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Revisiting Primordial Black Hole Evolution

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Abstract: Primordial black holes (PBHs) are the sensitive probe for physics and cosmology of very early Universe. The observable effect of their existence depends on the PBH mass. Mini PBHs evaporate and do not survive to the present time, leaving only background effect of products of their evaporation, while PBHs evaporating now can be new exotic sources of energetic particles and gamma rays in the modern Universe. Here we revisit the history of evolution of mini PBHs. We follow the aspects associated with *growth versus evaporation rate* of "a mini PBH being trapped inside intense local cosmological matter inhomogeneity". We show that the existence of baryon accretion forbidden black hole regime enables constraints on mini PBHs with the mass $M \leq 5.5 \times 10^{13}$ g. On the other hand, we propose the mechanism of delay of evaporation of primordial population of PBHs of primordial mass range 5.5×10^{13} g $\leq M \leq 5.1 \times 10^{14}$ g. It can provide their evaporation to be the main contributor to γ-ray flux distribution in the current Universe. At the final stage of evaporation these PBHs can be the source of ultrahigh energy cosmic rays and gamma radiation challenging probe for their existence in the LHAASO experiment.

Keywords: cosmology; particle physics; primordial black hole; baryon accretion; forbidden regime; high energy physics

1. Introduction

Ever since it was proposed, Hawking's idea of glowing black hole [1] remained a standout theoretical effort towards merging quantum mechanics with the general theory of gravity. The prediction was that a black hole of mass M should act like a black-body of temperature $6.2 \times 10^{-8} \times (M/M_{\odot})^{-1}$ Kelvins, having radiation consisting of photons, neutrinos, and even all sorts of massive particles as M gets small enough [1,2]. Hence the Luminosity of Hawking radiation, viz. $\mathcal{L}_H$, can be amendable for microscopic black holes:

$$\mathcal{L}_H \propto \left(\frac{M}{M_{\odot}}\right)^{-4}. \tag{1}$$

Profuseness of Hawking flux had led to the idea of a regime of black hole abandoned to matter accretions [3]. In this paper we shall point out a critical analytic incompleteness in the derivation of baryon accretion forbidden regime of [3] and hence put forward what appears to be at least a analytically complete analysis. Further we shall provide a rigorous basis for why a baryon accretion forbidden regime should be feasible in any physical circumstance. In accordance with the concept of matter accretion forbidden regime, a radiating, Hawking black hole of mass $\sim 10^{13}$ g makes the black hole to act as a strong shield against the gravitational inflow of external matter. We also recall that, as an immediate reveal in [3] it was pointed out that some analyses regarding accretion of ordinary baryonic matter onto (assumptive) mini black holes [4,5] are likely to be impractical. Since a lower bound to the energy scale of untested physics lies in the TeV range [6] whereas a regime of mini black hole that forbids ordinary matter accretion begins to work at a classical GeV energy scale, the works in [4,5] appear barely physical even in exotic physics arena. Here, through our present work, these issues on [4,5] will be more readily understood.

One key point for anticipating a real physical existence of microscopic black holes is the possibility of formation of black hole in primordial cosmology. The early cosmological energy density fluctuation inside the radiation dominated era of the Universe is conventionally regarded as the origin of primordial black holes (PBHs) [7,8]. The early Universe; gravitational collapse; high energy physics; and quantum gravity are the four issues of physics that the PBHs could probe [9]. Their origin reflects physics beyond the Standard model, governing very early Universe [10–14]. It provides the probe for the fundamental structure of the symmetry of microworld and the pattern of its breaking at the super high energy scale [13–15].

Possibility of PBHs demands contribution of radiating microscopic black holes to the thermal structures of the Universe [16,17]. Indeed, the frequently observed events of cosmic γ-ray bursts founded investigations on analyzing if the sources could be the tiny PBHs [18]. This adds to one exciting modern issue of probing any role of PBH relics in dark matter structure formation [9,19–21].

The PBH group of primordial masses $M < 5.1 \times 10^{14}$ g, kept (hypothetically) isolated from the surroundings, should have already completed evaporation by the present epoch of the universe. However, since a PBH in reality would always interact with its surroundings, in presence of intense environmental matter in-homogeneities a significant prolongation could be expected over the period of complete decay of a PBH that gets trapped inside some gas-matter lump in early Universe. In this context a baryon accretion forbidden regime could lead to a constrained picture of PBH-evaporation events and consequences, basically since the 'decay time prolongation' is possible only for a black hole that does not hold the accretion forbidden regime. Indeed several properties get influenced by the PBHs-evaportion, like, the extragalactic γ-ray background [22,23], antimatter in cosmic rays [24,25], production of light elements after BBN nucleosynthesis [26,27] reionization picture of pregallactic medium [28], distortion CMB background [27,29] and events of γ-ray bursts [30,31]. Hence these aspects lead to inferring the picture of PBH distribution [9,19]. In a recent work [20], one can find illustrations on what could be the various observational evidences of PBHs.

It is a fact that structure formation of the Universe requires in-homogeneity in the distribution of matter from the early matter dominated era, and indeed this essential essence of formation of galaxies and stars leads one to anticipate the possibility of such factor influencing PBH-evolution. So it appears clarifying to estimate the evolution of PBHs by modeling a scenario where a PBH gets trapped inside a patchy environment of high density matter. Understanding a current status of PBHs demands a well-defined idea on the evolution of PBHs over the entire age of the Universe. There are numerous works that exist in literature on providing constraints and making inferences [19,26,32–34]. However new illuminating ideas, analyses could still allow scope for further exploring the effect of Hawking radiation phenomenon on the evolution of PBHs. In this paper we shall try to find ways to evaluate the evolution of PBHs by taking into account 'ordinary baryon accretion forbidden regime of a black hole'. Emergence of baryon accretion forbidden black hole regime enables a few independent constraints on PBH evolution and probes cosmological γ-ray flux distribution in the sky.

The outline and the content of this paper can be divided into following three main segments: (i) By duly modifying the derivation in [3] we uphold the regime of black hole that forbids to ordinary baryonic matter accretion, and further argue on why the said regime should be feasible at any viable physical circumstance (Sections 2 and 3); (ii) In Section 4 we assess the context of the 'black hole regime forbidding baryon accretion' on aspects associated with the evolution of mini PBHs, and hence anticipate a few constrained pictures to probe observational evidences of such PBHs; (iii) Finally, we conclude in Section 5.

2. A Refined Analysis of Regime of Black Hole Forbidden to Baryon Accretion

A Hawking black hole may not always support accretion [3]—outward flux of Hawking radiation becomes amendable for a sufficiently mini black hole, and eventually the resultant pressure gradient appears large enough to prevent the black hole from accreting ordinary baryonic matter. Indeed the basic concept behind this possibility is analogous to what gets involved in the usual astrophysics for setting the well known Eddington limit [35], viz. $\mathcal{L}_{Edd}$, to accretion luminosity [3]. Once the Hawking luminosity of a black hole exceeds the Eddington limit, baryons get blown away, and the accretion process stops [3]. Since Hawking temperature varies being inversely proportional to the mass, M of a black hole, viz. $T_H \propto 1/M$, once gets restrained (conventionally) there will be no revival for the accretion process.

Following analysis will be a critically refined form of the first analysis, recently appeared in [3]. This refinement comes along with a due improvisation (as referred to a radiating, Hawking black hole) on the Eddington luminosity constraint for sustainability of accretion process. Only, thus we are able account the fact that different Hawking particle species would interact with different characteristic cross-sections with the same inflowing ambient gas-matter. Hence we should have an analytically complete analysis to follow in the present paper.

In the analysis to follow we use the subscript "$_{cs}$" to denote a critical black hole state such that for example for $M < M_{cs}$ there would be no baryon accretion into a black hole.

2.1. Fundamentals of the Analysis

Here are the bases of our analysis:

1. We assume a chargeless, nonspinning black hole. This assumption is reasonable since quantum particle emission process causes a black hole to lose its charge and angular momentum very rapidly as compared to its mass [36–38].

2. Black hole environment is baryonic, with m_p being the typical mass per particles in that environment.

3. Hawking luminosity is an integration of different distinguishable forms of radiation. Hence as we incorporate MacGibbon-Carr summarized formula [9,39] of Hawking luminosity, we must develop an improvised version of Eddington formula. This improvised Eddington formula would explicate all the minute details of interaction of Hawking particles with the black hole surroundings—a necessary treatment that was missing in [3].

Since the last point is really critical referred to a Hawking radiating black hole, present paper aims to provide a significantly refined analysis over [3].

2.2. Improvised Eddington Luminosity Limit

Radiation from a black hole consists of several species of emitted particles and Eddington luminosity has to distinguish these particles. Hence one requires an improvised form of Eddington formula. In terms of physically separable constituents of a radiation system flowing outward from a Hawking black hole, Eddington luminosity can be determined as

$$\mathcal{L}_{Edd} = \frac{2\pi c^3 r_H m_p}{\sum_i \mu_i \sigma_i} \equiv \frac{2\pi c^3 r_H m_p}{\sigma_E}, \tag{2}$$

where m_p equals the proton mass, $r_H = 2M$, while we include the prescription that

$$\mathcal{L}_H = \sum_i \mathcal{L}_i = \sum_i \mu_i \, \mathcal{L}_H, \tag{3}$$

in which the total Hawking luminosity, $\mathcal{L}_H$, is explicitly given as a sum of its distinguishable component luminosities, $\mathcal{L}_i$, such that

$$\sum_i \mu_i = 1. \tag{4}$$

One has also to note that $r_H = 2GM/c^2$, and

$$\sigma_E = \sum_i \mu_i \sigma_i, \tag{5}$$

with μ_i, σ_i being the relative luminosity and cross-section parameters associated with a distinguishable component of black hole radiation. Thus the improvised Eddington luminosity limit readily allows a non-trivial refinement on the analysis of [3]. Let us point out that it is not the individual entities μ_i, σ_i, but the products $\mu_i \sigma_i$ are of real importance in determining $\mathcal{L}_{Edd}$.

2.3. Determining μ_i

We must evaluate μ_i-s in context of Hawking radiation. Hence first we observe that corresponding to every one specie of Hawking particles

$$\mathcal{L}_i \;\propto\; M^{-2} \alpha_i, \tag{6}$$

where α_i is determined in terms of the standard model of physics and the degrees of freedom of the particle specie one is concerned of [19–21]:

$$\alpha_i = n_{dof,i} \, \chi_{s,i}, \tag{7}$$

where $n_{dof,i}$ denotes the degrees of freedom of one particle specie, and the entity $\chi_{s,i}$-s are determined by the standard model of physics as per the charge-notion and spin of different particle species (by following [20] readers would be able to obtain a complete comprehensive idea of the entity $\chi_{s,i}$). Now one defines the following identity

$$\alpha_\Sigma = \sum_i \alpha_i, \tag{8}$$

(which depends on the mass, M, of a Hawking black hole) to have

$$\mathcal{L}_H \;\propto\; M^{-2} \alpha_\Sigma. \tag{9}$$

The constant of proportionality in Equation (6) is identical to that of Equation (9). In literature, there exists several conventions on scaling the entity α_Σ along with the said constant of proportionality. In this paper we will follow the convention of [19]. Since the parameter μ_i implies

$$\mathcal{L}_i = \mu_i \, \mathcal{L}_H, \tag{10}$$

corresponding to distinguishable radiation components, μ_i -s are determined as

$$\mu_i = \frac{\alpha_i}{\alpha_\Sigma}, \tag{11}$$

such that Equation (4) is met. Obviously μ_i of a particle specie is set by its spin, degrees of freedom.

Let us now figure out some entities numerically in advance. Hence we consider Hawking radiation to consists of those species of particles that include all probable particles to be emitted by a radiating black hole that could just be able to forbid accretion of baryons [3,20]. To determine μ_i we must know α_i. So first we note the 'number of degrees of freedom,' n_{dof}, of the particle species [40]. It is $n_{dof} = 2$ for a photon; with two lepton species (electron and muon) the total number of degrees of freedom of lepton species is $n_{dof} = 8$; for neutrinos the number we have $n_{dof} = 6$; a gluon is associated with $n_{dof} = 16$, and finally (viable) 12 quarks hold $n_{dof} = 36$. These n_{dof}-numbers respectively now imply for the following α_i values [19,40]

$$\alpha_{ph} = 2 \times 0.06 \; ; \; \alpha_{lep} = 8 \times 0.142 \; ; \alpha_{\nu} = 6 \times 0.147 \; ; \alpha_{gluon} = 16 \times 0.06 \; ; \alpha_{quark} = 36 \times 0.14, \tag{12}$$

(note: ph=photon, lep=lepton, ν=neutrino, gluon, quark) with $\alpha_{\Sigma} = 8.138$. Of the emitted particles, quarks and gluons will combine subsequently after their emission to form observable hadrons [19]. Hence on using Equation (11) we have

$$\mu_{ph} = 0.015 \; ; \; \mu_{lep} = 0.140 \; ; \; \mu_{hdrn} \equiv \left(\mu_{gluon} + \mu_{quark}\right) = 0.737 \; ; \; \mu_{\nu} = 0.108 \, , \tag{13}$$

where we again imply 'hdrn' = hadron.

The above set of values will be one of the critical new inputs in our refined estimation of baryon accretion forbidden black hole regime.

2.4. Determining the Cross-Sections (σ_i)

Note that the relative contribution of hadrons to the net Hawking luminosity is highly greater than the contribution of photons. There are also leptons, muons, that could contribute significantly to the net Hawking luminosity. Hence we must specify σ_i -s due to all the significant particle species. It is reasonable to let the black hole environment be fixed in nature, being made of protons/neutrons or Hydrogen atoms/ions. However, as has been already emphasized, there are several independent particle species of Hawking radiation that would interact quite differently with the same ambient gas-matter system of a black hole.

For the energy regime of the present analysis baryon-photon interaction cross-section would be determined by $\sigma_{ph} \approx \sigma_{es} + \sigma_{pair} + \sigma_{pncl}$ [35] where σ_{es}, σ_{pair}, σ_{pncl} respectively denote cross-sections due to electron scattering, pair production and photonuclear interaction (see for detailed analysis [3]).

The photon interaction cross-section is determined as [3]

$$\left(\frac{\sigma_{ph}}{\sigma_T}\right) \simeq f(k_{cs}) + g^* \left(\frac{\sigma_{pair}}{\sigma_T}\right), \tag{14}$$

where

$$f(k_{cs}) = \frac{3}{4}\left[\frac{1+k_{cs}}{k_{cs}^2}\left\{\frac{2(1+k_{cs})}{1+2k_{cs}} - \frac{ln(1+2k_{cs})}{k_{cs}}\right\} + \frac{1}{2k_{cs}} ln(1+2k_{cs}) - \frac{1+3k_{cs}}{(1+2k_{cs})^2}\right], \tag{15}$$

while

$$k_{cs} = \frac{2.82 k_B T_{cs}}{m_e c^2} = \frac{2.82\hbar}{4\pi m_e c r_{cs}} \tag{16}$$

with $\sigma_T = 0.665 \times 10^{-24}$ cm^2. It is also to be noted that $\sigma_{pair} \simeq 0.02823\sigma_T$ and typically $\sigma_{pncl} \sim$ 0.2 mb [3], whereas g^* is a parameter that takes into account the possibility of enhancement of σ_{ph} basically due to interactions other than the dominating Compton scattering and pair production processes. Nevertheless, any enhancement would only be tiny, i.e., $g* \approx 1$. In the present paper we would like to present cross sections of the other particle species as followed.

The prescribed μ_i-values in Equation (13) already indicate that Hawking radiation will be quite dominated by the particle species leptons, hadrons and neutrinos other than the photons. An expected energy regime of the present problem suggests us to have

$$E_{lep} \sim k_B T_H \; ; \; E_{hdrn} \sim m_p c^2. \tag{17}$$

Hence we estimate the approximate cross-section of interaction of a lepton with the background ions/nuclei/nucleons by [41,42]

$$\sigma_{lep} \propto \frac{\alpha_f^2 \hbar^2 c^2}{E_e^2} \propto \frac{(2.82)^2 \alpha_f^2 \hbar^2 c^2}{m_e^2 c^4} \frac{1}{k_{cs}^2} \approx (2.82)^2 \pi r_e^2 \frac{1}{k_{cs}^2} \simeq 3.0 \sigma_T \frac{1}{k_{cs}^2}, \; k_{cs} \geq 1, \tag{18}$$

so that it recovers the classical cross-section value for a low lepton-energy regime. Note that $\alpha_f = 1/137.036$, is the fine structure constant and $r_e = 2.8 \times 10^{-13}$ cm [43,44] is the radius of the electron. It is also notable that of the lepton species muons subsequently decay into electrons. Thus cross-section of a lepton gets modified from a classical estimate.

Now, on the other hand, cross-section of interaction of a hadron specie (which would either be a meson or a baryon, having $\lambda_{hdrn} \sim r_p$) with the background ions/nucleons may be approximated as [45,46]

$$\sigma_{hdrn} \propto \frac{\alpha_f^2 \hbar^2 c^2}{E_p^2} \propto \frac{\alpha_f^2 \hbar^2 c^2}{m_p^2 c^4} \approx \xi^* \pi r_p^2 \simeq 0.036 \, \xi^* \sigma_T. \tag{19}$$

Here proton is approximated to be a typical hadron specie. Note that $r_p = 8.7 \times 10^{-14}$ cm [47]. In general hadrons correspond to both mesons and baryons. Thus a typical hadron cross-section agrees well with the classical estimate. Regarding the characteristic cross-section of a typical meson and a typical baryon we expect a possible uncertainty to the assumed ξ-value to be restricted by $0.6 < \xi^* < 1.2$. Nevertheless we may approximate: $\xi^* = 1$.

For neutrinos we can assume

$$\sigma_\nu \approx 0. \tag{20}$$

2.5. Final Step of Analysis

Now we require to consider a MacGibbon-Carr summarized picture [9,32] of Hawking black hole radiation. Equation (9) is equivalent to saying [3]:

$$\mathcal{L}_H \equiv \epsilon \mathcal{L}_{BB} = 4\pi \epsilon r_H^2 \sigma_B \left(T_H\right)^4, \tag{21}$$

where

$$\epsilon \simeq 13.47 \alpha_\Sigma, \tag{22}$$

such that $\mathcal{L}_{BB}$ corresponds to a 'perfect black body photon-radiation' and $\sigma_B = \pi^2 k_B^4 \hbar^{-3} c^{-2}/60$ is the Stefan-Boltzmann's black body constant. The standard model of physics provides a discontinuous functional behavior of $\alpha_\Sigma(T_H)$. As an alternative, an approximate continuous functional formula of MacGibbon [19,39] could also be used. Since we opt for an accurate value of $\alpha_\Sigma(T_H)$, it must be put by hand. Hence we shall fix the α_Σ-value at the time of our graphical, numerical analysis.

With respect to Equation (21) for $\mathcal{L}_H$, and the improvised Formula (2) for $\mathcal{L}_{Edd}$, the Eddington criterion, viz. $\mathcal{L}_H = \mathcal{L}_{Edd}$, yields a critical state for

$$\frac{\sigma_E(k_{cs})}{\sigma_T} = \left(\frac{1}{k_{cs}}\right)^3 \left\{\frac{1}{\epsilon(k_{cs})}\right\} \tilde{\mathcal{C}}, \tag{23}$$

where k_{cs} is, as has been already defined in Ref. [3] [i.e., Equation (16)] and here ϵ is given by Equation (22)

$$\tilde{C} = \frac{120\hbar^2 m_p}{(2.82)^{-3}\sigma_T \pi c^2 (m_e)^3}, \tag{24}$$

is a constant and

$$\begin{aligned} \sigma_E &= \sum_i \mu_i \sigma_i \\ &= \mu_{ph}\sigma_{ph} + \mu_{lep}\sigma_{lep} + \mu_{hdrn}\sigma_{hdrn} + \mu_\nu \sigma_\nu. \end{aligned} \tag{25}$$

Hence a modified equation determining the baryon accretion forbidden regime appears as

$$\mu_{ph}\left(\frac{\sigma_{ph}}{\sigma_T}\right) + \mu_{lep}\left(\frac{\sigma_{lep}}{\sigma_T}\right) + \mu_{hdrn}\left(\frac{\sigma_{hdrn}}{\sigma_T}\right) + \mu_\nu\left(\frac{\sigma_\nu}{\sigma_T}\right) = \left(\frac{1}{k_{cs}}\right)^3 \left\{\frac{1}{\epsilon(k_{cs})}\right\} \tilde{C}, \tag{26}$$

where μ_i-s are given by Equation (11) while σ_i-s are given by Equations (14) and (18)–(20). So, Equation (26), a transcendental equation in k_{cs}, refines the corresponding result in [3] while the refinement is here endowed to the LHS. The parameters "ϵ", "μ_i" will vary in accordance with the standard model of physics. In the present estimate, ϵ gets appropriately set in terms of $\epsilon \approx 109.6$ (for $\alpha_\Sigma \approx 8.14$, as referred to [40]). Values of the μ_i parameters are set by Equation (13) in our numerical analysis. Note that $\tilde{C} \simeq 3.52 \times 10^9$. Hence we find that (see Figure 1) $r_{cs} \approx 8.2 \times 10^{-15}$ cm. Critical temperature of black hole is therefore $T_{cs} \simeq 2.2 \times 10^{12}$ K ($\simeq$ 192 MeV), while the critical mass is estimated to be $M_{cs} \simeq 5.5 \times 10^{13}$ g. We may now freshly quote the accretion forbidden regime of a black hole. This regime corresponds to $r_H < r_{cs}$ for a classical singular Schwarzschild black hole. However for a quantum improved black hole that could rapidly cool down at a Planck scale [48,49], ordinary matter accretion forbidden regime corresponds to $l_{Planck} \leq r_H < r_{cs}$, i.e., 10^{-33}cm $\leq r_H < 8.2 \times 10^{-15}$ cm.

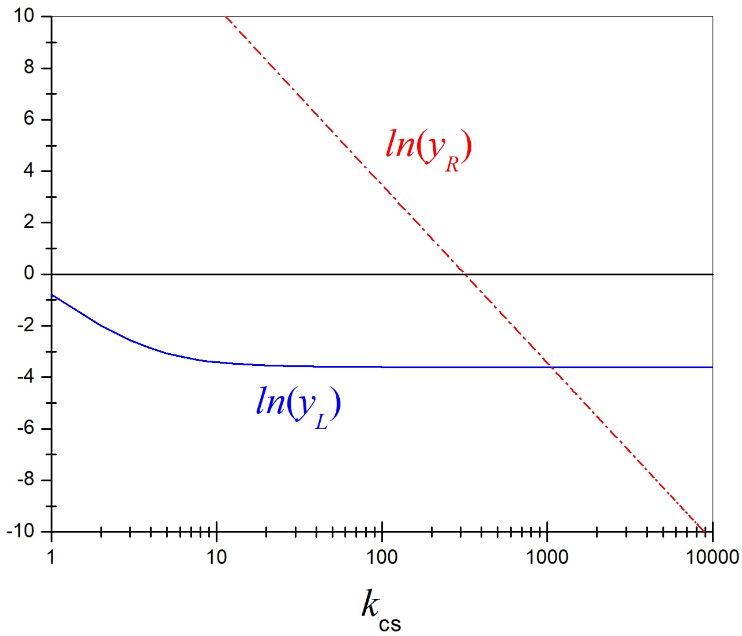

Figure 1. This figure depicts numerical analysis for Equation (26). Here we imply $y_R = \epsilon^{-1}(k_{cs})^{-3}\tilde{C}$ and $y_L = \sigma_E/\sigma_T$. The dash-dotted red line represents $ln(y_R)$ and the solid blue line represents $ln(y_L)$.

Let us mention that in the RHS of Equation (26), the competing as well as the dominating terms correspond to the photon and hadron species. Hence it seems that if there still leaves a scope for further refinement in estimating M_{cs}, that would come only along with a refinement in ξ^*. Nevertheless uncertainty is expected to be limited by 4.5×10^{13} g $< M_{cs} < 6 \times 10^{13}$ g. Note that the Equation (26) remains in anyway a complete analytic prescription for determining r_{cs}, and shows the incompleteness of the corresponding first analysis forwarded in [3].

3. There Is No Radiation Trapping

The existence of a black hole regime forbidden to baryon accretion, will here be tested based on the possibility of radiation trapping by inflowing accreting relativistic gas. Let us consider spherically symmetric Bondi accretion [50]. For maintaining brevity we start analysis with the Bernoulli equation for gas flow [51,52]

$$\left(1 + \frac{a^2/c^2}{\Gamma - 1 - a^2/c^2}\right)^2 \left(1 - \frac{2GM}{c^2 r} + v^2/c^2\right) = \left(1 + \frac{a_\infty^2/c^2}{\Gamma - 1 - a_\infty^2/c^2}\right)^2. \tag{27}$$

Hence for a standard polytropic gas flow, matter accretion rate turns out to be [51]

$$\dot{M}_{acn} = 4\pi r^2 v \rho = 4\pi \left(\frac{GM}{a_\infty^2}\right)^2 \lambda \rho_\infty a_\infty, \tag{28}$$

with

$$\lambda = \left(\frac{1}{2}\right)^{\Gamma+1/2(\Gamma-1)} \left(\frac{5-3\Gamma}{4}\right)^{3\Gamma-5/2(\Gamma-1)}$$

and $1 < \Gamma < 5/3$ (Γ being the adiabatic index); also ρ_∞ is the baryon density at infinity and $a = dp/d\rho$ is the speed of sound associated with the gas-matter inflow. For an inflowing gas that could trap radiation energy, we are to take the adiabatic index for the gas state to be $\Gamma = 4/3$. Taking $\Gamma = 4/3$, in the standard physical limit $a_\infty^2 < a^2 << c^2$, Equation (27) yields $v^2 \approx 2GM/r$. Hence Equation (28) provides the gas-density profile, namely [51]

$$\frac{\rho(r)}{\rho_\infty} \approx \frac{\lambda}{4}\left(\frac{c}{a_\infty}\right)^3 \left(\frac{r_H}{r}\right)^{3/2}, \tag{29}$$

where r_H denotes the Schwarzschild radius (i.e. the radius of black hole event horizon). This profile for density may be taken theoretically valid roughly for $r < r_B$, where r_B represents the well known Bondi radius or sonic radius, denoting the sonic point of gas inflow and running as

$$r_B \approx \frac{5-3\Gamma}{8}\left(\frac{c}{a_\infty}\right)^2 r_H.$$

For details of the calculation for the above radius we refer the following work [51]. Now, for $r > r_B$ there is no unique solution in literature; nevertheless it would be roughly like $\rho/\rho_\infty \propto (r_{acn}/r)^\beta$, where $\beta < 3/2$ and r_{acn} is the accretion radius (see Appendix A): $r_{acn} \simeq (c/a_\infty)^2 r_H$); β slowly runs towards *zero* as $r \to r_{acn}$.

To proceed in our discussion we shall now assume a scenario where the energy absorbing processes evade the possibility of re-emission of energy by the inflowing gas under accretion. We are basically presuming a scenario with maximal radiation trapping. In accordance with this scenario, Hawking luminosity would get attenuated as [35,53]

$$d\mathcal{L}(r) \simeq -\bar{\kappa}\rho(r)\mathcal{L}(r)dr, \tag{30}$$

where $\mathcal{L}(r)$ represents an attenuating Hawking luminosity, while $\bar{\kappa}$ (an average of opacities $\kappa \equiv \sigma/m_p$) stands for the opacity of gas medium under accretion. It is reasonable to assume that luminosity is attenuating in an isotropic manner: $Flux \propto Intensity$. Hence on using Equation (29) in Equation (30), and integrating for $r_H < r < r_{acn}$, we get

$$ln\left\{\frac{\mathcal{L}_\infty}{\mathcal{L}_H}\right\} \approx -\left[\frac{\lambda\bar{\kappa}}{2}\left(\frac{c}{a_\infty}\right)^3 \rho_\infty\right] r_H, \tag{31}$$

where $\mathcal{L}_H$ is the original Hawking luminosity and $\mathcal{L}_\infty$ is that reaches infinity (regarding the gas density profile for $r_B < r < r_{acn}$, a parametrization $\beta < 3/2$ is ineffective in seriously modifying this result). In order for obtaining the expression (31), it has been assumed that for a temperature much greater than 1 KeV, gas opacity $\bar{\kappa}$ does not respond much with the variations in the gas composition and the gas temperature, while it ($\bar{\kappa}$) also remains quite insensitive to the gas density [44]. The last result (Equation (31)) holds better while $r_{acn} >> r_H$. For an awkwardly relativistic case wherein r_{acn} gets very close to r_H, one may simply afford to take $\mathcal{L}_\infty/\mathcal{L}_H \sim exp(-\bar{\kappa}\rho\, r_H)$. Our presumed scenario had been that the outgoing radiation is getting trapped by the inflowing gas. In this scenario the ambient gas at infinity is expected to have a temperature (viz. T_∞) much less than the freely flowing Hawking temperature (viz. T_H). Now it is important to note that the ambient temperature at infinity would indirectly depend on the status of Hawking luminosity at infinity, viz. $\mathcal{L}_\infty$. That is,

$$\frac{\mathcal{L}_H}{\mathcal{L}_\infty} \approx \left(\frac{r_H}{r_{acn}}\right)^2 \left(\frac{T_H}{T_\infty}\right)^4 \tag{32}$$

$$\approx 10^{-18}\left(\frac{T_\infty}{10^4\,\mathrm{K}}\right)^2 \left(\frac{2.2\times 10^{12}\,\mathrm{K}}{T_\infty}\right)^4 \approx 2\times 10^{15}\left(\frac{10^4\,\mathrm{K}}{T_\infty}\right)^2, \tag{33}$$

where we actually model that every spherical thermal surface with radius r_H to radius r_{acn} holds a definite temperature which is gradually decreasing with the increasing radius. We take $T_H \sim T_{cs}$ since we are concerned with black holes with $T_H > T_{cs}$. We are now to test the requirement of gas-matter density in order that a well-defined accretion scenario like that mentioned above is producible. It is hence useful to rewrite Equation (31) as

$$ln\left\{\frac{\mathcal{L}_H}{\mathcal{L}_\infty}\right\} \approx \left[\frac{\lambda\bar{\kappa}}{2}\left(\frac{r_{acn}}{r_H}\right)^{3/2} \rho_\infty\right] r_H, \tag{34}$$

which may also be cast as

$$\mathcal{L}_\infty \approx e^{-\Omega}\,\mathcal{L}_H, \tag{35}$$

with Ω denoting a factor that represents the optical thickness of accreting medium. In regard to a gas-radiation system of an average temperature around MeV scale, the average opacity of a more/less ionized hot gas runs roughly like $\bar{\kappa} = 0.05\,\mathrm{cm}^2/\mathrm{gm}$ [44,53], which indeed also agrees with what one can observe by following a discussion on "κ" in [3]. It would be $\lambda = 0.71$ for a radiation dominated gas with $\Gamma = 4/3$. We are concerned of mini black holes with Schwarzschild radii $r_H < 8.2\times 10^{-15}$ cm.

To justify an accreting ambient gas of temperature $T_\infty \sim 10^6$ K, it has to be $ln(\mathcal{L}_H/\mathcal{L}_\infty) \approx 26$ (note Equation (33)). Hence the gas density at infinity is required to be $\rho_\infty \sim 10^7\,\mathrm{g/cm}^3$ (note Equation (31)), which corresponds to a near horizon density $\rho(r_H) \sim 10^{16}\,\mathrm{g/cm}^3$ (note Equation (29)). This limit for gas-density can easily be understood impossible simply by recalling that even a neutron star has a typical density of $10^{14}\,\mathrm{g/cm}^3$. It may also be noted that a neutron star is made of degenerate gas, not of ideal gas. Real accreting gas would have density $<< 10^{14}\,\mathrm{g/cm}^3$. Accreting gas density may in general be as high as what prevails inside a white dwarf stars, viz. $\sim 10^6\,\mathrm{g/cm}^3$. If we even check the gas density requirement directly via Equation (35), say, for a minimal considerable attenuation, i.e., $\mathcal{L}_\infty \approx 0.99\mathcal{L}_H$, it asks for an accreting gas density $\rho \sim 10^{14}\,\mathrm{g/cm}^3$ (we let $\bar{\kappa} \sim 0.01\,\mathrm{cm}^2/\mathrm{g}$, $\Omega \sim \bar{\kappa}\rho\, r_H$, as is appropriate to this case)—which is impractical. Thus it appears beyond of any doubt that radiation trapping is not viable with matter flowing into a mini black hole. Hence also the regime of black hole prohibiting baryonic accretion must really be existing. Further comments on the "no radiation trapping" can be found in Appendix A.

4. Where Does Bafrbh Apply?

Here "BAFRBH" is implied as an abbreviation for the term "baryon accretion forbidden regime of a black hole". Understanding BAFRBH can be clarifying in certain physical respects. We can address the following issues, e.g.,

(1). To test if there is always a practical relevance of considering baryon accretion by mini black holes of every mass-regime.
(2). To judge the growth versus evaporation for black holes trapped in local cosmological matter inhomogeneities, while dissociating the truly viable PBH mass-range for the possibility of prolongation of decay.
(3). To better anticipate and probe the potential 'cosmological, high energy photon background due to PBHs' in galactic and extragalactic scenario.

The first query gets immediately answered by the existence of "BAFRBH". No event of baryon accretion is allowed in presence of Hawking radiation phenomenon for a black hole with mass below 5.5×10^{13} g. The Point (2) is attributed mainly to Sections 4.2 and 4.3, whereas the Point (3) is summarized in Section 4.5. However for completeness readers should follow all the Subsections to follow. Let us anyway make an immediate, important note: *arguments justifying the ineffectiveness of dark matter and/or dark energy accretion(s) (as apply to the PBHs of our interest) are placed in Appendix B*. Again it is important to clarify a terminology: *we will use a term 'primordial mass' for a PBH to imply the 'PBH mass at birth state'*.

4.1. Pbhs by the End of Radiation Dominated Era and How Effective 'Bafrbh' Could Be!

This Subsection reviews an analysis of [33] (while including a quantitative, contextual revision). In view of the scenario of the Universe by the end of radiation dominated era, we shall first argue that 'BAFRBH' had to be perfectly effective from the very beginning of a matter dominated era of the Universe. So, let us consider an isotropic environment of radiation surrounding a black hole. In such a scenario, the black hole cross-section of radiation absorbency would be proportional to the square of the mass of black hole [54,55]:

$$\sigma_{BH}(M) = \frac{3\sqrt{3}}{2}\frac{16\pi G^2}{c^4}M^2, \tag{36}$$

such that $\dot{M} = \sigma_{BH}\left(pc^{-2} + \rho_{\rm rad}\right)$ where $\rho_{\rm rad}$ denotes the ambient radiation energy density. We may explicate this result as follows: In case of an accreting fluid characterized by $p/c^2 = w\rho$, while $\dot{M} = 4\pi A G^2 M^2(\rho + p/c^2)/c^3$, A is determined to be [55]

$$A = \frac{(1+3w)^{\frac{1+3w}{2w}}}{4w^{3/2}}, \tag{37}$$

which for a radiation system, viz. for $w = 1/3$, assumes the value

$$A = 6\sqrt{3}. \tag{38}$$

So one gets

$$\dot{M}_{\rm rad} = \sqrt{3} \times 32\pi\left(\frac{G^2M^2}{c^3}\right)\rho_{\rm rad}, \tag{39}$$

as the time rate of accretion of radiation energy.

Now what follows, understands the state of a PBH by the end of radiation dominated era and hence the effectiveness of BAFRBH. So first we have to consider the net rate of growth of a black hole inside the radiation dominated Universe [33]:

$$\dot{M} = -\frac{K_{evp}}{c^2M^2} + 32\sqrt{3}\pi\left(\frac{G^2M^2}{c^3}\right)\rho_{\rm rad}, \tag{40}$$

where $K_{evp} = \mathcal{L}_H M^2$.

Hence one finds a critical mass [33]

$$M_{*r} = \left\{ \frac{K_{evp} c}{32\sqrt{3}\pi G^2 \rho_{\rm rad}} \right\}^{1/4}, \tag{41}$$

such that there is net growth for black holes with $M > M_{*r}$. Let us next note that the density of relativistic particles in the radiation dominated Universe runs as [35]

$$\rho_{\rm rad} = \frac{2\sigma_B \mathcal{G}}{c^3} T_U^4, \tag{42}$$

where $\mathcal{G} \equiv \mathcal{G}(T_U)$ includes consideration of the degrees of freedom of the relativistic particles, while the cosmological temperature is estimated by [35]

$$T_U = \left(1.81 \times 10^{10}{\rm K}\right) \mathcal{G}^{-1/4} \left(\frac{t}{1\,{\rm s}}\right)^{-1/2}, \tag{43}$$

where t implies cosmological time. Now we can re-express Equation (41) as

$$M_{*r} = \left\{ \frac{1}{\sqrt{3}} \right\}^{1/4} \times 1.80 \times 10^{16} \left(\frac{t}{1\,{\rm s}}\right)^{1/2} \left(\frac{K_{evp}/c^2}{7.8 \times 10^{26}\,{\rm g}^3/{\rm s}}\right)^{1/4} {\rm g}. \tag{44}$$

This equation defines the mass below which there would be no effective growth of black holes via radiation accretion. Hence around the very beginning of a matter dominated era we find $M_{*r} \sim 10^{22}$ g, which is much greater than M_{cs}. This result in turn assesses that a black hole that holds baryon accretion forbidden regime will not have any significant growth due to radiation accretion at all times after the beginning of matter dominated era. Hence BARFBH regime will be effective all along the matter dominated era and afterward.

We shall again revisit [33] to demand that PBHs did not find enough opportunity to grow over the available period of the radiation dominated era. Mass spectrum of PBHs runs as (assuming that there must be cosmic/particle horizon) [19,33]

$$\dot{M} = \frac{\mathcal{N} c^3}{G}, \tag{45}$$

with $\mathcal{N}$ being a parameter. However PBHs can obviously also have masses lower than this particle-horizon mass. A precise value of $\mathcal{N}$ seems debatable [19], even though a likely value is around 0.4 [33,40], or smaller. Let us now consider $M >> M_{*r}$ where black hole growth due to radiation absorption dominates over evaporation. Hence on using Equations (40) and (45) one gets the following constraint for having relativistic growth of PBHs [33]

$$\rho_{rad} \geq \frac{1}{\sqrt{3}} \times \mathcal{N} \times 2.430 \times 10^{52} \left(\frac{M}{10^{15}\,{\rm g}}\right)^{-2} {\rm g/cm}^3. \tag{46}$$

Referred to Equations (42) and (43), corresponding time-scale for relativistic growth becomes

$$t_{rel} \leq 5.67 \times \frac{1}{\sqrt{\mathcal{N}}} \times 10^{-24} \left(\frac{M}{10^{15}\,{\rm g}}\right) {\rm s}. \tag{47}$$

Now, on the other hand, causality of horizon formation restricts PBH growth in accordance with the following time-scale (see Equation (45)):

$$t_{causal} \geq 2.50 \times \frac{1}{\mathcal{N}} \times 10^{-24} \left(\frac{M}{10^{15}\,\mathrm{g}} \right) s. \tag{48}$$

It is interesting that for $\mathcal{N} \approx 0.20$, last two inequalities merges for a unique point designating $t_{rel}|_{max} = t_{causal}|_{min}$. In fact $\mathcal{N} = 0.20$ goes on to define an upper bound for relativistic PBH growth via radiation accretion. Now by utilizing Equation (40), it can be shown that the growth of a PBH from an initial particle-horizon mass M_i to a final particle-horizon mass M_f assumes the striking form

$$\frac{M_i}{M_f} \simeq 1 - P \times \mathcal{N}$$

for $t_f >> t_i$ (viz. t_i is the timing of birth of a PBH and t_f is a later time) with P being a constant of an approximate value: 5 (in accordance with causality). Therefore, unless $\mathcal{N}$ is very close to a value 0.20, relativistic growth is negligible. For instance even if it only is $\mathcal{N} < 0.1$, relativistic growth is quite weak. For $\mathcal{N} << 0.20$ relativistic growth of a PBH becomes negligibly small. Causality criterion does not give scope for a relativistic growth of a black hole because of the rarefied status of background radiation energy density along with the evolution of time [33]. Note that $M \longrightarrow M_{ph}$ where M_{ph} denotes the particles-horizon mass. In a realistic scenario one must expect $M << M_{ph}$ (without this condition being met, creation of PBH population could not have complied with the future structure formation of the Universe), by which no relativistic growth is guaranteed. If this happens to be the case, then in turn due to the existence of BAFRBH, all the PBHs that was born for $t < 10^{-25}$ s, are bound to be prevailing as Planck mass relics in the today's Universe (provided it is physical to have a Planck relic formation).

Let us make an additional note: ref. [3] forwards a concept of radiation forbidden regime which transpires that for $t < 10^{-25}$ sec, the outward Hawking flux of a PBH in its birth state would act to resist an inflow of external radiation. Naturally, this idea is a boost to the convention of insignificant growth of PBHs in radiation dominated era.

4.2. Understanding Growth Versus Evaporation of a Marginally Accretive Mini Black Hole Trapped Inside Patchy, Dense Matter Cloud

Unlike in the radiation dominated era, in the matter dominated era (and, also afterward), consideration of inhomogeneity in matter distribution is essential to judge the growth versus evaporation rate of a PBH. Whereas an average baryon density is ever below $10^{-19}\,\mathrm{g/cm^3}$ after the radiation dominated era, the energy density for a patchy cloud of ordinary matter could well be practical to reach $10^6\,\mathrm{g/cm^3}$. This extent of matter in-homogeneity has its potential to have a profound effect on PBH evolution. Hence we analyze the headlined case while considering the existence of BAFRBH.

Let $\mathcal{L}_{acn}$ be the luminosity that results due to a matter accretion process. This luminosity is a result of release of gravitational binding energy from matter under accretion, and hence $\mathcal{L}_{acn}$ happens to be a fraction of the matter that gets absorbed by a black hole in an accretion process. So the mass growth rate of a black hole can be estimated as [35,56,57]

$$\dot{M}_{acn} = \frac{\mathcal{L}_{acn}}{c^2 \eta}. \tag{49}$$

Here η is the efficiency of (gravitational) energy-release as a black hole accretes matter. During accretion process, Hawking power would contribute to the "accretion power," and they unitedly constitute a luminosity which is constrained to lie below the Eddington order luminosity. That is, in general, for an ordinary matter accreting black hole, it is

$$(\mathcal{L}_H + \mathcal{L}_{acn}) < \mathcal{L}_{Edd}. \tag{50}$$

However, in order to have a smooth accretion process, it should be $\mathcal{L}_{acn} >> \mathcal{L}_H$. We shall now generalize the idea of η by letting

$$\eta \dot{M}_{acn} = \frac{\mathcal{L}_{acn}}{c^2} = \frac{\zeta \mathcal{L}_{Edd}}{c^2}, \quad \eta, \zeta < 1, \tag{51}$$

where ζ is a parameter that would in general depend both on the mass of black hole and ambient matter density. This leads to recast Equation (49) as [58–64]

$$\dot{M}_{acn} = \frac{\mathcal{L}_{Edd}}{\tilde{\eta}\, c^2}. \tag{52}$$

It is to note that $\tilde{\eta} = \eta/\zeta$ can in practice be both greater and less than *one*. The entity, viz. $\tilde{\eta}$, is interesting, because $\tilde{\eta}$ can lead to rightly judge whether a net evaporation of a black hole starts earlier to reaching the accretion forbidden regime or not.

Crudely, it happens to be $10^{-4} < \eta < 0.06$ for a typical accretion of ordinary matter onto black hole [57,58,62–64]. Hence η value is quite predictable. However ζ value depends on the size of black hole and the ambient matter density, and this dependence is real critical for a sufficiently mini black hole. Gigantic black holes have the ability to sustain Eddington accretion luminosity for a wide range of particle-density level in ambient matter. A mini black hole on the other hand requires a very high minimum level of ambient matter density to produce Eddington order luminosity. For example, in case of a black hole of radius $r \sim r_{cs}$ ambient matter density should at least be $\sim 2\ \mathrm{g/cm^3}$ in order that the black hole holds potential to produce Eddington order accretion luminosity. Thus for a mini black hole $\zeta \sim 1$ is a scenario that requires highly dense clouds of matter to mask the black hole. In any way it will be $\zeta < 1$ in general. So we shall put a lower bound to $\tilde{\eta}$, viz. $\tilde{\eta} \geq 10^{-4}$, whereas there is no upper bound. In accordance with Equation (52), the case $\tilde{\eta} > 1$ corresponds to $\dot{M}_{acn} < \mathcal{L}_{Edd}/c^2$, which implies that $\dot{M}_{acn} < \dot{M}_{evp}$ while a black hole is accretive. Hence a black hole may still shrink in size even while there continues matter accretion. If we let for example $\tilde{\eta} = 10^6$, then it turns out that a black hole (roughly) of radius $r_{cs} < r_H < 10^2 r_{cs}$ would shrink even while matter accretion process is active. However, if $\tilde{\eta}$ is constrained to be less than unity, the net effect of Hawking radiation and matter accretion would suit a black hole with $r_H > r_{cs}$ to grow.

In a recent paper, viz. [65], the growth rate of a black hole (which is active in accretion) was claimed to be positive precisely for $\dot{M}_{acn} - \dot{M}_{evp} > 0$, and accordingly a critical mass line below which a net black hole evaporation (decay) starts, was estimated. However $\dot{M} = 0$ may not always be an appropriate equation for determining the line separating a shrinking black hole from the growing one. Presence of a regime forbidden to (ordinary) matter accretion can curtail possibility of accretion generated growth. Let us introduce

$$r_* = 2GM_*/c^2$$

to designate the critical state of black hole determining the line separating a shrinking black hole from the growing one. Clearly then there will be no growth for a black hole with $r_H < r_{cs}$. That is the valid answers for r_* are bound by $r_* > r_{cs}$. Hence, even though the technique $\dot{M} = (\dot{M}_{acn} - \dot{M}_{evp}) = 0$ is efficient in determining r_* for $\tilde{\eta} > 1$, fails in the case $\tilde{\eta} < 1$.

In accordance with the setup of analysis employed in the paper [65], the critical mass state would be rightly given by

$$M_* \approx \left\{ \frac{G^{-2} c\, K_{evp}}{4\pi A \left(\rho + \frac{p}{c^2}\right)} \right\}^{1/4}, \tag{53}$$

restrictively for a spherically symmetric accretion of matter. Here ρ and p respectively denote the proper energy-density and the pressure of an ambient gas-matter system, and [55]

$$A = 4\frac{v^{(r)}}{c} r^2 r_H^{-2} exp\left[\int_{\rho_\infty}^{\rho} \frac{d\rho}{\left(\rho + \frac{p(\rho)}{c^2}\right)}\right] \tag{54}$$

$$= 4\frac{v^{(r)}|_H}{c}\frac{n_{\rho H}}{n_\infty} \sim \frac{n_{\rho H}}{n_\infty}, \tag{55}$$

is an integral for the accretion of isotropic fluid where n goes on to represent the particle number density, satisfying $d\rho/(\rho + pc^{-2}) = dn/n$. Further we should note that $v^{(r)}$ is the proper radial velocity of gas-matter inflow. Numerical result reveals that for ambient matter density, roughly below 1 g/cm^3, M_* could be rightly estimated by the above formula (see Appendix C). For higher densities the last formula due to [65] leads to a unphysical M_*.

Thus we observe a technical flaw, that associates with [65] because of treating accretion process to be uninterrupted by the Hawking radiation process. In reality there are regimes for which one would no more be in comfort of ignoring interruption in the accretion process due to Hawking radiation. No black holes with $r_H < r_{cs}$ would support the ordinary matter accretion process (in presence of an outward thrust of Hawking radiation), and precisely for $\tilde{\eta} < 1$, the Equation (53) fails in its prediction of the beginning of a net black hole evaporation. We have a clear understanding that judging the evolution of a black hole of $r_H < r_{cs}$ does not require consideration of accretion of ordinary matter since accretion would never happen there. Nevertheless, a black hole of radius $r_H > r_{cs}$ can well accrete matter around Eddington rate if it gets trapped inside a sufficiently dense environment of matter.

4.3. Time Scale of Growth of Marginally Accretive Mini Pbhs and Its Context

In this Subsection we shall explicate if there is a practical possibility of growth of marginally accretive mini PBHs of mass just exceeding 5.5×10^{13} g, subject to ordinary matter accretion and with respect to a required time-scale. Regarding the absolutely plausible primordial inhomogeneties in early matter dominated era of Universe it will be observed that it is indeed possible for a mini PBH of mass $M > M_{cs}$ to grow enough by matter accretion to markedly delay the decay. This aspect naturally isolates PBHs of mass $M < M_{cs}$ from the others. Such a factor is promising in developing constraints on the evolution of mini PBHs.

In accordance with Equations (9), (21) and (52) we have (note that $M_{evp} = \mathcal{L}_H/c^2$):

$$\dot{M} = \dot{M}_{acn} - \dot{M}_{evp} = \frac{\mathcal{L}_{Edd}}{c^2\tilde{\eta}}\left\{1 - \tilde{\eta}\frac{K_{evp}}{M^2\mathcal{L}_{Edd}}\right\} = h_1 M(1 - h_2 M^{-3}), \tag{56}$$

where $h_1 = 4\pi G m_p/(c\sigma_E\tilde{\eta})$ and $h_2 = \tilde{\eta}K_{evp}\sigma_E/(4\pi G c m_p)$, both of which are functions of $\tilde{\eta}$. Here, it is reasonable to approximate $\sigma_E \sim \sigma_T$ by following conventional Eddington prescription. One may think quite generally of any asymmetrically in-flowing matter onto a (primordial) black hole, and Equation (56) would still work well. As was already discussed, we have a constraint $\tilde{\eta} > 10^{-4}$. Through the expected co-relation between η and ζ we suggest a typical parametrization to be $\tilde{\eta} \sim 1$. This model is efficient enough in estimating an approximate time-scale for having substantial growth in PBHs. Hence Equation (56) yields us the required time-scale for a PBH of initial mass M_i to grow upto a mass M_f:

$$\tau_{grw} = \frac{1}{h_1}\left[ln\left\{\frac{M_f}{M_i}\right\} + \frac{1}{3}ln\left\{\frac{1 - h_2 M_f^{-3}}{1 - h_2 M_i^{-3}}\right\}\right]. \tag{57}$$

Note that $h_2 \approx 2\tilde{\eta} \times 10^{41}$ g^3, and therefore we may take $|ln(1 - h_2 M_i^{-3})| \sim 1$ for "M_i"-s of values slightly greater than M_{cs} where $M_{cs} = \sqrt[3]{h_2}$, and say, for $h_2 M_i^{-3} < 0.99$, we find a finite τ_{grw} (which fairly follows from Equation (58)), while $|ln(1 - h_2 M_f^{-3})| \sim 0$ for $M_f >> M_i$. Hence a time-scale for the growth of mini PBHs with mass $M > M_{cs}$ can be effectively determined as [66]

$$\tau_{grw} \approx 0.45\tilde{\eta} \times ln \left\{ \frac{M_f}{M_i} \right\} \text{Gyr}, \tag{58}$$

which can also be implied for

$$M_i = M_f \, exp \left\{ -\frac{\tau_{grw}}{0.45\tilde{\eta}\,\text{Gyr}} \right\}. \tag{59}$$

Hence for constituting any considerable growth in a PBH (say for $1.05 M_i M_f 10 M_i$) the required minimal time-scale is $\tau_{grw} > 10^3 - 10^5$ years which is indeed of a minimal order since we took $\tilde{\eta} \equiv \tilde{\eta}_{min} = 10^{-4}$. For a mini black hole this time-scale is subject to the criterion of continuous availability of matter-environments of density very much larger than the average matter density prevailing in the evolving Universe at any point of time. However, for a moderate efficiency $\tilde{\eta} = 1$, if we let for example $10 M_i < M_f < 10^9 M_\odot$, and $M_i > 5.5 \times 10^{13}$ g, the required time-scale of growth comes out to be $\tau_{grw} \sim 10^9 - 10^{10}$ years (i.e., one to ten billion years).

Therefore one may easily perceive that in order for ensuring a massive growth to a marginally accretive mini PBH, there must be a stably existing highly dense environment of accretable matter, masking the concerned black hole over a long period of time (perhaps, over a time period as large as the age of the present-day Universe). Hence massive growth of accretive mini PBHs requires substantial time. Unless PBHs were massive by birth, fulfilling this requirement is barely possible. In this context we need to note that PBHs of primordial mass $M > 10^3 \tilde{\eta}^{-1} M_\odot$ would necessarily produce accretion luminosity at Eddington level upto some period after decoupling, and large enough black holes might continue to do so until galaxy formation [56]. Hence a considerably macroscopic black hole gets more obvious chance compared to mini ones to become the supermassive black holes of present day galaxies. Timing of first galaxy formation (viz. $t \sim (1 - 10)$ Gyr) could comply with required timescale for a massive pre-growth of PBHs. However, apart from the PBHs of mass $M < 5.5 \times 10^{13}$ g possibility of massive growth remains much weaker also for a special PBH class of the mass range 5.5×10^{13} g $< M < 5.1 \times 10^{14}$ g.

Nevertheless we must also consider the fact that a minor growth of mini PBHs is always plausible inside early matter dominated era, since we have inference for this case to be realistic in terms of star structure formation [67] inside an early matter dominated era. So it appears that accretion in matter dominated era had certain opportunity to prolong the timescale of PBH decay to a fairly considerable extent. Hence there emerges a simple, but, important point: a black hole of primordial mass as small as 5.5×10^{13} g (yet critically never less than that) can in reality decay at the present epoch of the Universe. Moreover regarding the required large time-scale for massive growth of a mini PBH, following proposition is plausible: one expects a dominating fraction of PBHs with primordial mass 5.5×10^{13} g $<$ $M < 5.1 \times 10^{14}$ g to complete decay by the time interval 4.0×10^6 years $-$ 13.8×10^9 years. In fact such a proposition gets transpired by the present day probe of extragalactic, galactic photon flux distribution [19]. We shall recall and summarize the issue of this last point in Section 4.5 after following the very next subsection.

4.4. Time Scale of Evaporation of Bafrbh-Pbhs and Its Context

We have observed that the PBHs with a regime forbidden to accretion cannot accrete ordinary baryonic matter, and therefore these PBHs are only to get evaporated in Hawking's way, and eventually could leave cold Planck remnants. Time required for a black hole to lose its mass until it becomes closely a Planck mass relic, is found by using Equation (21) [9,20,40]:

$$\tau_{evp} = \left(\frac{c^2}{3K_{evp}} \right) M^3 = \frac{1}{16.02} \times 10^{-25} \alpha_\Sigma^{-1} \left(\frac{M}{1\,\text{gm}} \right)^3 \text{s}, \tag{60}$$

for $M >> M_{Planck}$. Hence for a black hole of mass 5.5×10^{13} g, the time of complete decay is $\tau_{evp} \approx 4.0 \times 10^{6}$ years (regarding $\alpha_{\Sigma} \approx 8.14$ [40]).

Certainty of decay-history of PBHs forbidden to accretion leads to strong prediction for evolution of this class PBH population. The PBH population of primordial mass under baryon accretion forbidden regime could be constrained in a few ways [9,19], e.g., by big bang nucleosynthesis (BBN) and baryon-photon ratio, extragalactic γ ray background, distortion and anisotropy in 'cosmic microwave background' (CMB), pre-galactic re-ionization picture. Since PBHs of mass $M < 5.5 \times 10^{13}$ g must decay without any delay, propositions of CMB distortion and anisotropy, BBN, re-ionization should lead to strong constraints for their abundance.

Photons, emitted sufficiently early are expected to be completely thermalized, and merely contribute to the photon-to-baryon ratio [26]. Thermalization will be prominent before recombination epoch. Note that the Universe became transparent for $t \geq 3.8 \times 10^{5}$ years, after recombination epoch. Still around the recombination era ($t \sim 10^{5} - 10^{6}$ years), partial thermalization is possible [19,26]. Hence PBH population of primordial mass range $2.5 \times 10^{13} g < M < 5.5 \times 10^{13}$ g, are regarded to have certain effects on controlling 'y-distortion' in CMB and anisotropies [19]. An abundance of this PBH class is estimated by the issue of allowable CMB distortion and anisotropy [26,29]. Also the issue of required damping of CMB anistropies leads to abundance-constraints [19,68]. Further constraints that BBN imposes on relative abundance of PBH mass-spectrum $10^{9}\text{g} < M < 5.5 \times 10^{13}$ g [19] gets a relevant support from the proposition of pure Hawking decay in BAFRBH.

The cosmological age of the Universe is about 13.8×10^{9} years. As it was illustrated earlier, it is reasonable to assume that PBHs did not find opportunity to grow much during the radiation dominated period of the evolving Universe - an idea which is in agreement with [16,33,69–71]. Hence we expect a finite abundance of PBHs with masses $M < 5.5 \times 10^{13}$ g, by the time of the beginning of the matter dominated era of Universe. Being led by the idea of BAFRBH, we may again assess that these PBHs should have reached the Planck mass ($\sim 10^{-5}$ g) by the dark ages (provided, of course, Planck relic formation is physical), well before the beginning of a re-ionization epoch of Universe. Naturally the entire population of this Planck mass relics is expected to prevail in the present-day Universe.

Present day cosmological photon flux distribution might be approximated due to PBHs with masses exceeding the BAFRBH limit. Observed present day γ-ray background (GRB) becomes consistent with an approximate 'monochromatic mass spectrum' model scenario of Hawking emission as one considers a maximal abundance of PBHs of mass greater than 5.5×10^{13} g [19], which are just not included in the PBH class holding baryon accretion forbidden regime. This point is encouraging. Satisfactory probe of galactic γ-ray explosions for a maximal primordial abundance of PBHs of mass $5.5 \times 10^{13}\text{g} < M < 5.1 \times 10^{14}$ g could be one important evidence of existence of PBHs. As stressed in [19,20], final explosive phase of PBH evaporation should show certain unique signatures in the emitted light spectra which might be observable in GeV/TeV γ ray observatories such as the HAWC observatory. Hence again it could be possible to differentiate PBH burst events from other usually speculated cosmological GRB sources.

Thus it seems valuable to put stress on the fact that the cosmological present-day photon-flux distribution/burst events of potential PBHs in galactic or extragalactic space, would be dominantly contributed by those with primordial mass greater than 5.5×10^{13} g. On the other hand PBHs of primordial mass less than 5.5×10^{13} g are more important regarding the issues of CMB, BBN, re-ionization, Planck PBH relics etc.

4.5. Summarizing the Constraints in Probing Observational Evidences of Pbhs

Profuse radiating state of a class of PBHs in the present epoch of the Universe corresponds to those PBHs that by the end of radiation dominated era were at such a state that the time required for them to complete decay from thereon happens to be [9,19]

$$\tau_{evp} \sim t_0 \approx 13.8 \times 10^{9} \text{ years}, \tag{61}$$

where t_0 stands for the present age of the Universe. Provided we do not consider possibility of growth of PBH by accretion of ordinary matter (also note that the radiation dominated time period is negligible compared to the present age of the Universe), scenario of a PBH completing decay in the present epoch implies a critical mass

$$M_{\mathrm{IGPR}} \sim \left\{ \frac{3K_{evp}t_0}{c^2} \right\}^{1/3}, \tag{62}$$

where "M_{IGPR}" stands for the mass of an initially grown PBH by the end of the era dominated by radiation. Hence, in accordance with standard evaporation model $M_{\mathrm{IGPR}} \sim 5.1 \times 10^{14}$ g [20,40] should set an upper mass-bound to PBHs that complete decay by the current epoch. Let us make a note that would be useful later: A distribution of potential PBH photon flux in the present-day Universe does not merely correspond to the PBHs that could be evaporating currently, but also includes contributions of PBHs from all the previous epochs of the Universe.

Subject to no considerable growth of PBHs inside radiation dominated era, a PBH population of primordial mass below(/above) the mass scale 5.5×10^{13} g, would be roughly equivalent to a PBH population of IGPR-mass below(/above) $M_{\mathrm{IGPR}} = 5.5 \times 10^{13}$ g. It is also essential for us to note that every PBH with a mass $M_{\mathrm{IGPR}} \leq 5.1 \times 10^{14}$ g is not bound to be evaporated down to the Planck scale by now. Given a condition that a PBH of mass 5.5×10^{13} g $< M_{\mathrm{IGPR}} < 5.1 \times 10^{14}$ g gets trapped inside some patchy dense matter-environment, decay time of that PBH would get prolonged. We now recall that radiation from PBHs of sufficiently early Universe is expected to be thermalized. Hence the population of PBHs with 5.5×10^{13} g $< M_{\mathrm{IGPR}} < 5.1 \times 10^{14}$ g becomes a truly effective source of observable distribution of γ-ray-flux in the present day sky. Again regarding the possibility of growth of PBHs due to matter accretion, the PBHs with mass just exceeding the scale 5.5×10^{13} g might be a dominating contributor to present-day photon flux distribution.

The PBHs with $M_{\mathrm{IGPR}} < 5.5 \times 10^{13}$ g are by now either left to be at Planck scale or else have vanished completely through PBH bursts events by the early dark ages. Hence these PBHs could have significant contribution to nucleosynthesis and CMB anistropies, distortions. Since the PBHs under the mass-scale M_{cs} will complete decay by $t \approx 4 \times 10^6$ years, radiation they cause is quite expected to be thermalized soon, and they may be excluded while probing the present-day high energy photon flux distribution due to potential PBHs from all the previous epochs of Universe. Nevertheless if it could be practically possible to only consider photons from $t > 4 \times 10^6$ years, since there exits BAFRBH, that would completely cut contributions of PBHs of mass below the scale $M_{\mathrm{IGPR}} = 5.5 \times 10^{13}$ g in today's photon-flux probe.

Profuse evaporation scenario of PBHs after the beginning of reionization era only belongs to the PBHs of a primordial mass above 5.5×10^{13} g. On the other hand no PBHs of primordial mass considerably greater than 5.1×10^{14} g are to radiate profusely at the present time. Hence it seems crucial to recognize a special class of PBHs: 5.5×10^{13}g $< M_{\mathrm{IGPR}} < 5.1 \times 10^{14}$ g. Distribution of present day galactic and extragalactic gamma-ray burst events should be constrained by a primordial abundance of this very definite class PBHs. Thus we have a very definite class of PBHs to probe through observation. Probing galactic γ-ray burst events in the present epoch should reflect evolution history of PBHs of an IGPR-mass-range (5.5×10^{13} g $< M_{\mathrm{IGPR}} < 5.1 \times 10^{14}$ g [19].

We shall now briefly accumulate the emerged points. One essential point is that matter accretion is capable of prolonging the time of complete decay of a PBH of mass $M_{\mathrm{IGPR}} > 5.5 \times 10^{13}$ g. On the other hand, since the PBHs with mass $M_{\mathrm{IGPR}} < 5.5 \times 10^{13}$ g must evaporate, and also because they evaporate quite quickly in regard to cosmological standard (viz. $\tau_{evp} \approx 4 \times 10^6$ years $<< t_0$), subject to no relativistic growth in radiation dominated era, it essentially implies that an entire population of PBHs that born (The mechanisms of PBH formation may be related with early matter dominance and phase transitions [12,13]. In this case the relationship between mass of PBH and time of their formation differs from the case of PBH production at the RD stage) for $t < 10^{-25}$ s, should be available as Planck relics in the present-day Universe. The PBHs that complete evaporation inside the initial transparent Universe around 3.8×10^5 years $< t < 4 \times 10^6$ years, must belong to BAFRBH, and hence these PBHs would be responsible for pure Hawking energy emission. Emitted PBH radiation for 3.8×10^5 years $< t < 4 \times 10^6$ years might only be partially thermalized, and hence the corresponding PBH population becomes associated with a wide range of issues on PBHs (such issues are, e.g., source of cosmic γ-rays, CMB distortion, anisotropy, reionization etc.) [26,32,72]. There emerges a critical time-scale, viz. 4×10^6 years: while earlier to this time, Hawking radiations are distributed due to PBHs with mass $M_{\rm IGPR} \leq 5.5 \times 10^{13}$ g, at later times Hawking radiations must be born due to PBHs of mass $M_{\rm IGPR} \geq 5.5 \times 10^{13}$ g. Evidently, a satisfactory probe of the potential present-day Hawking, photon fluxes with a maximal abundance of PBHs for the mass range 5.5×10^{13} g $< M < 5.1 \times 10^{14}$ g could be an encouraging point of existence of both PBHs and Hawking radiation phenomenon.

Great primordial abundance of PBHs with mass $M_{\rm IGPR} < 5.5 \times 10^{13}$ g could enable Planck mass PBH relics to be a significant component of dark matter. Apart from that a Planck mass relic of black hole has every right characteristics to act as dark matter particle [17,32,40,73,74]. However a stable existence of Planck relic of black hole must be physical.

Only macroscopic PBHs of a primordial mass $M \geq 10^3 \tilde{\eta}^{-1} M_\odot$ are realistic sources for supermassive galactic black holes [56]. Matter accretion by black hole could affect ionization and thermal history of the Universe [56,75]. Accreting black holes could act as x-ray emitter leading to contribution to CMB rays and anisotropies. However these aspects are associated with really macroscopic black holes [56].

5. Conclusions

As a central issue of this paper, we follow up and duly refine [3] to uphold an unavoidable existence of a regime of black hole that would forbid accretion of baryonic matter (BAFRBH regime). First we identified that the analysis in [3] involves a crude picture of not distinguishing different forms of radiation in accordance with the distinguishable ways of their interaction. This analytical incompleteness is rectified in the present analysis by improvising the formula of Eddington luminosity limit and distinguishing Hawking radiation species in accordance with their proportions and cross-sections of interaction with the black hole's ambient gas-baryons. Hence a thorough analysis leads us to a refined estimate that ordinary matter can never accrete onto a radiating semi-classical Hawking black hole of mass $M < M_{cs}$ (where $M_{cs} \approx 5.5 \times 10^{13}$ g) (i.e., equivalently a black hole of radius $r_H < r_{cs}$ (where $r_{cs} \approx 8.2 \times 10^{-15}$ cm) will hold baryon accretion forbidden regime). Next we provide a pertinent argument for why should BARFBH be true at any feasible physical circumstance. Even if a quantum black hole cools down at a Planck mass state, energy absorption remains no more significant there. Note that the presently provided refined analytical analysis for deriving baryon accretion forbidden regime could well be used in a refined estimation of radiation accretion forbidden regime.

We have considered the simplest cosmological scenario, assuming dark matter as nearly homogeneous medium, playing no role in the considered effect. It would be interesting to extend our analysis to the models, predicting strong primordial inhomogeneity of some dark matter components, like archioles in axion models [14,15,76–78], dissipating mirror or shadow matter [15] or clustering of primordial black holes [79].

Hence we discuss physics contexts of BAFRBH. Since there exists BAFRBH, there could be no delay in evaporation of a black hole of mass less than 5.5×10^{13} g, however, matter accretion could well have led to prolonging the duration of decay of PBHs of primordial mass greater than 5.5×10^{13} gm. It is understood that an entire primordial population of black holes with primordial mass $M < 5.5 \times 10^{13}$ g would contribute to the scenario of Planck scale PBH relics by $t = 4 \times 10^6$ years. Hence, existence of BAFRBH gives rise to a way of imposing a lower bound for the abundance of cold Planck relics of PBHs in the present-day Universe through the primordial abundance of the PBH population of primordial mass $M \leq 5.5 \times 10^{13}$ g. PBHs did probably not have opportunity to grow considerably during the radiation dominated period, which implies that a primordial population, $M \leq 5.5 \times 10^{13}$ g, would be effectively equivalent to a PBH population of IGPR mass $M_{\rm IGPR} \leq 5.5 \times 10^{13}$ g ($M_{\rm IGPR}$ is a PBH mass-state by the end of radiation dominated era). So the PBHs of primordial mass less than 5.5×10^{13} g, that are born at times earlier than 10^{-25} s, are certain to get evaporated into a Planck relic state by the time 4×10^6 years after big bang. Consequences of PBH evaporation events for 3.8×10^5 years $< t < 4 \times 10^6$ years are critical in reflecting the proportion of PBH population earlier to 10^{-25} s. It appears that the mass line 5.5×10^{13} g separates the PBHs contributing majorly to CMB, BBN, Planck relics, from the PBHs that would dominate in contributing to galactic, extragalactic cosmic rays, γ-ray explosions etc. Present day high energy photon flux distribution due to potential PBHs should be majorly contributed by the PBH population of primordial mass 5.5×10^{13} g $< M < 5.1 \times 10^{14}$ g. Hence present-day γ-flux distribution is expected to probe primordial abundance of the PBHs of primordial mass 5.5×10^{13} g $< M < 5.1 \times 10^{14}$ g. At the final stages of evaporation they become the source of ultra high energy γ-radiation, challenging the search of this effect of new physics in the LHAASO experiment.

In this paper we further anticipate that a major contributor to the potential high energy Hawking flux in today's cosmological scenario is the PBH population of primordial mass spectrum just exceeding the scale 5.5×10^{13} g . Thus we propose that under the projection of no relativistic growth of PBHs in radiation dominated era, BAFRBH could be one key idea for undertaking issues related with mini PBHs.

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Appendix A. There Is No Radiation Trapping (Extended Discussion)

Following discussion would be another way of perceiving that radiation trapping by the inflowing gas is not viable for black holes with $r_H < r_{cs}$. Let us first recast Equation (27) as

$$1 - \frac{2GM}{c^2 r} + v^2/c^2 = \left(\frac{\Gamma - 1 - a^2/c^2}{\Gamma - 1 - a_\infty^2/c^2} \right)^2 . \quad \text{(A1)}$$

There exists a hypothetical sphere of radius $r = r_{acn}$, on crossing which the ambient gas gets compelled to fall inward being driven by gravity and a negative gas-density gradient [58,62–64]. The Equation (A1), as given above, reveals this accretion sphere. It is to note that while $r > r_{acn}$, inherent kinetic ability of gas makes it capable of ignoring inward drag; the entities ρ, a roughly equal their ambient values ρ_∞, a_∞ for all $r > r_{acn}$. While the inflow velocity "v" of gas approaches *zero* for $r >> r_{acn}$, it ("v") increase only very slowly with r decreasing and reaches a value of the order of a_∞ at $r = r_{acn}$. For $r > r_{acn}$, the sound speed of a gas inflow, "a" would be $a_\infty \approx \sqrt{k_B T_\infty / m_p}$. Therefore, for inflowing gas at large r ($r > r_{acn}$), it would be

$$\frac{c}{a_\infty} \sim 3 \times 10^4 \left\{ \frac{10^4\,\text{K}}{T_\infty} \right\}^{1/2} . \quad \text{(A2)}$$

Hence, Equation (A1) provides [58,62–64]

$$r_{acn} \simeq \left(\frac{c}{a_\infty} \right)^2 r_H \approx 10^9 r_H \left(\frac{10^4\,\text{K}}{T_\infty} \right) . \quad \text{(A3)}$$

So accretion radius depends on the temperature of ambient gas environment (note that it is the ambient temperature T_∞ which sets up a_∞), and on the mass of the accretor. For $r < r_{acn}$ the entities ρ, a begin to increase above their ambient values with a decreasing radius r.

In order for denying accretion process, the critical Eddingtonian radiation flux needs only to get read of the accreting gas residing inside the sphere active under accretion. Gas prevailing outside of the radius r_{acn} would act rather sensitively to any little excess outward pressure applied there, and would immediately escape outward. An imaginatively accreting mini black hole with potential accretion forbidden regime would have $r_{acn} < 10^{-6}$ cm (with respect to the standard cases of accretion with ambient temperature $T_\infty > 10^4$ K). It really appears as an incredibly tiny accretion gas-sphere for making a highly energetic γ-ray type Hawking radiation flux attenuated. Due to this tiny size of accretion sphere, indeed the required trapping of Hawking radiation that *justifies* an ambient with temperature $<< 10^{12}$ K and maintains the process of a gas accretion, is not possible with any feasible ambient gas-matter. In fact even if we think of an ambient temperature $\sim 10^{12}$ K, one may easily find that the radius of accretion sphere shrinks down to $r_{acn} \approx 10\, r_H \sim 10^{-13}$ cm, and as is being expected, there can be no radiation trapping (gas with a temperature 10^{12} K is so much relativistic that even profound gravity gets easily ignored). Thus it gets indicated that the radiation trapping by inflowing gas is likely to be inefficient in case of the mini black holes of our interest.

Appendix B. Are the Dark Matter and/or Dark Energy Accretion(s) Significant?

Let us here illustrate how the accretion of ordinary baryonic matter has to be a dominating phenomenon compared to the dark matter accretion. This idea is led by one simple point: Dark matter is expected to be quite uniformly distributed in space as astrophysical probes demand (e.g., it may be drawn out of the rotational curves at great distances around galaxies), and its density was less than 10^{-19} g/cm^3 inside the evolving Universe from the very beginning of matter dominated era onward. Now let us note that even though on an average over a cosmologically large scale of space, ordinary matter density is less as compared to the dark matter density, in places, ordinary matter density overwhelms the dark matter density. These scenarios of patchy dense baryonic matter cloud enclosing black holes are what we are concerned of; without the scenarios of this kind no significant accretion of matter would anyway take place, and mini black holes will therefore get evaporated simply in accordance with the Hawking prescribed evaporation process. For explicitness we may note that a realistic ordinary gas-matter system may be as dense as 10^6 g/cm^3. Hence the status of dark matter density says it all. Therefore no ambiguities should be there in constraining evolution of Hawking-radiating mini black holes by the sole consideration of ordinary baryonic matter accretion into them.

Let us next take into consideration the possibility of association of dark energy accretion. In accordance with the orthodox theory, dark energy density is $\rho_D \sim 10^{-9}$ erg/cm^3 having a precise equation of state $p/c^2 = -\rho_D$ (with a fine agreement with observations). Hence dark fluid leads to null accretion, being evident by [54,55]

$$\dot{M} = 4\pi A \frac{G^2 M^2}{c^3}\left(\frac{p}{c^2} + \rho_D\right), \qquad \text{(A4)}$$

with A being a constant. Even for an exotic version of equation $p/c^2 = w\rho_D$, dark energy accretion will be insignificant with respect to mini black holes of mass less than M_{cs} [80]. Note that A is suggested to be of value, 4, in [54,55,80] for quiescent type dark energy with $w < 0$. It may be checked numerically that for $w < 0$:

$$\dot{M}|_{max} \sim c r_{cs}^2 \rho_D \sim 10^{10} \times 10^{-28} \times 10^{-29}\ \text{gm/sec}. \qquad \text{(A5)}$$

This amount is extremely negligible. Dark energy accretion can be significant only for gigantic black holes, which are out of concern in our present paper.

Thus we conclude that both the dark matter and dark energy accretions are perfectly negotiable for the estimates made inside the present paper.

However, this conclusion may be strongly influenced by strong primordial inhomogeneities of dark matter, like large scale correlations of energy density of primordial oscillations of axion field [14,15,76–78] or by inhomogeneity of dissipating mirror or shadow matter [15].

Appendix C. Growth Versus Evaporation of a Black Hole (Extended Discussion)

An alternative useful form of Equation (53) may be obtained by using Michel's formula of accretion rate [52] for a Bondi type accretion process [50]. In accordance with Michel's calculation matter accretion rate for a polytropic fluid is determined as [51,52]

$$\dot{M}_{acn} = 4\pi \left(\frac{GM}{a_\infty^2}\right)^2 \lambda_s m_b n_\infty a_\infty$$

with

$$\lambda = \left(\frac{1}{2}\right)^{\Gamma+1/2(\Gamma-1)} \left(\frac{5-3\Gamma}{4}\right)^{3\Gamma-5/2(\Gamma-1)}$$

and $1 < \Gamma < 5/3$; n_∞ is the baryon number density at infinity (one may note that $m_b n_\infty \approx \rho_\infty$) and $a = dp/d\rho$ is the sound speed of gas-matter flow. Hence the net effect of accretion and Hawking radiation gives a growing black hole for

$$\frac{K_{evp}}{c^2 M^2} < 4\pi \left(\frac{GM}{a_\infty^2}\right)^2 \lambda m_b n_\infty a_\infty, \tag{A6}$$

provided $\tilde{\eta} > 1$. Thus for $\tilde{\eta} > 1$, the mass-line separating a growing black hole from evaporating one would be accordingly given by

$$r_* = 2GM_*/c^2 \quad ; \quad M_* \approx \left\{\frac{G^{-2} a_\infty^3 K_{evp}}{4\pi \lambda_s m_b n_\infty c^2}\right\}^{1/4}. \tag{A7}$$

While $\tilde{\eta} < 1$, Equation (A7) represents a defective result, and the correct result is simply $r_* \approx r_{cs}$. With a particular choice of accretion parameters, viz. $m_b \approx m_p = 1.67 \times 10^{-24}$ g, $a_\infty = 3 \times 10^6$ cm/sec, $\Gamma = 4/3$ and $n_\infty = 10^{16}\,\mathrm{cm}^{-3}$, we get $M_* \approx 10^{17}$ g (by taking $K_{evp}/c^2 \sim 10^{26}\,\mathrm{gm}^3\mathrm{s}^{-1}$). This result is meaningful since here $M_* \geq M_{cs} (\approx 5.5 \times 10^{13}\,\mathrm{g})$. However a different choice of accretion parameters can alter the scenario. For example if we choose $n_\infty \approx 10^{28}\,\mathrm{cm}^{-3}$ and keep the choices of the other parameters unaltered, then Equation (A7) gives $M_* \approx 10^{13}$ g, which is a forbidden result. It shows the insecurity involved in Equation (A7).

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