FROM STRINGS TO BRANES: A PRIMER

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We give a pedestrian introduction to recent advances in string theory. We discuss T- and S-duality and introduce D-branes. The close relation between D-branes and gauge theories is explained. Finally, recent applications such as the microscopic understanding of black hole thermodynamics and the realization of the holographic principle are developed.

Table 1: The five perturbatively consistent super string theories. The first one contains both open and closed strings while the latter four were formulated in terms of closed strings only. The low-energy sector consists each time of a gravitational sector sometimes combined with an additional Yang-Mills sector. Each of these is supersymmetric and N denotes the number of supersymmetries. Note that one supersymmetry in ten dimensions contains 16 fermionic charges (compare this to four dimensions where one supersymmetry has only 4 fermionic charges). The type HE and HO string theories are better known as heterotic string theories.

name	low energy description
type I	$N = 1$ supergravity + $N = 1$ $SO(32)$ super Yang-Mills
type IIA	$N = 2$ non-chiral supergravity
type IIB	$N = 2$ chiral supergravity
type HE	$N = 1$ supergravity + $N = 1$ $E_8 \times E_8$ super-Yang-Mills
type HO	$N = 1$ supergravity + $N = 1$ $SO(32)$ super Yang-Mills

1 Introduction

Three of the four fundamental interactions are very well understood. Indeed, the Standard Model provides us with a qualitatively and quantitatively excellent description of their behaviour at long and short distances. However, the from dayly experience most familiar interaction, gravity, stays out of the picture. Its structure at long distances is well described by general relativity but its behaviour at short distances is unknown. The main reason for this is that novel properties of gravity due to quantum mechanical effects are generally expected to appear at energy scales more than fifteen orders of magnitude higher than what is experimentally attainable at the moment.

Despite the lack of experimental input, there is one, and only one, framework known which provides us with a quantum mechanically consistent description of gravity: string theory. Instead of viewing an elementary particle as a point like object, one takes it to be a tiny string. The low energy behaviour of the theory is such that it corresponds to a supersymmetric version of Einstein's general relativity, called supergravity, coupled to (non-)abelian supersymmetric gauge theories. In this way, string theory supplies us with a serious candidate for a unified description of all fundamental interactions.

Before 1995 however, string theory could hardly be called a theory. It existed in the form of five seemingly different sets of Feynman rules allowing for the calculation of string scattering amplitudes. As string theory naturally lives in ten dimensions, a compactification down to four dimension was called for as well. Though severe consistency conditions were derived which guaranteed a classically stable compactification, many models were found to satisfy these requirements. So a candidate *theory of everything* existed in five different flavours each of which allowed for numerous compactifications down to four dimensions. If one realizes that the details of four dimensional physics depend largely on the precise structure of the compactification, we had what one could call *l'embarras du choix* and it is not unexpected that the phenomenological use of string theory remained rather limited.

In 1995, the situation changed abruptly. For the first time, certain non-perturbative aspects of string theory could be probed. The solitonic excitations of string theory were discovered [a], the so-called D-branes and they allowed for the formulation of various *dualities* relating seemingly different string theories. In this way a unified picture of string theory emerged: the five known string theories were special limits of a single underlying theory which goes under the name of *M-theory*.

In the present paper, I will review the essential ideas which emerged since 1995 in string theory and discuss some of the main results and applications. Because of the very introductory nature of this paper, I will mostly limit the references to review papers. References to the original papers can be found in them. The interested reader is referred to two excellent, but technical books [1], and two more popular introductions [2].

[a] Solitons are finite energy solutions to the classical equations of motion whose masses typically go as g^{-2} or g^{-1} where g is the coupling constant. So in the perturbative regime, where g is necessarily small, they are very massive and as a result very "invisible". *E.g.* think about monopoles in gauge theories.

2 Dualities

2.1 Introduction

Duality relates various theories with sometimes very different appearances. The weakly coupled regime of one theory is then identified with the strongly coupled regime of the other theory and vice-versa. When doing this identification, the elementary excitations of one theory are identified with the solitonic states of the other one and vice-versa.

A prototype example of this occurs in two dimensions, where it turns out that the Sine-Gordon model is equivalent to the massive Thirring model [3]. The former has a single scalar field ϕ, and is described by the action

$$\mathcal{S} = \int d^2x \left(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{m^2}{g}\left(\cos(\sqrt{g}\phi) - 1\right) \right). \tag{1}$$

Expanding the potential, one recognizes m as the mass of the scalar field and g as the coupling constant. Besides the fundamental scalar excitations, this model has solitonic solutions as well. They interpolate between different vacua and have a mass given by $8m/g$. The solitons are very heavy in the perturbative regime (when g is very small) of the theory. The massive Thirring model describes a Dirac fermion with mass m' and interacting through a four-fermion interaction. It is defined by the action

$$\mathcal{S} = \int d^2x \left(i\bar{\psi}\partial\!\!\!/\psi - m'\bar{\psi}\psi + \frac{g'}{2}\bar{\psi}\gamma_\mu\psi\bar{\psi}\gamma^\mu\psi \right). \tag{2}$$

Under the duality transformation, one identifies

$$\frac{g}{4\pi} = \frac{1}{1+\frac{g'}{\pi}}, \tag{3}$$

in other words the strong coupling regime of one theory is related to the weak coupling regime of the other. Under this identification the solitons of the Sine-Gordon theory are identified with the fundamental Dirac fermions in the Thirring models, while the fundamental scalar is identified with a fermion–anti-fermion bound state.

In the next, I will introduce two essential stringy dualities.

2.2 From T-duality to D-branes

We consider a scalar field theory in d dimensions and compactify one of the spatial dimensions on a circle of radius R. Consider now *e.g.* the massless Klein-Gordon equation, $\Box\phi = 0$. We expand the scalar field in terms of a complete set of functions on the circle,

$$\phi(x) = \sum_{m\in\mathbb{Z}} \phi_m(z)e^{\frac{imy}{R}}, \tag{4}$$

where z denotes the dependence on the non-compact coordinates and $y = y + 2\pi R$ is the coordinate parametrizing the circle. Inserting this in the Klein-Gordon equation, we obtain

$$\left(\Box + \frac{m^2}{R^2}\right)\phi_m(z) = 0, \qquad \forall m \in \mathbb{Z}, \tag{5}$$

where the d'Alambertian acts now on the non-compact coordinates only. Comparing this to the Klein-Gordon equation for a scalar field with mass M, $(\Box + M^2)\phi = 0$, we conclude that from the point of view of the non-compact directions, the system describes a countable set of scalar fields with a mass spectrum given by

$$M^2 = \frac{m^2}{R^2}, \qquad m \in \mathbb{Z}. \tag{6}$$

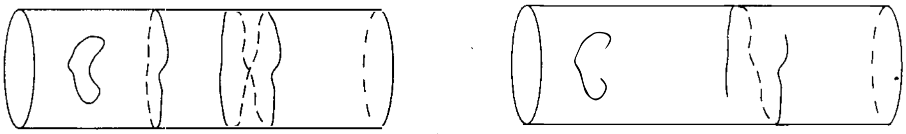

Figure 1: Closed strings can wind themselves non-trivially around a compact direction (left), while open strings cannot (right).

When sending the radius to zero, we see from Eq. 6 that the states with $m \neq 0$ (called Kaluza-Klein states) become very massive and eventually decouple from the system. In other words, putting the radius to zero, we end up with a theory in one dimension less. Once strings are present, the situation changes. Indeed, closed strings can wrap around the circle. A string wrapped n times around the circle is said to have winding number n. It is clear that a string wrapped a number of times around the circle cannot continously be deformed into a string which is not wrapped at all. It is also intuitively clear that the mass dependence of winding states should be different from the one given in Eq. 6 as the larger the radius gets, the more energy it will cost to wrap the string around the circle. The correct dependence can be shown to be,

$$M^2 = \frac{m^2}{R^2} + \frac{n^2 R^2}{\alpha'^2} + \text{ oscillator contributions,} \qquad m,\, n \in \mathbb{Z}, \tag{7}$$

where $1/(2\pi\alpha')$ is the string tension or the energy per unit length of the string (alternatively, you can view $\sqrt{\alpha'}$ as the typical string length) and the oscillator contributions take the excitations of the string into account. Sending the radius R to zero now shows a surprising behaviour. While the Kaluza-Klein states decouple, we see that a new continuum opens in the winding states! In fact it can be shown that this is a limiting case of an exact perturbative equivalence: *the theory on a circle with radius R is completely equivalent to the theory on a circle with radius $\hat{R} = \alpha'/R$, provided one interchanges the Kaluza-Klein states for the winding states and vice-versa.* This is the simplest example of how T-duality acts [b]!

Note that the previous observations hold for closed strings. What about open strings? Open strings do not allow for non-trivial windings (cfr Fig. 1). Indeed, any open string configuration can continously be shrunk to a point. As a result, one would guess that the spectrum is of the Kaluza-Klein type, Eq. 6. However, as demonstrated by Fig. 2, a theory containing only open strings is not unitary, it must necessarily contain closed strings as well. In this way, we arrive at a very unplausible situation: sending the radius to zero, the open string sector ends up in 8+1 dimensions, while the closed strings continue to live in 9+1 dimensions.

Here Polchinski made a great leap forward! Looking at open versus closed strings, he realized that the difference is only two points, the end-points of the open string. This led Polchinski to formulate the correct behaviour of open string theory: sending the radius of the circle of compactification to zero, both open and closed strings will end up living in the 9+1 dimensional dual space, however the end-points of the open string are confined to live on a 8 dimensional hypersurface, called a D8-brane, perpendicular to the direction along which we T-dualize! In other words, the open strings get Dirichlet boundary conditions in one dimension, hence the name *D(irichlet)-branes.* Dirichlet boundary conditions seem to violate energy-momentum conservation, therefore we are led to conclude that these hypersurfaces are dynamical themselves.

This procedure can be repeated. Compactifying on a circle longitudinal to the 8-brane, we will end up, after T-duality, with a 7-brane (a 7-dimensional hypersurface). On the other hand, compactifying on a circle transversal to the 8-brane, *i.e.* along the direction along which we originally dualized, we end up with the original theory. In fact the original theory can be viewed as containing a 9-brane. Indeed, the open strings are confined to move on a 9-dimensional hypersurface which is nothing but space itself.

[b] This simple example of T-duality can be generalized to higher dimensional compact spaces with more complicated topologies.

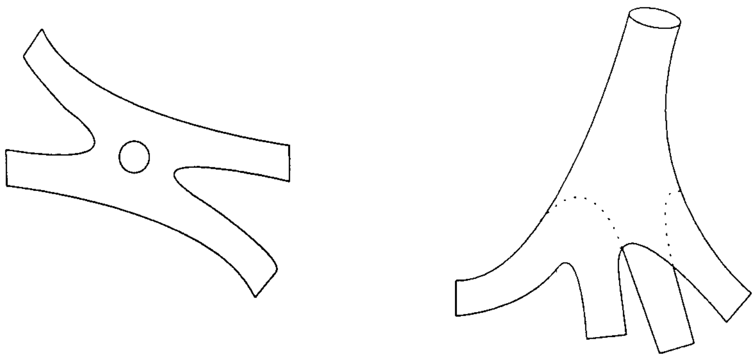

Figure 2: The one loop two open strings → two open strings amplitude can be continously deformed to the tree level 4 open strings → 1 closed string amplitude.

Continuing like this, we can generate Dp-branes which are D-branes having p spatial dimensions. E.g. a D0-brane is a point, a D1-brane is string-like (a fundamental string differs from a D1-brane however), a D2-brane is a membrane, etc. These D-branes interact by open strings ending on them. Not all Dp-branes appear in a given string theory. *E.g.* in type IIA string theory p is even while in type IIB p is odd. From the previous, one also deduces the behaviour of a Dp-brane under T-duality. If the brane is wrapped around the circle, it becomes a D$(p-1)$-brane transversal to the dual circle in the T-dual theory. If the Dp-brane was transversal to the circle its T-dual is a D$(p+1)$-brane wrapped around the dual circle.

Various properties of these D-branes can be explicitely calculated. One of those is the brane tension, defined as the energy of the Dp-brane per unit of volume. It is proportional to $g^{-1}(\alpha')^{-(1+p)/2}$ with g the string coupling constant. The solitonic nature of D-branes is clear from this formula: for small values of the string coupling constant, the D-branes are very heavy.

When a Dp-branes evolves it sweeps out a $(p+1)$-dimensional volume in space-time which is called the world volume. The effective action for type II string theory in a background containing D-branes is of the form

$$\mathcal{S}_{bulk} + \mathcal{S}_{brane} + \mathcal{S}_{bulk-brane}, \tag{8}$$

where $\mathcal{S}_{bulk}$ is the ten-dimensional type II supergravity action, $\mathcal{S}_{brane}$ is a $p+1$-dimenional field theory describing the brane dynamics and $\mathcal{S}_{bulk-brane}$ is again a $p+1$-dimensional field theory describing the interactions between the bulk and brane degrees of freedom.

An easy way to identify the brane degrees of freedom uses supersymmetry. Though type II string theory has an $N=2$ supersymmetry, inserting a Dp-brane, breaks this to $N=1$. Indeed, once a D-brane is present, we have open strings as well and they have only an $N=1$ supersymmetry. This spontanous breaking of half the supersymmetry generates 16 Goldstino's which gives 8 propagating fermionic degrees of freedom. Supersymmetry requires them to be matched by eight bosonic degrees of freedom. We can choose a gauge such that the $p+1$ world volume coordinates are identified with $p+1$ space-time coordinates (think about a point particle where you work in a gauge where its proper time is identified with the time coordinate). Obvious candidates for the bosonic degrees of freedom are the $10-(p+1)$ transversal coordinates of the brane. In the $p+1$-dimensional brane theory they appear as $9-p$ massless scalar fields. So, we miss $p-1$ massless bosonic degrees of freedom. The little group in $p+1$ dimensions is $SO(p-1)$. This implies that the missing bosonic coordinates precisely match up with a massless vector in $p+1$ dimensions! This vector is the lowest mass state of a string beginning and ending on the brane. Summarizing, the degrees of freedom of a Dp-brane are effectively described by a $p+1$ dimensional field theory containing $9-p$ massless scalar fields which determine the transversal position of the brane and a $U(1)$ gauge field which is due to the open strings ending on the brane.

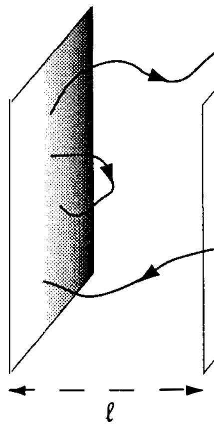

Figure 3: D-branes interact by open strings ending on them. The mass of the strings is proportional to the shortest distance l between the two branes it connects.

When more D-branes are present, the situation changes. The mass of an open string ending on two D-branes is proportional to the shortest distance between the two branes. If we consider a situation with two parallel D-branes, as in Fig. 3, we would expect a $U(1) \times U(1)$ gauge theory. However if the distance between the branes reduces to zero, two additional massless states appear. It can be shown that they complete the $U(1) \times U(1)$ abelian gauge multiplet to a non-abelian $U(2)$ multiplet! The same reasoning can be repeated for n D-branes. Well separated they correspond to a $(U(1))^n$ gauge theory while when they coincide we get a $U(n)$ gauge theory. This provides a very geometric realization of the Higgs mechanism!

Note that one can also study configurations involving multiple Dp-branes with different values of p, branes at angles, intersecting branes, bound states of branes and strings, ... These situations provide geometric settings for various properties of gauge theories.

2.3 S-duality

S-duality is the name given to a duality between a strongly coupled theory and a weakly coupled theory. It is well known that the Maxwell equations in the absence of sources are invariant under the exchange $\vec{E} \to \vec{B}$ and $\vec{B} \to -\vec{E}$. Turning on sources, this invariance can be kept provided one introduces besides electrically charged sources, magnetically charged sources as well. Under the duality transformation the electric and magnetic fields are interchanged and electric and magnetic sources as well. Dirac showed that the product of electric and magnetic charges is quantized. This implies that the fine structure constant in the dual theory is now inversely proportional to the fine structure constant of the original theory. In other words this relates a weakly coupled to a strongly coupled theory.

Does this happen in reality? The answer is yes, but sufficient supersymmetry should be present! The presence of supersymmetry makes the radiative corrections controllable. In fact it was argued that strongly coupled type I string theory is S-dual to the weakly coupled heterotic $SO(32)$ string theory. Type IIB string theory turns out to be self-dual under S-duality. Note that S-duality acts highly non-trivial on the states of the theory. *E.g.* under S-duality in type IIB string theory, the fundamental string gets mapped to the D1-brane and vice-versa.

What about the strong coupling limit of type IIA string theory and the heterotic $E_8 \times E_8$ string theory?

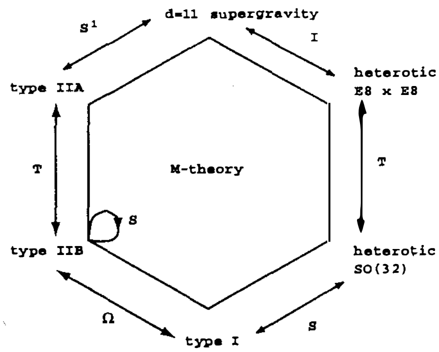

Figure 4: The 5 perturbatively defined string theories and eleven dimensional supergravity are all related by various duality transformations. Here T and S denote T- and S-duality resp., S^1 and I stand for compactification on a circle or a line segment resp. and Ω stands for an orientifold construction. The latter, because of its more technical nature, has been omitted from the discussion in the text.

2.4 From ten to eleven dimensions?

The D0-branes in type ten-dimensional IIA string theory show a remarkable behaviour: they allow for marginal bound (read: zero-force) bound states. I.e. a bound state of n D0-branes has a mass given by $n\,M_{D0}$, where M_{D0} is the mass of a D0-brane, explicitely

$$M_{nD0} = \frac{n}{g\sqrt{\alpha'}}. \tag{9}$$

Assume now that we have an underlying eleven-dimensional theory. Compactifying one space-like dimension on a circle with radius R_{11} gives a Kaluza-Klein like spectrum of the form

$$M = \frac{n}{R_{11}} \quad \text{with } n \in \mathbb{Z}. \tag{10}$$

Comparing Eq. 9 to Eq. 10, one is tempted to make the identification

$$R_{11} = g\sqrt{\alpha'}. \tag{11}$$

As the radius of the eleventh dimension goes as g, with g the string coupling constant, the eleventh dimension remains hidden in the string perturbative regime. While the coupling constant grows, an extra dimension opens up. The resulting effective low energy theory is nothing but eleven dimensional supergravity. Eleven dimensions is the highest number of dimensions which allows for a consistent supersymmetric field theory and the resulting theory is unique.

This bold step can be justified! All objects in type IIA string theory originate in eleven dimensional supergravity. Eleven dimensional supergravity contains besides gravitons, 2- and 5-dimensional solitonic states (called M2- and M5-branes resp.) as well. Upon compactifying eleven dimensional supergravity on a circle, an M2-brane longitudinal to the circle is identified with the fundamental string in type IIA supergravity. If the M2-brane is transversal to the circle, it becomes the D2-brane in IIA string theory. This can be continued not only in a qualitative fashion but in a quantitative manner as well.

A similar construction can be made for the heterotic $E_8 \times E_8$ theory: it corresponds to eleven dimensional supergravity compactified on a line segment whose length is again proportional to the string coupling constant.

2.5 The big picture

The strong coupling behaviour of all five string theories is now known. Furthermore, it can be shown that type IIA and type IIB string theory are T-dual to each other [c]. A similar remark can be made for the two heterotic string theories. In Fig. 4, we summarize the present situation. All five ten-dimensional string theories and eleven dimensional supergravity are in some way dual to each other. Note that the web of dualities is much richer than this picture indicates. Indeed, there are many more duality transformations between various string involving diverse compactifications.

From this the picture emerged of an underlying theory, called M-theory. Though its structure is largely unknown, we know that it does not contain any dimensionless parameters. It has many massless scalar fields, called moduli, whose vacuum expectation values (vev's) appear in the low energy effective theory as dimensionless parameters. When certain of these vev's are very small, a perturbative formulation in terms of a string theory exists. These perturbative formulations are related to each other through duality transformations.

3 Applications

3.1 Extra dimensions and world-on-a-brane

Recently it was realized that the compact dimensions in string theory do not necessarily have to be very small [4]. In addition, regarding our universe as a D3-brane in ten dimensional space-time where the gauge interactions are confined to the brane while gravity lives in the bulk as well, could answer questions such as why gravity is so weak compared to the gauge interactions and why the cosmological constant is so extremely small. I will not dwell on these very interesting developments as they are extensively covered by the contributions of K. Benakli, E. Dudas and T. Han in this volume.

3.2 Black hole physics

The most successful results of string theory have been obtained in the study of black holes [5]. General relativity predicts that the line element changes to

$$ds^2 = \left(1 - \frac{2GM}{r}\right) dt^2 - \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 - r^2 d\Omega^2, \tag{12}$$

in the presence of an isotropic static source of mass M and centered at $r = 0$. Newton's constant is denoted by G and this expression is only valid *outside* the source. The special value $r_h = 2GM$ corresponds to the event horizon. For most objects, r_h lies deep inside the source itself (*e.g.* for the sun $r_h \approx 2.95\ km$) and there Eq. 12 is not valid anymore. A source whose radius is inferior to r_h is called a black hole. Any object coming from outside and crossing the horizon is trapped inside forever, hence its name.

Hawking [6] discovered that black holes are not really black. Quantizing a field theory in background containing a black hole, he found that to an external observer the hole is radiating as a black body with temperature

$$T_H = \frac{1}{8\pi k G M}. \tag{13}$$

The mechanism behind this can be understood as follows. A virtual particle-anti-particle pair popping up in the neighborhood of the horizon can have such a dynamics that one of the two crosses the horizon. The other one, being forced by energy conservation to become a real particle will do so by absorbing and carrying away part of the gravitational energy of the black hole. Using the second law of thermodynamics, we can associate an entropy to a black hole [7]

$$\frac{1}{k} S_H = 4\pi G M^2 = \frac{1}{4} A_H l_p^{-2}, \tag{14}$$

[c] T-duality modifies the chirality of the fermions as well.

where A_H is the area of the horizon and l_p is the Planck length $l_p = \sqrt{G\hbar/c^3} \approx 1.6\ 10^{-33}\ cm$. This expression is known as the Bekenstein-Hawking formula and it has a universal behaviour: the entropy of any black hole is one quarter of the area of the horizon in Planck units. Several questions arise here:

- As anything crossing the horizon disappears forever leaving only thermal radiation behind, the S-matrix of a system containing a black hole seems not unitary anymore. This is known as the information paradox.
- Entropy is a measure for the degeneracy of microstates in some underlying microscopic description of the system. The entropy of a black hole is very large, so can we find a microscopic physical system exhibiting such a wealth of microstates?
- Eq. 13 clearly shows that the more mass is radiated away from the black hole, the hotter it becomes. So, what is the endpoint of black hole evaporation?

A simple class of black holes where some of these problems can be tackled are the so-called extremal black holes. Consider a source which is also electrically charged with charge Q. Solving the Maxwell-Einstein equations gives the line element

$$ds^2 = \left(1 - \frac{2GM}{r} + \frac{Q^2}{r^2}\right) dt^2 - \left(1 - \frac{2GM}{r} + \frac{Q^2}{r^2}\right)^{-1} dr^2 - r^2 d\Omega^2, \tag{15}$$

which generalizes Eq. 12. The event horizon is now

$$r_H = GM + \sqrt{(GM)^2 - Q^2}. \tag{16}$$

A source with a radius smaller than r_H is a Reissner-Nordstrom black hole which has temperature and entropy given by

$$\begin{aligned} T_H &= \frac{\sqrt{(GM)^2 - Q^2}}{2\pi k(GM + \sqrt{(GM)^2 - Q^2})^2}, \\ \frac{S_H}{k} &= \frac{\pi}{G}(GM + \sqrt{(GM)^2 - Q^2})^2 = \frac{1}{4} A_H l_p^{-2}. \end{aligned} \tag{17}$$

For a given value of Q we now take $M \to Q/G$ and we find that the temperature vanishes. In other words, the black hole behaves as if it was an elementary particle. Such a black hole is called extremal: its mass is tuned such that the gravitational collapse is precisely compensated by the electrostatic repulsion.

Extremal black holes are easily described in terms of string theory [8]. One of the simplest configurations consists of type IIB string theory with 5 dimensions compactified on a 5-dimensional torus and a collection of D5- and D1-branes wrapped a number of times around the torus. This is a stable configuration in type IIB string theory. One can calculate the number of excitations of this configuration (they are described by open strings ending on the branes and wrapping in various ways around the torus) and hence the entropy. Subsequently one takes the supergravity theory describing the low-energy dynamics of this configuration and one calculates the line-element, the event horizon, the temperature and finally the entropy. Comparing both results in the limit where both descriptions are valid, one finds exact agreement! Since then many other black hole configurations were studied and the analyis was successfully extended to arbitrary (read non-extremal) black holes [9].

For near extremal black holes, the information paradox was solved as well [10]. Studying a configuration slightly away from extremality, it was found that Hawking radiation arises through the *annihilation* of two open strings, each ending on a D-brane, thereby forming an open string which remains on the brane and emitting a closed string. The radiation turns out to be exactly thermal with both the temperature and the radiation rate in perfect agreement with the Hawking like calculation! Almost by construction, the procedure is unitary and so the information must reside on the D-branes.

3.3 *The holographic principle*

No physical system filling a certain volume can have an entropy larger than a black hole [11]. Very roughly, this can be understood by imagining a system filling a volume and having an entropy larger than that of a black hole. Throwing particles into it, it eventually reaches its critical mass and becomes a black hole, thereby lowering, by assumption, its entropy. This is excluded by the first law of thermodynamics. From this point of view, it is highly surprising that the entropy of a black hole goes as the area of the horizon and not as its volume. Crudely speaking, if one has a certain volume, the maximal amount of information we can store into it is a little more than one bit per unit of Planck area. This led 't Hooft and later Susskind to formulate the so-called holographic principle: *any theory containing gravity should somehow be equivalent to a theory without gravity living on its boundary.*

Maldacena [12] found an explicit realization of this using string theory. He started from type IIB string theory in an $AdS_5 \times S^5$ background[d] containing a stack of n D3-branes and conjectured that this is equivalent to an $N = 4$, $U(n)$ supersymmetric gauge theory. This conjecture has been well tested in the classical limit at the string theory side, which is nothing but the corresponding supergravity theory. At the gauge theory side this corresponds to the limit $n \to \infty$, $g^2 n \to \infty$ with the latter the 't Hooft coupling constant. In other words on the one hand we have a classical theory of gravity, on the other a gauge theory in its deep non-perturbative regime, and both are dual to each other! Some partial results outside this limit have been obtained as well together with several other examples of the holographic principle. This sheds a completely new light on both gravity and gauge theories and fulfills, albeit in an unexpected way, the dream to formulate gauge interactions in terms of a string theory.

4 Conclusions

String theory is the only known consistent quantum theory for gravity. Not only does it describe gravity but the other gauge interactions as well. The past few years have seen enormous progress. The microscopic understanding, not only qualitatively but quantitatively as well, of black hole thermodynamics impressively demonstrates that though the final theory might look very different from what we are doing now, string theory does contain important grains of truth. Furthermore, a fascinating interplay between gauge theories and gravity emerged, teaching us not only about gravity but giving us an alternative and powerful way to investigate properties of gauge theories.

The question which will immediately come to mind of the present audience is: *What are the phenomenological implications of string theory?* In honesty, I have to admit that besides some qualitative issues there are no firm quantitative predictions which can be tested at present or near future accelerators yet. Though we have gained a deep understanding of certain non-perturbative aspects of string theory, we still have no way to address its fine structure. Right now, we have an enormous collection of classically stable vacua around which we can do perturbation theory. Some of these vacua are related through duality transformations. We still lack a background independent description of string theory. This would enable us to study quantum effects which would presumably lift the huge vacuum degeneracy, thereby breaking supersymmetry and introducing small masses and perhaps explain us why our world is as it is. Being an optimist, I think this means that a thrilling trip still lies ahead of us!

Acknowledgments

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[d]We denoted by AdS_5, 5-dimensional anti-de Sitter space, an Einstein manifold with a negative cosmological constant and which has 4-dimensional (conformally compactified) Minkowski space as its boundary. The remainder of the space is the five dimensional sphere, S^5.

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