

Baryogenesis and Leptogenesis

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Abstract. We summarize the state of the art of baryogenesis with particular attention to thermal leptogenesis, according to which the baryon asymmetry in the Universe is produced at very high temperatures from the decay of heavy right-handed neutrinos, and to electroweak baryogenesis, where the baryon asymmetry is generated at the electroweak phase transition.

1. Introduction

The symmetry between particles and antiparticles, firmly established in collider physics, naturally leads to the question of why the observed universe is composed almost entirely of matter with little or no primordial antimatter.

Outside of particle accelerators, antimatter can be seen in cosmic rays in the form of a few antiprotons, present at a level of around 10^{-4} in comparison with the number of protons. However, this proportion is consistent with secondary antiproton production through accelerator-like processes, $p + p \rightarrow 3p + \bar{p}$, as the cosmic rays stream towards us. Thus there is no evidence for primordial antimatter in our galaxy. Also, if matter and antimatter galaxies were to coexist in clusters of galaxies, then we would expect there to be a detectable background of γ -radiation from nucleon-antinucleon annihilations within the clusters. This background is not observed and so we conclude that there is negligible antimatter on the scale of clusters.

More generally, if large domains of matter and antimatter exist, then annihilations would take place at the interfaces between them. If the typical size of such a domain was small enough, then the energy released by these annihilations would result in a diffuse γ -ray background and a distortion of the cosmic microwave radiation, neither of which is observed. A careful numerical analysis of this problem demonstrates that the universe must consist entirely of either matter or antimatter on all scales up to the Hubble size. It therefore seems that the universe is fundamentally matter-antimatter asymmetric.

While the above considerations put an experimental upper bound on the amount of antimatter in the universe, strict quantitative estimates of the relative abundances of baryonic matter and antimatter may also be obtained from the standard cosmology. The baryon number density does not remain constant during the evolution of the universe, instead scaling like a^{-3} , where a is the cosmological scale factor [1]. It is therefore convenient to define the baryon asymmetry of the universe in terms of the quantity

$$Y_B \equiv \frac{n_B}{s}, \quad (1)$$

where $n_B = n_b - n_{\bar{b}}$ is the difference between the number of baryons and antibaryons per unit volume and $s = (2\pi^2/45)g_\star$ is the entropy density at temperature T when the thermal plasma

contained g_* relativistic degrees of freedom.

The observed baryon asymmetry of the Universe is by now accurately determined by Cosmic Microwave (CMB) Anisotropy measurements [2]

$$Y_B^{\text{CMB}} = (8.75 \pm 0.23) \times 10^{-11}. \quad (2)$$

The physics behind the determination of Y_B and the CMB anisotropy would require another paper. It suffices to say that the baryon number plays a crucial role in determining the relative amplitude of the even peaks to the odd ones. Measuring this ratio gives a determination of the baryon asymmetry of the Universe [3]. Primordial nucleosynthesis (for a review see [4]) is also one of the most powerful predictions of the standard cosmological model. The theory allows accurate predictions of the cosmological abundances of all the light elements, H, ^3He , ^4He , D, B and ^7Li , while requiring only the single input parameter Y_B which has been constant since nucleosynthesis. The range of Y_B consistent with nucleosynthesis agrees with the one provided by CMB anisotropies, even though they are relevant on various ranges of temperatures, about MeV for nucleosynthesis and a fraction of eV for the CMB anisotropies.

To see that the standard cosmological model cannot explain the observed value of Y_B , suppose that initially we start with $Y_B = 0$ [5, 6]. We can compute the final number density of nucleons b that are left over after annihilations have frozen out. At temperatures $T < 1$ GeV the equilibrium abundance of nucleons and antinucleons is [1]

$$\frac{n_b}{n_\gamma} \approx \frac{n_{\bar{b}}}{n_\gamma} \approx \left(\frac{m_p}{T}\right)^{3/2} e^{-\frac{m_p}{T}}. \quad (3)$$

When the universe cools off, the number of nucleons and antinucleons decreases as long as the annihilation rate $\Gamma_{\text{ann}} \approx n_b \langle \sigma_A v \rangle$ is larger than the expansion rate of the universe $H \approx 1.66 g_*^{1/2} \frac{T^2}{M_{\text{Pl}}}$. The thermally averaged annihilation cross section $\langle \sigma_A v \rangle$ is of the order of m_π^2 , so at $T \approx 20$ MeV, $\Gamma_{\text{ann}} \approx H$, and annihilations freeze out, nucleons and antinucleons being so rare that they cannot annihilate any longer. Therefore, from (3) we obtain

$$\frac{n_b}{n_\gamma} \approx \frac{n_{\bar{b}}}{n_\gamma} \approx 10^{-18}, \quad (4)$$

which is much smaller than the value required by nucleosynthesis. In conclusion, in the standard cosmological model there is no explanation for the value of the observed baryon asymmetry, if we start from $Y_B = 0$. An initial asymmetry may be imposed by hand as an initial condition, but this would violate any naturalness principle. Rather, the guiding principle behind *modern* cosmology is to attempt to explain the initial conditions required by the standard cosmology on the basis of quantum field theories of elementary particles in the early universe. In this context, the generation of the observed value of Y_B is referred to as *baryogenesis*.

The goal of this paper is to present the thermal leptogenesis scenario. As we shall see, it is strictly related to neutrino physics, the main subject of this School. As such, one would open, through low energy neutrino measurements, to open a window on the early Universe dynamics. For more details about the subject, the reader is forwarded to the excellent recent review of the subject [7]. Other useful reviews are [5, 6].

A note about our conventions. Throughout we use a metric with signature +2 and, unless explicitly stated otherwise, we employ units such that $\hbar = c = k = 1$.

2. The Sakharov criteria

A small baryon asymmetry Y_B may have been produced in the early universe if three necessary conditions are satisfied [8]: *i*) baryon number (B) violation; *ii*) violation of C (charge conjugation

symmetry) and CP (the composition of parity and C) and *iii*) departure from thermal equilibrium. The first condition should be clear since, starting from a baryon symmetric universe with $Y_B = 0$, baryon number violation must take place in order to evolve into a universe in which Y_B does not vanish. The second Sakharov criterion is required because, if C and CP are exact symmetries, then one can prove that the total rate for any process which produces an excess of baryons is equal to the rate of the complementary process which produces an excess of antibaryons and so no net baryon number can be created. That is to say that the thermal average of the baryon number operator B , which is odd under both C and CP , is zero unless those discrete symmetries are violated. CP violation is present either if there are complex phases in the lagrangian which cannot be reabsorbed by field redefinitions (explicit breaking) or if some Higgs scalar field acquires a VEV which is not real (spontaneous breaking). We will discuss this in detail shortly.

Finally, to explain the third criterion, one can calculate the equilibrium average of B

$$\begin{aligned}\langle B \rangle_T &= \text{Tr} (e^{-\beta H} B) = \text{Tr} [(CPT)(CPT)^{-1} e^{-\beta H} B] \\ &= \text{Tr} (e^{-\beta H} (CPT)^{-1} B (CPT)) = -\text{Tr} (e^{-\beta H} B) ,\end{aligned}\quad (5)$$

where we have used that the Hamiltonian H commutes with CPT . Thus $\langle B \rangle_T = 0$ in equilibrium and there is no generation of net baryon number.

Of the three Sakharov conditions, baryon number violation and C and CP violation may be investigated only within a given particle physics model, while the third condition – the departure from thermal equilibrium – may be discussed in a more general way, as we shall see. Let us discuss the Sakharov criteria in more detail.

3. B -violation in the Electroweak theory.

It is well-known that the most general Lagrangian invariant under the SM gauge group and containing only color singlet Higgs fields is automatically invariant under global abelian symmetries which may be identified with the baryonic and leptonic symmetries. These, therefore, are accidental symmetries and as a result it is not possible to violate B and L at tree-level or at any order of perturbation theory. Nevertheless, in many cases the perturbative expansion does not describe all the dynamics of the theory and, indeed, in 1976 't Hooft realized that nonperturbative effects (instantons) may give rise to processes which violate the combination $(B + L)$, but not the orthogonal combination $(B - L)$. The probability of these processes occurring today is exponentially suppressed and probably irrelevant. However, in more extreme situations – like the primordial universe at very high temperatures – baryon and lepton number violating processes may be fast enough to play a significant role in baryogenesis. Let us have a closer look. At the quantum level, the baryon and the lepton symmetries are anomalous

$$\partial_\mu j_B^\mu = \partial_\mu j_L^\mu = n_f \left(\frac{g^2}{32\pi^2} W_{\mu\nu}^a \tilde{W}^{a\mu\nu} - \frac{g'^2}{32\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu} \right) , \quad (6)$$

where g and g' are the gauge couplings of $SU(2)_L$ and $U(1)_Y$, respectively, n_f is the number of families and $\tilde{W}^{\mu\nu} = (1/2)\epsilon^{\mu\nu\alpha\beta} W_{\alpha\beta}$ is the dual of the $SU(2)_L$ field strength tensor, with an analogous expression holding for $\tilde{F}$. To understand how the anomaly is closely related to the vacuum structure of the theory, we may compute the change in baryon number from time $t = 0$ to some arbitrary final time $t = t_f$. For transitions between vacua, the average values of the field strengths are zero at the beginning and the end of the evolution. The change in baryon number may be written as

$$\Delta B = \Delta N_{CS} \equiv n_f [N_{CS}(t_f) - N_{CS}(0)] . \quad (7)$$

where the Chern-Simons number is defined to be

$$N_{CS}(t) \equiv \frac{g^2}{32\pi^2} \int d^3x \epsilon^{ijk} \text{Tr} \left(A_i \partial_j A_k + \frac{2}{3} ig A_i A_j A_k \right) . \quad (8)$$

Although the Chern-Simons number is not gauge invariant, the change ΔN_{CS} is. Thus, changes in Chern-Simons number result in changes in baryon number which are integral multiples of the number of families n_f . Gauge transformations $U(x)$ which connects two degenerate vacua of the gauge theory may change the Chern-Simons number by an integer n , the winding number. If the system is able to perform a transition from the vacuum $\mathcal{G}_{\text{vac}}^{(n)}$ to the closest one $\mathcal{G}_{\text{vac}}^{(n\pm 1)}$, the Chern-Simons number is changed by unity and $\Delta B = \Delta L = n_f$. Each transition creates 9 left-handed quarks (3 color states for each generation) and 3 left-handed leptons (one per generation). However, adjacent vacua of the electroweak theory are separated by a ridge of configurations with energies larger than that of the vacuum. The lowest energy point on this ridge is a saddle point solution to the equations of motion with a single negative eigenvalue, and is referred to as the *sphaleron*. The probability of baryon number nonconserving processes at zero temperature has been computed by 't Hooft and is highly suppressed by a factor $\exp(-4\pi/\alpha_W)$, where $\alpha_W = g^2/4\pi$. This factor may be interpreted as the probability of making a transition from one classical vacuum to the closest one by tunneling through an energy barrier of height ≈ 10 TeV corresponding to the sphaleron. On the other hand, one might think that fast baryon number violating transitions may be obtained in physical situations which involve a large number of fields. Since the sphaleron may be produced by collective and coherent excitations containing $\approx 1/\alpha_W$ quanta with wavelength of the order of $1/M_W$, one expects that at temperatures $T \gg M_W$, these modes essentially obey statistical mechanics and the transition probability may be computed via classical considerations. Analogously to the case of zero temperature and since the transition which violates the baryon number is sustained by the sphaleron configuration, the thermal rate of baryon number violation in the *broken* phase is proportional to $\exp(-S_3/T)$, where S_3 is the three-dimensional action computed along the sphaleron configuration,

$$S_3 = E_{sp}(T) \equiv (M_W(T)/\alpha_W)\mathcal{E} , \quad (9)$$

with the dimensionless parameter $\mathcal{E}$ lying in the range $3.1 < \mathcal{E} < 5.4$ depending on the Higgs mass. The prefactor of the thermal rate reads

$$\Gamma_{sp}(T) = \mu \left(\frac{M_W}{\alpha_W T} \right)^3 M_W^4 \exp \left(-\frac{E_{sph}(T)}{T} \right) , \quad (10)$$

where μ is a dimensionless constant. Although the Boltzmann suppression in (10) appears large, it is to be expected that, when the electroweak symmetry becomes restored at a temperature of around 100 GeV, there will no longer be an exponential suppression factor. Although calculation of the baryon number violating rate in the high temperature *unbroken* phase is extremely difficult, a simple estimate is possible. The only important scale in the symmetric phase is the magnetic screening length given by $\xi = (\alpha_W T)^{-1}$. Thus, on dimensional grounds, we expect the rate per unit volume of sphaleron events to be

$$\Gamma_{sp}(T) = \kappa (\alpha_W T)^4 , \quad (11)$$

with κ another dimensionless constant. The rate of sphaleron processes can be related to the diffusion constant for Chern-Simons number by a fluctuation-dissipation theorem. In almost all numerical calculations of the sphaleron rate, this relationship is used and what is actually evaluated is the diffusion constant. The first attempts to numerically estimate κ in this way yielded $\kappa \approx 0.1 - 1$, but the approach suffered from limited statistics and large volume

systematic errors. Nevertheless, more recent numerical attempts found approximately the same result. However, these approaches employ a poor definition of the Chern-Simons number which compromises their reliability.

This simple scaling argument leading to (11) is not correct however. Damping effects in the plasma suppress the rate by an extra power of α_W to give $\Gamma_{sp} \approx \alpha_W^5 T^4$. Indeed, since the transition rate involves physics at soft energies $g^2 T$ that are small compared to the typical hard energies $\approx T$ of the thermal excitations in the plasma, the simplest way of analyzing the problem is to consider an effective theory for the soft modes, where the hard modes have been integrated out, and to keep the dominant contributions, the so-called hard thermal loops. It is the resulting typical frequency ω_c of a gauge field configuration immersed in the plasma and with spatial extent $(g^2 T)^{-1}$ that determines the change of baryon number per unit time and unit volume. This frequency ω_c has been estimated to be $\approx g^4 T$ when taking into account the damping effects of the hard modes. Using the effective dynamics of soft nonabelian gauge fields at finite temperature one can find that $\Gamma_{sp} \approx \alpha_W^5 T^4 \ln(1/\alpha_W)$. Lattice simulations with hard-thermal loops seem to indicate the $\Gamma_{sp} \approx 30\alpha_W^5 T^4$, which is not far from $\alpha_W^4 T^4$.

4. The standard out-of-equilibrium decay scenario

Out of the three Sakharov's conditions, the baryon number violation and C and CP violation may be investigated thoroughly only within a given particle physics model, while the third condition – the departure from thermal equilibrium – may be discussed in a more general way. Very roughly speaking, the various models of baryogenesis that have been proposed so far fall into two categories [5]:

- models where the out-of-equilibrium condition is attained thanks to the expansion of the Universe and the presence of heavy decaying particles;
- models where the departure from thermal equilibrium is attained during the phase transitions which lead to the breaking of some global and/or gauge symmetry.

In this paper we will analyse the first category – the standard out-of-equilibrium decay scenario [1].

4.1. The conditions for the out-of-equilibrium decay scenario

It is obvious that in a static Universe any particle, even very weakly interacting, will attain sooner or later thermodynamical equilibrium with the surrounding plasma. The expansion of the Universe, however, introduces a finite time-scale, $\tau_U \approx H^{-1}$. Let suppose that X is a *baryon number violating* superheavy boson field (vector or scalar) which is coupled to lighter fermionic degrees of freedom with a strength $\alpha_X^{1/2}$ (either a gauge coupling α_{gauge} or a Yukawa coupling α_Y).

In the case in which the couplings are renormalizable, the decay rate Γ_X of the superheavy boson may be easily estimated to be

$$\Gamma_X \approx \alpha_X M_X, \quad (12)$$

where M_X is the mass of the particle X . In the opposite case in which the boson is a gauge *singlet* scalar field and it only couples to light matter through gravitational interactions – this is the case of singlets in the hidden sector of supergravity models – the decay rate is from dimensional arguments

$$\Gamma_X \approx \frac{M_X^3}{M_{\text{P}}^2}. \quad (13)$$

At very large temperatures $T \gg M_X$, it is assumed that all the particles species are in thermal equilibrium, *i.e.* $n_X \approx n_{\bar{X}} \approx n_\gamma$ (up to statistical factors) and that $B = 0$. At $T < M_X$ the

equilibrium abundance of X and $\bar{X}$ relative to photons is given by

$$\frac{n_X^{\text{EQ}}}{n_\gamma} \approx \frac{n_{\bar{X}}^{\text{EQ}}}{n_\gamma} \approx \left(\frac{M_X}{T} \right)^{3/2} e^{-\frac{M_X}{T}}, \quad (14)$$

where we have neglected the chemical potential μ_X .

For the X and $\bar{X}$ particles to maintain their equilibrium abundances, they must be able to diminish their number rapidly with respect to the Hubble rate $H(T)$. The conditions necessary for doing so are easily quantified. The superheavy X and $\bar{X}$ particles may attain equilibrium through decays with rate Γ_X , inverse decays with rate Γ_X^{ID}

$$\Gamma_X^{\text{ID}} \approx \Gamma_X \begin{cases} 1 & T > M_X, \\ (M_X/T)^{3/2} \exp(-M_X/T) & T < M_X, \end{cases} \quad (15)$$

and annihilation processes with rate $\Gamma_X^{\text{ann}} \propto n_X$. The latter, however are “self-quenching” and therefore less important than the decay and inverse decay processes. They will be ignored from now on. Of crucial interest are the B -nonconserving scattering processes $2 \leftrightarrow 2$ mediated by the X and $\bar{X}$ particles with rate Γ_X^{S}

$$\Gamma_X^{\text{S}} \approx n\sigma \approx \alpha^2 T^3 \frac{T^2}{(M_X^2 + T^2)^2}, \quad (16)$$

where $\alpha \approx g^2/4\pi$ denotes the coupling strength of the X boson. At high temperatures, the $2 \leftrightarrow 2$ scatterings cross section is $\sigma \approx \alpha^2/T^2$, while at low temperatures $\sigma \approx \alpha^2 T^2/M_X^4$.

For baryogenesis, the most important rate is the decay rate, as decays (and inverse decays) are the mechanism that regulates the number of X and $\bar{X}$ particles in the plasma. It is therefore useful to define the following quantity

$$K \equiv \left. \frac{\Gamma_X}{H} \right|_{T=M_X} \quad (17)$$

which measures the effectiveness of decays at the crucial epoch ($T \approx M_X$) when the X and $\bar{X}$ particles must decrease in number if they are to stay in equilibrium. Note also that for $T < M_X$, K determines the effectiveness of inverse decays and $2 \leftrightarrow 2$ scatterings as well: $\Gamma_X^{\text{ID}}/H \approx (M_X/T)^{3/2} \exp(-M_X/T) K$ and $\Gamma_X^{\text{S}}/H \approx \alpha(T/M_X)^5 K$.

Now, if $K \gg 1$, and therefore

$$\Gamma_X \gg H|_{T=M_X}, \quad (18)$$

then the X and $\bar{X}$ particles will adjust their abundances by decaying to their equilibrium abundances and no baryogenesis can be induced by their decays –this is simply because out-of-equilibrium conditions are not attained. Given the expression $H \sim g_*^{1/2} T^2/M_{\text{P}}$ for the expansion rate of the Universe, the condition (18) is equivalent to

$$M_X \ll g_*^{-1/2} \alpha_X M_{\text{P}} \quad (19)$$

for strongly coupled scalar bosons, and to

$$M_X \gg g_*^{1/2} M_{\text{P}}, \quad (20)$$

for gravitationally coupled X particles. Obviously, this last condition is never satisfied for $M_X < M_{\text{P}}$.

However, if the decay rate is such that $K \ll 1$, and therefore

$$\Gamma_X < H|_{T=M_X}, \quad (21)$$

then the X and $\bar{X}$ particles cannot decay on the expansion time-scale τ_U and so *they remain as abundant as photons* for $T < M_X$. In other words, at some temperature $T > M_X$, the superheavy bosons X and $\bar{X}$ are so weakly interacting that they cannot catch up with the expansion of the Universe and they decouple from the thermal bath when still *relativistic*, $n_X \approx n_{\bar{X}} \approx n_\gamma$ at the time of decoupling. Therefore, at temperature $T \approx M_X$, they will populate the Universe with an abundance which is much larger than the equilibrium one. This *overabundance* with respect to the equilibrium abundance is precisely the departure from thermal equilibrium needed to produce a final nonvanishing baryon asymmetry. Condition (21) is equivalent to

$$M_X > g_*^{-1/2} \alpha_X M_P \quad (22)$$

for strongly coupled scalar bosons, and to

$$M_X < g_*^{1/2} M_P, \quad (23)$$

for gravitationally coupled X particles. It is clear that this last condition is always satisfied, whereas the condition (22) is based on the smallness of the quantity $g_*^{-1/2} \alpha_X$. In particular, if the X particle is a gauge boson, $\alpha_X \approx \alpha_{\text{gauge}}$ can span the range $(2.5 \times 10^{-2} - 10^{-1})$, while g_* is about 10^2 . In this way we obtain from (22) that the condition of out-of-equilibrium can be satisfied for

$$M_X > (10^{-4} - 10^{-3}) M_P \approx (10^{15} - 10^{16}) \text{ GeV}. \quad (24)$$

If X is a scalar boson, its coupling α_Y to fermions f with mass m_f is proportional to the squared mass of the fermions

$$\alpha_Y \approx \left(\frac{m_f}{m_W} \right)^2 \alpha_{\text{gauge}}, \quad (25)$$

where m_W is the W -boson mass and α_Y is typically in the range $(10^{-2} - 10^{-7})$, from where

$$M_X > (10^{-8} - 10^{-3}) M_P \approx (10^{10} - 10^{16}) \text{ GeV}. \quad (26)$$

Obviously, condition (26) is more easily satisfied than condition (24) and we conclude that baryogenesis is more easily produced through the decay of superheavy scalar bosons. On the other hand, as we have seen above, the condition (23) tells us that the out-of-equilibrium condition is automatically satisfied for gravitationally interacting particles.

4.2. The production of the baryon asymmetry

Let us now follow the subsequent evolution of the X and $\bar{X}$ particles. When the Universe becomes as old as the lifetime of these particles, $t \approx H^{-1} \approx \Gamma_X^{-1}$, they start decaying. This takes place at a temperature T_D defined by the condition

$$\Gamma_X \approx H|_{T=T_D}, \quad (27)$$

i.e. at

$$T_D \approx g_*^{-1/4} \alpha_X^{1/2} (M_X M_P)^{1/2} < M_X, \quad (28)$$

where the last inequality comes from (22) and is valid for particles with unsuppressed couplings. For particles with only gravitational interactions

$$T_D \approx g_*^{-1/4} M_X \left(\frac{M_X}{M_P} \right)^{1/2} < M_X, \quad (29)$$

the last inequality coming from (23). At $T \approx T_D$, X and $\bar{X}$ particles start to decay and their number decrease. If their decay violate the baryon number, they will generate a net baryon number per decay.

Suppose now that the X particle may decay into two channels, let us denote them by a and b , with different baryon numbers B_a and B_b , respectively. Correspondingly, the decay channels of $\bar{X}$, $\bar{a}$ and $\bar{b}$, have baryon numbers $-B_a$ and $-B_b$, respectively. Let $r(\bar{\tau})$ be the branching ratio of the $X(\bar{X})$ in channel $a(\bar{a})$ and $1 - r(\bar{\tau})$ the branching ratio of $X(\bar{X})$ in channel $b(\bar{b})$,

$$\begin{aligned} r &= \frac{\Gamma(X \rightarrow a)}{\Gamma_X}, \\ \bar{r} &= \frac{\Gamma(\bar{X} \rightarrow \bar{a})}{\Gamma_X}, \\ 1 - r &= \frac{\Gamma(X \rightarrow b)}{\Gamma_X}, \\ 1 - \bar{r} &= \frac{\Gamma(\bar{X} \rightarrow \bar{b})}{\Gamma_X}, \end{aligned} \quad (30)$$

where we have been using the fact that the *total* decay rates of X and $\bar{X}$ are equal because of the *CPT* theorem plus unitarity. The average net baryon number produced in the X decays is

$$rB_a + (1 - r)B_b, \quad (31)$$

and that produced by $\bar{X}$ decays is

$$-\bar{r}B_a - (1 - \bar{r})B_b. \quad (32)$$

Finally, the mean net baryon number produced in X and $\bar{X}$ decays is

$$\Delta B = (r - \bar{r})B_a + [(1 - r) - (1 - \bar{r})]B_b = (r - \bar{r})(B_a - B_b). \quad (33)$$

Equation (33) may be easily generalized to the case in which $X(\bar{X})$ may decay into a set of final states $f_n(\bar{f}_n)$ with baryon number $B_n(-B_n)$

$$\Delta B = \frac{1}{\Gamma_X} \sum_n B_n [\Gamma(X \rightarrow f_n) - \Gamma(\bar{X} \rightarrow \bar{f}_n)]. \quad (34)$$

At the decay temperature, $T_D < M_X$, because $K \ll 1$ both inverse decays and $2 \leftrightarrow 2$ baryon violating scatterings are impotent and can be safely ignored and thus the net baryon number produced per decay ΔB is not destroyed by the net baryon number $-\Delta B$ produced by the inverse decays and by the baryon number violating scatterings.

At $T \approx T_D$, $n_X \approx n_{\bar{X}} \approx n_\gamma$ and therefore the net baryon number density produced by the out-of-equilibrium decay is

$$n_B = \Delta B n_X. \quad (35)$$

The three Sakharov ingredients for producing a net baryon asymmetry can be easily traced back here:

- If B is not violated, then $B_n = 0$ and $\Delta B = 0$.
- If C and CP are not violated, then $\Gamma(X \rightarrow f_n) = \Gamma(\bar{X} \rightarrow \bar{f}_n)$, and also $\Delta B = 0$.
- In thermal equilibrium, the inverse processes are not suppressed and the net baryon number produced by decays will be erased by the inverse decays.

Since each decay produces a mean net baryon number density $n_B = \Delta B n_X \approx \Delta B n_\gamma$ and since the entropy density is $s \approx g_\star n_\gamma$, the net baryon number produced is

$$B \equiv \frac{n_B}{s} \approx \frac{\Delta B n_\gamma}{g_\star n_\gamma} \approx \frac{\Delta B}{g_\star}. \quad (36)$$

Taking $g_\star \approx 10^2$, we see that only tiny C and CP violations are required to generate $\Delta B \approx 10^{-8}$, and thus $B \approx 10^{-10}$.

To obtain (36) we have assumed that the entropy release in X decays is negligible. However, sometimes, this is not a good approximation (especially if the X particles decay very late, at $T_D \ll M_X$, which is the case of gravitationally interacting particles). In that case, assuming that the energy density of the Universe at T_D is dominated by X particles

$$\rho_X \approx M_X n_X, \quad (37)$$

and that it is converted entirely into radiation at the reheating temperature T_{RH}

$$\rho = \frac{\pi^2}{30} g_\star T_{RH}^4, \quad (38)$$

we obtain

$$n_X \approx \frac{\pi^2}{30} g_\star \frac{T_{RH}^4}{M_X}. \quad (39)$$

We can therefore write the baryon number as

$$B \approx \frac{3 T_{RH}}{4 M_X} \Delta B. \quad (40)$$

We can relate T_{RH} with the decay rate Γ_X using the decay condition

$$\Gamma_X^2 \approx H^2(T_D) \approx \frac{8\pi\rho_X}{3M_P^2} \quad (41)$$

and so we can write

$$B \approx \left(\frac{g_\star^{-1/2} \Gamma_X M_P}{M_X^2} \right)^{1/2} \Delta B. \quad (42)$$

For the case of strongly decaying particles (through renormalizable interactions) we obtain

$$B \approx \left(\frac{g_\star^{-1/2} \alpha M_P}{M_X} \right)^{1/2} \Delta B. \quad (43)$$

while for the case of weakly decaying particles (through gravitational interactions) we obtain

$$B \approx \left(\frac{g_\star^{-1/2} M_X}{M_P} \right)^{1/2} \Delta B. \quad (44)$$

In the other extreme regime $K \gg 1$, one expects the abundance of X and $\bar{X}$ bosons to track the equilibrium values as $\Gamma_X \gg H$ for $T \approx M_X$. If the equilibrium is tracked precisely enough, there will be no departure from thermal equilibrium and no baryon number may evolve. The intermediate regime, $K \approx 1$, is more interesting and to address it one has to invoke numerical analysis involving Boltzmann equations for the evolution of B . The numerical analysis essentially confirms the qualitative picture we have described so far and its discussion is beyond the scope of this paper.

5. Baryogenesis via leptogenesis: one-flavour approximation

Baryogenesis through Leptogenesis [9] is a simple mechanism to explain this baryon asymmetry of the Universe. As we said, a lepton asymmetry is dynamically generated and then converted into a baryon asymmetry due to $(B + L)$ -violating sphaleron interactions which exist in the SM. The reader is forwarded to Refs. [11] and [12] for more details.

A simple model in which this mechanism can be implemented is “Seesaw”(type I) [13], consisting of the SM plus two or three right-handed (RH) Majorana neutrinos. In this simple extension of the SM, the usual scenario that is explored (referred to as “thermal leptogenesis”) consists of a hierarchical spectrum for the RH neutrinos, such that the lightest of the RH neutrinos is produced by thermal scattering after inflation, and subsequently decays out-of-equilibrium in a lepton number and CP-violating way, thus satisfying Sakharov’s constraints.

This section introduces notation and reviews the calculation of the lepton asymmetry when the charged lepton Yukawa couplings are neglected. As we shall see, the commonly used formulae for the final lepton asymmetry, which we report here, may not be appropriate once flavours are considered. The reader should be patient, flavour effects will be discussed later.

Our starting point is the Lagrangian of the SM with the addition of three right-handed neutrinos N_i ($i = 1, 2, 3$) with heavy Majorana masses $M_3 > M_2 > M_1$ and Yukawa couplings $\lambda_{\alpha i}$. Working in the basis in which the Yukawa couplings for the charged leptons are diagonal, the Lagrangian reads

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \left(\frac{M_i}{2} N_i^2 + \lambda_{\alpha i} L_\alpha H N_i \right) + \text{h.c.} . \quad (45)$$

Here L_α is the lepton doublet with flavour ($\alpha = e, \mu, \tau$), and H is the Higgs doublet whose neutral component has a vacuum expectation value (VEV) equal to $v = 175$ GeV. After spontaneous symmetry breaking, a Dirac mass term $m_D = \lambda v$, is generated by the VEV of the Higgs boson.

In the see-saw limit, $M \gg m_D$, the spectrum of neutrino mass eigenstates splits in two sets: three very heavy neutrinos, N_1, N_2 and N_3 respectively with masses $M_1 \leq M_2 \leq M_3$ almost coinciding with the eigenvalues of M , and three light neutrinos with masses $m_1 \leq m_2 \leq m_3$, the eigenvalues of the light neutrino mass matrix given by the see-saw formula [13],

$$m_\nu = -m_D \frac{1}{M} m_D^T . \quad (46)$$

Neutrino oscillation experiments measure two neutrino mass-squared differences. For normal schemes one has $m_3^2 - m_2^2 = \Delta m_{\text{atm}}^2$ and $m_2^2 - m_1^2 = \Delta m_{\text{sol}}^2$, whereas for inverted schemes one has $m_3^2 - m_2^2 = \Delta m_{\text{sol}}^2$ and $m_2^2 - m_1^2 = \Delta m_{\text{atm}}^2$. For $m_1 \gg m_{\text{atm}} \equiv \sqrt{\Delta m_{\text{atm}}^2 + \Delta m_{\text{sol}}^2} = (0.050 \pm 0.001)$ eV [14] the spectrum is quasi-degenerate, while for $m_1 \ll m_{\text{sol}} \equiv \sqrt{\Delta m_{\text{sol}}^2} = (0.00875 \pm 0.00012)$ eV [14] it is fully hierarchical (normal or inverted). Here we will restrict ourselves to the case of normal schemes. The most stringent upper bound on the absolute neutrino mass scale comes from cosmological observations. Recently, a conservative upper bound on the sum of neutrino masses, $\sum_i m_i \leq 0.61$ eV (95% CL), has been obtained by the WMAP collaboration combining CMB, baryon acoustic oscillations and supernovae type Ia observations [2]. Considering that it falls in the quasi-degenerate regime, it straightforwardly translates into

$$m_1 < 0.2 \text{ eV (95\% CL)} . \quad (47)$$

It proves sometimes useful to adopt the bi-unitary parametrization

$$m_D = V_L^\dagger D_{m_D} U_R , \quad (48)$$

where V_L and U_R are two unitary matrices that diagonalize m_D and D_{m_D} is the diagonal matrix whose elements are the eigenvalues of m_D : $D_{m_D} \equiv \text{diag}(l_{D1}, l_{D2}, l_{D3})$. This shows that in the

process of see-saw, 9 parameters are lost: at high energies there 18 real parameters (3 from the eigenvalues of m_D , 3 from M_i , and 12 from V_L and V_R). At low energy there are only 9 (3 from m_i and 6 from the matrix U diagonalizing m_ν). We now assume that right-handed neutrinos are hierarchical, $M_{2,3} \gg M_1$ so that studying the evolution of the number density of N_1 suffices. The final amount of $(B - L)$ asymmetry can be parametrized as $Y_{B-L} = n_{B-L}/s$, where $s = 2\pi^2 g_\star T^3/45$ is again the entropy density and $g_\star$ counts the effective number of spin-degrees of freedom in thermal equilibrium ($g_\star = 217/2$ in the SM with a single generation of right-handed neutrinos). After reprocessing by sphaleron transitions, the baryon asymmetry is related to the L asymmetry by [5]

$$Y_B = - \left(\frac{8n_G + 4n_H}{14n_G + 9n_H} \right) Y_L, \quad (49)$$

where n_H is the number of Higgs doublets, and n_G the number of fermion generations (in equilibrium). It is also useful to define an efficiency factor η which tells how efficient is the production of the baryon asymmetry, *e.g.* how much of the asymmetry per RH neutrino decay remains after wash-out processes are account for:

$$Y_B \approx 1.38 \cdot 10^{-3} \epsilon_1 \eta, \quad (50)$$

where we have assumed $n_H = 1$. Now, the idea of thermal leptogenesis is that RH neutrinos decay in the early Universe out-of-equilibrium, thus producing a lepton asymmetry.

One defines the CP asymmetry generated by N_1 decays as

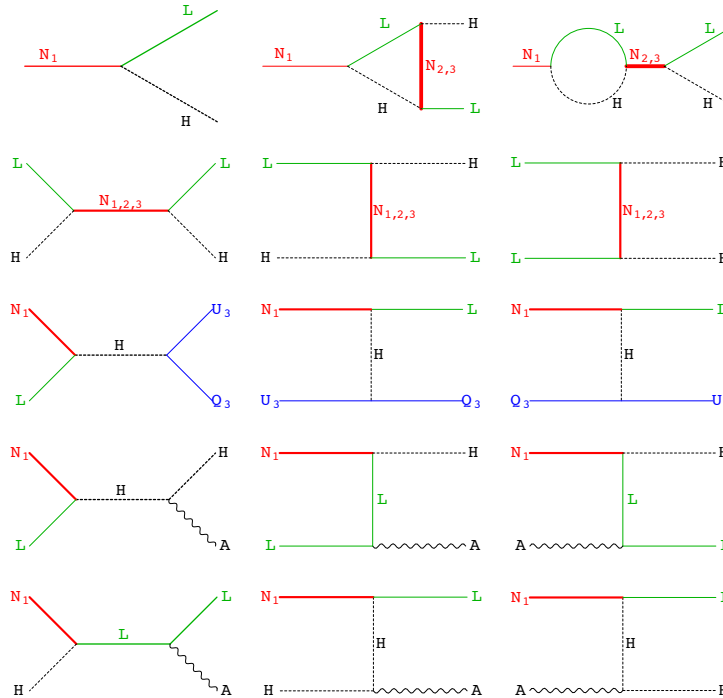


Figure 1. Feynman diagrams contributing to thermal leptogenesis.

$$\epsilon_1 \equiv \frac{\sum_\alpha [\Gamma(N_1 \rightarrow H\ell_\alpha) - \Gamma(N_1 \rightarrow \overline{H}\ell_\alpha)]}{\sum_\alpha [\Gamma(N_1 \rightarrow H\ell_\alpha) + \Gamma(N_1 \rightarrow \overline{H}\ell_\alpha)]} = \frac{1}{8\pi} \sum_{j \neq 1} \frac{\text{Im}[(\lambda\lambda^\dagger)_{j1}^2]}{[\lambda\lambda^\dagger]_{11}} g \left(\frac{M_j^2}{M_1^2} \right). \quad (51)$$

It is the sum of two contributions, the vertex and the wavefunction ones. A calculation similar to the one performed in the toy model previously discussed leads to

$$g(x) = \sqrt{x} \left[\frac{1}{1-x} + 1 - (1+x) \ln \left(\frac{1+x}{x} \right) \right] \xrightarrow{x \gg 1} -\frac{3}{2\sqrt{x}}. \quad (52)$$

Besides the CP parameter ϵ_1 , the final baryon asymmetry depends on a single wash-out parameter,

$$K \equiv \frac{\sum_{\alpha} \Gamma(N_1 \rightarrow H \ell_{\alpha})}{H(M_1)} \equiv \left(\frac{\tilde{m}_1}{\tilde{m}^*} \right), \quad (53)$$

where $H(M_1)$ denotes the value of the Hubble rate evaluated at a temperature $T = M_1$ ($\tilde{m}^* \approx 10^{-3}$ eV) and

$$\tilde{m}_1 \equiv \frac{(\lambda \lambda^{\dagger})_{11} v^2}{M_1} \quad (54)$$

is proportional to the total decay rate of the right-handed neutrino N_1 . One could now proceed as in the previous sections by estimating the baryon asymmetry. However, we want to do a better job and we resort to the Boltzmann equations. By defining the variable $z = M_1/T$, the Boltzmann equations for the lepton asymmetry $Y_{\mathcal{L}}$, and the right-handed neutrino number density Y_{N_1} (both normalised to the entropy s), may be written in a compact form as

$$\frac{d(Y_{N_1} - Y_{N_1}^{\text{EQ}})}{dz} = -\frac{z}{sH(M_1)} (\gamma_D + \gamma_{\Delta L=1}) \left(\frac{Y_{N_1}}{Y_{N_1}^{\text{EQ}}} - 1 \right) - \frac{dY_{N_1}^{\text{EQ}}}{dz}, \quad (55)$$

$$\frac{dY_{\mathcal{L}}}{dz} = \frac{z}{sH(M_1)} \left[\left(\frac{Y_{N_1}}{Y_{N_1}^{\text{EQ}}} - 1 \right) \epsilon_1 (\gamma_D + \gamma_{\Delta L=1}) - \frac{Y_{\mathcal{L}}}{Y_L^{\text{EQ}}} (\gamma_D + \gamma_{\Delta L=1}) \right]. \quad (56)$$

The processes taken into account in these equations are decays and inverse decays with rate γ_D , $\Delta L = 1$ scatterings such as $(qt^c \rightarrow N\ell)$, and $\Delta L = 2$ processes mediated by heavy neutrinos. The first three modify the abundance of the lightest right-handed neutrinos. The $\Delta L = 2$ scatterings mediated by $N_{2,3}$ are neglected in our analysis for simplicity. The various γ are thermally averaged rates, including all contributions summed over flavour (s , t channel interference etc); explicit expressions can be found in the literature. Notice that in this “usual” analysis, $\Delta L = 1$ scattering contributes to the creation of N_1 ’s and not to the production of a lepton asymmetry, only to the washout

Approximate analytic solutions for $Y_{\mathcal{L}}$ and $\Delta_{N_1} \equiv Y_{N_1} - Y_{N_1}^{\text{EQ}}$ can be obtained from simplified equations. Calculating in zero temperature field theory for simplicity, one obtains

$$\gamma_D \approx s Y_{N_1}^{\text{EQ}} \frac{K_1(z)}{K_2(z)} \Gamma_D, \quad Y_{N_1}^{\text{EQ}} \approx \frac{1}{4g_{\star}} z^2 K_2(z). \quad (57)$$

The Boltzmann equations can be approximated

$$\Delta'_{N_1} = -z K \frac{K_1(z)}{K_2(z)} f_1(z) \Delta_{N_1} - Y_{N_1}^{\text{EQ}'} \quad (58)$$

$$Y'_{\mathcal{L}} = \epsilon_1 K z \frac{K_1(z)}{K_2(z)} \Delta_{N_1} - \frac{1}{4} z^3 K K_1(z) f_2(z) Y_{\mathcal{L}} \quad (59)$$

where K_1 and K_2 are modified Bessel functions of the second kind. The function $f_1(z)$ accounts for the presence of $\Delta L = 1$ scatterings, and $f_2(z)$ accounts for scatterings in the washout term of the asymmetry. They can be approximated, in interesting limits, as

$$f_1(z) \approx \begin{cases} 1 & \text{for } z \gg 1 \\ \frac{N_c^2 m_t^2}{4\pi^2 v^2 z^2} & \text{for } z < 1, \end{cases} \quad (60)$$

and

$$f_2(z) \approx \begin{cases} 1 & \text{for } z \gg 1 \\ \frac{a_K N_c^2 m_t^2}{8\pi^2 v^2 z^2} & \text{for } z < 1, \end{cases} \quad (61)$$

where $\frac{N_c^2 m_t^2}{8\pi^2 v^2} \equiv K_s/K \approx 0.1$ parametrizes the strength of the $\Delta L = 1$ scatterings and $a_K = 4/3(2)$ for the weak (strong) wash out case. A good approximation to the rate $Kz(K_1(z)/K_2(z))f_1(z)$ is given by the function $(K_s + Kz)$ while the wash out term $-(1/4)z^3 K K_1(z)f_2(z)Y_L$ is well approximated at small z by $-a_K K_s Y_L$.

5.1. Strong wash-out regime

In the strong wash-out regime, the parameter $K \gg 1$ and the right handed neutrinos N_1 's are nearly in thermal equilibrium. Under these circumstances, one can set $\Delta'_{N_1} \approx 0$ and $\Delta_{N_1} \approx (zK_2/4g_\star K)$. Exploiting a saddle-point approximation in Eq. (59) we find that the lepton asymmetry is given by

$$Y_L \approx \epsilon_1 \int_0^\infty dz \frac{K}{4g_\star} z^2 e^{-\int_z^\infty dz' ((z')^3/4)K_1(z')K}. \quad (62)$$

Using the steepest descent method to evaluate the integral, one finds that it gets the major contribution at $\bar{z}$ such that $\bar{z} = \log K + (5 \ln \bar{z}/2)$ when inverse decays become inefficient. The lepton asymmetry in the flavour α becomes

$$Y_L \approx 0.3 \frac{\epsilon_1}{g_\star} \left(\frac{0.55 \times 10^{-3} \text{ eV}}{\tilde{m}_1} \right)^{1.16}. \quad (63)$$

5.2. weak wash-out regime

In this case all the $K \ll 1$. We assume that right-handed neutrinos are not initially present in the plasma, but they are generated by inverse decays and scatterings. The equation of motion for Y_{N_1} is well approximated by

$$Y'_{N_1} = -(K_s + Kz) (Y_{N_1} - Y_{N_1}^{\text{EQ}}), \quad (64)$$

We split the solution in two pieces. Let us define z_{EQ} the value of z at which $Y_{N_1}(z_{\text{EQ}}) = Y_{N_1}^{\text{EQ}}(z_{\text{EQ}})$. This value has to be found a posteriori. For $z \ll z_{\text{EQ}}$, we may suppose that $Y_{N_1} \ll Y_{N_1}^{\text{EQ}}$ and Eq. (64) is solved by

$$\begin{aligned} Y_{N_1}^-(z) \approx \int_0^z dz' (K_s + Kz') Y_{N_1}^{\text{EQ}} &= \frac{1}{4g_\star} \int_0^z dz' (K_s + Kz') (z')^2 K_2(z') \\ &= \frac{K}{4g_\star} \left(\frac{K_s}{K} I_1(z) + I_2(z) \right) \end{aligned} \quad (65)$$

With I_1 and I_2 integral involving the modified Bessel functions :

$$I_1(z) = \int_0^z x^2 K_2(x) dx \approx f(z) + z^3 K_2(z) \quad (66)$$

where

$$f(z) = \frac{3\pi z^3}{((9\pi)^c + (2z^3)^c)^{1/c}}, c = 0.7 \quad (67)$$

The integral I_2 is well known, and equals

$$I_2(z) = \int_0^z x^3 K_2(x) dx = 8 - z^3 K_3(z) \quad (68)$$

Therefore

$$Y_{N_1}^-(z) \approx \frac{K}{4g_\star} \left(\frac{K_s}{K} (f(z) + z^3 K_2(z)) + 8 - z^3 K_3(z) \right) \quad (69)$$

As expected for weak washout, we find that the maximum number density of N_1 is proportional to K (recall $K_s \propto K$).

Let us now compute the value of z_{EQ} . We expect it to be $\gg 1$ and we therefore approximate, up to $\mathcal{O}(z^{-3/2})$: $K_2(z) \approx K_3(z) \approx \sqrt{\frac{\pi}{2}} z^{-1/2} e^{-z}$. Imposing $Y_{N_1}^-(z_{\text{EQ}}) = Y_{N_1}^{\text{EQ}}(z_{\text{EQ}})$, we find

$$z_{\text{EQ}} \approx \frac{3}{2} \ln z_{\text{EQ}} - \ln \left(\frac{8}{\sqrt{\pi/2}} K + 3\sqrt{\frac{\pi}{2}} K_s \right). \quad (70)$$

This solution is a good approximation to the real value for $K \ll 1$.

For $z > z_{\text{EQ}}$, we have $(K_s + Kz) \approx Kz$ and

$$Y_{N_1}(z) \approx Y_{N_1}^{\text{EQ}}(z_{\text{EQ}}) e^{K/2(z_{\text{EQ}}^2 - z^2)}. \quad (71)$$

Notice that we have included CP violation in $\Delta L = 1$ scattering, unlike the usual analysis, so we expect our solution for Y_L to have a different scaling with K than in the traditional literature: if $\mathcal{CP}$ in scattering is neglected, then $Y_N \propto K$, and the N_1 decay out of equilibrium, so one expects $Y_L \propto K\epsilon_1$. However, if $\mathcal{CP}$ in N_1 production ($\approx$ scattering) is included, and washout is neglected, then the equations for Y_{N_1} and Y_L are identical, so $Y_L(z \rightarrow \infty)$ vanishes. That is, for every $|1/\epsilon_1|$ N_1 's that are created, be it by inverse decay or scattering, an (anti)-lepton is produced. This (anti)-asymmetry will approximately cancel against the lepton asymmetry generated later on, when the N_1 decay. However the cancellation will be imperfect, because the anti-asymmetry has more time to be washed out, so the final asymmetry should scale as K^2 . After integrating by parts, this is what we find for the lepton asymmetry, which is given by

$$\begin{aligned} Y_L &\approx \epsilon_1 \int_0^\infty dz' Y_{N_1}(z') g_1(z') e^{-\int_z^\infty dz'' g_1(z'')} , \quad g_1(z) = \frac{1}{4} z^3 K K_1(z) f_2(z), \\ &\approx 1.5 \frac{\epsilon_1}{g_\star} \left(\frac{\tilde{m}_1}{3.3 \times 10^{-3} \text{ eV}} \right)^2. \end{aligned} \quad (72)$$

Our findings hold provided that the non-resonant $\Delta L = 2$ scattering rates, in particular those mediated by the N_2 and N_3 heavy neutrinos, are slower than decays and $\Delta L = 1$ scatterings when most of the asymmetry is generated. We estimate that this applies when

$$\left(\frac{M_1}{10^{14} \text{ GeV}} \right) < 10^{-1}. \quad (73)$$

This means that K should be larger than 10^{-4} .

Our results can be summarized with simple analytical fits for the efficiency factor

$$\frac{1}{\eta} \approx \left(\frac{3.3 \times 10^{-3} \text{ eV}}{\tilde{m}_1} \right)^2 + \left(\frac{\tilde{m}_1}{0.55 \times 10^{-3} \text{ eV}} \right)^{1.16} \quad (74)$$

valid for $M_{N_1} < 10^{14}$ GeV and when one starts with no RH neutrinos in the plasma. Numerical results are presented in Fig. 5.

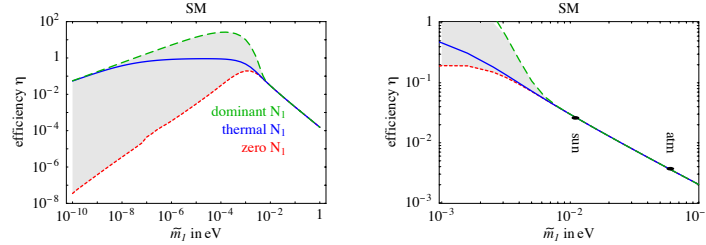


Figure 2. The efficiency factor as a function of $\tilde{m}_1$. Notice that for small values of $\tilde{m}_1$ the slope is $\tilde{m}_1^{-1}$ as the CP asymmetry from scattering is not accounted for. On the left-hand side three cases are analyzed: zero N_1 abundance, thermal abundance ($n_{N_1} \approx n_{N_1}^{\text{eq}}$) and dominant N_1 . On the right hand side, there is a zoom. From [11].

This enables the reader to study leptogenesis in neutrino mass models without setting up and solving the complicated Boltzmann equations.

5.3. Implications of one-flavour leptogenesis

Experiments have not yet determined the mass m_3 of the heaviest mainly left-handed neutrino. We assume $m_3 = \max(\tilde{m}_1, m_{\text{atm}})$. A crucial assumption we have so far is that right-handed neutrinos are very hierarchical. Under this hypothesis the CP asymmetry is bounded by the expression [15] that in the hierarchical and quasi-degenerate light neutrino limit simplifies as follows

$$|\epsilon_1| \leq \frac{3}{16\pi} \frac{M_1(m_3 - m_1)}{v^2} \times \begin{cases} 1 - m_1/\tilde{m}_1 & \text{if } m_1 \ll m_3 \\ \sqrt{1 - m_1^2/\tilde{m}_1^2} & \text{if } m_1 \approx m_3 \end{cases} \quad (75)$$

where all parameters are renormalized at the high-energy scale $\approx M_1$. The 3σ ranges of m_{atm} and of Y_B imply the lower bound

$$M_1 > \frac{4.5 \times 10^8 \text{ GeV}}{\eta} > 2.4 \times 10^9 \text{ GeV} \quad (76)$$

in the case in which no RH neutrinos are present in the plasma at the beginning of the dynamics and we have assumed that the efficiency is maximal (and therefore η minimal). This bound is relevant because it tells us that a working model of thermal leptogenesis (in the one-flavour approximation) needs enough heavy RH neutrinos. This has implications for $SO(10)$ grand-unified theories, commonly regarded as the most attractive way to embed the see-saw mechanism. Indeed, in a traditional version of leptogenesis, where the spectrum of right-handed (RH) neutrinos is hierarchical and the asymmetry is produced from the decays of the lightest ones, there is a stringent lower bound on their mass [15], $M_1 > \mathcal{O}(10^9)$ GeV, for a sufficiently large baryon asymmetry to be produced. On the other hand, $SO(10)$ grand-unified theories typically yield, in their simplest version and for the measured values of the neutrino mixing parameters, a hierarchical spectrum with the RH neutrino masses proportional to the squared of the up-quark masses, leading to $M_1 = \mathcal{O}(10^5)$ GeV and to a final asymmetry that falls a few orders of magnitude below the observed one.

We can also work out a bound on the light neutrino masses, $\sum_i m_i < 0.15$ eV. It can be understood to arise from the lower bound on the total decay rate $m_1 \leq \tilde{m}_1$, and the upper

bound on the total CP asymmetry (75). We assume the light neutrinos are degenerate, so $|m_1| \approx |m_2| \approx |m_3| \equiv \bar{m}/\sqrt{3}$ — but the masses can have different Majorana phases. Leptogenesis takes place in the strong washout regime, due to the lower bound on the total decay rate. The final baryon asymmetry can be roughly approximated as $Y_B \approx 10^{-4}\epsilon/K \propto \Delta m_{\text{atm}}^2/\bar{m}^2$. As the light neutrino mass scale is increased, M_1 and the temperature of leptogenesis must increase to compensate the $\Delta m_{\text{atm}}^2/\bar{m}^2$ suppression. However, this temperature is bounded from above, from the requirement of having the $\Delta L = 2$ processes out of equilibrium when leptogenesis takes place:

$$\frac{\bar{m}^2 T^3}{12\pi v^4} < \frac{10T^2}{M_P}, \quad (77)$$

so $M_1 < 10^{10}(\text{eV}/\bar{m})^2 \text{ GeV}$. There is therefore an upper bound on the baryon asymmetry which scales as $1/\bar{m}^4$, and with our rough estimates, one finds

$$\bar{m}/\sqrt{3} < 0.1 \text{ eV}. \quad (78)$$

This result is confirmed by a full numerical analysis [11].

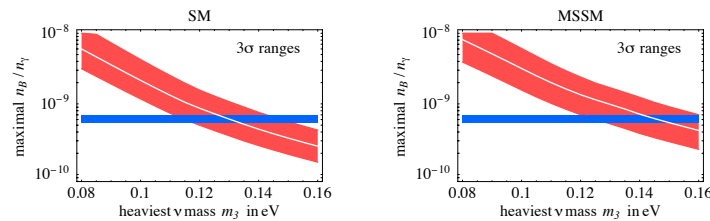


Figure 3. The bound on the heaviest light neutrino mass for the SM and for the Minimal Supersymmetric Standard Model (MSSM).

6. Comments on baryogenesis via leptogenesis when flavours accounted for

In recent years, a lot of work has been devoted to a thorough analysis of thermal leptogenesis giving limited attention to the issue of lepton flavour [16]. The dynamics of leptogenesis is usually addressed within the ‘one-flavour’ approximation, where Boltzmann equations are written for the abundance of the lightest RH neutrino, responsible for the out of equilibrium and CP asymmetric decays, and for the total lepton asymmetry. However, this ‘one-flavour’ approximation is rigorously correct only when the interactions mediated by charged lepton Yukawa couplings are out of equilibrium. Flavour effects have not been included in leptogenesis calculations till very recently [17, 18, 19]. This is perhaps because perturbatively, they seem to be a small correction. For instance, if the asymmetry is a consequence of the very-out-of-equilibrium decay of an initial population of right-handed neutrinos, then the total lepton asymmetry is of order ϵ/g_* , where ϵ is the total CP asymmetry in the decay, and g_* counts for the entropy dilution factor. Clearly the small charged lepton Yukawa couplings have no effect on ϵ . However, realistic leptogenesis is a drawn-out dynamical process, involving the production and destruction of right-handed neutrinos, and of a lepton asymmetry that is distributed among *distinguishable* flavours. The processes which wash out lepton number are flavour dependent, *e.g* the inverse decays from electrons can destroy the lepton asymmetry carried by, and only by, the electrons. The asymmetries in each flavour are therefore washed out differently, and will appear with different weights in the final formula for the baryon asymmetry. This is physically inequivalent to the treatment of washout in the one-flavour approximation, where

indistinguishable leptons propagate between decays and inverse decays, so inverse decays from all flavours are taken to wash out asymmetries in any flavour.

We define $Y_{\alpha\alpha}$ to be the lepton asymmetry in flavour α , where the α are the lepton mass eigenstates at the temperature of leptogenesis. The $Y_{\alpha\alpha}$ are the diagonal elements of a matrix $[Y]$ in flavour space, whose trace is the total lepton asymmetry. In this paper the off-diagonal elements are neglected. The equations of motion for the matrix $[Y]$ are more complicated than the Boltzmann equations, but at most temperatures are equivalent to Boltzmann equations written in the mass eigenstate basis of the leptons in the plasma. The off-diagonal elements of $[Y]$ could have some effect on the lepton asymmetry, if leptogenesis takes place just as a charged lepton Yukawa coupling is coming into equilibrium (so the mass eigenstate basis is changing).

The mass eigenstates for the particles in the Boltzmann equations (BE) are determined by the interactions which are fast compared to those processes included in the BE. The interaction rate for Yukawa coupling h_α can be estimated as

$$\Gamma_\alpha \approx 5 \times 10^{-3} h_\alpha^2 T, \quad (79)$$

so interactions involving the tau (μ) Yukawa coupling are out-of-equilibrium in the primeval plasma if $T > 10^{12}$ GeV ($T > 10^9$ GeV)¹. Thermal leptogenesis takes place at temperatures on the order of M_1 , and the asymmetry is generated when the rates $< H$, so we conclude that the τ (μ) lepton doublet is a distinguishable mass eigenstate, for the purposes of leptogenesis, at $T < 10^{12}$ (10^9) GeV.

Suppose therefore that M_1 is below 10^9 GeV and we forget for the time being about the bound (76). All flavours are distinguishable.

The Boltzmann equations for the flavour asymmetries $Y_{\alpha\alpha}$, are as follows. The Equation for the N_1 number density remains unchanged, and the equation for the flavoured lepton asymmetry is

$$\frac{dY^{\alpha\alpha}}{dz} = \frac{z}{sH(M_1)} \left[\left(\frac{Y_{N_1}}{Y_{N_1}^{\text{EQ}}} - 1 \right) \epsilon^{\alpha\alpha} (\gamma_D + \gamma_{\Delta L=1}) - \frac{Y^{\alpha\alpha}}{Y_L^{\text{EQ}}} (\gamma_D^{\alpha\alpha} + \gamma_{\Delta L=1}^{\alpha\alpha}) \right]. \quad (80)$$

To obtain analytic solutions we could simplify this, with the approximations introduced in the previous section, to

$$Y'_{\alpha\alpha} = \epsilon_{\alpha\alpha} K z \frac{K_1(z)}{K_2(z)} f_1(z) \Delta_{N_1} - \frac{1}{4} z^3 K_1(z) f_2(z) K_{\alpha\alpha} Y_{\alpha\alpha} \quad (81)$$

$$\Delta'_{N_1} = -z K \frac{K_1(z)}{K_2(z)} f_1(z) \Delta_{N_1} - Y_{N_1}^{\text{EQ}'}, \quad (82)$$

where

$$K_{\alpha\alpha} Y_{\alpha\alpha} \quad (83)$$

$$\Delta'_{N_1} = -z K \frac{K_1(z)}{K_2(z)} f_1(z) \Delta_{N_1} - Y_{N_1}^{\text{EQ}'}, \quad (84)$$

where

$$K_{\alpha\alpha} = K \frac{\lambda_{1\alpha} \lambda_{1\alpha}^*}{\sum_\gamma |\lambda_{1\gamma}|^2} = \left(\frac{\tilde{m}_{\alpha\alpha}}{10^{-3} \text{ eV}} \right), \quad K = \sum_\alpha K_{\alpha\alpha}. \quad (85)$$

$K_{\alpha\alpha}$ parametrizes the decay rate of N_1 to the α -th flavour, and the trace $\sum_\alpha K_{\alpha\alpha}$, coincides with the K parameter defined in the previous section.

¹ The electron Yukawa coupling mediates interactions relevant in the early Universe only for temperatures beneath $\approx 10^5$ GeV and can be safely disregarded.

Notice in particular that the dynamics of the right-handed neutrinos is always set by the total K .

The CP asymmetry in the α -th flavour is $\epsilon_{\alpha\alpha}$ and is normalised by the total decay rate

$$\epsilon_{\alpha\alpha} = \frac{1}{(8\pi)} \frac{1}{[\lambda\lambda^\dagger]_{11}} \sum_j \text{Im} \left\{ (\lambda_{1\alpha})(\lambda\lambda^\dagger)_{1j} \lambda_{j\alpha}^* \right\} g \left(\frac{M_j^2}{M_1^2} \right) \quad (86)$$

$$\rightarrow \frac{3}{(16\pi)[\lambda\lambda^\dagger]_{11}} \text{Im} \left\{ \lambda_{1\beta} \frac{[m^*]_{\beta\alpha}}{v^2} \lambda_{1\alpha} \right\} \quad (87)$$

where the second line is in the limit of hierarchical N_J , and $m = U^* D_m U^\dagger = v^2 \lambda^T M^{-1} \lambda$ is the light neutrino mass matrix. If m_3 is the heaviest light neutrino mass ($= m_{\text{atm}}$ for the non-degenerate case) and we define $\epsilon^{\text{max}} = 3\Delta m_{\text{atm}}^2 M_1 / (8\pi v^2 m_{\text{max}})$ [15], then the flavour dependent CP asymmetries are bounded by

$$\epsilon_{\alpha\alpha} \leq \frac{3M_1 m_3}{16\pi v^2} \sqrt{\frac{K_{\alpha\alpha}}{K}} = \epsilon^{\text{max}} \frac{m_3^2}{\Delta m_{\text{atm}}^2} \sqrt{\frac{K_{\alpha\alpha}}{K}} \quad (88)$$

so the maximum CP asymmetry in a given flavour is unsuppressed for degenerate light neutrinos [17], but decreases as the square root of the branching ratio to that flavour $= K_{\alpha\alpha}/K$. The first consequence of this result is that there no upper bound on the light neutrino masses when flavours are accounted for.

The CP asymmetry $\epsilon_{\alpha\alpha}$ can be written in terms of the diagonal matrix of the light neutrino mass eigenvalues $m = \text{Diag}(m_1, m_2, m_3)$, the diagonal matrix of the the right handed neutrino masses $M = \text{Diag}(M_1, M_2, M_3)$ and an orthogonal complex matrix [20]

$$R = v M^{-1/2} \lambda U m^{-1/2}, \quad (89)$$

where the matrix U diagonalizes the light neutrino mass matrix m_ν , so that

$$U^\dagger m_\nu U^* = -D_m \quad (90)$$

and it can be identified with the lepton mixing matrix in a basis where the charged lepton mass matrix is diagonal. The asymmetry reads

$$\epsilon_{\alpha\alpha} = -\frac{3M_1}{16\pi v^2} \frac{\text{Im} \left(\sum_{\beta\rho} m_\beta^{1/2} m_\rho^{3/2} U_{\alpha\beta}^* U_{\alpha\rho} R_{\beta 1} R_{\rho 1} \right)}{\sum_{\beta} m_\beta |R_{1\beta}|^2}. \quad (91)$$

For a real R matrix, the individual CP asymmetries $\epsilon_{\alpha\alpha}$ may not vanish because of the presence of CP violation in the U matrix. On the contrary, the total CP asymmetry $\epsilon_1 = \sum_{\alpha} \epsilon_{\alpha\alpha}$ vanishes. This is the second implication of the flavours: leptogenesis work even if the matrix R is real. Let us say his in other words. The neutrino Yukawa coupling can be written in its singular value decomposition, $\lambda = V_L^\dagger \text{Diag}(\lambda_1, \lambda_2, \lambda_3) U_R$. Hence, the CP violation in the right-handed neutrino sector is encoded in the phases in V_R , that can be extracted from diagonalizing the combination $\lambda\lambda^\dagger = U_R^\dagger \text{Diag}(\lambda_1^2, \lambda_2^2, \lambda_3^2) U_R$. In the one-flavour regime, only the phases of the matrix V_R count, in the flavour regime the phase of the matrix V_L also count and the final lepton asymmetry does not vanish even if the matrix V_R is real. Implications of this result can be found in [21, 22, 23].

7. Electroweak baryogenesis

So far, we have been assuming that the departure from thermal equilibrium, necessary to generate any baryon asymmetry, is attained by late decays of heavy particles. In this section, we

will focus on a different mechanism, namely the departure from equilibrium during first order phase transitions.

A first order phase transition is defined to occur if some thermodynamic quantities change discontinuously. This happens because there exist two separate thermodynamic states that are in thermal equilibrium at the time of the phase transition. The thermodynamic quantity that undergoes such a discontinuous change is generically called the order parameter ϕ . Whether a phase transition is of the first order or not depends upon the parameters of the theory and it may happen that, changing those parameters, the order parameter becomes continuous at the time of the transition. In this case, the latter is said to be of the second order at the point at which the transition becomes continuous and a continuous crossover at the other points for which all physical quantities undergo no changes. In general, we are interested in systems for which the high temperature ground state of the theory is at $\phi = 0$ and the low temperature phase is at $\phi \neq 0$ [1].

For a first order phase transition, the extremum at $\phi = 0$ becomes separated from a second local minimum of the potential by an energy barrier. At the critical temperature T_c both phases are equally favoured energetically and at later times the minimum at $\phi \neq 0$ becomes the global minimum of the theory. The phase transition proceeds by nucleation of bubbles. Initially, the bubbles are not large enough for their volume energy to overcome the competing surface tension, they shrink and disappear. However, at the nucleation temperature, critical bubbles form, *i.e.* bubbles which are just large enough to be nucleated and to grow. As the bubble walls separating the broken from the unbroken phase pass each point in space, the order parameter changes rapidly, leading to a significant departure from thermal equilibrium.

The critical bubbles of the broken (Higgs) phase have a typical profile

$$\phi(r) = \frac{\langle \phi(T_c) \rangle}{2} \left[1 + \tanh \left(\frac{r}{L_\omega} \right) \right], \quad (92)$$

where r is the spatial coordinate, L_ω is the bubble wall width and $\langle \phi(T_c) \rangle$ is the VEV of the Higgs field inside the bubble.

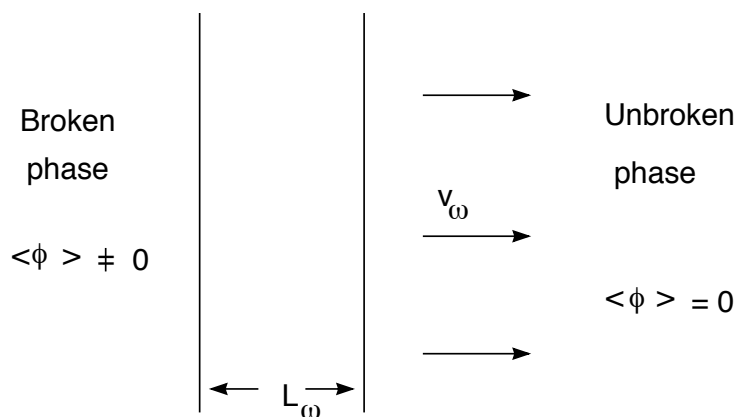


Figure 4. Schematic picture of the propagating bubble separating the broken from the unbroken phase during the electroweak phase transition.

Bubbles expand with velocity v_ω until they fill the Universe; local departure from thermal equilibrium takes place in the vicinity of the expanding bubble walls, see Fig. 10.

The fundamental idea of electroweak baryogenesis is to produce asymmetries in some local charges which are (approximately) conserved by the interactions inside the bubble walls, where

local departure from thermal equilibrium is attained. These local charges will then diffuse into the unbroken phase where baryon number violation is active thanks to the unsuppressed sphaleron transitions. The latter convert the asymmetries into baryon asymmetry, because the state of minimum free energy is attained for nonvanishing baryon number. Finally, the baryon number flows into the broken phase where it remains as a remnant of the electroweak phase transition if the sphaleron transitions are suppressed in the broken phase. The recipe for electroweak baryogenesis is therefore the following:

- Look for those charges which are approximately conserved in the symmetric phase, so that they can efficiently diffuse in front of the bubble where baryon number violation is fast, and non-orthogonal to baryon number, so that the generation of a non-zero baryon charge is energetically favoured.

- Compute the CP violating currents of the plasma locally induced by the passage of the bubble wall.

- Write and solve a set of coupled differential diffusion equations for the local particle densities, including the CP violating source terms derived from the computation of the current at the previous step and the particle number changing reactions. The solution to these equations gives a net baryon number which is produced in the symmetric phase and then transmitted into the interior of the bubbles of the broken phase, where it is not wiped out if the first transition is strong enough. Since C and CP are known to be violated by the electroweak interactions, it is possible – in principle – to satisfy all Sakharov's conditions within the SM if the electroweak phase transition. CP violation is however already deadly. A naive estimate suggests that, since CP violation vanishes in the SM if any two quarks of the same charge have the same mass, the measure of CP violation should be the Jarlskog invariant

$$\frac{A_{CP}}{J} = \frac{(M_t^2 - M_c^2)(M_c^2 - M_u^2)(M_u^2 - M_t^2)}{(M_b^2 - M_s^2)(M_s^2 - M_d^2)(M_d^2 - M_b^2)}, \quad (93)$$

where J is twice the area of the unitarity triangle. The quantity A_{CP} has dimension twelve. In the limit of high temperature, T much larger than the quark masses M_q , the only mass scale in the problem is the temperature itself. Therefore, the dimensionless quantity δ_{CP} is

$$\delta_{CP} \simeq \frac{A_{CP}}{T_c^{12}} \simeq 10^{-20}, \quad (94)$$

far too small for the SM to explain the observed baryon asymmetry.

7.1. Electroweak baryogenesis in the MSSM

Electroweak baryogenesis in the framework of the Minimal Supersymmetric Standard Model (MSSM) has attracted much attention in the past years, with particular emphasis on the strength of the phase transition and the mechanism of baryon number generation. A thorough analysis has shown that the electroweak phase transition is strong enough if the mass of the lightest Higgs is less than about 120 GeV and the right-handed stop is lighter than the top [25]. The new contributions to explicit violation of CP are given in the phases of the complex trilinear coupling A , the B parameter, the gaugino masses M_i ($i = 1, 2, 3$) and by the parameter μ in the superpotential. Two phases may be removed by redefining the phase of one of the two Higgs superfields, $\hat{H}_2$, in such a way that the phase of μ is opposite to that of B . The product μB is therefore real. It is also possible to remove the phase of the gaugino mass M by an R symmetry transformation. The latter leaves all the other supersymmetric couplings invariant and only modifies the trilinear ones, which get multiplied by $\exp(-\phi_M)$ where ϕ_M is the phase of M .

The phases which are left are therefore

$$\phi_A = \arg(AM) \quad \text{and} \quad \phi_\mu = -\arg(B). \quad (95)$$

The two new phases ϕ_A and ϕ_μ will be crucial for the generation of the baryon asymmetry. These new phases are essential for the generation of the baryon number since large CP violating sources may be locally induced by the passage of the bubble wall separating the broken from the unbroken phase during the electroweak phase transition. Baryogenesis is fuelled when transport properties allow the CP violating charges to efficiently diffuse in front of the advancing bubble wall where anomalous electroweak baryon violating processes are not suppressed. The new phases appear in the soft supersymmetry breaking parameters associated to the stop mixing angle and to the gaugino and neutralino mass matrices; large values of the stop mixing angle are, however, strongly restricted in order to preserve a sufficiently strong first order electroweak phase transition. Therefore, an acceptable baryon asymmetry from the stop sector may only be generated through a delicate balance between the values of the different soft supersymmetry breaking parameters contributing to the stop mixing parameter. As a result, the contribution to the final baryon asymmetry from the stop sector turns out to be negligible. On the other hand, charginos and neutralinos may be responsible for the observed baryon asymmetry if the phase of the parameter μ is large enough [24]. Yet, this is true within the MSSM. If the strength of the electroweak phase transition is enhanced by the presence of some new degrees of freedom beyond the ones contained in the MSSM, *e.g.* some extra standard model gauge singlets, light stops (predominantly the right-handed ones) and charginos/neutralinos are expected to give quantitatively the same contribution to the final baryon asymmetry.

As mentioned, a strongly first order electroweak phase transition can be achieved in the presence of a top squark lighter than the top quark [25]. In order to naturally suppress its contribution to the parameter $\Delta\rho$ and hence preserve a good agreement with the precision measurements at LEP, it should be mainly right handed. This can be achieved if the left handed stop soft supersymmetry breaking mass m_Q is much larger than M_Z . Indeed, the preservation of the baryon number asymmetry requires the order parameter $\langle\phi(T_c)\rangle/T_c$ to be larger than one (in the one Higgs limit, valid if the mass of the pseudoscalar A is high enough). The latter is bounded from above

$$\frac{\langle\phi(T_c)\rangle}{T_c} < \left(\frac{\langle\phi(T_c)\rangle}{T_c} \right)_{\text{SM}} + \frac{2 m_t^3}{\pi v m_h^2}, \quad (96)$$

where $m_t = \overline{m}_t(m_t)$ is the on-shell running top quark mass in the $\overline{\text{MS}}$ scheme. The first term on the right hand side of expression (96) is the Standard Model contribution

$$\left(\frac{\langle\phi(T_c)\rangle}{T_c} \right)_{\text{SM}} \simeq \left(\frac{40}{m_h[\text{GeV}]} \right)^2, \quad (97)$$

and the second term is the contribution that would be obtained if the right handed stop plasma mass vanished at the critical temperature. The difference between the SM and the MSSM is that light stops may give a large contributions to the effective potential in the MSSM.

In order to overcome the Standard Model constraints, the stop contribution must be therefore large. The stop contribution strongly depends on the value of the soft breaking right-handed mass squared $\tilde{m}_U^2$, which must be small in magnitude, and negative, in order to induce a sufficiently strong first order phase transition. Indeed, large stop contributions are always associated with small values of the right handed stop plasma mass

$$m_t^{\text{eff}} = -\tilde{m}_U^2 + \Pi_R(T), \quad (98)$$

where $\tilde{m}_U^2 = -m_U^2$, $\Pi_R(T) \simeq 4g_3^2 T^2/9 + h_t^2/3T^2$ is the finite temperature self-energy contribution to the right-handed squarks. Although large values of $\tilde{m}_U$, of order of the critical temperature,

are useful to get a strongly first order phase transition, they may also induce charge and color breaking minima. Indeed, if the effective plasma mass at the critical temperature vanished, the universe would be driven to a charge and color breaking minimum at $T \geq T_c$. Hence, the upper bound on $\langle \phi(T_c) \rangle / T_c$, Eq. (96) cannot be reached in realistic scenarios. A conservative bound on $\tilde{m}_U$ may be obtained by demanding that the electroweak symmetry breaking minimum should be lower than any color-breaking minima induced by the presence of $\tilde{m}_U$ at zero temperature, which yields the condition

$$\tilde{m}_U \leq \left(\frac{m_H^2 v^2 g_3^2}{12} \right)^{1/4}. \quad (99)$$

It can be shown that this condition is sufficient to prevent dangerous color breaking minima at zero and finite temperature for any value of the mixing parameter $\tilde{A}_t$.

In order to obtain values of $\langle \phi(T_c) \rangle / T_c$ larger than one, the Higgs mass must take small values, close to the present experimental bound. Numerically, an upper bound, of order 80 GeV, can be derived. For small mixing, the one-loop Higgs mass has a very simple form

$$m_h^2 = M_Z^2 \cos^2 2\beta + \frac{3}{4\pi^2} \frac{\overline{m}_t^4}{v^2} \log \left(\frac{m_t^2 m_T^2}{\overline{m}_t^4} \right), \quad (100)$$

where $m_T^2 \simeq m_Q^2 + m_t^2$, is the heaviest stop squared mass. Hence, $\tan \beta$, which is the ratio of the vacuum expectation values of the two Higgses, must take values as small as possible. The larger the left handed stop mass, the closer to one $\tan \beta$ must be. This implies that the left handed stop effects decouple at the critical temperature and hence, different values of m_Q mainly affect the baryon asymmetry through the resulting Higgs mass. Values of the CP -odd Higgs mass $m_A < 200$ GeV are associated with a weaker first order phase transition.

Those charges which are approximately conserved in the symmetric phase, so that they can efficiently diffuse in front of the bubble where baryon number violation is fast, and non-orthogonal to baryon number, are good charges to generate a baryon number.

Charges with these characteristics in the MSSM are the axial stop charge and the Higgsino charge, which may be produced from the interactions of squarks and charginos and/or neutralinos with the bubble wall, provided a source of CP -violation is present in these sectors. This is exactly the case, since both the parameters A_t and μ may carry a physical phase [24]. The idea is that, if nonvanishing CP violating sources for the right-handed stop and higgsino numbers are induced in the bubble wall, the scattering among particles as well as diffusion will generate an asymmetry in the left-handed fermion asymmetries in the unbroken phase. The asymmetry – in turn – will fuel baryogenesis because sphaleron transitions will push the system towards the state of minimum free energy, which is the one with nonvanishing baryon asymmetry. In the next subsection, we will give some indications of how to compute the CP -violating sources.

The final baryon asymmetry turns out to be [24]

$$\frac{n_B}{s} \simeq \left(\frac{|\sin(\phi_\mu)|}{10^{-3}} \right) 4 \times 10^{-11}, \quad (101)$$

for $v_\omega \simeq 1$. A more complete analysis has shown that there exist regions of the MSSM parameter space that are consistent with both present two-loop electric dipole moment limits and the relic density and that allow for successful baryogenesis through resonant chargino and neutralino processes at the electroweak phase transition [26]. Under certain conditions the lightest neutralino may be simultaneously responsible for both the baryon asymmetry and relic density. Furthermore, requiring a sufficiently large asymmetry imposes lower limits on the size of the electric dipole moments which are typically on the same order, or above, the expected sensitivity of the next generation of experimental searches, implying that MSSM electroweak baryogenesis will be soon conclusively tested.

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