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Big Bang Nucleosynthesis Constraints and Indications for Beyond Standard Model Neutrino Physics

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Abstract: We use Big Bang Nucleosynthesis (BBN) to probe Beyond Standard Model physics in the neutrino sector. Recently, the abundances of primordially produced light elements D and He-4 were determined from observations with better accuracy. The good agreement between the theoretically predicted abundances of primordially produced light elements and those derived from observations allows us to update the BBN constraints on Beyond Standard Model (BSM) physics. We provide numerical analysis of several BSM models of BBN and obtain precise cosmological constraints and indications for new neutrino physics. Namely, we derive more stringent BBN constraints on electron neutrino–sterile neutrino oscillations corresponding to 1% uncertainty of the observational determination of the primordial He-4. The cosmological constraints are obtained both for the zero and non-zero cases of the initial population of the sterile neutrino state. Then, in a degenerate BBN model with neutrino $\nu_e \leftrightarrow \nu_s$ oscillations, we analyze the change in the cosmological constraints in case lepton asymmetry L is big enough to suppress oscillations. We obtain constraints on the lepton asymmetry L. We discuss a possible solution to the dark radiation problem in degenerate BBN models with $\nu_e \leftrightarrow \nu_s$ oscillations in case L is large enough to suppress neutrino oscillations during the BBN epoch. Interestingly, the required value of L for solving the DR problem is close to the value of L indicated by the EMPRESS experiment, and also it is close to the value of lepton asymmetry that is necessary to relax Hubble tension.



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Keywords: BSM physics; BBN constraints; neutrino oscillations; sterile neutrino; lepton asymmetry; dark radiation

1. Introduction

Cosmology presents complimentary and often unique knowledge about neutrino physics. It constrains neutrino characteristics, its mass, number density, the number of different light neutrino types, oscillation parameters, lepton asymmetry in the neutrino sector, etc. This paper is dedicated to the cosmological influence of the right-handed neutrino, referred to as the sterile neutrino ν_s , to $\nu_e \leftrightarrow \nu_s$ neutrino oscillations and lepton asymmetry in the neutrino sector, all of which represent BSM neutrino physics.

The study of sterile neutrino is motivated by the important role of ν_s in many models of BSM physics and the beyond standard cosmological model. The sterile neutrino is predicted by several Grand Unified Theories. Theoretical models use ν_s to explain small non-zero neutrino masses, to build models of natural baryogenesis through leptogenesis, etc. Sterile neutrino is the preferred particle candidate for solving the dark matter problem in cosmology; it plays a role in models of large scale structure formations, etc.

In addition, combined neutrino oscillations data of the reactor experiments + LSND + MiniBooNe + Gallium experiments GALLEX, SAGE, and BEST [1–14] provide hints of the possible presence of light right-handed neutrinos participating in oscillations with flavor neutrinos. The neutrino oscillation parameters suggested experimentally, however, contradict the cosmological constraint on additional radiation during early Universe stage (this is the so-called dark radiation problem).

In this paper, based on contemporary BBN and recent precise data on primordial He-4, we update several cosmological constraints on the physical characteristics of sterile neutrino. We analyze the following BSM of BBN, namely BBN with late $\nu_e \leftrightarrow \nu_s$ neutrino oscillations, BBN with $\nu_e \leftrightarrow \nu_s$ with non-zero initial population of sterile neutrino and BBN with late $\nu_e \leftrightarrow \nu_s$ with lepton asymmetry in the neutrino sector. (For other possibilities of the conversion of active–sterile neutrinos such as spin flip oscillations in early universe and their effect on the BBN, see ref. [15–17]). We derive cosmological constraints on $\nu_e \leftrightarrow \nu_s$ oscillations parameters for different degrees of sterile neutrino population during BBN. In the BBN model with $\nu_e \leftrightarrow \nu_s$ oscillations and neutrino–antineutrino lepton asymmetry, we discuss the interplay between the neutrino oscillations and the asymmetry. In case of big asymmetry values, the BBN constraints on the oscillation parameters are changed. In this BBN model, the constraints on the value of the lepton asymmetry can be obtained. We discuss the cosmological and observational indications for non-zero lepton asymmetry during the BBN epoch. Cosmological information about the lepton asymmetry is valuable because—unlike the case of baryon asymmetry, which has already been measured with good precision—the lepton asymmetry value is still not known.

We obtain updated cosmological constraints on BSM neutrino derived in several previous publications; see refs. [18–21]. First, in Section 2, after a brief review of BBN and the new precise measurements of primordially produced He-4, we derive updated BBN constraints on electron neutrino–sterile neutrino oscillation parameters corresponding to the recent precise determination of He-4. We discuss the change in the BBN constraints on the oscillation parameters in case of non-zero initial population of the sterile neutrino. Then, in Section 3, we discuss lepton asymmetry in degenerate BBN model with $\nu_e \leftrightarrow \nu_s$ neutrino oscillations. We also derive a stringent cosmological constraint on the lepton asymmetry in the model of BBN nucleosynthesis with neutrino oscillations. We discuss the change in the BBN constraints on oscillation parameters in the case of initially present lepton asymmetry and discuss a solution to the dark radiation problem in such models. Finally, we discuss a recently found observational hint for neutrino–antineutrino asymmetry in the electron neutrino sector by the EMPRESS experiment, which is close to the value of the lepton asymmetry required to solve the DR problem and close to the value of lepton asymmetry necessary to relax Hubble tension.

2. BBN and Updated Constraints on Electron Neutrino–Sterile Neutrino Oscillations Parameters

BBN is a theoretically and experimentally well established and observationally confirmed model explaining the synthesis of the light elements during the early epoch corresponding to the Universe cooling from $T \sim 1$ MeV until 0.1 MeV. Precise experimental data on nuclear processes rates relevant for BBN epoch exist; see NACRE-I [22] and NACRE-II [23]. Over 400 nuclear reactions are included in the precise BBN codes, like PArthENoPE, AlterBBN, PRIMAT, etc. [24–28].

The three parameters, on which production of elements depend during BBN are determined precisely, namely: the baryon-to-photon ratio η , measured independently by CMB $\eta_{\text{CMB}} = (6.104 \pm 0.055) \times 10^{-10}$, the neutron lifetime $\tau_n = 881.5 \pm 0.7 \pm 0.6$ s (see ref. [29]), and the number of the effective degrees of freedom of light particles N_{eff} during the BBN epoch $\Delta N_{\text{eff}} < 0.2$. It is important to note that in the 90ies LEP experiments at CERN provided the stringent measurement of the number of weakly interacting neutrino: $N_\nu = 2.98 \pm 0.008$. This constraint does not concern sterile neutrinos, which do not participate into weak interactions.

There exists a remarkable agreement between the predicted abundances of light elements produced during BBN for $\eta = 6.10^{-10}$ and those derived from observations values. During recent years, observational data on deuterium and helium-4 Y_p have reached high precision [30–34]. In particular, the precision of Y_p observational data improved

considerably due to the inclusion of a He10830 infrared emission line of the extremely metal poor galaxy Leo P [35]:

$$Y_p = 0.2453 \pm 0.0034.$$

More details can be found in [35–37]. The theoretically predicted using BBN mass fraction of primordial He-4 is [27]

$$Y_t = 0.24709 \pm 0.00017.$$

Hence, BBN is considered to be one of the most reliable precision probes for physical conditions in the early Universe and a unique test for new physics.

The determination of primordial He-4 with 1% accuracy allows us to update and strengthen the cosmological constraints on BSM physics. Here, we present the BBN constraints on BSM neutrino physics, corresponding to 1% Y_p precision. In the following subsections the BBN constraints on electron–sterile neutrino oscillation parameters are derived in the cases of zero and of non-zero initial population of the sterile neutrino.

2.1. BBN with Electron–Sterile Neutrino Oscillations: BBN Constraints on Neutrino Oscillation Parameters in Case of Zero Initial Population of the Sterile Neutrino

We considered the model of BBN with non-equilibrium electron neutrino–sterile neutrino oscillations, $\nu_e \leftrightarrow \nu_s$, with small mass differences, effective after the electron neutrino decoupling at 2 MeV.

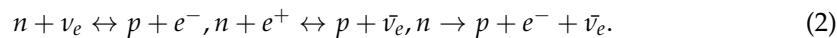
$$\begin{aligned} \nu_1 &= \cos(\theta)\nu_e + \sin(\theta)\nu_s \\ \nu_2 &= -\sin(\theta)\nu_e + \cos(\theta)\nu_s, \end{aligned} \quad (1)$$

ν_1 and ν_2 are particles with masses m_1 and m_2 and θ is the mixing angle.

This model was proposed and numerically studied in refs. [38–40]. For oscillations to become effective after 2 MeV, the following condition between the neutrino squared mass differences expressed in eV² and neutrino mixing holds:

$$\delta m^2 \sin^4 2\theta \leq 10^{-7}.$$

In case when the neutrino oscillations proceed after the electron neutrino decoupling, the neutrino electron–sterile neutrino oscillations deplete ν_e state for the expense of the ν_s state; hence, there is no increase in the radiation density and of the expansion rate of the Universe. (In BBN with fast $\nu_a \leftrightarrow \nu_s$ oscillations which proceed before active neutrino decoupling oscillations can bring the sterile neutrino state into equilibrium; thus, lead to higher expansion rate of the Universe and higher production of He-4. This is the well-known dynamical effect of active–sterile neutrino oscillations [41–43]). Nevertheless, neutrino energy distribution and the number density of electron neutrinos in this model may considerably differ from its equilibrium BBN values—for a large number of oscillation parameters, the number density of the electron neutrino is reduced, and the energy spectrum distribution is distorted from its equilibrium Fermi–Dirac form; this leads to a considerable reduction in both the number density of the electron neutrino and its energy. These changes in ν_e and correspondingly $\bar{\nu}_e$ influence the kinetics of nucleons participating in the reactions in the pre-BBN epoch:



Thus, nucleon densities and their values at nucleon freeze-out differ from the standard BBN ones; hence, the primordial yields of He-4 and other light elements are changed. We refer to this effect on BBN as the kinetic effect of neutrino oscillations.

Qualitatively, the change in primordial He-4 production is as follows: both the reduction in the number density of electron neutrino and the energy spectrum distortion lead to reduced weak reaction rates of nucleons interactions in Equation (2) in comparison with the

standard BBN case; hence, their freezing out occurs earlier. This causes an overproduction of primordially synthesized ${}^4\text{He}$.

We have provided a precise numerical analysis of neutrino evolution and nucleons freezing in the presence of neutrino oscillations. The evolution of the oscillating ν_e and ν_s is described numerically, simultaneously accounting for the Universe expansion, the neutrino oscillations, and the neutrino forward scattering. The governing kinetic equations for the density matrix of neutrino ρ and antineutrino $\bar{\rho}$ in the momentum space used in our analysis to describe the evolution of the neutrino and antineutrino ensembles in the early Universe (see also refs. [40,44,45]) are:

$$\begin{aligned}\partial\rho(t)/\partial t &= Hp_v (\partial\rho(t)/\partial p_v) + \\ &+ i[\mathcal{H}_o, \rho(t)] + i\sqrt{2}G_F (\mathcal{L} - Q/M_W^2) N_\gamma [\alpha, \rho(t)],\end{aligned}\quad (3)$$

$$\begin{aligned}\partial\bar{\rho}(t)/\partial t &= Hp_v (\partial\bar{\rho}(t)/\partial p_v) + \\ &+ i[\mathcal{H}_o, \bar{\rho}(t)] + i\sqrt{2}G_F (-\mathcal{L} - Q/M_W^2) N_\gamma [\alpha, \bar{\rho}(t)].\end{aligned}\quad (4)$$

These equations describe the neutrino and antineutrino ensembles evolution and account simultaneously for the Universe expansion (first term), neutrino oscillations (second term), and neutrino forward scattering. Here, $\alpha_{ij} = U_{ie}^* U_{je}$, $\nu_i = U_{il} \nu_l$ ($l = e, s$). \mathcal{H}_o is the free neutrino Hamiltonian. Q arises as an W/Z propagator effect, $Q \sim E_V T$. $\mathcal{L} \sim 2L_{\nu_e} + L_{\nu_\mu} + L_{\nu_\tau}$, $L_{\nu_\mu, \nu_\tau} \sim (N_{\nu_\mu, \nu_\tau} - N_{\bar{\nu}_\mu, \bar{\nu}_\tau})/N_\gamma$, $L_{\nu_e} \sim \int d^3p (\rho_{LL} - \bar{\rho}_{LL})/N_\gamma$. H is the Hubble parameter, N_γ denotes the number density of photons, N_{ν_μ} is the number density of muon neutrino and N_{ν_τ} of tau neutrino; for antineutrinos this is $N_{\bar{\nu}_\mu}$, $N_{\bar{\nu}_\tau}$, respectively.

The kinetic equation for the neutron number densities in the momentum space is:

$$\begin{aligned}\partial n_n/\partial t &= Hp_n (\partial n_n/\partial p_n) + \\ &+ \int d\Omega(e^-, p, \nu) |\mathcal{A}(e^- p \rightarrow vn)|^2 [n_{e^-} n_p (1 - \rho_{LL}) - n_n \rho_{LL} (1 - n_{e^-})] \\ &- \int d\Omega(e^+, p, \tilde{\nu}) |\mathcal{A}(e^+ n \rightarrow p \tilde{\nu})|^2 [n_{e^+} n_n (1 - \bar{\rho}_{LL}) - n_p \bar{\rho}_{LL} (1 - n_{e^+})].\end{aligned}\quad (5)$$

By n we denoted the particle number densities in momentum space, where the index shows the corresponding type of particles, for example for electrons density n_{e^-} . Here, we provide precise numerical analysis for the evolution of neutrino and the number density of nucleons as a function of time in the pre-BBN period in the presence of $\nu_e \leftrightarrow \nu_s$ oscillations, which become effective after neutrino freezing, for numerous sets of oscillation parameters, accounting for the distortion in electron neutrino energy distribution due to neutrino oscillations. Furthermore, both neutrino oscillations cases corresponding to non-resonant $\delta m^2 < 0$ and resonant $\delta m^2 > 0$ neutrino oscillations were considered. We provide a numerical analysis for a large number of neutrino oscillation parameters—135 sets of δm^2 and mixing $\sin^2 2\theta$. The evolution of the oscillating neutrino and the number density nucleons as a function of time was simultaneously numerically followed during the pre-BBN and BBN epoch, for temperatures from 2 MeV to 0.3 MeV. We calculated the primordially synthesized ${}^4\text{He}$ in this model. The iso-helium contours corresponding to 1–3% ${}^4\text{He}$ overproduction were calculated, corresponding to the contemporary accuracy of the observational determination of primordial ${}^4\text{He}$. BBN constraints on neutrino oscillation parameters—squared mass differences δm^2 and mixing $\sin^2 2\theta$ —were obtained for different initial populations of the sterile neutrino. (In previous papers [46–48], the iso-helium contours corresponding to 3–7% ${}^4\text{He}$ overproduction were calculated, which corresponded to the accuracy of determination of helium-4 at that time.)

As far as the sterile neutrino ν_s does not have the usual weak interactions, it decouples earlier than the active neutrinos; hence, it is expected that its number density is lower than the electron neutrino one $n_{\nu_s} \ll n_{\nu_e}$ at the decoupling time of the active neutrino. We discuss different possibilities for the initial population of the sterile neutrino state, namely

initially empty sterile neutrino state, $N_s = 0$, and the partially filled sterile neutrino state, $0 \leq N_s < 1$.

In the cases when the distortion of neutrino momentum distribution by oscillations is considerable, the numerical analysis is heavy because precise description of neutrino momenta distribution is needed. We have found that for proper description of the distortion in the neutrino momenta distribution in the non-resonant oscillations case 5000 bins are enough, while in the resonant case, up to 10,000 bins may be necessary. This complicates the numerical task severely and increases the calculation time considerably, because the number of integro-differential equations to be solved is multiplied by 5000 and 10,000, respectively.

In Figure 1, we present the results for iso-helium contours corresponding to 1% and 3% Y_p overproduction. The initial sterile neutrino state was assumed empty, $N_s = 0$. BBN constrains the neutrino oscillation parameters corresponding to the area above the 1% iso-helium contour. BBN constraints are considerably strengthened for 1% helium overproduction in comparison with 3% ones.

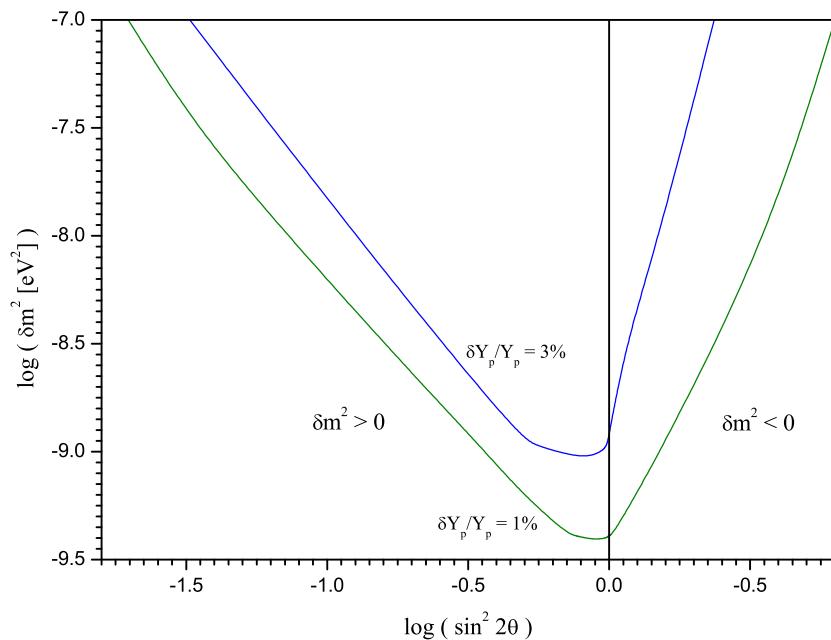


Figure 1. BBN constraints on electron neutrino–sterile neutrino oscillation parameters correspond to 1% (lower curve) Y_p uncertainty. The 3% (upper curve) Y_p uncertainty curve is given for comparison with previous constraints.

The analytical fit to the exact numerical constraints corresponding to 1% helium uncertainty reads:

$$\delta m^2 (\sin^2(2\theta))^{2.9} \leq 10^{-9.45} \quad dm^2 < 0$$

$$\delta m^2 \leq 3.9 \times 10^{-10} \quad dm^2 > 0, \quad (6)$$

where, δm^2 is in eV^2 . These BBN constraints update and strengthen the previously existing ones; see refs. [47–50]. They are half an order of magnitude more stringent at maximal mixing than previous 3% constraints. They excluded totally the low mixing angle electron-sterile solution to the solar neutrino problem in addition to the large mixing angle solution excluded in previous papers. Furthermore, these constraints exclude the oscillation parameters range discussed by several neutrino experiments; that have obtained hints for electron sterile oscillations with large mixings and squared mass differences in the range $10^{-2}\text{--}O(1) \text{ eV}^2$. Namely, several small base line experiments—reactor experiments, LSND, MiniBooNe, Gallium expt, SAGE, and recently Ice Cube and NU4—hint to a presence of

sterile neutrino participating into oscillations with flavor neutrinos with large mixing and squared mass difference in the range from $\delta m_{41}^2 = 10^{-2}$ eV² to several eV² [51–56].

In the next subsection, we discuss the general case of partially filled initially sterile neutrino state.

2.2. BBN Constraints on Electron–Sterile Neutrino Oscillations in Case of Non Zero Initial Population of the Sterile Neutrino

The effective number of relativistic species N_{eff} is given by $\rho_\nu = 7/8(T/T_\nu)^4 N_{eff} \rho_\gamma(T)$. Additional relativistic species or additional light sterile neutrino types increase the expansion rate of the universe $H \sim (G_N \rho)^{1/2}$, where $\rho = \rho_\gamma + \rho_\nu$ is the relativistic density. Therefore, it has considerable cosmological effect, in particular it influences BBN synthesis of light elements. Hence, BBN, and in particular He-4 is a sensitive probe to additional species and it tests and constrains new physics, which can be parameterized by ΔN_{eff} .

Although the indicated by BBN and CMB effective number of light neutrino types are consistent with the predicted value by the standard cosmological model $N_{eff} = 3.045$ within uncertainties, small extra relativistic component $\Delta N_{eff} = N_{eff} - 3.045 < 0.2$ is still allowed. Cosmological constraints on ΔN_{eff} based on BBN+CMB considerations [57] read:

$$N_{eff} = 2.898 \pm 0.141, N_{eff} < 3.18(95\%)$$

For comparison, BBN only based constraint from different analyzes, based on D and He-4 data [27,31], gives:

$$N_{eff} = 2.8 \pm 0.154(95\%).$$

While a maximum likelihood analysis on η and N_{eff} provides the limit [58]:

$$N_{eff} = 2.843 \pm 0.27(95\%), \eta = 6.09 \pm 0.055(95\%).$$

For comparison, the constraint based on Planck CMB data [27] reads:

$$N_{eff} = 3.01 \pm 0.15(95\%).$$

In what follows, we assume the cosmological constraint $\Delta N_{eff} \leq 0.2$. We consider the case when the sterile neutrino state was initially partially filled, corresponding to $\delta N_s = 0.2$, and study how its presence changes the BBN constraints on $\nu_e \leftrightarrow \nu_s$ oscillation parameters, presented in the previous subsection.

The presence of additional sterile neutrino in the BBN model with late $\nu_e \leftrightarrow \nu_s$ oscillations leads to the increase in the expansion rate of the Universe (dynamical effect) and it suppresses the kinetic effects of the non-equilibrium neutrino oscillations (a kinetic effect) [44,59].

Hence, depending on which effect dominates for a certain set of oscillation parameters, the additional sterile neutrino population has a non-trivial effect—it may correspondingly increase or decrease the primordial production of He-4. For more details, see refs [44,50,59].

We provided a numerical analysis of the BBN model with late electron–sterile neutrino oscillations in case of non-zero initial population of the sterile neutrino $\delta N_s = 0.2$. In this case, our analysis proved that the dynamical effect of additional δN_s dominates, leading to increased primordial production of He-4 and hence to stronger BBN limits on the neutrino oscillation parameters. Figure 2 presents our results. The lower curve gives the BBN constraints corresponding to $\delta N_s = 0.2$, the upper curve is given for comparison (it shows the BBN constraints corresponding to initially empty sterile state, i.e., $\delta N_s = 0$).

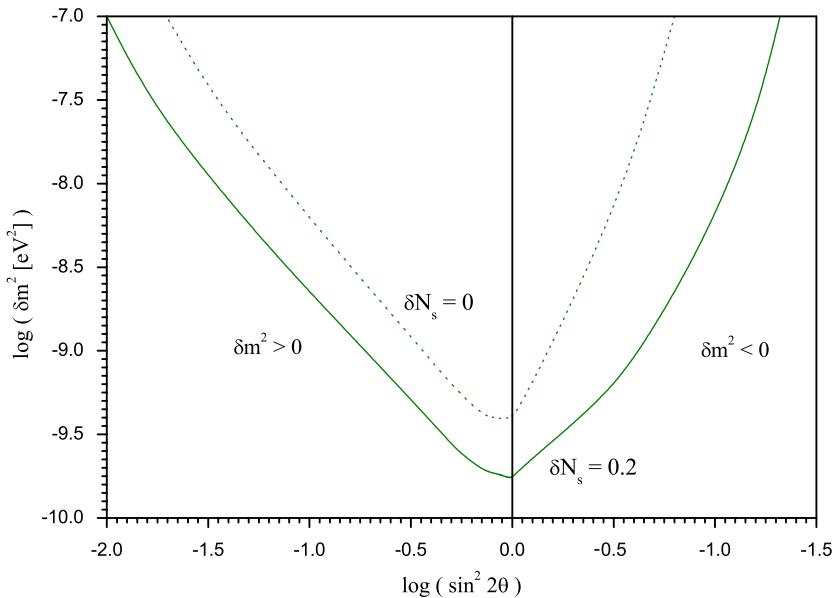


Figure 2. BBN constraints on electron neutrino–sterile neutrino oscillation parameters corresponding to 1% He-4 overproduction and initially empty sterile state (upper dashed curve) and $\delta N_s = 0.2$ (lower solid curve).

As illustrated in the figure, BBN constraints are considerably strengthened in case of non-zero initial population of the sterile neutrino $\delta N_s = 0.2$. This result is in agreement with our preliminary analysis, discussed in ref. [20].

The obtained BBN constraints, corresponding to $\delta N_s = 0.2$, forbid the presence of thermalized BBN light sterile neutrinos. The existence of such light sterile neutrinos have been suggested by several neutrino oscillations experiments. The oscillation parameters of these sterile neutrinos would lead to their thermalization during BBN. A possible solution to this so-called dark radiation problem will be shortly discussed in the next section.

3. BBN with Lepton Asymmetry and Neutrino Oscillations

BBN with non-zero lepton asymmetry L has been studied in numerous publications since ref. [60]. L is given by the difference between the number of leptons and antileptons over the number of photons: $L = (n_l - n_{\bar{l}})/n_{\gamma}$. L increases the radiation energy density, leading to faster universe expansion, thus influencing BBN. In particular, it leads to the overproduction of primordially produced He-4. This dynamical effect can be described by changing N_{eff} , namely:

$$\Delta N_{eff} = 15/7((\xi/\pi)^4 + 2(\xi/\pi)^2)$$

Here, $\xi = \mu/T$ is the chemical potential. ξ is used as a qualitative measure of the lepton asymmetry in case of equilibrium, then $L = 1/12\xi(3) \sum_i T_{\nu_i}^3/T_{\gamma}^3(\xi_{\nu_i}^3 + \pi^2\xi_{\nu_i})$. Large enough L in the electron neutrino sector $|L_{\nu_e}| > 0.01$ changes the electron neutrino and antineutrino number densities in reactions with nucleons from Equation (2) in the pre-BBN epoch and, hence, influences the neutron–proton kinetics during pre-BBN epoch. This allows us to use degenerate BBN and on the basis of observational data to constrain L . The BBN conservative constraint for all neutrino sectors reads as follows:

$$|\xi| < 0.1, |L| < 0.07.$$

Due to neutrino flavor oscillations, the equalization of degeneracies in different neutrino sectors occurs in the early Universe plasma [61,62].

In BSM of BBN with late $\nu_e \leftrightarrow \nu_s$, yet another indirect kinetic effect of L is possible. Namely, tiny L in the range $10^{-8} < L << 0.01$, that cannot effect directly BBN kinetics, can influence BBN through oscillations. It was shown that such tiny L can considerably

effect neutrino number density, neutrino energy spectrum distribution, neutrino oscillation patterns, and thus influence n/p kinetics in the pre-BBN epoch and consequently BBN of light elements [18,45,47].

We have found in our previous numerical analyses that, in addition to L 's ability to suppress neutrino oscillations [47,63], a tiny L is capable of enhancing neutrino oscillations, i.e., leading to the resonant enhancement of neutrino transfers [45,47]. The parameter range for which L is able to enhance, suppress, or inhibit oscillations has been determined numerically in refs. [18,45]. Thus, depending on its value, L can relax or strengthen BBN constraints on neutrino oscillations, or eliminate the constraints. The latter is possible for

$$L > (0.01\delta m^2/eV^2)^{3/5}. \quad (7)$$

Here δm^2 is in eV². This relation is an analytical fit to the exact constraints obtained numerically in refs. [18,45,47]. It can be used to estimate roughly the value of L necessary to solve the dark radiation problem.

3.1. *L* and the Possible Solution of the Dark Radiation Puzzle

Oscillations between sterile neutrino and flavor neutrinos with large mixing and squared mass difference in the range predicted by the small baseline experiments, $\delta m_{41}^2 = 10^{-2}$ eV² to several eV², lead to thermalization of the sterile neutrino at the BBN epoch. However, as discussed in the previous section, a fully thermalized light inert state is not allowed by BBN $\Delta N_{eff} \leq 0.2$. Various solutions to the DR problem have been proposed. Here, we will discuss the solution proposed in our works [45,64]. (See also similar solutions in refs. [21,65–67].) The idea is as follows:

If L present during BBN is large enough to suppress neutrino oscillations, it prevents the full thermalization of ν_s ; thus, BBN cosmological constraints on the dark radiation predicted from the small baseline experiments date can be avoided. From relations found in ref. [18], we estimate the necessary value of L for that solution to hold: $|L| > 0.016$ is required to prevent neutrino oscillations between sterile and active neutrinos with $\delta m^2 \sim 0.1$ eV², $|L| > 0.063$ is needed for the mass difference of 1 eV². For bigger mass differences, predicted using data from some neutrino oscillations experiments, $L \sim 0.03$ is able to suppress only partially the oscillations. In this case, a more sophisticated numerical analysis is necessary to calculate the exact degree of thermalization of ν_s state.

Thus, it is remarkable that the eventual future detection of sterile to active neutrino oscillations at a higher confidence level may be used to obtain a lower limit on L value and might point to the presence of lepton asymmetry during BBN epoch.

Vice versa, measuring L in the early universe, it is possible to obtain an upper limit on the squared mass differences according to Equation (7). Thus, below, we use the value of L obtained in recent survey to estimate squared mass differences of neutrino oscillations. Recently, the EMPRESS survey of extremely metal poor systems [68] has reported 3σ smaller primordial He-4 abundance than the standard BBN-predicted one and than the previous measurements. Namely,

$$Y_p = 0.2379 + 0.0031 - 0.003.$$

This value can be explained through the presence of L in the electron neutrino sector:

$$\xi_{\nu_e} = 0.03 \pm 0.014(3\sigma).$$

Using Equation (7), we estimate that L corresponding to this ξ_{ν_e} , $L \sim 0.027 \pm 0.01$, can suppress neutrino oscillations with $\delta m^2 < 0.3$ eV².

Implications of lepton asymmetry for interpreting EMPRESS results have been discussed in refs. [69,70].

3.2. Discussion on L and the Hubble Tension

Local measurements (corresponding to observations at low-red shift) of the Hubble constant by different methods indicate a larger value $H_0 \sim 73\text{--}74 \text{ km/s Mpc}$ than the value inferred at large red-shifts, from temperature anisotropy of cosmic microwave background (CMB) by Planck (2018) $H_0 = 67.36 \pm 0.54 \text{ km/s/Mpc}$ [71]. This discrepancy is called the Hubble tension. Analyzing the data from Planck, baryon acoustic oscillation, BBN, and type-Ia supernovae, it was shown [72] that the Hubble tension can be resolved for $\xi \sim 0.04$ and $0.3 < \Delta N_{\text{eff}} < 0.6$. This value is very close to the estimated value of L required to solve the DR problem and is also close to the recently found observational hint by EMPRESS for neutrino–antineutrino asymmetry in the electron neutrino sector.

All these estimations of the L value point to much larger lepton asymmetry than the baryon asymmetry of the local universe.

4. Conclusions

We discuss and numerically analyze several BSM models of BBN and derive cosmological constraints and indications for new neutrino physics. Namely, we study BBN with late $\nu_e \leftrightarrow \nu_s$ neutrino oscillations, BBN with $\nu_e \leftrightarrow \nu_s$ and non-zero initial population of sterile neutrino and BBN with late $\nu_e \leftrightarrow \nu_s$ and with lepton asymmetry in the neutrino sector. We account simultaneously for the universe expansion, neutrino oscillations and forward neutrino scattering. A precise description of the distortion of the neutrino energy distribution due to these non-equilibrium oscillations is provided. We derive stringent BBN constraints on $\nu_e \leftrightarrow \nu_s$ oscillation parameters corresponding to 1% uncertainty of the primordial abundance of He-4. Both the case of zero and non-zero initial population of the sterile neutrino state is considered. These BBN constraints strengthen the previously existing cosmological constraints on the $\nu_e \leftrightarrow \nu_s$ oscillation parameters.

In the degenerate BBN model with late neutrino $\nu_e \leftrightarrow \nu_s$ oscillations, we discuss the interplay between neutrino oscillations and lepton asymmetry. We present the change in the cosmological constraints on the oscillation parameters in cases where the lepton asymmetry is big enough to suppress the oscillations. We derive cosmological constraints on the lepton asymmetry L for different values of oscillation parameters. We discuss a possible solution to the dark radiation problem in cases where L is large enough to suppress the neutrino oscillations during the BBN epoch. The eventual future detection of sterile to active neutrino oscillations at a higher confidence level may be used to obtain a lower limit on the L value and might point to the presence of lepton asymmetry during the BBN epoch.

Interestingly, the required value of L for solving DR problem is close to L indicated by EMPRESS experiment and close to the value of lepton asymmetry necessary to relax Hubble tension. The discussed cosmological and observational indications for non-zero lepton asymmetry during BBN epoch point to lepton asymmetry orders of magnitude bigger than the baryon asymmetry of the universe. Cosmological indications about the lepton asymmetry and estimations of its value are valuable because—unlike the case of baryon asymmetry, which has already been measured with good precision—the lepton asymmetry value is not measured.

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Abbreviations

The following abbreviations are used in this manuscript:

BSM: Beyond Standard Model
 BBN: Big Bang Nucleosynthesis
 DR: Dark Radiation

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