Random matrix ensembles with parity preserving random interactions

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Introduction

Parity ratio of nuclear level densities is an important ingredient in nuclear astrophysical applications [1]. It is now well understood that at moderate excitation energies the parity ratio is not close to unity as assumed in the past. Recently, a Fermi gas model has been developed and used for tabulating parity ratios as a function of excitation energy in large number of nuclei of astrophysical interest. However, a good ab-initio theory for parity ratios is not yet available. With the success of random interaction matrix ensembles (RIMM) [2, 3], one can argue that the ensembles generated by parity preserving random interaction (here after called RIMM-PTY) may provide some generic results for parity ratios. In addition, there is also the important recognition in the past few years that random interactions generate regular structures [4]. Then a question is: why ground states of even-even nuclei are always of +ve parity. A simple RIMM-PTY has been identified and analyzed recently [5] to address the question of 'abundance of ground states with positive parity'. Going beyond this, we have constructed more general RIMM-PTY to address the two issues mentioned above and also to examine the form of fixed parity state densities. Here we will give the definition of RIMM-PTY, a method for its construction and some first results.

RIMM-PTY Ensemble

Given N_+ number of positive parity single particle (sp) states and similarly N_- number of negative parity states, let us assume for simplicity that the +ve and −ve parity states are degenerate and separated by energy Δ (see Fig. 1). This defines the one-body part $h(1)$ of the Hamiltonian H with $N = N_+ + N_-$ sp states. The matrix for the two-body part $V(2)$ of H will be a 3×3 block matrix in two particle spaces as there are three possible ways to generate two particle states with definite parity: (i) both in +ve parity states; (ii) both in −ve parity states; (iii) one in +ve and other in −ve parity states. They will give the matrices A, B and C respectively in Fig. 1. For parity preserving interactions only the states (i) and (ii) will be mixed and mixing matrix is D in Fig. 1.

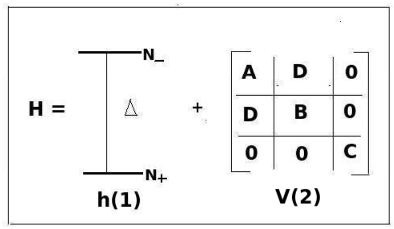

FIG. 1: Parity preserving H with sp spectrum defining $h(1)$ with the $V(2)$ matrix. Dimension of the matrices A, B and C are $N_+(N_+ - 1)/2$, $N_-(N_- - 1)/2$ and N_+N_- respectively.

Many particle states for m fermions in the N sp states can obtained by distributing m_1 fermions in N_+ +ve parity sp states and similarly m_2 fermions in the N_- states with $m = m_1 + m_2$. Let us denote each distribution of m_1 fermions by $\widetilde{m_1}$ and similarly $\widetilde{m_2}$. In the many particle basis defined by $(\widetilde{m_1}, \widetilde{m_2})$ the H matrix construction reduces to the well known spinless fermion problem [2]. The matrix dimensions (d_+) for +ve parity states follows from the dimensions for all (m_1, m_2) with m_2 even and similarly for −ve parity states (d_-) with m_2 odd. For example: (i) for $N_+ = N_- = 7$ and $m = 6$, $d_+ = 1484$

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and $d_- = 1519$; (ii) for $N_+ = N_- = 8$ and $m = 6$, $d_+ = 3976$ and $d_- = 4032$.

The RIMM-PTY is defined by choosing the matrices A, B and C to be independent GOE's with matrix elements variances v_a^2, v_b^2 and v_c^2 respectively. Similarly the matrix elements of the mixing D matrix are chosen to be independent (independent of A, B and C matrix elements) zero centered Gaussian variables with variance v_d^2. Without loss of generality we choose $\Delta = 1$ so that all the v's are in Δ units. Defining the matrix elements variances of the diagonal blocks A, B and C to be same (to reduce number of free parameters in RIMM-PTY), we have the RIMM-PTY model defined by two parameters (τ,α) where $\frac{v_a^2}{\Delta^2} = \frac{v_b^2}{\Delta^2} = \frac{v_c^2}{\Delta^2} = \tau^2$ and $\frac{v_d^2}{\Delta^2} = \alpha^2\tau^2$. In the limit $\tau^2 \to \infty$ (with $\alpha^2 = 1$) the model reduces to the simple model analyzed in [5].

Results and Discussion

Firstly, for $N_+ = N_- = m = 6$ system with 200 members, we have verified (using large values for τ and putting $\alpha = 1$) that $R_+ \sim 20\%$ as given in [5]. Going beyond this, calculations with 100 members for $N_+ = N_- = 7$ and $m = 6$ system are performed using different values for τ and α parameters defined above. We have numerically studied : (i) percentage of +ve parity ground states R_+ ; (ii) shapes of +ve (ρ_+) and −ve parity (ρ_-) state densities; (iii) parity ratio ρ_-/ρ_+. The first results are as follows. It is seen that with (τ, α) variation, R_+ shows variation and for example: for $\alpha = 0.2$, R_+ changes from 100% to 75% as τ varies from 0.03 to 0.2 and for $\alpha = 2$, $R_+ \sim 100\%$ as τ varies from 0.03 to 0.2. The state densities ρ_+ and ρ_- and the ratio ρ_-/ρ_+ are shown in Fig. 2 for some examples. For small τ values, the densities are multi-modal and as τ increases to 0.2, they approach Gaussian form. This is verified for α= 0.2 to 2. We observe considerable structure in ρ_-/ρ_+ for small τ values. For $\tau \sim 0.2$ and larger, it is seen that $\rho_- \sim \rho_+$ for $E - E_{gs} > \sigma$. Here gs stands for ground state and σ is average width over the ensemble for +ve and −ve parity state densities. Similarly in Fig.

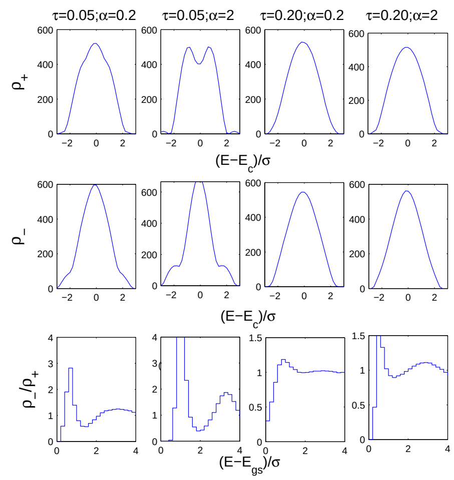

FIG. 2: Density of eigenvalues for +ve and −ve parity states and the corresponding parity ratio.

1, for ρ_+ and ρ_- densities, E_c is energy centroid and σ is spectral width for the corresponding densities. We are attempting to derive analytical results for spectral variances to understand R_+ variation and also the variation in ρ_+, ρ_- and ρ_-/ρ_+. Also calculations are being carried out for many different values of (N_+, N_-, m) values and for a much larger range of (τ, α). It is important to identify the range of (N_+, N_-, m) and (τ,α) values appropriate for some typical nuclei and then determine R_+, ρ_+, ρ_- and ρ_-/ρ_+ for these system using RIMM-PTY. This exercise is being attempted.

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