Quantum Field Theory in Doubly Special Relativity: A Geometrical Approach

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Abstract. Doubly special relativity is a theory that deforms the kinematics of special relativity by preserving a relativity principle, thus capturing the residual effects of a quantum gravity theory. This work explores the connection between doubly special relativity and non-local quantum field theory, which has been proposed as a way to reconcile general relativity and quantum theories. We demonstrate how a geometrical interpretation of doubly special relativity in a curved momentum space can lead to a formulation of a quantum field theory with infinite derivatives. The work establishes that only Snyder kinematics (and not κ-Poincaré) are compatible with the proposed framework, which maintains linear Lorentz invariance while introducing deformations through a modified derivative operator. We present deformed equations for scalar, fermionic, and electromagnetic fields, demonstrating how these formulations naturally lead to nonlocality effects that are characteristic of doubly special relativity.

1 Introduction

A major challenge in modern theoretical physics is the development of a quantum gravity (QG) theory. Several inconsistencies arise when attempting to merge general relativity (GR) and quantum field theory (QFT). One of the main issues is the role of spacetime in the theory. In QFT, the spacetime is assumed to be fixed, and the focus is on studying the properties and movement of the particles within it. In contrast, GR assumes that the properties of matter and radiation are determined and describes the resulting spacetime, specifically examining its curvature. However, in GR spacetime is considered classical, not quantum. It is a classical theory, whereas QFT is a quantum theory. While one could attempt to develop a QG theory based on an interaction mediated by a spin-2 particle (the graviton), which would reproduce the same equations as those in GR, this approach faces the issue of being non-renormalizable, limiting its ability to make precise predictions except at low energies [1].

Numerous efforts have been made to overcome the inconsistencies between GR and QFT, such as string theory [2], loop quantum gravity [3], supergravity [4], and causal set theory [5]. In many of these theories, a minimum length is introduced [6], often linked to the Planck length, which leads to the Planck energy scale. The main issue with these theories is the lack of experimental data that could guide us in determining the correct approach to the fundamental quantum gravity theory [7].

We know that spacetime in a QG theory should change its classical nature leading to a quantum structure, and precisely, a way to characterize it would be to consider a minimum length, in such a way that at small distances spacetime does not take a well-defined shape. In addition, we can wonder about the symmetries of this theory. Although we do not know them for sure, we do know that Lorentz invariance should be broken or at least deformed at small scales. Because of these quantum fluctuations

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in spacetime, new effects that would be unthinkable in a classical spacetime could emerge, such as the potential formation of micro black holes. Subsequently, virtual particles and antiparticles are constantly created and destroyed as the bubbles in a foam [8]. For instance, this means that the movement of particles could be modified or a nonlocality of interactions could arise.

So, how can a QG theory be constructed? One approach that has been extensively explored in the literature involves adding new terms proportional to the Ricci scalar in the Einstein-Hilbert action. However, these $f(R)$ theories [9] are not renormalizable. Another option is to include terms proportional to the squared Ricci and Riemann tensors [10]. While these theories are renormalizable at the perturbative level, the Hamiltonian is unbounded from below, which leads to vacuum instabilities in the physical system and the loss of unitarity in QFT. This issue can be addressed by considering not just terms proportional to the Ricci scalar and Ricci and Riemann tensors, but by incorporating infinite derivatives of these quantities. This results in theories of infinite derivative gravity [11].

An entirely different approach involves modifying the kinematics of special relativity (SR) by introducing a high-energy scale Λ. When examining symmetries, there are two potential ways to go beyond the kinematics of SR. One approach is to assume that Lorentz symmetry holds only at low energies, leading to Lorentz invariance violation [12]. In this framework, there is a preferred observer that removes the relativity principle that is crucial in both special and general relativity. Additionally, the dispersion relation is modified, but the conservation of momentum remains unchanged. Another possibility is to suggest that the symmetries are not broken, but rather deformed. This idea, known as doubly/deformed special relativity (DSR) [13], preserves a relativity principle. The basic elements of DSR [14] are a deformed dispersion relation (different from the quadratic expression of SR), and a deformed conservation/composition law of momenta (which is no longer the sum of the individual momenta of particles). In addition, there are some deformed Lorentz transformations that make the two previous ingredients compatible with the relativity principle (i.e., invariance of the physical measurements for any observer). In addition, in DSR theories, a noncommutative spacetime is usually associated with the kinematics [15].

Let us now discuss how nonlocality appears in DSR theories. It is well known that momentum can be considered as the generator of translations on spacetime. Because we have introduced a deformed composition law to define the total momentum of the system, translations should be different. In fact, when considering an action that takes into account a deformed conservation law for momenta, one sees that interactions are nonlocal. They will only be local for observers who are just placed where the interaction takes place. This phenomenon is known as relative locality [16]. The interactions are only local for an observer placed at the vertex of the interaction, and as one moves away, one sees that the particles do not cross. To explain this better, we use the following pictures by F. Mercati:

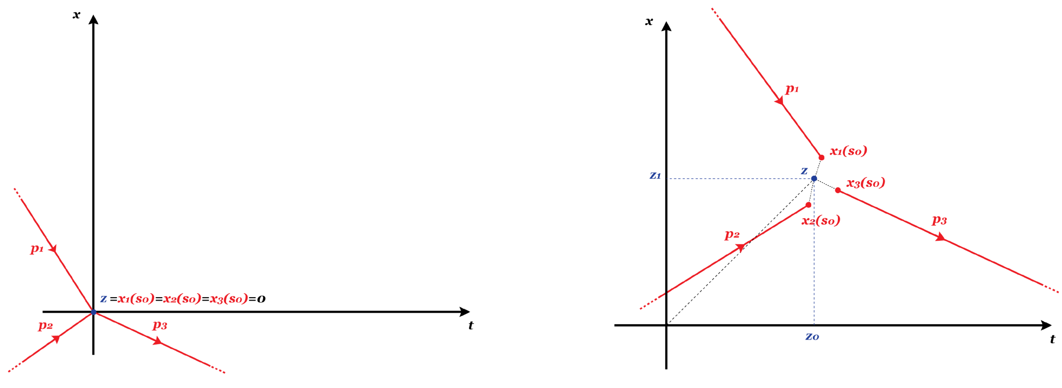

Figure 1: Figures concerning relative locality designed by F. Mercati for a talk at the Perimeter Institute in 2011 [17].

If we observe an interaction in our origin of coordinates, we would see that the worldlines of the particles cross at the same point. However, if the same interaction were to occur at some distance from our origin, we would see that the particles do not cross, and this effect would be greater the farther away the interaction occurs. Consequently, there are two sources of nonlocality in DSR theories: one due to the noncommutativity of space-time coordinates and the other coming from relative locality.

In this paper, we establish a link between a QFT with infinite derivatives (which results in a non-local QFT such as the ones at [18–20]) and a deformation of SR kinematics. Before presenting our proposal, Section 2 reviews previous findings related to the geometric connection between relativistic deformed kinematics (RDK) and a curved momentum space. In Section 3, we develop a QFT in DSR for scalar, fermionic, and electromagnetic fields. From this, we derive the modified Maxwell's equations, as well as the electric and magnetic potentials for a point particle and a magnetic dipole. Finally, we conclude with a discussion of the results and future directions.

2 Geometrical interpretation of relativistic deformed kinematics

One might ask: can RDK have a geometric interpretation? Recall that gravitational interactions can be described as a curvature of spacetime. Similarly, just as the transition from SR to GR involves considering curved spacetime instead of flat spacetime, could the transition from SR to DSR be represented by a curved momentum space? The concept of curved momentum space was introduced by Born in the 1930s. He proposed a duality between space-time and momentum variables. According to this idea, just as one can have a nontrivial geometry in spacetime, there could also be a nontrivial geometry in the momentum space.

In RDK, there exists a deformed dispersion relation, a conservation law defined through a deformed composition of momenta (referred to as the deformed composition law), and modified Lorentz transformations, all of which are consistent with the relativity principle. From an algebraic perspective [15], the deformed dispersion relation can be identified with the Casimir operator of the Poincaré algebra. The coproduct of momenta is regarded as the deformed composition law, whereas the coproduct of boosts describes how one momentum changes under Lorentz transformations in the presence of another momentum. Now, let's explore a geometric interpretation of RDK [21, 22]. In our framework, we treat the dispersion relation as the square of the distance from the origin to a point k in momentum space. This is because the distance is invariant under coordinate transformations in geometry, and the dispersion relation must also remain invariant under transformations between reference frames if the relativity principle holds. Both the composition law and Lorentz transformations act as isometries of the metric and together they form a Lie algebra. The conservation law corresponds to isometries that leave the origin unchanged, whereas the Lorentz generators are associated with the isometries corresponding to the Lorentz group.

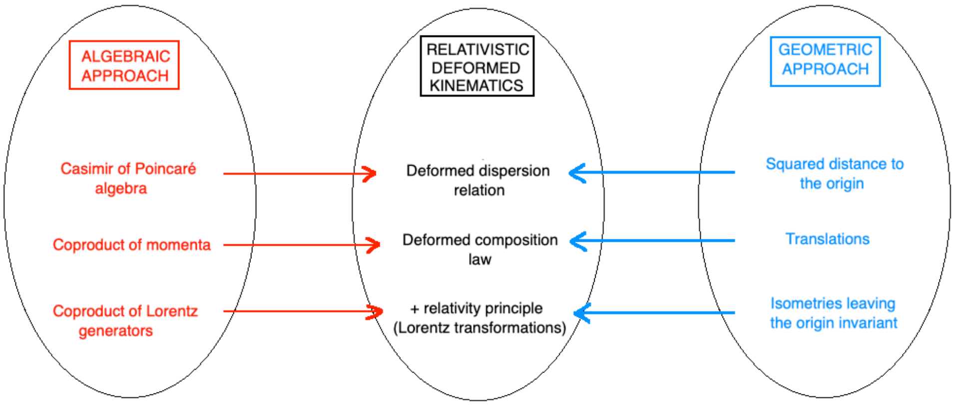

Figure 2: Algebraic interpretation of DSR versus the geometric one.

To make it clearer, consider a momentum space of four dimensions. We want four translations and six Lorentz generators; therefore, we need the momentum space to have ten isometries to define a relativistic kinematics. The only spaces with such a property are the maximally symmetric spaces: Minkowski, de Sitter (dS), and anti-de Sitter (AdS). Starting from a maximally symmetric momentum space, one is able to construct a RDK. The Casimir can be calculated from

$$C(p) = f^{\mu}(p) g_{\mu\nu}(p) f^{\nu}(p), \qquad f^{\mu}(p) := \frac{1}{2}\frac{\partial C(p)}{\partial p_{\mu}}. \tag{1}$$

The composition law

$$g_{\mu\nu}\left(p \oplus q\right) = \frac{\partial \left(p \oplus q\right)_{\mu}}{\partial q_{\rho}} g_{\rho\sigma}(q) \frac{\partial \left(p \oplus q\right)_{\nu}}{\partial q_{\sigma}} \tag{2}$$

is associated to the translation isometries of the momentum metric. This composition law defines a tetrad in momentum space

$$e^{\mu}{}_{\nu}(p) := \left.\frac{\partial\,(p\oplus q)_{\nu}}{\partial q_{\mu}}\right|_{q\to 0}, \tag{3}$$

being different for different kinematics, although they have the same dispersion relation. In order to illustrate this, we can consider the following dS momentum metric

$$g_{\mu\nu}(p) = \eta_{\mu\nu} + p_\mu p_\nu/\Lambda^2. \tag{4}$$

We can find all the isometries of this metric. The six generators of rotations and boosts can be univocally determined when imposing that the isometry generators close a Lorentz algebra:

$$\mathcal{J}^{\mu\nu} = p_\rho(\delta^{\nu}_{\lambda}\eta^{\mu\rho} - \delta^{\mu}_{\lambda}\eta^{\nu\rho})\frac{\partial}{\partial p_\lambda}, \quad \text{satisfying} \quad [\mathcal{J}^{\alpha\beta},\mathcal{J}^{\gamma\delta}] = \eta^{\beta\gamma}\mathcal{J}^{\alpha\delta} - \eta^{\alpha\gamma}\mathcal{J}^{\beta\delta} - \eta^{\beta\delta}\mathcal{J}^{\alpha\gamma} + \eta^{\alpha\delta}\mathcal{J}^{\beta\gamma}. \tag{5}$$

Now, there are infinite choices of the generators of translations. If we choose the ones for Snyder kinematics [23],

$$\mathcal{T}^{\lambda}_{S} = \sqrt{1+\frac{p^2}{\Lambda^2}}\,\frac{\partial}{\partial p_\lambda}, \tag{6}$$

that satisfy

$$[\mathcal{T}^{\alpha}_{S},\mathcal{T}^{\beta}_{S}] = \frac{\mathcal{J}^{\alpha\beta}}{\Lambda^2}, \qquad [\mathcal{T}^{\alpha}_{S},\mathcal{J}^{\beta\gamma}] = \eta^{\alpha\beta}\mathcal{T}^{\gamma}_{S} - \eta^{\alpha\gamma}\mathcal{T}^{\beta}_{S}, \tag{7}$$

they lead to the following composition law

$$(p\oplus q)^{S}_{\mu} = p_\mu\left(\sqrt{1+\frac{q^2}{\Lambda^2}} + \frac{p_\mu\eta^{\mu\nu}q_\nu}{\Lambda^2\left(1+\sqrt{1+p^2/\Lambda^2}\right)}\right) + q_\mu. \tag{8}$$

Now, we can identify the translation generators in momentum space defined by the deformed composition law with some noncommutative coordinates

$$[x^\mu, x^\nu] = \frac{\mathcal{J}^{\mu\nu}}{\Lambda^2}. \tag{9}$$

Hence, we have the non-locality effect of the coordinates mentioned previously. However, choosing the generators of translations which defines κ–Poincaré algebra [24]

$$\mathcal{T}^{\mu}_{\kappa} = \mathcal{T}^{\mu}_{S} + n_\alpha\frac{\mathcal{J}^{\mu\alpha}}{\Lambda}, \qquad \text{with} \qquad n_\mu := (1,0,0,0), \tag{10}$$

one finds

$$[\mathcal{T}^{\alpha}_{\kappa},\mathcal{T}^{\beta}_{\kappa}] = \frac{n_\gamma}{\Lambda}\left(\mathcal{T}^{\alpha}_{\kappa}\eta^{\beta\gamma} - \mathcal{T}^{\beta}_{\kappa}\eta^{\alpha\gamma}\right), \tag{11}$$

$$[\mathcal{T}^{\alpha}_{S},\mathcal{J}^{\beta\gamma}] = \eta^{\alpha\beta}\mathcal{T}^{\gamma}_{\kappa} - \eta^{\alpha\gamma}\mathcal{T}^{\beta}_{\kappa} + \frac{n_\delta}{\Lambda}\left(\eta^{\delta\beta}\mathcal{J}^{\alpha\gamma} - \eta^{\delta\gamma}\mathcal{J}^{\alpha\beta}\right), \tag{12}$$

$$[\mathcal{J}^{\alpha\beta},\mathcal{J}^{\gamma\delta}] = \eta^{\beta\gamma}\mathcal{J}^{\alpha\delta} - \eta^{\alpha\gamma}\mathcal{J}^{\beta\delta} - \eta^{\beta\delta}\mathcal{J}^{\alpha\gamma} + \eta^{\alpha\delta}\mathcal{J}^{\beta\gamma}. \tag{13}$$

We can see that either the composition law

$$(p\oplus q)^{\kappa}_{\mu} = p_\mu\,\left(\sqrt{1+\frac{q^2}{\Lambda^2}} + \frac{q_0}{\Lambda}\right) + q_\mu + n_\mu\left[\frac{\sqrt{1+p^2/\Lambda^2} - p_0/\Lambda}{1-\vec{p}^2/\Lambda^2}\left(q_0 + \frac{q_\alpha\eta^{\alpha\beta}p_\beta}{\Lambda}\right) - q_0\right], \tag{14}$$

or the coordinates of the non-commutative associated space

$$[x^0, x^i] = -\frac{x^i}{\Lambda}, \tag{15}$$

are different from those of Snyder. However, from a geometrical perspective, both relativistic kinematics share the same momentum metric, which is a dS space.

It is important to mention that the Klein–Gordon and Dirac equations were previously obtained in DSR using the Hopf algebra scheme [25, 26]. However, thanks to the geometrical construction mentioned above, we obtain exactly the same equations [22]:

$$\text{Klein–Gordon:} \qquad \left(f^\mu(p) g_{\mu\nu}(p) f^\nu(p) - m^2\right) \tilde{\phi}(p) = 0, \tag{16}$$

$$\text{Dirac:} \qquad \left(\gamma^\mu \eta_{\mu\rho} e^\rho{}_\nu(p) f^\nu(p) - m\right) \tilde{\psi}(p) = 0, \tag{17}$$

being $\tilde{\phi}(p)$ the Fourier transform of the scalar field $\phi(x)$

$$\phi(x) = \frac{1}{(2\pi)^3} \int \mathrm{d}^4 p \, e^{ix^\lambda p_\lambda} \tilde{\phi}(p) \, \delta(C(p) - m^2), \tag{18}$$

and similarly for the Dirac field $\psi(x)$. Hence, we can conclude that our geometrical construction is equivalent to the algebraic one.

3 DSR QFT in position space

Our aim is to formulate a DSR QFT in position space, following the geometrical construction in momentum space discussed in the previous section. A simple possibility of generalizing the usual Klein–Gordon equation is to replace the momentum dependency of the metric and the derivatives of the Casimir by an operator ℓ such that:

$$S = \int \frac{\mathrm{d}^4 x}{2} \left\{ -\ell^\mu(-i\partial_x) \, \phi(x) \eta_{\mu\nu} \ell^\nu(-i\partial_x) \, \phi(x) - m^2 \phi^2(x) \right\} \quad \text{where} \quad \ell^\mu(-i\partial_x) = e^\mu{}_\nu(p) f^\nu(p) \mid_{p \to -i\partial_x} . \tag{19}$$

Notice that the Minkowski metric is at the base of our proposal. Then, the same linear Lorentz invariance that characterizes the usual QFT should also be present in our scheme. This fact restricts the possible bases of kinematics, leaving only space for those that possess such invariance. It implies that the tetrad must be quadratic in momentum, because $\ell_\mu(-i\partial) = \ell_\mu(i\partial)$ must hold [27, 28]. Consequently, only Snyder kinematics are allowed in our scheme (remember that in κ-Poincaré kinematics, some terms of the deformed composition law of momenta consist of the momentum times a fixed time-like vector). Due to the particular form of the tetrad and the dispersion relation, we see that our construction relies on the replacement of usual derivatives by a function of them, which involves the usual derivative times a function of the d'Alembertian operator. Thus, one can easily write the Klein–Gordon action, the equations of motion, and the energy-momentum tensor simply by performing the replacement $\ell^\mu(-i\partial_x) \to -i\partial^\mu \Omega(-\partial^\nu \partial_\nu) \equiv -i\tilde{\partial}^\mu$ in the usual formulae in QFT. Thus, for instance, the action becomes

$$S = \int \mathrm{d}^4 x \, \frac{1}{2} \left\{ \tilde{\partial}^\mu \phi(x) \tilde{\partial}_\mu \phi(x) - m^2 \phi^2(x) \right\}. \tag{20}$$

Because our proposal of DSR QFT leads to an infinite derivative QFT, it leads to non-local effects. However, nonlocality is already present in DSR through a noncommutative spacetime and relative locality, as we have seen. Therefore, one of the most interesting aspects of DSR is recovered when considering our QFT proposal. Moreover, the QFTs of causal set [29] and string theories [30] can be embedded in our geometrical approach in momentum space if we choose the appropriate Casimir operators:

$$C^{\text{causal}}(-i\partial_x) = -\Box + \frac{3}{2\pi\sqrt{6}} \frac{\Box^2}{\Lambda^2} \left[3\gamma - 2 + \ln\left(\frac{3\Box^2}{2\pi\Lambda^4} \right) \right] + \cdots , \tag{21}$$

$$C^{\text{string}}(-i\partial_x) = -\Box \, e^{\Box/\Lambda^2}. \tag{22}$$

We now discuss the fermionic and electromagnetic fields. The Dirac and Maxwell equations can be obtained just by replacing the usual partial derivative by the tilde derivative as follows:

$$\begin{aligned} &\left(i\gamma^\mu \tilde{\partial}_\mu - m \right) \psi(x) = 0, \\ &\tilde{\partial}^\mu \tilde{F}_{\mu\nu} = j_\nu, \qquad \tilde{\partial}^\mu {}^*\tilde{F}_{\mu\nu} = \tilde{\partial}^\mu \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} \tilde{F}_{\rho\sigma} = 0, \text{ with } \tilde{F}_{\mu\nu} = \Omega(-\Box) F_{\mu\nu}. \end{aligned} \tag{23}$$

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It is important to note that when considering only the fermionic case, the equations derived from the variational principle impose no restrictions on the possible kinematics. However, it does for bosons. Then, since we aim to develop a complete QFT that includes both fermions and bosons, we will adopt a kinematic framework that respects Lorentz invariance (specifically, Snyder models), similar to the scalar case, as this represents the most restrictive scenario. Indeed, this restriction can be avoided for bosons if the starting deformed Klein–Gordon action is

$$S = \int \frac{\mathrm{d}^4 x}{2} \left\{ \ell^\mu(-i\partial_x)\, \phi(x) \eta_{\mu\nu} \ell^\nu(i\partial_x)\, \phi(x) - m^2 \phi^2(x) \right\}, \tag{24}$$

which only requires that the condition $\ell_\mu(-i\partial_x)\ell^\mu(-i\partial_x) = \ell_\mu(i\partial_x)\ell^\mu(i\partial_x)$ be satisfied when one imposes the dispersion relation in the variational principle. Since in that case $C(p) = C(-p)$, the dispersion relation (and not also the tetrad) must be quadratic in the momentum. Then, the κ-Poincaré model is allowed, being linear Lorentz invariance the only requirement of the allowed kinematics in this framework. However, the condition $\ell_\mu(-i\partial) = \ell_\mu(i\partial)$ derived from the action of bosons in Eq. (19) can also be recovered for the electromagnetic sector. If we define the electromagnetic tensor as

$$\bar{F}_{\mu\nu} = \ell_\mu(-i\partial_x)A_\nu - \ell_\nu(-i\partial_x)A_\mu, \tag{25}$$

the electromagnetic Lagrangian will be

$$S_{EM} = -\int \mathrm{d}^4 x \left(\frac{1}{4} \bar{F}_{\mu\nu} \bar{F}^{\mu\nu} \right) = -\int \mathrm{d}^4 x \frac{1}{2} \left(\ell_\mu(-i\partial_x)A_\nu\, \ell^\mu(-i\partial_x)A^\nu - \ell_\mu(-i\partial_x)A_\nu\, \ell^\nu(-i\partial_x)A^\mu \right). \tag{26}$$

Note that this Lagrangian is invariant under the gauge transformation

$$A'_\mu = A_\mu + \ell_\mu(-i\partial_x)\theta. \tag{27}$$

By applying the variational principle, one obtains the following equations of motion

$$\ell^\mu(i\partial_x)\bar{F}_{\mu\nu} = 0. \tag{28}$$

Moreover, when using the gauge

$$\ell^\mu(i\partial_x)\ell_\mu(-i\partial_x)\theta = 0, \qquad \text{implying} \qquad \ell^\mu(i\partial_x)A_\mu = 0, \tag{29}$$

one finds the equation

$$\ell^\mu(i\partial_x)\ell_\mu(-i\partial_x)A^\nu = 0, \tag{30}$$

which does not satisfy the dispersion relation. Then, $\ell_\mu(-i\partial) = \ell_\mu(i\partial)$ must hold, and the same restriction as for bosons is recovered.

A different option would be to consider the next electromagnetic Lagrangian

$$S_{EM} = -\int \mathrm{d}^4 x \left(\frac{1}{4} \bar{F}_{\mu\nu} \bar{\bar{F}}^{\mu\nu} \right), \tag{31}$$

where

$$\bar{\bar{F}}^{\mu\nu} = \ell^\mu(i\partial_x)A^\nu - \ell^\nu(i\partial_x)A^\mu. \tag{32}$$

In this case, the variational principle leads to $\ell_\mu(-i\partial_x)\ell^\mu(-i\partial_x) = \ell_\mu(i\partial_x)\ell^\mu(i\partial_x)$, instead of the more restrictive condition $\ell_\mu(-i\partial_x) = \ell_\mu(i\partial_x)$. Then, κ-Poincaré kinematics can be accommodated in this scheme. However, the electromagnetic Lagrangian will lose the gauge invariance that it has in usual QFT, since $\bar{\bar{F}}^{\mu\nu}$ is not invariant under the gauge of Eq. (27).

Let's see now a pair of simple examples. Consider the electric scalar potential of a point particle. We can prove that for the Casimirs obtained from the following maximally symmetric metrics (the sign plus stands for dS while the minus for AdS)

$$g^{(1)}_{\mu\nu} = \eta_{\mu\nu} \left(1 \pm \frac{p^2}{4\Lambda^2} \right)^2, \qquad g^{(2)}_{\mu\nu} = \eta_{\mu\nu} \pm \frac{p_\mu p_\nu}{\Lambda^2}, \tag{33}$$

the electric potential

$$A^0(r) \;=\; -\frac{q}{2\pi^2} \int_0^\infty \mathrm{d}k\, \frac{k^2}{C(-\vec{k}^2)} \frac{\sin(kr)}{kr}, \tag{34}$$

is finite at the origin when considering AdS momentum spaces. However, in the cases of dS, it diverges. This fact seems to favor AdS spaces over dS spaces.

Consider a static magnetic dipole localized at the origin and calculate the magnetic vector potential

$$\vec{A}(r) = \vec{m} \times \frac{\vec{r}}{2\pi^2 r^3} \int_0^\infty \frac{k\,\mathrm{d}k}{C(-\vec{k}^2)} \left(kr\cos(kr) - \sin(kr)\right). \tag{35}$$

We can verify that the magnetic vector potential for AdS tends to zero when $r \to 0$. In contrast, for both the undeformed model (Minkowski spacetime) and the dS case, the magnetic vector potential becomes infinite at the origin. It is evident that for larger distances from the origin, all the models (dS and AdS) exhibit the same behavior as the Minkowski space, where the vector potential vanishes at sufficiently large distances. Further details on the calculations, along with some figures, are presented in [27, 28].

4 Conclusions

We have demonstrated how to build a QFT within the DSR framework, drawing inspiration from a geometric approach to RDK. This method results in a nonlocal QFT due to the infinite derivatives operating on the fields. Our framework supports Snyder kinematics but excludes κ-Poincaré, because the composition law of the latter depends on a fixed vector. We have derived a deformation of the Maxwell equations that allowed us to calculate the electric potential of a point particle and the magnetic field of a magnetic dipole, which become finite at the origin in certain cases. Our current work focuses solely on the free components of the theory. In future research, we aim to investigate how to incorporate interactions using a modified composition law within our DSR QFT proposal.

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