SEARCH FOR NEUTRAL

CURRENTS IN THE PROCESS

$e^+e^- \to \pi^+\pi^-\pi^0$. *

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Abstract: We propose a new electron-positron colliding beam experiment for the confirmation of the existence of the weak neutral current which at the same time provides an alternative for understanding the nature of its hadronic part.

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INTRODUCTION

The electron-positron annihilation processes represent a valuable source of information on weak neutral currents. The very fundamental question whether neutral currents couple to charged leptons as gauge theories suggest[1] can be elucidated through the study of such reactions.

In general, reactions of the type $e^+e^- \rightarrow$ anything can provide information on the properties of weak neutral currents in the three following ways:

a) Through the study of the asymmetries in the angular distribution of the produced particles.

b) Through the study of the state of polarization of the produced particles.

c) Through the study of the behavior of the total cross section for a total center-of-mass energy of the order of the mass of the intermediate neutral vector boson Z.

Several authors have considered the experimental implications of weak neutral currents for the process $e^+e^- \rightarrow \mu^+\mu^-$ the three different aspects mentioned above have been under continuing theoretical study during the last few years. Nowadays there exist numerical estimates for the theoretical expectations of the proposed experiments.[2] All such estimates were made within the context of the Weinberg-Salam type models.

The contribution I will present today has been performed in collaboration with A. Zepeda. The aim of this contribution is twofold: first, to propose a new electron-positron colliding beam experiment for the confirmation of the existence of weak neutral currents, and, second, to provide a possibility for understanding the nature of its hadronic part.

For definitness we shall restrict ourselves to the Weinberg-Salam model, so that we will assume that the neutral current is of the V-A form. Furthermore, in order to facilitate the discussion we restrict ourselves to I=0 and 1 currents. This last assumption which will be very relevent for our purposes is in agreement with present-day models which generally "build up" the currents in a bilinear fashion out of a set of fundamental objects (quarks) with isospin restricted to I=0 and 1/2.[3] So we are assuming that the hadronic part of the weak neutral current carries the quantum numbers $I^G(J^P)C = 1^+(1^-)-;\ 0^-(1^-)-$ and $1^-(1^+)+$.

II. The Proposed Experiment

With the above assumptions in mind let us consider the process:

$$e^+e^- \rightarrow \pi^+\pi^-\pi^o \qquad (1)$$

G-parity conservation tells us that the first piece of the hadronic weak neutral current does not contribute to this process. The second piece provides a contribution which has the same quantum numbers as the electromagnetic current, finally the third piece provides an axial vector contribution with charge conjugation $C = +1$.

Therefore, because of the different charge conjugation properties and different parities of the electromagnetic current and the axial part of the weak neutral current, the interference of the amplitudes:

$$e^+e^- \rightarrow \gamma \rightarrow \pi^+\pi^-\pi^o$$

and

$$e^+e^- \rightarrow Z \rightarrow \pi^+\pi^-\pi^o$$

gives rise to charge asymmetries as well as to parity violating effects in the angular distribution of the produced pions. Obviously, the same asymmetries should arise from the product of the vector and axial vector parts of the weak neutral current in the square of the weak amplitude.

It is clear that if the hadronic neutral current does not contain an axial part, (as in the case of the Beg-Zee model), then there are no asymmetries at all since the quantum numbers of its vector part are the same as those of the electromagnetic current.

Let q_-, q_+, p_-, p_+, p_0 be the four-momenta of the electron, positron and the pions respectively in the center-of-mass frame of the electron and positron. Then the amplitude for the reaction (1) is as follows, assuming that it proceeds via one photon and one vector boson exchange only (Bjorken and Drell conventions[4] are used throughout this work):

$$M = \frac{1}{S} \; l^{\mu} L_{\mu} + (v_{\mu}+a_{\mu}) \frac{1}{S-M_2^2} \left(g^{\mu\nu} - \frac{p_r^{\mu} p^{\nu}}{S-M^2{}_2}\right) (V_{\nu}+ A_{\nu})$$

where

$$P_r = p_+ + p_- + p_0 \, , \qquad S = p_r^2$$

$$l^{\mu} = em_e i \, \bar{v}\gamma_{\mu} u$$

$$v_{\mu} = g_{\nu} m_e \, e \, \bar{v} \, \gamma_{\mu} u$$

$$a_{\mu} = g_a \, m_e \, i \, \bar{v} \, \gamma_{\mu}\gamma_5 u$$

L_{μ} describes the $\gamma\to\pi^+\pi^-\pi^0$ vertex, V_{ν} and A_{ν} describe respectively the vector and axial vector parts of the $Z\to\pi^+\pi^-\pi^0$ vertex, e being the electric charge and g_v and g_a. The weak vector and axial vector couplings to the electrons

L_μ and V_μ are axial vectors antisymmetric in their dependence upon p_+, p_- and p_o. That is

$$L_\mu = iee\epsilon_{\mu\nu\rho\sigma} p_+^\nu p_-^\rho p_o^\sigma F_1$$

$$V_\mu = ig_v \epsilon_{\mu\nu\rho\sigma} p_+^\nu p_-^\rho p_o^\sigma F_2$$

where F_1 and F_2 are Lorentz scalars symmetric in their dependence upon p_+, p_- p_o. On the other hand A is a vector symmetric in p_+ and p_-

$$A_\mu = g_a \left[(p_+ + p_-)_\mu F_3 + (p_+ - p_-)_\mu F_4 + P_{r\mu} F_5 \right]$$

where F_3 and F_5 are symmetric in their dependence upon p_+ and p_- and F_4 is antisymmetric. g_v and g_A are the weak vector and axial vector couplings of the weak neutral current to hadrons.

In the extreme relativistic limit and for the case of unpolarized initial particles standard calculation leads (after integration over the variables of the unobserved π^0), to the following expression for the parity conserving part of the differential cross section (1):

$$\frac{d\sigma}{d\Theta_+ d\Theta_-} = \frac{1}{8(2\pi)^4 s} \sin\Theta_+ \sin\Theta_- \int_M^{E_{max}} dE_+ \int_M^{E_{max}} dE_- \Theta(1-\cos^2\Theta_{+-})$$

$$\times\ \Theta\left[\cos\Theta_{+-} - \cos(\Theta_+ + \Theta_-)\right]\Theta\left[\cos(\Theta_+ - \Theta_-) - \cos\Theta_{+-}\right]$$

$$\times \Big[\left[\cos\Theta_{+-} - \cos(\Theta_+ + \Theta_-)\right]^{-1/2} \left[\cos(\Theta_+ - \Theta_-) - \cos\Theta_{+-}\right]^{-1/2}$$

$$\times \{ \vec{p}_+^{\,2}\, \vec{p}_-^{\,2} \left[1 + \cos(\Theta_+ + \Theta_-)\cos(\Theta_+ - \Theta_-) - 2\cos\Theta_+ \cos\Theta_- \cos\Theta_{+-}\right]$$

$$\times \left[8\pi^2\alpha^2 |F_1|^2 + \frac{s}{s - M_Z^2} 4\pi\alpha g_v g_V \,\mathrm{Re}\, F_1^* F_2 \right.$$

$$\left. + \frac{s^2}{2(s - M_Z^2)^2} (g_v^2 + g_a^2)\, g_V^2 \,|F_2|^2 \right]$$

$$+ \frac{s}{4(s - M_Z^2)^2} (g_v^2 + g_a^2) g_A^2 \left[(|F_3 + F_4|^2\, \vec{p}_+^{\,2} + |F_3 - F_4|^2\, \vec{p}_-^{\,2}) \right.$$

$$\times (\sin^2\Theta_+ + \sin^2\Theta_-) + 4(|F_3|^2 - |F_4|^2)|\vec{p}_+||\vec{p}_-|$$

$$\left. \times (\cos\Theta_{+-} - \cos\Theta_+ \cos\Theta_-) \right] \}$$

$$+ \frac{g_a g_A \sqrt{s}}{s - M_Z^2} (\cos\Theta_+ - \cos\Theta_-) \left[\cos\Theta_{+-} - \cos(\Theta_+ + \Theta_-)\right]^{-1/2}$$

$$\times \left[\cos(\Theta_+ - \Theta_-) - \cos\Theta_{+-}\right]^{-1/2} (1 + \cos\Theta_{+-}) |\vec{p}_+||\vec{p}_-|$$

$$\times \mathrm{Re} \{ \left[2\pi\alpha F_1^* + \frac{s g_v g_V}{s - M_Z^2} F_2^* \right] \left[F_3 (|\vec{p}_+| + |\vec{p}_-|) \right.$$

$$\left. + F_4 (|\vec{p}_+| - |\vec{p}_-|) \right.$$

Where $E_+ = p_+^0$, $E_- = p_-^0$, Θ_+ and Θ_- are the orbital angles of the π^+ and π^- respectively, $\Theta_{\pm}$ being the angle between them.

E_{max} is the maximum energy which can be carried by either $\pi^+\pi^-$:

$$E_{max} = \frac{s-3M^2}{a\sqrt{s}}$$

M being the mass of the pion.

It can be seen easily that the former expression can be splitted into two parts:

a) A part which is charge symmetric.
It contains α^2, $\alpha g_v g_V$, $(g^2_v + g^2_a)g^2_V$ and $(g_v^2+g_a^2)g^2$.
We shall call it $d\sigma^S$

b) A part which is charge antisymmetric.
It is proportional to $g_a g_A$ and $g_a g_A g_v g_V$ We shall call it d^{ca}

Now let us note that in the former expression for the differential cross section parity violating terms do not appear. The reason is that we have integrated over the aximuthal angles of the π^+ and π^-. If we integrate one of the two azimuthal variables, say ϕ_+ either from o to π or from π to 2π, then the parity violating part of the cross section remains and we obtain the following expression:

$$d\tilde{\sigma}_{\substack{E\\W}} = d\tilde{\sigma}^{S} + d\tilde{\sigma}^{ca} + d\tilde{\sigma}_{\substack{E\\W}}^{pv}$$

where

$$d\tilde{\sigma}^{S} = \frac{1}{2}\, d\sigma^{S}, \quad d\tilde{\sigma}^{ca} = \frac{1}{2}\, d\sigma^{ca}$$

$$d\tilde{\sigma}_{\substack{E\\W}}^{pv} = \pm \frac{d\Theta_{+}\, d\Theta_{-}}{16(2\pi)^{4} s} \sin\Theta_{+}\sin\Theta_{-} \int_{M}^{E_{max}} dE_{+} \int_{M}^{E_{max}} dE_{-}\, \Theta(1-\cos^{2}\Theta_{+-})$$

$$\times\ \Theta\left[\cos\Theta_{+-} - \cos(\Theta_{+}+\Theta_{-})\right]\Theta\left[\cos(\Theta_{+}-\Theta_{-}) - \cos\Theta_{+-}\right]$$

$$\times\ \frac{\sqrt{s}}{s-M_Z^2}(\cos\Theta_{+} + \cos\Theta_{-})\, |\vec{p}_{+}||\vec{p}_{-}|\ \mathrm{Im}\,\{\left[2\pi\alpha g_V g_A F_1^{*}\right.$$

$$\left. + \frac{s(g_v^2+g_a^2)g_V g_A}{2(s-M_Z^2)} F_2^{*}\right]\left[F_3(|\vec{p}_{+}|+|\vec{p}_{-}|)+F_4(|\vec{p}_{+}|-|\vec{p}_{-}|)\right]\}$$

The subscript E(W) and the +(-) sign refers to the integration of ϕ_+ from o to π (π to 2π). In both cases ϕ_- is integrated from o to 2π. In Eq. (14) there are no parity violating terms proportional to $\alpha g_a g_V$, $g_V^2 g_a g_v$ or $g_a g_v g_A^2$; the first two are identically zero while the third does not contribute due to the symmetry of the (E_+, E_-) domain of integration.

$d\sigma^S$, $d\sigma^{ca}$ and $d\tilde{\sigma}^{pv}$ can also be recognized by their dependence on $\cos\Theta_+$ and $\cos\Theta_-$. This sturcture implies that $d\sigma^s$, $d\sigma^{ca}$ and $d\tilde{\sigma}^{pv}$ have different "parities" under the angular inversions listed in table 1. From this table we can see that the maximum information on the presence of the neutral currents is obtained by measuring the following asymmetry parameters

$$A_c = \frac{\sigma_{+-} - \sigma_{-+}}{\sigma_{+-} + \sigma_{-+}}$$

$$A_p = \frac{\Delta\sigma_N - \Delta\sigma_S}{\sigma_N + \sigma_S}$$

where

$$\sigma_{\pm\mp} = \int_{\Theta_o}^{\pi/2} d\Theta_\pm \int_{\pi/2}^{\pi-\Theta_o} d\Theta_\mp \frac{d\sigma}{d\Theta_+ d\Theta_-}$$

$$\sigma_N = \int_{\Theta_o}^{\pi/2} d\Theta_+ \int_{\Theta_o}^{\pi/2} d\Theta_- \frac{d\sigma}{d\Theta_+ d\Theta_-} \qquad (2)$$

$$\sigma_S = \int_{\pi/2}^{\pi-\Theta_o} d\Theta_+ \int_{\pi/2}^{\pi-\Theta_o} d\Theta_- \frac{d\sigma}{d\Theta_+ d\Theta_-} \qquad (3)$$

$\Delta\sigma_N$ ($\Delta\sigma_S$) being defined by an integration, analogue to that in eq. 2 (3), of $\Delta d\sigma \equiv d\sigma_E - d\sigma_W$. Θ_o is some given angular cut-off determined by the measuring apparatus.

As we have already indicated above, both A_c and A_p are identically zero if the neutral current does not couple axially to hadrons. If it does couple, then the following takes place: A_c is non-zero if and only if the neutral current couples axially to the e^+e^- state. A_p is zero if and only if the neutral current does not couple vectorially to both leptons and hadrons. However for low energies, such that

$$s(g_v^2 + g_a^2)|g_v g_A| << 4\pi\alpha|g_v g_A||s-M_Z^2|,$$

the last premise reduces to the neutral current being coupled vectorially to leptons. The above statements follow from the following relations obtained from table 1:

$$\sigma^S_{-+} = \sigma^S_{+-} \text{ , } \sigma^{ca}_{-+} = - \sigma^{ca}_{+-}$$

$$\sigma^S_S = \sigma^S_N, \ \sigma^{ca}_S = \sigma^{ca}_N = 0, \ \Delta\sigma^{pv}_S = - \Delta\sigma^{pv}_N$$

Thus

$$A_c = \frac{\sigma^{ca}_{+-}}{\sigma^S_{+-}} \text{ , } \quad A_p = \frac{\Delta\sigma^{pv}_N}{\sigma_N{}^S}$$

In order to obtain a numerical estimate we will use[1]

$$g_v = - (G/2\sqrt{2})^{1/2} M_Z(1-4 \sin^2\theta), \ g_a = - (G/2\sqrt{2})^{1/2} M_Z$$

$$g_V = - (8G/\sqrt{2})^{1/2} M_Z \sin^2\theta, \ g_A = - (\sqrt{2}G)^{1/2} M_Z \text{ ,}$$

$\sin^2\Theta = 0.35$, $M_Z = 75$ GeV, as well as simplified models for the form factors F_{1-4}. Saturating L_μ and V_μ with the ω resonance coupled to one pion and a ρ meson in all possible ways we get

$$F_1 = F_2 = \frac{2m_\omega^2 g_{\rho\omega\pi}}{s-m_\omega^2-i\Gamma_\omega m_\omega}\{[(p_++p_-)^2-m_\rho^2-i\Gamma_\rho m_\rho]^{-1}$$

$$+[(p_++p_0)^2-m_\rho^2-i\Gamma_\rho m_\rho]^{-1}+[(p_0+p_-)^2-m_\rho^2-i\Gamma_\rho m_\rho]^{-1}\}$$

where $g_{\rho\omega\pi} = 29\ \mathrm{GeV}^{-1}$, as obtained from a fit to the width. Assuming that the axial current couples to the three-pion state through the f resonance $(Z \to \pi^0 + f,\ f \to \pi^+\pi^-)$ we get the following expressions for F_3 and F_4.

$$F_3 = \frac{g_{f\pi\pi}(p_+-p_-)^2 p_0\cdot(p_++p_-)}{3m_f^3\,[(p_++p_-)^2-m_f^2-i\Gamma_f m_f]} \tag{4}$$

$$F_4 = \frac{g_{f\pi\pi}p_0\cdot(p_+-p_-)}{m_f[(p_++p_-)^2-m_f^2-i\Gamma_f m_f]} \tag{5}$$

where $g_{f\pi\pi} = 6.9$ fits the f width. To obtain expressions (4) and (5) we have used for the f propagator

$$\frac{\frac{1}{2}(\hat{P}_{\mu\rho}\hat{P}_{\nu\sigma}+\hat{P}_{\mu\sigma}\hat{P}_{\nu\rho})-\frac{1}{3}\hat{P}_{\mu\nu}\hat{P}_{\rho\sigma}}{(p_++p_-)^2-m_f^2-i\Gamma_f m_f}$$

with $\hat{P}_{\mu\nu} = - g_{\mu\nu}$

instead of

$$\hat{P}_{\mu\nu} = - g_{\mu\nu} + (p_+ + p_-)_\mu (p_+ + p_-)_\nu / m^2_f$$

This step has been taken in order to attenuate the quick rise with the energy which F_3 and F_4 show if we use the full spin 2 propagator. Since after this improvement F_3 and F_4 still rise too quickly, we have made a second modification to these form factors which consists in multiplying expressions (4) and (5) by a factor m^2_f/s. Without these changes the weak amplitudes rise too quickly.

III Numerical results.

Numerical integration of the above formulae has been carried out by means of a Monte Carlo program on a UNIVAC-1106 computer. The kinematical constraints assumed in the calculation correspond to present SPEAR experimental conditions namely: The total azimuthal angle spanned by either $\pi^+\pi^-$ is taken as 2π. The minimal angle between either $\pi^+\pi^-$ and the beam axis is given by $\cos\Theta_o = 0.65$. In addition, a minimal value of 200 MeV was set for the energy of either $\pi^+\pi^-$.

The numerical results for the beam energy dependence of the various parts of the cross section defined above are shown in Figures 1-4. In these figures we also show the contributions to the corresponding cross sections coming from the electromagnetic interaction, from the weak interaction and from the interference of both.

Figure 5 shows the beam energy dependence of the parameters A_p and A_c. As we can see from this figure these parameters reach their maximum value (3-4%), at beam energies of the order of ∿17 GeV. This is a very fortunate fact since such beam energies will be available in proposed electron-positron colliding beam facilities. (The electron-positron colliding beam facility PETRA at DESY will provide a maximum total energy of 38 GeV).

The measurement of the parameter A_p for the reaction

$e^+e^- \rightarrow \pi^+\pi^-\pi^0$ will provide unambiguos information about parity violating effects. It represents an alternative to the use of polarized beams proposed by some authors[2] for the detection of this same effect. For the case of leptons in the final state.

Now, the question arises whether the measurement of A_c provides an unambiguos signal of the axial coupling of the neutral current to both leptons and hadrons. It is evident that the interference between the annihilation via one and two photons will contribute also to this effect--due to the opposite charge conjugation porperties of the one and two-photon states--and then the elimination of such background remains an open question.

IV Conclusions and Remarks

Electron-positron colliding beam experiments turn out to be very desirable for understanding the nature of the hadronic part of the weak neutral current, as the above arguments show.

It must be emphasized, however, that some assumptions go into the calculation of the cross section which we are proposing here beyond the postulate of the existence of a neutral intermediate vector boson. Such assumptions are explicitly mentioned in the text and are, of course, in general agreement with present day models. Hence the numerical values for the theoretical expectations of the experiment which we propose must be taken only as an indication that it should be possible to at least set limits on the magnitude of the weak interaction effects.

TABLE I

Transformation properties of the three terms in the differential cross section.

	$\theta_+ \leftrightarrow \theta_-$	$\theta_+ \leftrightarrow \pi-\theta_-$	$\theta_\pm \to \pi-\theta_\pm$
$d\sigma^{e}/d\theta_+ d\theta_-$	+	+	+
$d\sigma^{ca}/d\theta_+ d\theta_-$	-	+	-
$d\sigma^{pv}/d\theta_+ d\theta_-$	+	-	-

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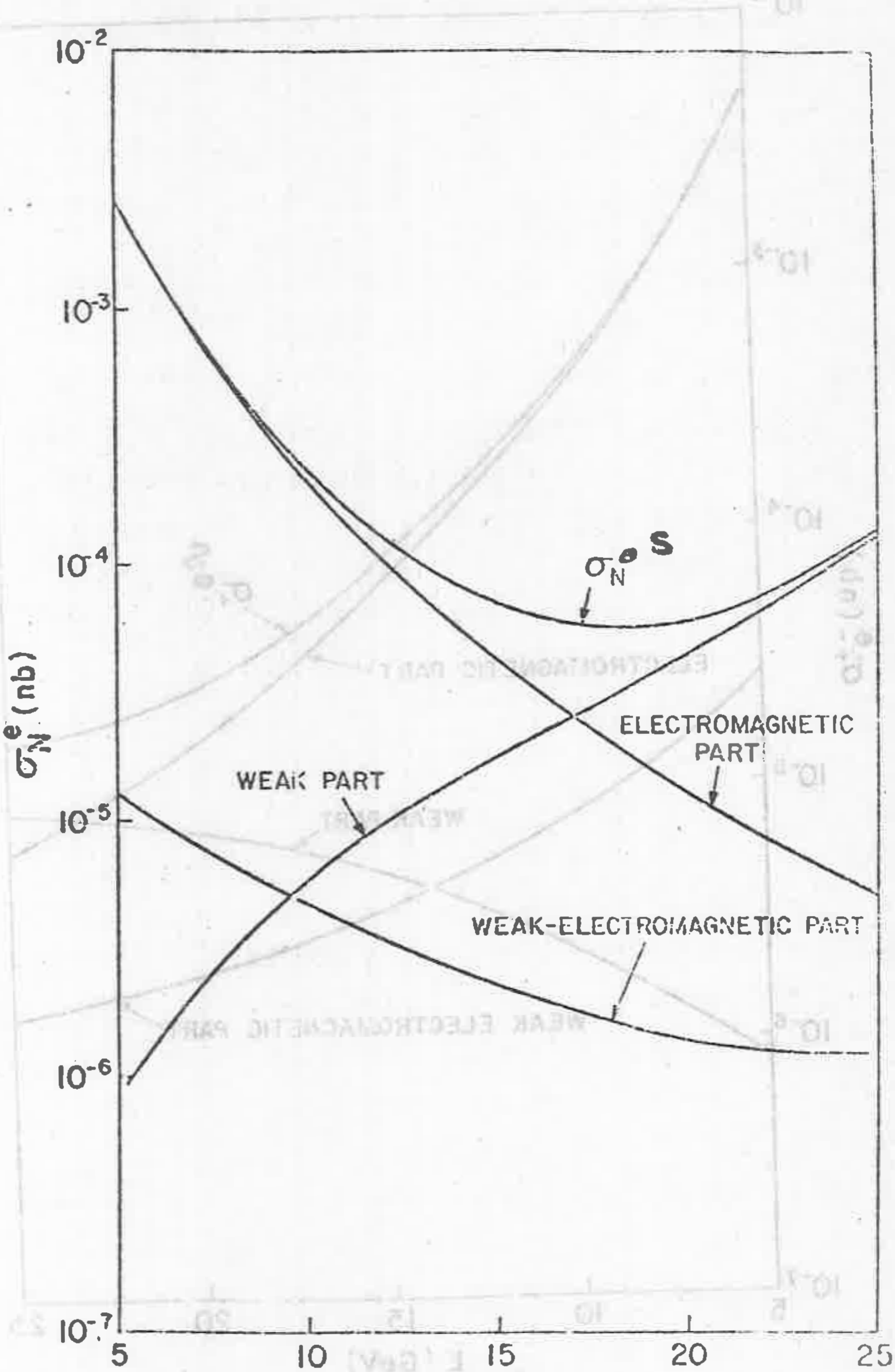

Fig. 1.- σ_N^{e} S and its three contributions

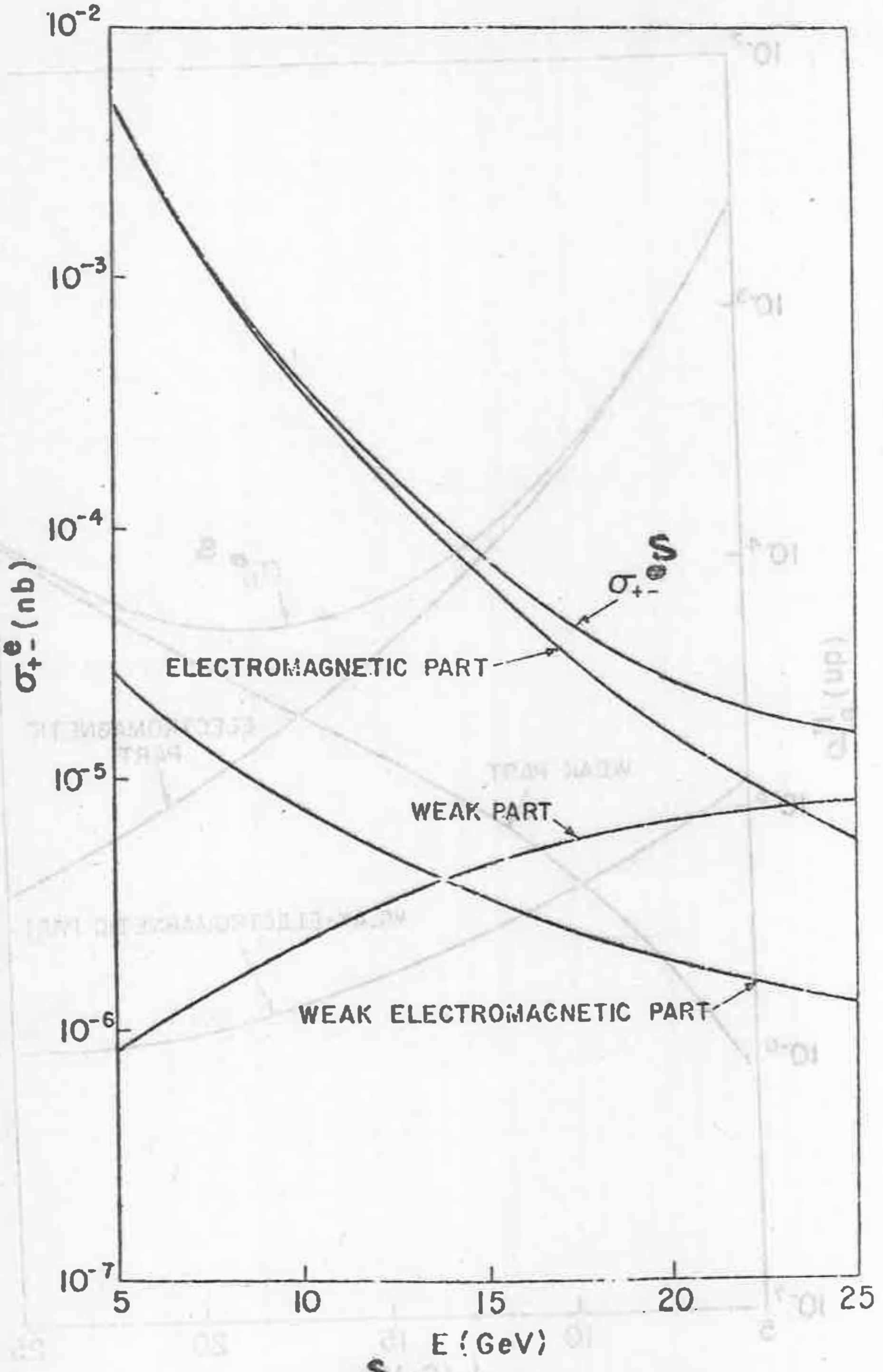

Fig. 2.- σ_{+-}^{eS} and its three contributions

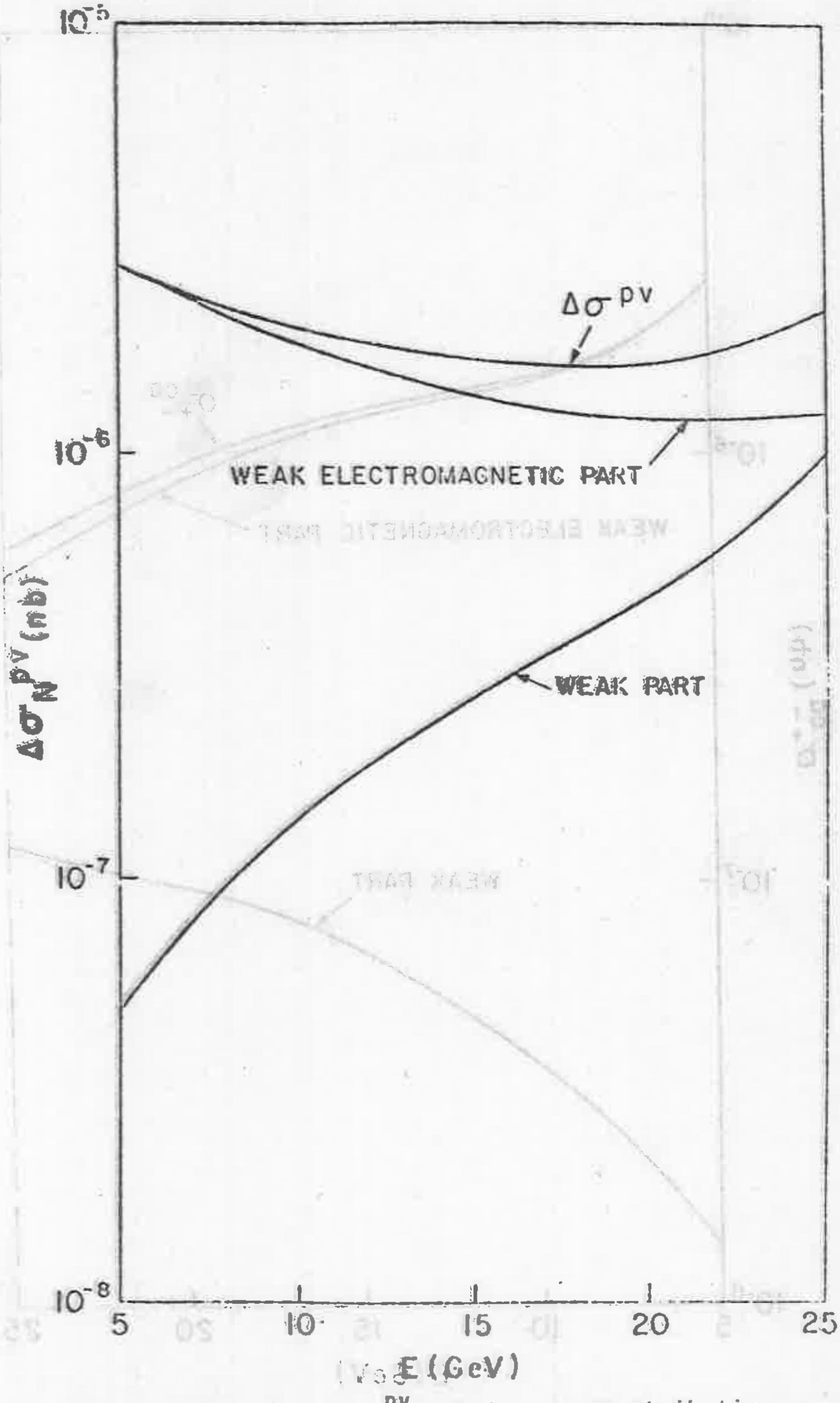

Fig. 3 - $\Delta\sigma_N^{pv}$ and its two contributions

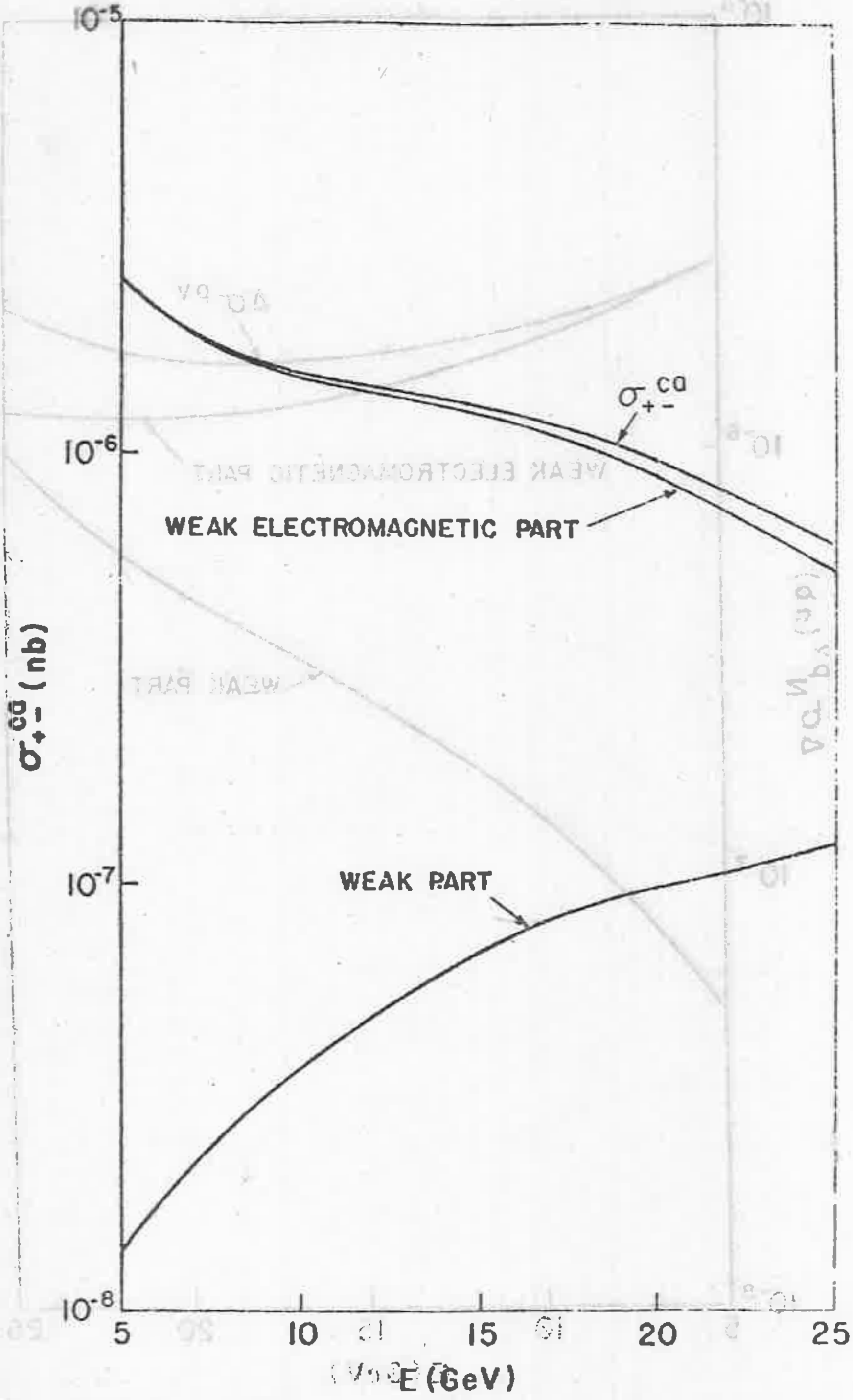

Fig. 4.- σ_{+-}^{ca} and its two contributions

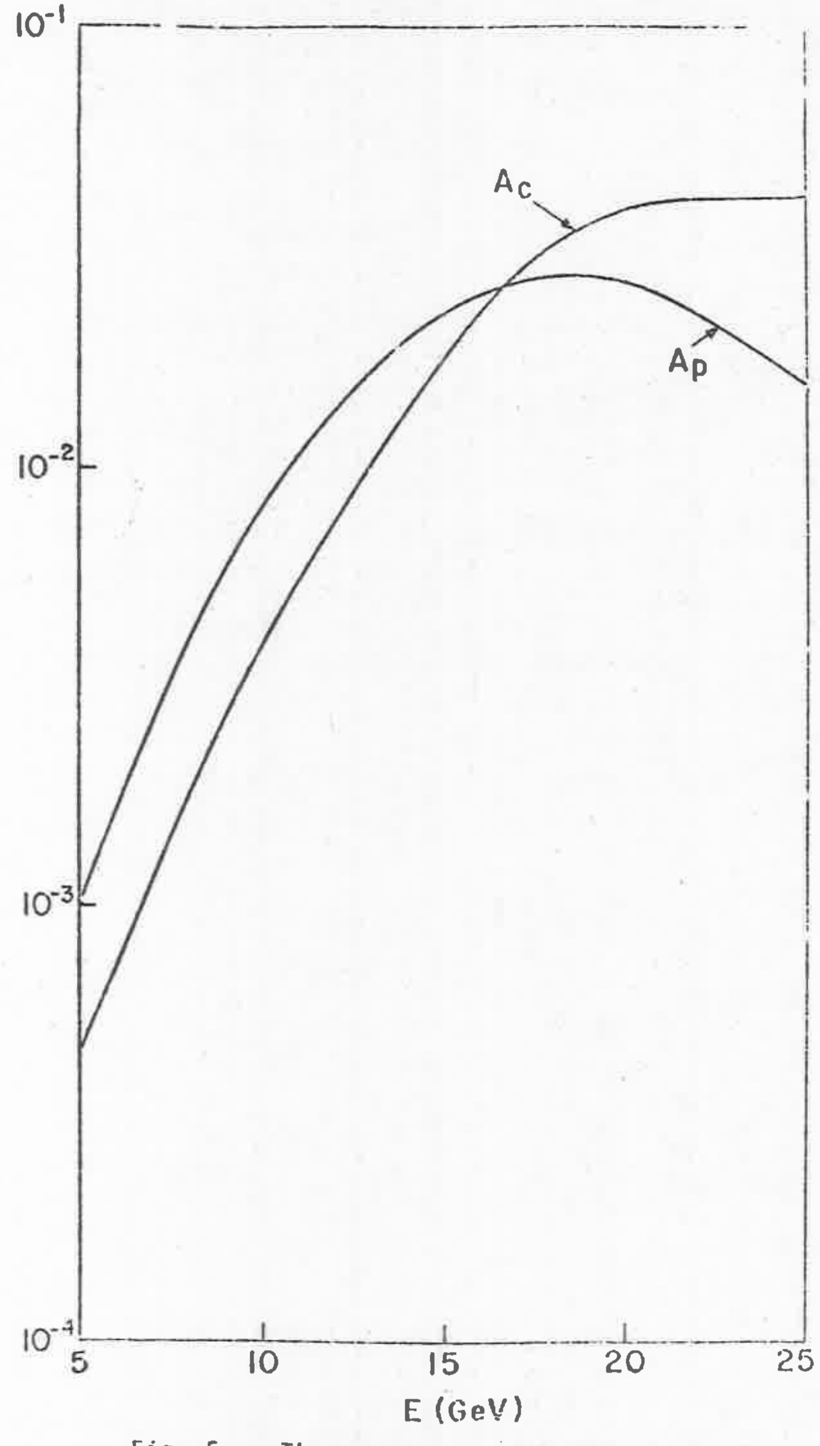

Fig. 5.- The asymmetry parameters A_c and A_p.