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On the Compensation of Solenoid Field Effects by Quadrupole Lenses

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Abstract

A theoretical possibility is shown of compensating the influence of solenoid models on particle motion in storage rings by means of quadrupole lenses only, which is indispensable in case that solenoids are used for longitudinal particle polarization within the region of interaction.

A method is proposed to construct various compensation schemes, in which the perturbation of motion is localized on a small part of the storage ring perimeter and particle motion remains unchanged outside of this part. The described method permits to obtain longitudinally polarized colliding beams by relatively simple means for facilities, which are already in operation.

Some versions of solenoid field compensation were checked, which might serve for longitudinal polarization.

I N T R O D U C T I O N

A series of designs for longitudinal polarization of particles within the interaction region in electron-positron storage rings ⁽¹⁾ implies the application of strong solenoids flipping the particle spin about an angle of $\varphi = \pi, \pi/2$ with a sum integral of the longitudinal field B_s along the trajectory S equal to $\int B_s ds (\text{kGauss}\cdot\text{m}) \sim 100 \cdot \frac{1}{\gamma} \cdot E$ (GeV), where E is the energy of this experiment. Such solenoids bring in a strong connection between horizontal and vertical betatron oscillations and influence considerably on the focussing properties of the magnetic structure. In spite of these disadvantages the method of longitudinal polarization by means of solenoids has the advantage, however, not to require any special measurements in the equilibrium orbit position and, therefore, is a most attractive method to be used on machines already in operation. The local compensation of the influence of the solenoid field on particle motion, as proposed in our paper, allows to focus the coupling of oscillations on a small part of the storage ring perimeter without changing the direction of polarization. This makes the longitudinal polarization of particles by means of solenoids much easier and the given method becomes comparatively simple and convenient.

For the particular case of $\varphi = \pi$, A. M. Kondratenko first proposed a compensation scheme consisting of four quadrupole lenses turned about an angle of 45° , with two of them arranged on each side of the solenoid. You will find a detailed description in ref. ⁽²⁾. This design allows to avoid the coupling of oscillations but it does not solve the problem involved in the compensation of additional solenoid focussing. For this purpose eight quadrupole lenses have been applied for instance in ref. ⁽²⁾, which, of course, makes the operation of the whole facility more difficult.

In this paper, we propose methods for different compensation of solenoid fields; they are complying with the condition of local perturbation as described above. In the general case, the approach to a solution of the problem does not depend on the choice of a concrete angle of particle spin flip in the solenoid system and may be applied not only for $\varphi = \pi$ but also for any other arbitrary angle φ . The described method has been applied in the search for a number of versions to compensate the solenoid field, which are to be applied for longitudinal polarization at the collider VEPP-4 and other facilities.

1. Transport Matrices of a Solenoid and of a turned Quadrupole Lens

We learnt from ref. (3) that the magnetic field of a solenoid is described completely by the dependence of the field on the axis on the longitudinal s-coordinate.

$$B_s = B_s(s) \quad (1)$$

The vector potential has only one component in the given case and is easily expressed by B_s :

$$A_\varphi(s, r) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \left(\frac{r}{2} \right)^{2n} B_s^{(n)} \left(\frac{s}{2} \right)^{2n+1} \quad (2)$$

The particle motion in such a field will be described by the Lagrange function, which takes the form in the cylindrical system of the (s, r, φ)-coordinates:

$$\mathcal{L} = -mc^2 \sqrt{1 - \frac{v^2}{c^2}} + \frac{e}{c} A_\varphi \dot{\varphi} \quad (3)$$

The following integral of motion results from axial symmetry ($\frac{\partial \mathcal{L}}{\partial \varphi} = 0$):

$$p_\varphi = \frac{\partial \mathcal{L}}{\partial \dot{\varphi}} = \frac{m v^2}{c^2} \frac{r}{2} + \frac{e}{c} A_\varphi = \frac{m v^2}{c^2} \frac{r}{2} \quad (4)$$

Hence we obtain (passing over to differentiation over s):

$$\dot{\varphi} - \dot{\varphi}_0 = -\frac{e}{mc} \frac{A_\varphi}{c} \quad (5)$$

In paraxial approximation ($r \ll \frac{\partial \ln B_s}{\partial s}$), the equations of motion in direction of r and φ are "decoupled", therefore:

$$\ddot{\varphi} - \ddot{\varphi}_0 = -\frac{e}{mc} B_s(s) \quad (6)$$

Here $-\frac{Q}{p} \frac{1}{c} = \overline{BR}$, where $\overline{BR}$ is the stiffness. It follows from (6) directly that the solenoid turns the plane of particle oscillations about an angle of $\Delta \varphi = \frac{b_s}{2 \overline{BR}} d_s$. In order that the solenoid will not bring in a coupling, the following condition must be fulfilled:

$$\int_{-l/2}^{l/2} B_s ds = 0 \quad (7)$$

In this case, x-oscillation passes over to x and z to z.

According to (4), we shall derive the equation of motion in a solenoid most easily by passing over to a rotating coordinate system (torsion $\mathcal{X}(s) = B_s(s)/2 \overline{BR}$):

$$\ddot{y} + \mathcal{X}(s)^2 y = 0, \quad y = x, z \quad (8)$$

It is easy to note that the transport matrix of a solenoid with arbitrary dependence of the field B_s on the longitudinal s-coordinate takes the form for the general case:

$$M_s = R(\theta) \begin{pmatrix} F & 0 \\ 0 & F \end{pmatrix}, \quad R = \begin{pmatrix} I \cos \theta & I \sin \theta \\ -I \sin \theta & I \cos \theta \end{pmatrix}, \quad \theta = \int_{-l/2}^{l/2} \mathcal{X} ds, \quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (9)$$

Here F is the 2 x 2 matrix of solenoid focussing, R is the matrix transforming the turning and l_s is the solenoid length. In the particular case of a solenoid with $B_s = \text{const}$ in the center and with sharp edges, we derive from (8) the well-known result (5):

$$F = \begin{pmatrix} \cos \theta & \frac{\sin \theta}{\omega} \\ -\frac{\sin \theta}{\omega} & \cos \theta \end{pmatrix}, \quad \omega = \frac{e}{mc}, \quad \theta = \frac{\omega l_s}{2} \quad (10)$$

We note that the solenoid matrix M_s and the matrix of the drift space L are commuting with the matrix transforming the turning R:

$$2 M_s = M_s R, \quad RL = LR \quad (11)$$

The transport matrix of the turned quadrupole lens can be easily expressed by the matrix of the standard quadrupole lens T and the matrix R:

$$Q = R^{-1}(\alpha) T R(\alpha) \quad (12)$$

where α is the angle of turning the lens.

2. Methods Constructing Compensation Schemes

An arbitrary 4 x 4 transport matrix has 10 independent parameters due to its symplectic property. In order to avoid the coupling of oscillations (antidiagonal 2 x 2 matrix blocks set equal to zero), it is sufficient to set four of these 10 parameters equal to zero. Therefore not more than four quadrupole lenses are required for the compensation of coupling. If we want to remove the influence of focussing introduced by the solenoid, it is necessary to increase the number of lenses.

We noticed that it is possible to take advantage of the mutual arrangement of lenses as an additional free parameter in the compensation design.

It is convenient to begin the description of methods to compensate the influence of solenoid fields with the scheme of fig. 1. Here two equal solenoids with an arbitrary integral of field were separated by a space, where 6 standard quadrupole lenses were mounted. The total transport matrix M from the beginning of the first solenoid up to the end of the second one, may be described according to (9) and (11) as follows:

$$M = R \begin{pmatrix} A_0 & 0 \\ 0 & B \end{pmatrix} R \quad (13)$$

It is obvious that the following condition should be fulfilled for the block diagonal form of the matrix M

$$B \equiv -A. \quad (14)$$

In this case three lenses are sufficient; the remaining 3 lenses are necessary for the transformation of matrix A into any previously given form (for instance into matrices of the type $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $\begin{pmatrix} 1 & p \\ 0 & 1 \end{pmatrix}$).

The second method of compensating the influence of solenoid fields on particle motion also consists finally in the reduction of the transport matrix of the whole system to the form (13). Here the mutual arrangement of quadrupole lenses and solenoids as well as the number of solenoids may be arbitrary. The fundamental idea of the given method lies in a correct choice of the angle of turning the quadrupole lenses, which follow the solenoids.

As we see from (8), the motion in direction of X and Z in the rotating coordinate system ($\varphi(s) = B_s/2 BR$) is not coupled. Therefore we obtain the following equations of motion, if we introduce the focusing by a quadrupole lens with true direction of the axis on any azimuth

$$\begin{aligned} x'' + [g(s) + \varphi'(s)^2]x &= 0 \\ z'' + [g(s) - \varphi'(s)^2]z &= 0 \end{aligned} \quad (15)$$

where $g(s)$ is the parameter of quadrupole focussing. The transport matrix in new coordinates apparently takes the partitioned diagonal form and in the initial coordinate system:

$$(16)$$

l_{tot} is the length of the compensation scheme. The matrix $\begin{pmatrix} A_0 & 0 \\ 0 & B \end{pmatrix}$ is a simple product of matrices of all elements involved (F, T, L).

In order to bring the matrix v to the form (13), we have to turn the whole system about the angle $(-\theta/2)$:

$$M = R^{-1} V R = R(\theta/2) \begin{pmatrix} A_0 & 0 \\ 0 & B \end{pmatrix} R(\theta/2) \quad (17)$$

Thus the system of quadrupole lenses and solenoids will be described by a transport matrix of the form (13), if we arrange the quadrupole lenses so that they are turned about an angle of $d = \int_0^{\theta} \varphi(s) ds = \theta/2$, where s is the azimuth of the arrangement of lenses.

We see that from this position all other considerations concerning the first method of compensating the influence of solenoid fields, fit completely the second method, too.

The most simple compensation scheme, constructed in accordance with the second method was illustrated in fig. 2. It only enables to remove any coupling of oscillations. Here the solenoid turns the plane of oscillations about the angle θ ; the first two lenses were turned about the angle $-\theta/2$; the second two lenses were turned about the angle $\theta/2$. The particular case of such a system for $\theta = \pi/2$ was made out in ref. (2). More complicated designs, avoiding the coupling of oscillations and allowing to obtain an arbitrary form of the matrix A , are illustrated in fig. 3 and 4.

3. Examples

It is clear that we must find for each concrete problem its scheme, which compensates the influence of the solenoid field. Above all it is necessary to find the most adequate form of the matrix A for the given problem. Quite often it is convenient to construct the compensation scheme in such a way that the matrix A has the form of a unit matrix $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ or the matrix of the drift space $\begin{pmatrix} 1 & \ell_{\text{tot}} \\ 0 & 1 \end{pmatrix}$, where ℓ_{tot} is the total length occupied by solenoids and 0 1 quadrupole lenses of the compensation scheme. According to (14), the transport matrix for the other coordinate should be equal to (-A). In order to fit the dispersion function, it is necessary that the matrix A corresponds to the motion in direction of X. Here the frequency of betatron oscillations in direction of X rises about a whole number and in direction of Z about half a number.

As an example of the efficiency of these methods, we calculated several versions of schemes compensating the influence of solenoid fields. We applied the program ORTI⁽⁶⁾ for these calculations and the search for definite solutions.

The compensation scheme of fig. 1 has the form of the matrix $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ for arbitrary values of the solenoid fields. The magnetic field gradients in quadrupole lenses are illustrated in fig. 5 as a function of the longitudinal field integral. The advantage of the given scheme is the standard position of quadrupole lenses and the possibility of an even connection of the field in the solenoids, which permits to vary adiabatically the direction of polarization. In order not to intersect the depolarizing spin resonances, it is necessary to carry out the rearrangement procedure at a spin frequency of $\nu_R = 1/2$.

Figs. 3 and 4 are showing compensation schemes with the matrix $A = \begin{pmatrix} 1 & \ell_{\text{tot}} \\ 0 & 1 \end{pmatrix}$. The theoretical values of gradients and angles of the turning of lenses for the given schemes are listed in table 1 and/or 2. We note that a mechanical turning of the lenses is required besides the alteration of the lens gradients in these schemes in order to secure a stepless connection of the magnetic field in the solenoids.

The possibility of a compact execution is another advantage of these schemes, since parts with a longitudinal field may be used as drift spaces here.

Conclusion

In this paper, we designed schemes to compensate the influence of solenoid fields on particle motion mainly on the basis of consideration of their application in order to obtain longitudinally polarized colliding beams. However, above mentioned methods of compensation might also be used for other purposes. One of the most obvious applications is for instance the compensation of field effects of solenoid detectors, which has usually been realized by additional solenoids. Since the field integral in these detectors is small enough ($\theta \sim 0.1 \pm 0.01$), it would be as well to use the scheme of fig. 2 (lens gradients $\sim \theta$). Due to the arrangement of quadrupole lenses closer to the point of interaction, we may suppose to obtain these smaller values of β -functions.

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Fig. 1



Fig. 2



Fig. 3

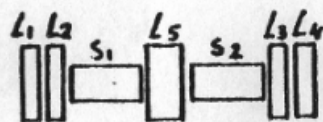


Fig. 4

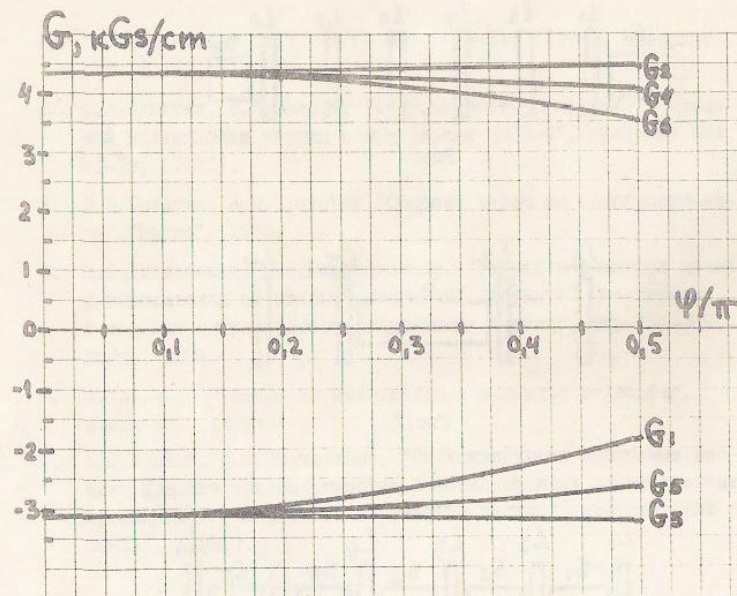


Рис.5 Зависимость градиентов линз от интеграла продольного поля. Энергия - 3 ГэВ, длина каждого соленоида - 82 см, длина каждой линзы - 20 см, расстояние между ними - 180 см, полная длина - 12 м.

Таблица 1

Энергия - 2 ГэВ, длина каждого соленоида - 50 см, поле соленоида - 83,78 кГс, длина линз ($L_1 \div L_4$) - 30 см, расстояние между соседними элементами - 10 см.

Название линзы	L_1	L_2	L_3	L_4	L_5	L_6
Градиент кГс/см	- 0.331	0.089	- 0.349	- 3.826	1.532	- 1.103
Угол	- 45°	- 27°	- 9°	9°	27°	45°

Таблица 2

Энергия - 2 ГэВ, длина каждого соленоида - 125 см, поле соленоида - 83.78 кГс, длина линз ($L_1 \div L_4$) - 20 см, длина линзы L_5 - 40 см, расстояние между соседними элементами - 10 см

Название линзы	L_1	L_2	L_3	L_4	L_5
Градиент кГс/см	- 3.252	2.949	2.949	- 3.252	- 3.123
Угол	- 45°	- 45°	45°	45°	0°