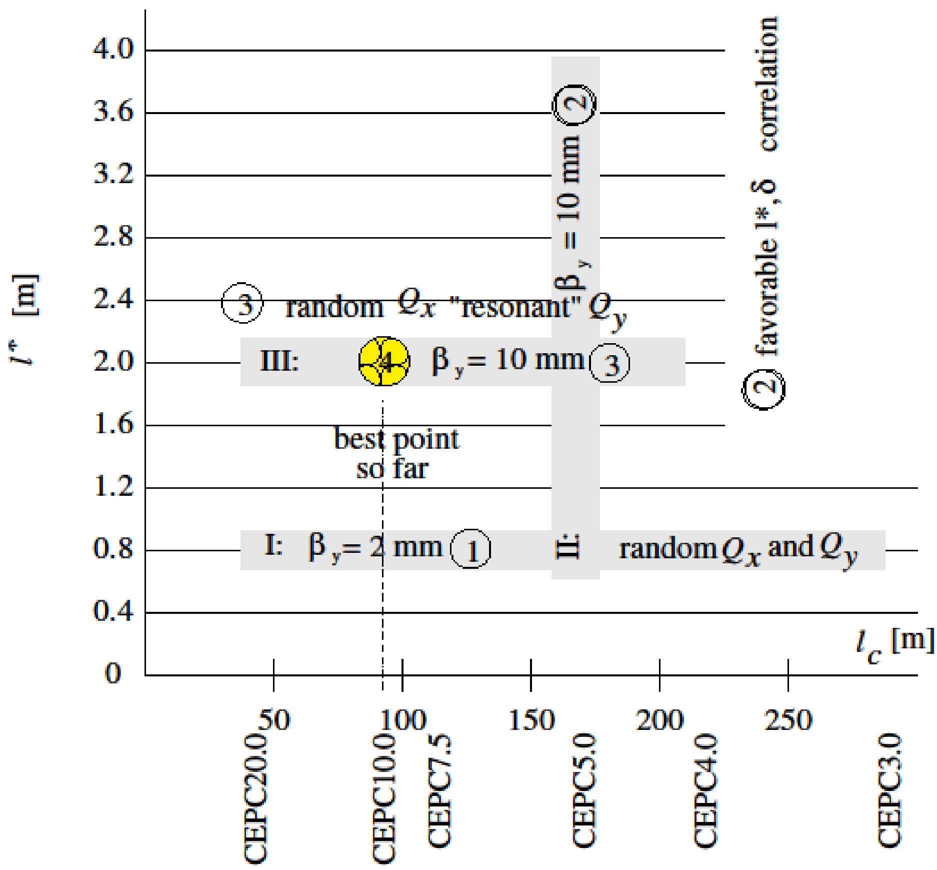

Figure 3. Parameter scans performed so far. Qualitative comment points mentioned in the text are tagged by circled numbers.

3.1.2.3 *Parameters of the six test lattices*

Parameters for lattices used in Scan I are given, above the double line, in Table 2. The shaded row represents nominal ``constant dispersion'' radiation-dominated extrapolation from LEP. Fixed Scan I parameters are β^*_y =2 mm, l^*=0.8 m. Parameters for Scan IV, varying β^*_y with l^*=2.0 m fixed, are given below the double line in Table 2.

In column 4, num5 is the total number of five-cell chromatic modules in each of the two main ring arcs; values of num5 were adjusted to keep the total ring circumference (more or less) constant for all lattices. Vertical β^*_y, ring and (nominally 90 degree) phase advance per cell, were held constant by tweaking the phase advance per cell, as the cell length was changed. Both integer and fractional parts of the *Q_x* and *Q_y* tunes were established in the process. *Ideally, for this study, the fractional parts would have been held fixed, but there was no fine tuning provision for this.*

In all cases the two sextupole currents, for the sext1 and sext2 families were adjusted for zero chromaticity in both planes.

Table 2. Parameters for Scan I and, below the double line, for scan IV. The shading in the CEPC5.0 row, in this and some subsequent tables, indicates that this row is the result of "nominal", constant dispersion, extrapolation from LEP.

Lattice name unit	circum m	cell length m	num5	l^* m	Q_x	Q_y	β_x^* m	β_y^* m
CEPC3.0	76061	281.7	26	0.8	69.14	68.51	0.489	0.0020
CEPC4.0	76131	211.3	35	0.8	92.08	91.52	0.333	0.0020
CEPC5.0	74483	169.0	43	0.8	112.54	111.02	0.208	0.0020
CEPC7.5	74540	112.7	65	0.8	169.06	166.02	0.137	0.0020
CEPC10.0	74568	84.5	87	0.8	223.83	227.02	0.172	0.0020
CEPC20.0	80949	42.3	190	0.8	494.10	496.03	0.046	0.0020
CEPC10.0 (best so far)	74618.6	84.5	87	2.0	223.841	221.504	0.981	0.010
CEPC10.0 (better?)	74618.6	84.5	87	2.0	223.848	226.501	0.958	0.0014

3.1.2.4 *Nonlinear ring optics*

Sextupole strength dependent parameters for Scan I, with β^*_y =2 mm are plotted in Figure 4.

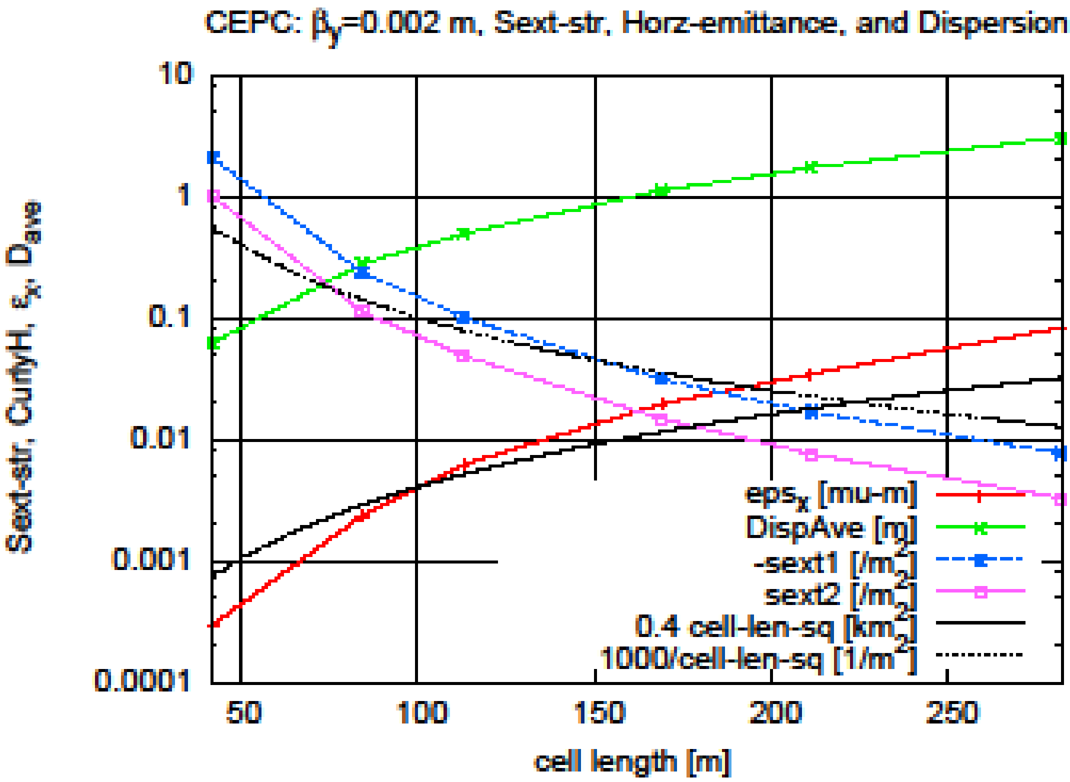

Figure 4. Parameter depemdencies imposed by chromatic correction for β^*_y =2 mm.

The ring vertical beta function could not be reduced below 0.004 m for **42** m cell length. Note that the achromatic sextupole strengths are independent of β^*_y. This means the chromaticity due to IR optics is relatively unimportant. This permits β^*_y to be made ``arbitrarily" small, without much detuning the ring optics**.** (It can be observed that) sextupole strengths vary inversely with horizontal emittance. As a result the dynamic aperture tends to ``track" the emittance. This dependence limits the ability to increase luminosity by decreasing the cell length---increasing the luminosity necessarily decreases the dynamic aperture.

3.1.3 **Ring emittance and acceptance performance**

Acceptance and emittance are directly commensurate. *Emittance must be less than acceptance for an injected beam to be stored without loss*. Raw acceptance plots (irrespective of beam emittances) for the six test lattices are shown in Figure 5.

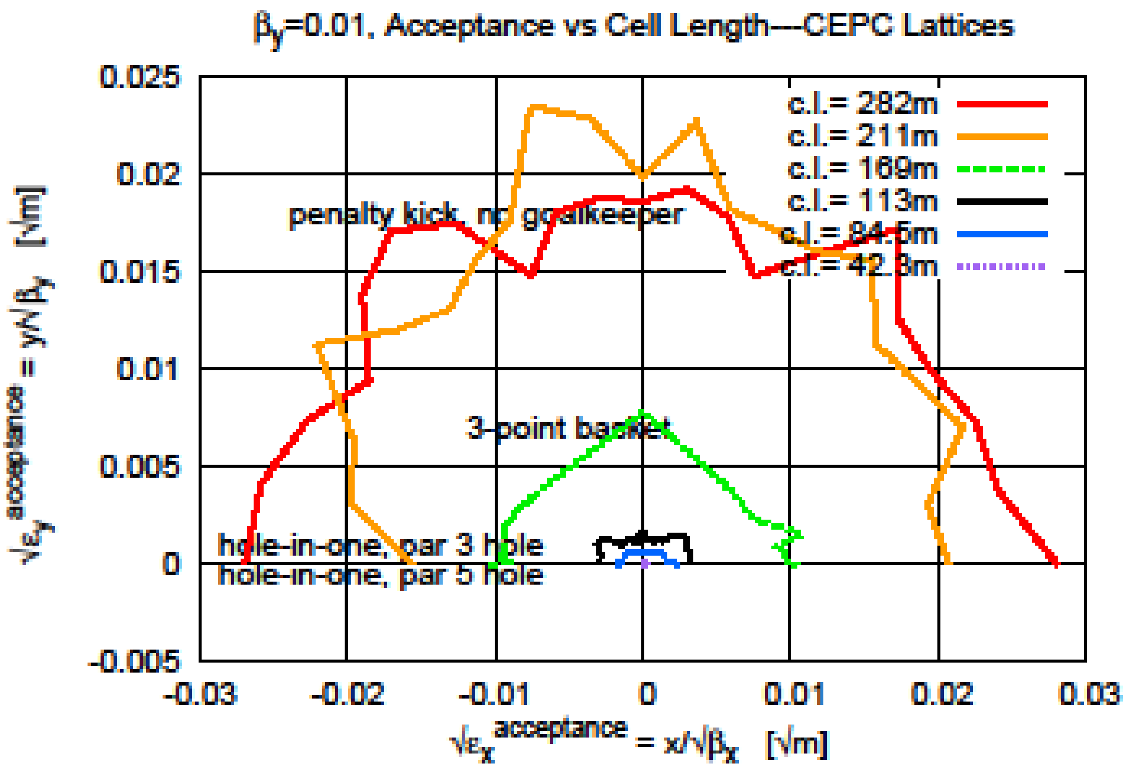

Figure 5. Raw acceptance plots (irrespective of beam emittances) for the six toy lattices. ``Cartoon" annotations are mnemonics indicating the challenges of ``putting things" in small containers.

The dynamic *x, y* product aperture is many orders of magnitude greater with 282 m cell length than 42 m cell length. For a large ε^(emittance) beam, for example for a muon collider, the cell length would be chosen as large as possible. This plot shows, however, and it is born out by tuning experience, that decreasing l_c causes the lattice to be harder to tune. This is reflected in the fine tuning penalty FOD increasing strongly as *l_c* is reduced. This is easily understandible in terms of lattice dispersion, which scales as *l_c*^2. Since the sextupole strengths needed for chromatic correction scale inversely with dispersion, the dynamic aperture decreases strongly with decreasing *l_c*.

But radiation damping shrinks our electron beams to micron scale transverse size at the IP, mm scale elsewhere, allowing our acceptance to be much smaller. To account for this it is conventional to plot the acceptances in units of the equilibrium beam sigmas, which is done in Figure 6. (This is made risky, especially as regards vertical aperture, by the fact that the vertical emittance itself is the least reliably known beam parameter.)

It can be seen that large cell length is still strongly favored. But the values of σ_x and σ_y are different for the six test lattices. To maximize the luminosity we need to minimize σ_x and σ_y (by reducing the cell-length) consistent with maintaining

acceptably small fine tuning penalty FOD. Acceptances are plotted in units of beam sigmas for the six toy lattices in Figure 6.

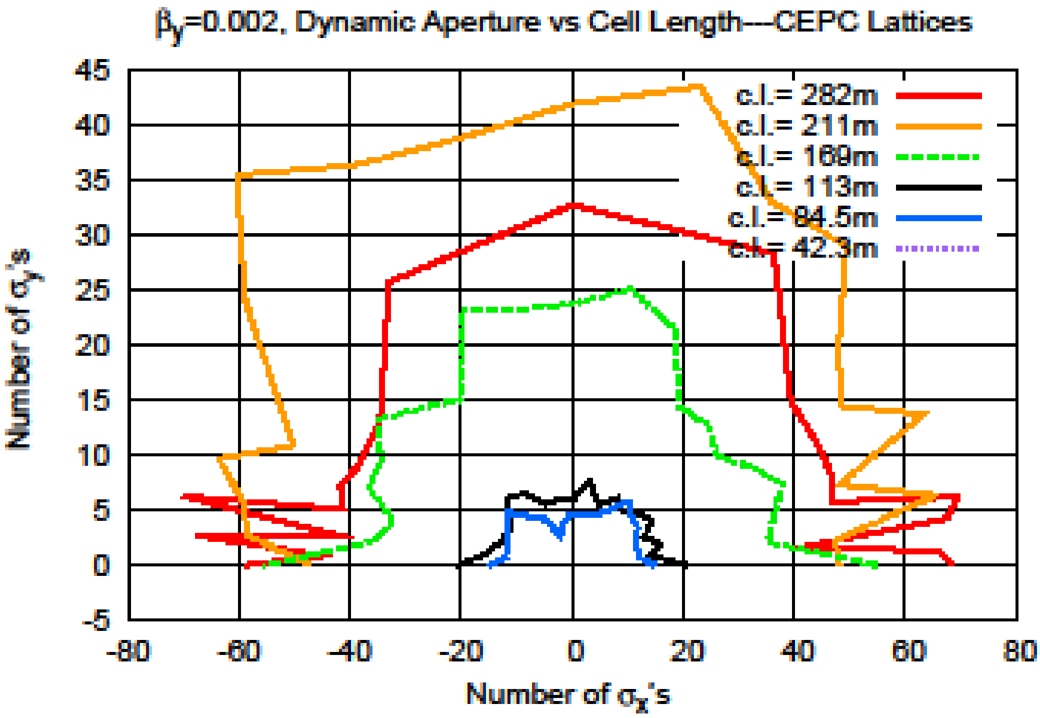

Figure 6. Acceptances plotted in units of beam sigmas for the six toy lattices.

Emittance and acceptance parameters for the six lattices are tabulated in Table 3. CEPC20.0 approximates the August 2015 CEPC design. (As indicated by the shaded row). CEPC5.0 approximates to constant dispersion scaling from LEP and the linear fit in Figure 1. The emittance ratio in these tables, ε_y/ε_x=0.068 is determined from the beam-beam saturated-tuneshift model. (Not by *ad hoc* assignement of a numerical value, such as m=0.003, to a ``coupling coefficient" which, in theory, scales as 1/γ, and should be completely negligible.)

As explained previously (and in greater detail in my earlier CEPC white paper[1], for conservative transverse insensitivity, the fine tuning penalty FOD should be less than 50. Values of FOD for the six test lattices are evaluated in Table 4.

Table 3. Emittance and acceptance parameters for the six test lattices; above the double line for Scan I, below for Scan IV.

Lattice name	cell length	l^*	ϵ_x	$\epsilon_x^{\text{accept.}}$	ϵ_y	$\epsilon_y^{\text{accept.}}$
unit	m	m	nm	nm	nm	nm
CEPC3.0	281.7	0.8	82.3	3.4e+06	5.61	1.184E+07
CEPC4.0	211.3	0.8	34.8	1.7e+07	2.36	4.504E+07
CEPC5.0	169.0	0.8	19.4	1.086e+07	1.32	1.917E+07
CEPC7.5	112.7	0.8	9.27	1.52e+06	0.630	4.346E+06
CEPC10.0	84.5	0.8	2.33	6.19e+05	0.158	1.714E+06
CEPC20.0	42.3	0.8	0.293	1.3E+05	0.0199	9.82E+04
CEPC10.0 (best so far)	84.5	2.0	2.25		0.153	$@\beta_y = 0.01$ m
CEPC10.0 (better?)	84.5	2.0	2.26		0.154	$@\beta_y = 0.0014$ m

Table 4. Factors entering the fine tuning penalty function FOD for the six test lattices; above the double line for Scan I, below for Scan IV.

Lattice name	cell-length	l^*	β_x^{max}	β_y^{max}	$\langle D \rangle$	transverse FOD $\beta_y^{\text{max}}/(\langle D \rangle \times c.l.)$
unit	m		m	m	m	1/m
CEPC3.0	281.7	0.8	500	3510	3.044	4.1
CEPC4.0	211.3	0.8	412	3080	1.703	8.6
CEPC5.0	169.0	0.8	420	3220	1.108	17.7
CEPC7.5	112.7	0.8	615	3005	0.488	33.8
CEPC10.0	84.5	0.8	190	3100	0.278	132
CEPC20.0	42.3	0.8	170	2780	0.0618	1060
CEPC10.0 (best so far)	84.5	2.0	310	1850	0.277	79 $@\beta_y = 0.01$ m
CEPC10.0 (better?)	84.5	2.0	312	9900	0.278	421 $@\beta_y = 0.0014$ m

3.1.3.1 ***Scan I cell length optimization***

Scan I results are plotted in Figure 7. This plot is shown more as an example than as a definitive result. It shows how maximizing the luminosity while limiting FOD is supposed to work. Superficially the maximum luminosity is for the CEPC10.0 case. But the maximum fine tuning penalty is badly exceeded in this case. The nominal optimum is where the green dashed FOD curve crosses the black dotted FOD=50/m constant line.

Table 5. FOMs, FODs, luminosities and other parameters for the six test lattices, six test lattices; above the double line for Scan I, below for Scan IV.

Lattice name	cell-length	l^*	ϵ_x	ϵ_y	$\Delta\delta$	$\frac{\Delta\delta}{\sqrt{\epsilon_x \epsilon_y}}$ FOM/l^*	$\mathcal{L}$
unit	m	m	nm	nm		10^9	$10^{34}\,\mathrm{m}^{-2}\mathrm{s}^{-1}$
CEPC3.0	281.7	0.8	82.3	5.61	0.022	1.023E-3	
CEPC4.0	211.3	0.8	34.8	2.36	0.020	2.21E-3	
CEPC5.0	169.0	0.8	19.4	1.32	0.020	3.95E-3	
CEPC7.5	112.7	0.8	9.27	0.630	0.015	6.21E-3	
CEPC10.0	84.5	0.8	2.33	0.158	0.006	9.89E-3	
CEPC20.0	42.3	0.8	0.293	0.0199	0.0005	6.55E-3	
CEPC10.0 (best so far)	84.5	2.0	2.25	0.146	0.031	0.0540	1.5 @$\beta_y = 0.005\,\mathrm{m}$
CEPC10.0 (better?)	84.5	2.0	2.25	0.146	0.031	0.0540	2.5 @$\beta_y = 0.003\,\mathrm{m}$
CEPC10.0 (better?)	84.5	2.0	2.26	0.147	0.033	0.0572	4.9 @$\beta_y = 0.0015\,\mathrm{m}$

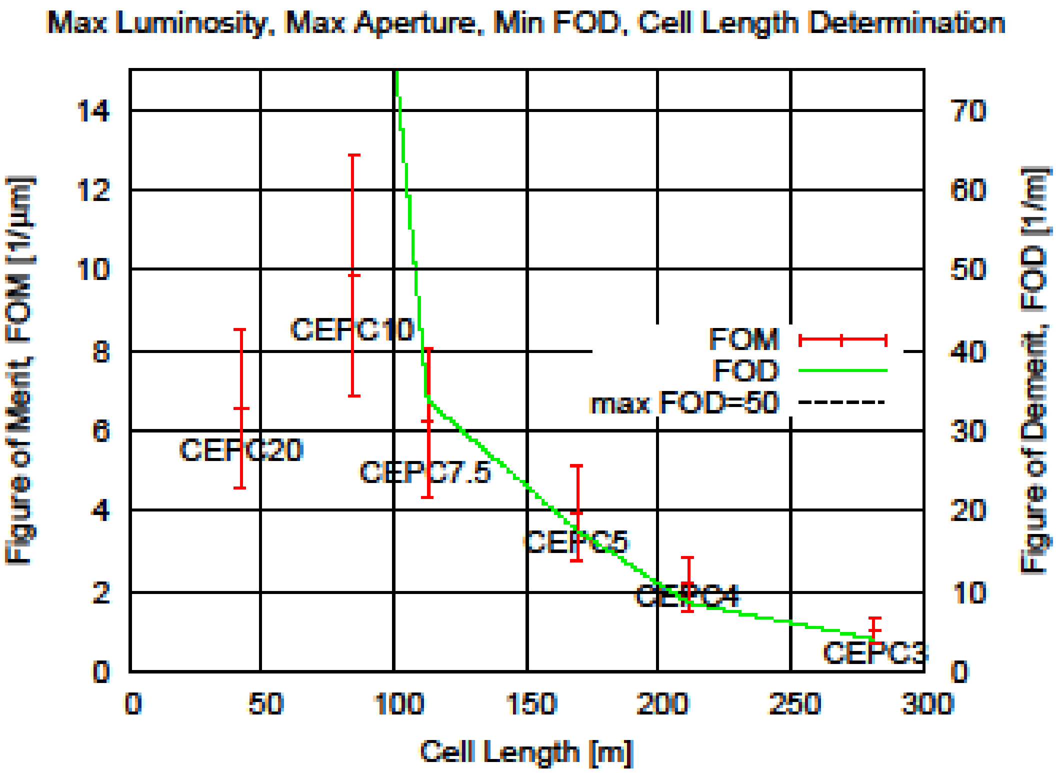

Figure 7. Plot of luminosity (data points) and fine tuning penalty function FOD (smooth dashed green curve) for Scan I. The maximum value of luminosity, consistent with keeping FOD below its limiting value is given by the point where the dashed green curve crosses the black dotted line.

3.1.3.2 *Scan IV dependence of momentum acceptance on β^*_y.*

Figure 8 shows, as expected, comfortably large acceptances for β_y^* =10 mm. This is promising for ``top-off'' injection. More surprising is Figure 9, which shows acceptances for the same lattice, but with β^*_y =2 mm. The jagged contours are indicative of nearby nonlinear resonances. But the range of momenta for which the aperture is acceptably large is as great as the β^*_y =10 mm range shown in the previous figure. This means one can reduce β^*_y almost arbitrarily without seriously harming the momentum acceptance. Of course β_y^(max) α $1/\beta_y^*$, which ``blows''  the ``fine tuning penalty'' budget for small β^*_y.

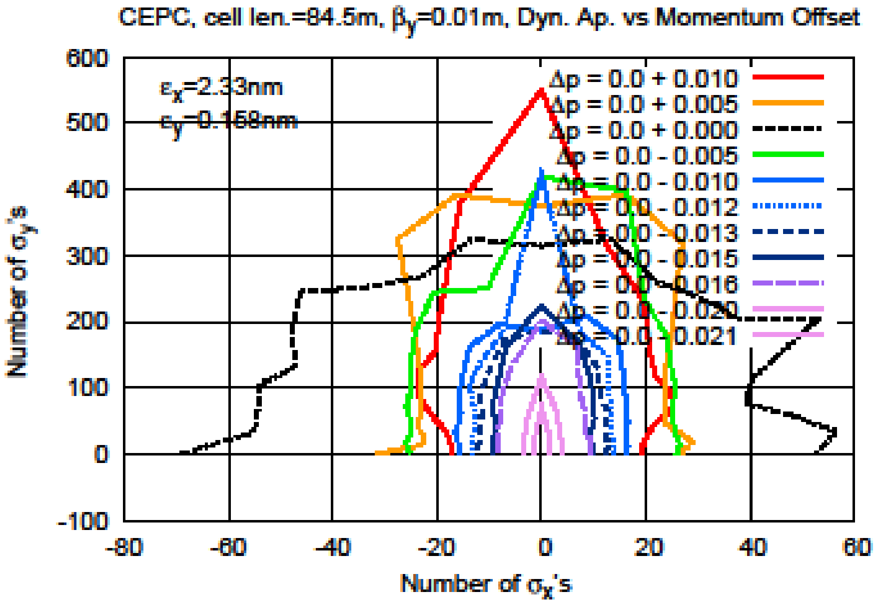

Figure 8. Dynamic aperture plots for lattice CEPC10.0 with β^*_y =10 mm for a range of beam momentum offsets.

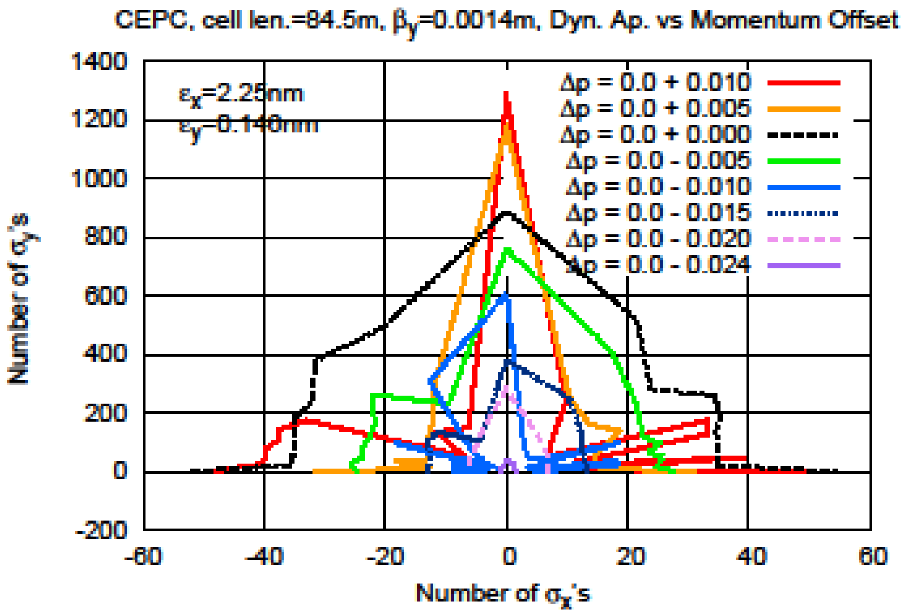

Figure 9. Dynamic aperture plots for lattice CEPC10.0 with β^*_y =2 mm for a range of beam momentum offsets. Though jagged, indicating nearby nonlinear resonances, the momentum acceptance is as good as in the previous β^*_y =10 mm case.

3.1.3.3 *Best so far lattice functions; l_c = 85 m, l^*=2.0 m*

Lattice functions for the CEPC10.0 lattice with β^*_y=10 mm are shown in Figure 10.For increased luminosity β^*_y would need to be decreased from this value. But the FOD value is 1800/(85 x 0.278)=76 m which already exceeds the nominal 50/m maximum. If the FOD=50/m limit is too conservative, then the luminosity can be increased by reducing β^*_y.

The left column of graphs in Figure 10 show a short lattice section starting at the IP. The graphs on the right show the entire ring. The middle figure on the right indicates the beta function mismatch mentioned earlier. This mismatch has seemed to be harmless in tracking studies. This has been the basis for my phrase ``we may as well have the game as the name'', meaning that having large beta functions at locations in every cell is not essentially worse than having large beta functions at just a few locations (for example in a local chromatic correction section).

3.1.4 **Predicted CEPC10.0 Luminosities: Single Ring Optics**

Luminosity predictions for the CEPC10.0 lattice are shown in Table 6. The entries in this (and following) tables ignore the FOD fine tuning penalty by assuming that β^*_y can be reduced arbitrarily. As such they are appropriate for comparisons with luminosity predictions that assume the FOM<50/m fine tuning penalty limit is too conservative (perhaps simply replacing this FOM by β_y(max)). Seemingly favorable choices (e.g. because the number of bunches is not too large) are indicated by shaded rows. Two beams in one ring is assumed. Also the possibility of bunch separation tricks, such as bunch trains with crossing angle, is not exploited. Pretzel beam separation requires the number of bunches *N_b* not to exceed half of the horizontal tune, which is 223/2=110 for CEPC10.0. For the Higgs energy (120 GeV) and above, this excludes entries with β^*_y <2 mm (at the top of the table). Since the fine tuning penalty function FOD limit is not respected for many entries in this table, some luminosities are overly optimistic.

Table 7 is a less busy table, showing only luminosities with the number of bunches required to not exceed 110. Where appropriate the luminosities are de-rated to account for the reduced number of bunches.

3.1.4.1 ***Low energy luminosities***

Luminosity at energies below the Higgs energy are given in Table 8. Contrary to common lore, two-beams-in-one-ring operation at the Z_0 pole, can yield very large luminosity, such as L=4.3 x 10^{35}/cm^2/s.

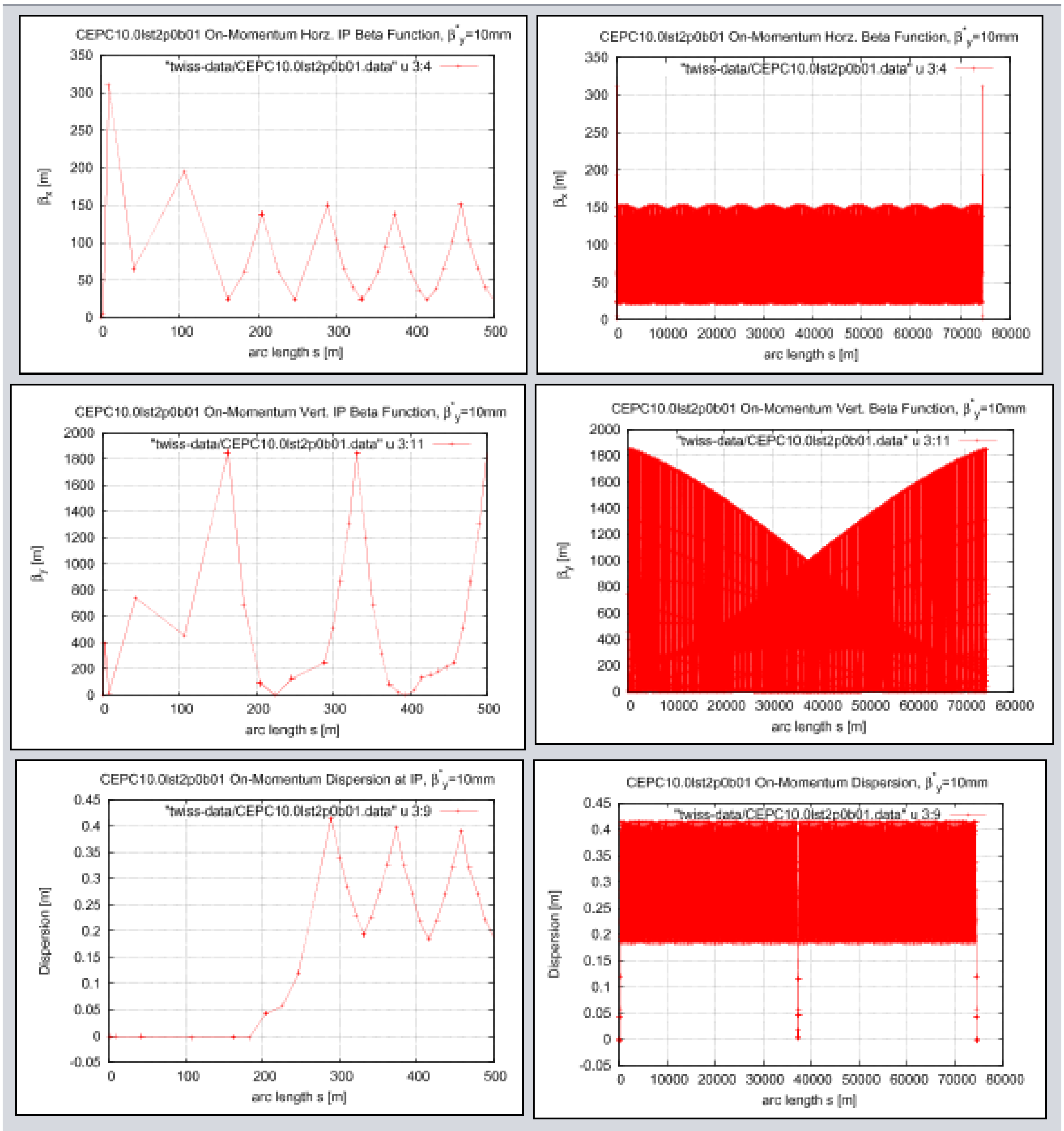

Figure 10. Lattice functions β_x, β_y, and dispersion D plots for the CEPC10.0 lattice. Short ranges starting at the IP are on the left, full ring plots are on the right. For these plots β^*_y =10 mm, which is undesirably large for maximizing luminosity, but comfortably small for limiting the fine tuning penalty function.

Table 6. Luminosity predictions for the CEPC10.0 lattice. Beam energy increases from row to row between the horizontal lines, between which the IP beta function β^*_y is held fixed. Ideally tuned, the entries in the three luminosity columns (corresponding to RF power (L^{RF}) beam-beam tune shift (L^{bb}), and beamstrahlung (L^{bs}_(trans) limits) would be equal. When unequal, the lowest of the three values has to be accepted as the actual luminosity.

E GeV	ϵ_x μm	β^*_y m	ϵ_y μm	ξ_{sat}	N_{tot}	σ_y μm	σ_x μm	$\mathcal{L}^{RF}$ 10^{34}	$\mathcal{L}^{bs}_{trans}$ 10^{34}	$\mathcal{L}^{bb}$ 10^{34}	N_b	β^*_x m	P_{RF} MW
120	2.40e-09	0.0015	1.09e-11	0.101	4.9e+13	0.128	28.12	4.9	6.94	4.903	333	0.33	50.0
150	1.02e-09	0.0015	4.63e-12	0.110	2.0e+13	0.0833	18.33	2.51	3.52	2.510	257	0.33	50.0
175	5.64e-10	0.0015	2.56e-12	0.115	1.1e+13	0.062	13.65	1.58	2.2	1.581	215	0.33	50.0
200	3.38e-10	0.0015	1.54e-12	0.120	6.4e+12	0.048	10.57	1.06	1.46	1.059	184	0.33	50.0
225	2.16e-10	0.0015	9.80e-13	0.129	4.0e+12	0.0383	8.43	0.744	1.02	0.744	160	0.33	50.0
120	5.96e-09	0.0020	2.71e-11	0.101	4.9e+13	0.233	51.20	3.68	4.94	3.677	134	0.44	50.0
150	2.53e-09	0.0020	1.15e-11	0.110	2.0e+13	0.152	33.40	1.88	2.5	1.883	103	0.44	50.0
175	1.41e-09	0.0020	6.39e-12	0.115	1.1e+13	0.113	24.87	1.19	1.56	1.186	86	0.44	50.0
200	8.43e-10	0.0020	3.83e-12	0.120	6.4e+12	0.0876	19.26	0.794	1.04	0.794	74	0.44	50.0
225	5.38e-10	0.0020	2.44e-12	0.129	4.0e+12	0.0699	15.38	0.558	0.726	0.558	64	0.44	50.0
120	2.16e-08	0.0030	9.80e-11	0.101	4.9e+13	0.542	119.28	2.45	3.06	2.451	37	0.66	50.0
150	9.18e-09	0.0030	4.17e-11	0.110	2.0e+13	0.354	77.86	1.26	1.55	1.255	29	0.66	50.0
175	5.10e-09	0.0030	2.32e-11	0.115	1.1e+13	0.264	58.00	0.79	0.966	0.790	24	0.66	50.0
200	3.06e-09	0.0030	1.39e-11	0.120	6.4e+12	0.204	44.94	0.529	0.642	0.529	20	0.66	50.0
225	1.95e-09	0.0030	8.88e-12	0.129	4.0e+12	0.163	35.90	0.372	0.447	0.372	18	0.66	50.0
120	1.09e-07	0.0050	4.97e-10	0.101	4.9e+13	1.58	346.79	1.47	1.67	1.471	7	1.1	50.0
150	4.67e-08	0.0050	2.12e-10	0.110	2.0e+13	1.03	226.57	0.753	0.841	0.753	6	1.1	50.0
175	2.59e-08	0.0050	1.18e-10	0.115	1.1e+13	0.768	168.88	0.474	0.524	0.474	5	1.1	50.0
200	1.56e-08	0.0050	7.09e-11	0.120	6.4e+12	0.595	130.96	0.318	0.348	0.318	4	1.1	50.0
225	9.96e-09	0.0050	4.53e-11	0.129	4.0e+12	0.476	104.65	0.223	0.242	0.223	3	1.1	50.0
120	9.97e-07	0.0100	4.53e-09	0.101	4.9e+13	6.73	1480.93	0.589	0.905	0.735	1	2.2	50.0
150	4.27e-07	0.0100	1.94e-09	0.110	2.0e+13	4.4	968.88	0.231	0.594	0.377	1	2.2	50.0
175	2.38e-07	0.0100	1.08e-09	0.115	1.1e+13	3.29	722.94	0.121	0.445	0.237	1	2.2	50.0
200	1.43e-07	0.0100	6.50e-10	0.120	6.4e+12	2.55	561.09	0.069	0.346	0.159	1	2.2	50.0
225	9.15e-08	0.0100	4.16e-10	0.129	4.0e+12	2.04	448.75	0.042	0.277	0.112	1	2.2	50.0

Table 7. Stripped down version of Table 6, with bunch number limit imposed.

E GeV	β^*_y m	ξ_{sat}	$\mathcal{L}^{bb}$ $10^{34}\mathrm{cm}^{-2}\mathrm{s}^{-1}$	N_b	P_{RF} MW
120	0.0015	0.1	1.62	110	50.0
150	0.0015	0.11	1.07	110	50.0
175	0.0015	0.12	0.81	110	50.0
200	0.0015	0.12	0.63	110	50.0
225	0.0015	0.13	0.51	110	50.0
120	0.0020	0.1	3.02	110	50.0
150	0.0020	0.11	1.88	103	50.0
175	0.0020	0.12	1.19	86	50.0
200	0.0020	0.12	0.79	74	50.0
225	0.0020	0.13	0.56	64	50.0
120	0.0030	0.1	2.45	37	50.0
150	0.0030	0.11	1.26	29	50.0
175	0.0030	0.12	0.79	24	50.0
200	0.0030	0.12	0.53	20	50.0
225	0.0030	0.13	0.37	18	50.0
120	0.0050	0.1	1.47	7	50.0
150	0.0050	0.11	0.75	6	50.0
175	0.0050	0.12	0.47	5	50.0
200	0.0050	0.12	0.32	4	50.0
225	0.0050	0.13	0.22	3	50.0
120	0.0100	0.1	0.74	1	50.0
150	0.0100	0.11	0.38	1	50.0
175	0.0100	0.12	0.24	1	50.0
200	0.0100	0.12	0.16	1	50.0
225	0.0100	0.13	0.11	1	50.0

Table 8. Luminosities at low energies.

E GeV	β_y^* m	ξ_{sat}	$\mathcal{L}^{bb}$ $10^{34}cm^{-2}s^{-1}$	N_b	P_{RF} MW
46	0.0015	0.038	9.65	110	50.0
60	0.0015	0.05	5.79	110	50.0
80	0.0015	0.067	3.41	110	50.0
100	0.0015	0.084	2.26	110	50.0
120	0.0015	0.1	1.62	110	50.0
46	0.0020	0.038	17.93	110	50.0
60	0.0020	0.05	10.77	110	50.0
80	0.0020	0.067	6.35	110	50.0
100	0.0020	0.084	4.21	110	50.0
120	0.0020	0.1	3.02	110	50.0
46	0.0030	0.038	43.02	110	50.0
60	0.0030	0.05	19.61	83	50.0
80	0.0030	0.067	8.27	60	50.0
100	0.0030	0.084	4.24	46	50.0
120	0.0030	0.1	2.45	37	50.0
46	0.0050	0.038	26.98	23	50.0
60	0.0050	0.05	11.77	17	50.0
80	0.0050	0.067	4.96	12	50.0
100	0.0050	0.084	2.54	9	50.0
120	0.0050	0.1	1.47	7	50.0
46	0.0100	0.038	13.49	3	50.0
60	0.0100	0.05	5.88	2	50.0
80	0.0100	0.067	2.48	1	50.0
100	0.0100	0.084	1.27	1	50.0
120	0.0100	0.1	0.74	1	50.0

3.1.5 Recapitulation

3.1.5.1 *Qualifications*

- A ring with only one IR has been investigated (to make tuning easier). Luminosities per IP are likely to be about twenty percent smaller with two IP's. Also tuning will be more difficult with two IP's.

- Only zero length quadrupoles have been used. This is an issue only for l^*=0.8, which is too small for a practical detector in any case.

- A major uncertainty concerns the fine tuning penalty FOD figure of demerit. The FOD<50/m used in this study has been very conservative. If it were valid to simply use an 10,000 m upper limit on β_y^(max) as FOD (which is what existing CEPC and FCC-ee designs seem to assume) then luminosity approaching $10^{35}/cm^2/s$ would be predicted.

- The parameters in this study are not entirely self-consistent. The worst discrepancy is a factor of 3 difference between β^*_x used in calculating the luminosity and the value actually provided by the lattice optimization procedure.

- Though pretzel separation of the two beams in one ring has been assumed, the simulations have not, in fact, had pretzel orbits.

3.1.6 **Conclusions**

The original intent of this white paper was to develop a ``ground up'' design methodology. The mere testing of this methodology has led to significantly improved understanding and the following tentative conclusions:

- *Local chromaticity compensation is unnecessary.* Two families of non-interleaved sextupoles in the arcs are sufficient to correct both IR and arc chromaticity while keeping acceptably large momentum acceptance.
- With proper choice of vertical tune, momentum acceptances in excess of 3 percent are achievable.
- Optimal values of vertical tune Q_y are close to half integers. Improved momentum acceptance there seems to be due to the detuning of off-momentum particles of pulling $Q\_y(\delta)$ away from (rather than accross) the necessarily-nearby $|\cos(\mu\_y)|>1$ precipice, as δ deviates from zero with either sign.
- With no need for local chromaticity compensation there is no need for finite dispersion nor bends near the IP, vastly reducing synchrotron radiation incident on the detector.
- The optimal cell length so far is 82\,m.

3.1.7 **References**

1. R. Talman, *Scaling Behavior of Circular Colliders Dominated by Synchrotron Radiation*, International Journal of Modern Physics A, Vol. 30 (2015) 1544003 (91 pages) World Scientific Publishing Company.
2. V.I. Telnov, *Restriction on the energy and luminosity of e+e- storage rings due to beamstrahlung*, arXiv:1203.6563v1 [physics.acc-ph] 29 Mar 2012.