DISCUSSION

WENZEL: Is the correlation that you predict for the like pions in the right direction?

SUDARSHAN: It qualitatively tends to make two pions come out with momenta that are very well correlated. At the moment, however, we have not computed $\langle \cos\theta \rangle$. We have no specific view as to the origin of the resonance. It may be a ρ_0 meson as Sakurai and others would like it to be or it may be just a " plain old resonance " or both. I also want to remark that the same method can also be used for a three-pion resonance. All you have to do is to calculate the effective mass distribution for three pions.

WATAGHIN: Have you made use in your calculations of the interaction volume which is used in some of these statistical formulations?

SUDARSHAN: Perhaps I should have mentioned this before. We do not have a volume in the covariant model because in the covariant model the dimensions are a little different. In the three dimensional case, people write the phase space as d^3p and multiply it by an Ω_0, so that this becomes dimensionless so that you can compare transition probabilities. In our model we have d^4p with a delta function after it and the delta function takes off two powers of the momentum and therefore the quantity we have is the square of the cube root of a volume, or an area. Therefore we should really write down κ^2 in front, where κ is a parameter, the hidden parameter I said we used. So any real comparison is a bit " dishonest ".

PRODUCTION OF PARTICLE BEAMS AT VERY HIGH ENERGIES

S. D. Drell

Stanford University, Stanford, California

We consider the production of beams of very high energy strongly interacting nuclear particles. By studying the transition amplitudes near their poles corresponding to " real " one-particle intermediate states we show that photons are very effective in initiating collimated beams of high energy charged pions and K-mesons, (anti-)nucleons, etc. A photon of energy k, incident on a nuclear target, produces a high energy charged pion, say, of energy $\omega_q > \frac{1}{2}k$ in a forward cone of opening angle $\theta_{\frac{1}{2}} \sim \mu/\omega_q$ with a cross section (that is reduced by roughly the fine structure constant, $\alpha = 1/137$, from geometric), $1/\mu^2 = 20$ mb. For nucleon initiated processes, on the other hand, although one avoids the fine structure constant, the statistical model predicts that very high energy secondaries emerge in only a very small fraction of the collisions.

This result is of significance for predicting and comparing yields from very high energy electron and proton accelerators. Experiments which can be performed on existing machines are proposed to check the validity of this " pole analysis " which, as applied here to general inelastic processes, is an extension of the work of Chew, Goebel, and Chew and Low [1]. In addition, feasible coincidence experiments are proposed to measure the π-π scattering length and the (γ-3π) coupling strength.

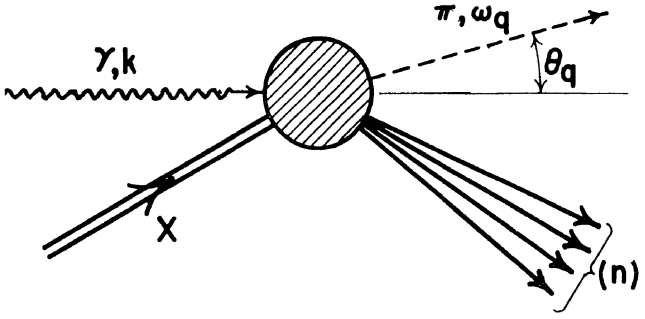

Fig. 1 Photon-initiated multiple production process.

Fig. 1 illustrates the process being considered, with X the nuclear target, and (n) anything else produced in addition to the high energy pion. Our basic approximation now in calculating the cross section when the pion is produced with energy $\omega_q \approx k$ and within the forward angular cone at an angle $\theta_q \leq \theta_{\frac{1}{2}} \sim \sim \mu/\omega_q$ is to assume that the amplitude corresponding to Fig. 2 makes the major contribution to this process. In Fig. 2 the photon produces a pair of charged pions, one of which emerges directly with ω_q, θ_q, while the other ploughs into X initiating reactions to various final states (n). The cross section of interest is the sum of the squares of matrix elements for all possible states (n). We expect this diagram to dominate since we are near the pole for one pion to propagate from (a) to (b):

$$(k-q)^2-\mu^2 = -2k\cdot q = -2k\omega_q \times$$
$$\times(1-\beta_q\cos\theta_q) \approx -\mu^2\frac{k}{\omega_q}\left\{1+\left[\frac{\omega_q\theta_q}{\mu}\right]^2\right\},$$

where $k = (k, \mathbf{k})$ and $q = (\omega_q, \mathbf{q})$ are the photon and pion four momenta, respectively. Also since all the inelastic channels are open at (b) in Fig. 2, the pion ploughs into the target X with a large amplitude corresponding to a total pion cross section at energy $k-\omega_q$. In contrast as we approach the one pion pole in single photoproduction shown in Fig. 3, we must absorb a low momentum pion at (b') and the amplitude for this is very small for p-wave absorption of pseudoscalar mesons. The cross section derived for Fig. 2 is

$$d^2\sigma_{\gamma,\pi}(k,\omega_q,\theta_q) = \frac{\alpha}{2\pi}\frac{\sin^2\theta_q}{[1-\beta_q\cos\theta_q]^2}\frac{d\Omega_q}{4\pi}\times$$
$$\times\left[\frac{\omega_q(k-\omega_q)d\omega_q}{k^3}\right]\sigma_{\pi+X,\,\mathrm{total}}(k-\omega_q)\,. \qquad (1)$$

This is larger than the Pauli-Weisskopf result for production in a Coulomb field of the target X and also exceeds the Behr-Hagedorn [2] predictions based on the statistical model. To get some idea of the numbers we compute from (1) that the cross section for 25 BeV gammas to produce 20 BeV π^+ or π^- mesons at $\theta = 1°$ is $\approx 0.01[\sigma_{\pi+X,\,\mathrm{total}}$ (5 BeV)], ster-BeV$\approx$ $\approx$0.3 mb/ster-BeV. This is a factor of $\approx$10 larger than the statistical model calculations of Behr and Hagedorn for p-p collisions at the same energies and angles. The predictions of (1) are in qualitative agreement with very recently reported Cornell measurements [3] and can be checked quantitatively not only by further studies of multiple photo-pion production but also by measuring the contribution of the same mechanism in inelastic nucleon cross sections as shown in Fig. 4. An incident proton p of energy

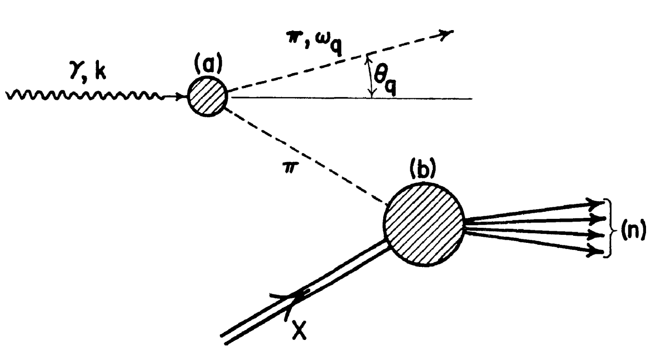

Fig. 2 One-pion exchange multiple photoproduction amplitude.

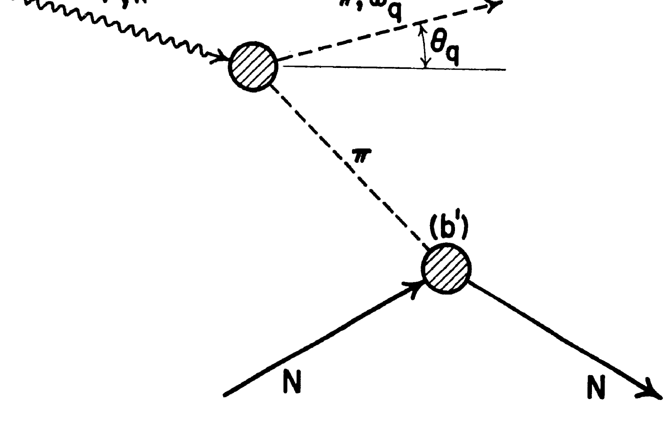

Fig. 3 One-pion exchange amplitude for single photoproduction.

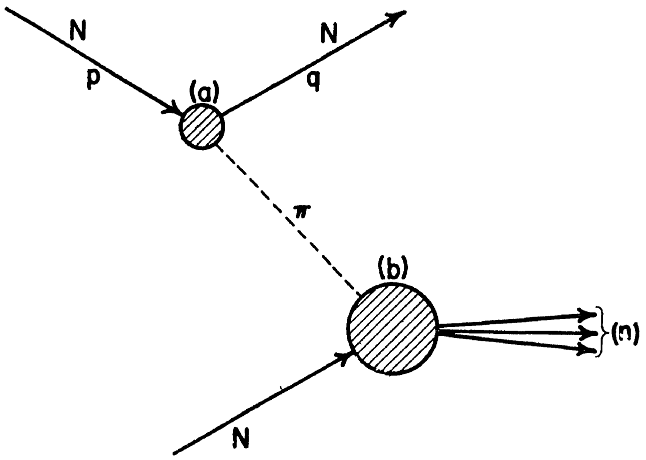

Fig. 4 One-pion exchange amplitude for nucleon-initiated multiple production process.

$E_p \gg M$ is incident on a target X producing a nucleon N with energy $E_Q \sim E_p$ and anything else (n). For $\Delta = E_p - E_Q \lesssim (\mu/M)E_p$, the high energy nucleon N will again emerge within a forward cone of half-angle $\theta_{\frac{1}{2}} \sim \mu/E_p$ when one "almost real" pion is exchanged between (a) and (b). The corresponding cross section is

$$d^2\sigma(E_p, E_Q, \theta_q) = \frac{1}{2\pi}\left(\frac{G^2}{4\pi}\right)\frac{p\cdot q - M^2}{\{p\cdot q - M^2 + \frac{1}{2}\mu^2\}^2} \times$$

$$\times \frac{d\Omega_q}{4\pi}\,\frac{\Delta E_Q dE_Q}{E_p}\sigma_{\pi+X,\,\mathrm{total}}(\Delta)\,, \qquad (2)$$

where $(G^2/4\pi) = (2M/\mu)^2 f^2$ with $f^2 = 0.08$, the pion nucleon coupling constant, and

$$p\cdot q = E_p E_q (1 - \beta_p \beta_q \cos\theta_q)\,.$$

For $E_p = 6$ BeV $E_q = 5$ BeV, and $\theta_q = \mu/E_p = 1.4°$, this gives $(d^2\sigma/d\Omega_q dE_q) = 150$ mb/ster-BeV and should be readily measurable.

Equation (1) may be transcribed directly to the process of K^+K^- pair production. It is only necessary to change the masses from pion (μ) to K-meson (μ_K) and the total cross section $\sigma_{\pi^\pm+X}(k-\omega_q)$ to $\sigma_{K^\pm+X}(k-\omega_q)$. Since K-mesons are appreciably heavier than pions, up to several pions can accompany the K-meson from (a) to (b) in Fig. 2 without appreciably broadening the cone of emission of the high-energy K-meson. Therefore in this application the results are to be given only qualitative significance. However they indicate large, well collimated K-beams at high energies [4)].

Finally, we can compute with our approximation the cross section to produce *two* high energy pions of energies ω_1 and ω_2 in the forward cone, either by an incident gamma or pion of energy k, as shown in Fig. 5. The cross section is

$$d^6\sigma = \frac{16}{\pi^2}\,\frac{|A(S_1, S_2, S_3)|^2}{[S_1+S_2+S_3-3\mu^2-k^2]^2}\,\frac{d\Omega_1}{4\pi}\,\frac{d\Omega_2}{4\pi}\times$$

$$\times\left\{\frac{(k-\omega_1-\omega_2)\omega_1\omega_2 d\omega_1 d\omega_2}{k}\right\}\sigma_{\pi+X,\,\mathrm{total}}(k-\omega_1-\omega_2)\,,$$

where $k^2 = \mu^2$ for an incident pion and $k^2 = 0$ for a gamma; $S_3 = (p_1+p_2)^2$, $S_1 = (k-p_1)^2$; $S_2 = (k-p_2)^2$ and $A(S_1, S_2, S_3)$ is the invariant amplitude at vertex (a). Since all invariant momentum transfers are $\approx \mu^2$ it is reasonable to make an effective range analysis of A for the ($\pi\pi\pi\pi$) or ($\gamma\pi\pi\pi$) vertex [or even for the ($\pi\pi\kappa\kappa$) vertex if a $K\bar{K}$ pair is detected]. Other related applications of one pion exchange effects in pion-proton and proton-proton collisions have been reported to this conference by F. and G. Salzman and by Chernovski, Dremin and co-workers (see report of Veksler, Session S1).

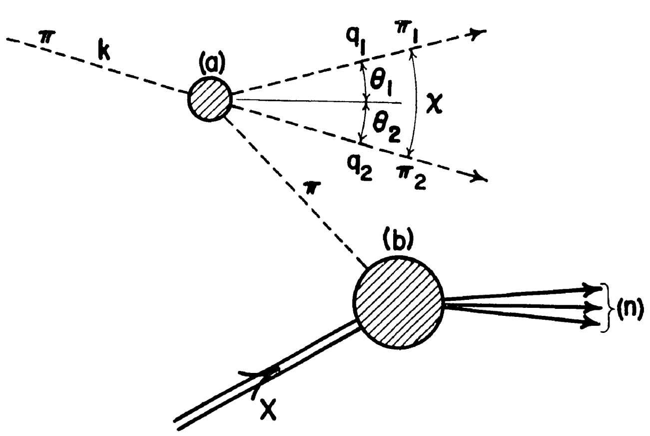

Fig. 5 One-pion exchange amplitude for multiple production involving two high energy pions.

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4. Strong anti-nucleon beams are also produced by this mechanism, as can be calculated readily. Since we are not close to the pole for a heavy nucleon intermediate state this result deserves only crude qualitative attention; it exceeds the statistical theory predictions by close to two orders of magnitude.

DISCUSSION

COCCONI : There is another support for what you say from the experiments of von Dardel and his group. They measured secondary protons coming out from 25 GeV protons hitting a target. At very high momenta, for instance at 18 GeV/c, there are many more protons than predicted by the statistical model, perhaps 10 or 20 times more, thus suggesting that quasi elastic collisions are much more frequent than we had thought before.

W. D. WALKER : Since Good is not here, I might add that he has pointed out at numerous occasisons that this latter process has probably already been observed at Berkeley in the case of the associated production of sigma and neutral K in proton-nucleus collisions. It was never understood why the K^0's had such a very strange angular distribution in the Moyer experiment. It seems likely that this kind of process is the likely explanation of this very peculiar angular distribution. Good and myself have been thinking along very similar lines to this. Our diagram is one, however, in which the interchanged pion is missing, and you have a diffraction scattering of an incoming strongly interacting particle. We think in such a case for the outgoing products of the reaction the quantum numbers of the products are the same as the quantum numbers of the incoming particle.

R. L. WALKER : In your photoproduction of pion pairs since the cross section is proportional to the scattering of the one pion by the proton, you might expect to find this larger for the production of negative pions in the forward direction than for positive pions. I wondered if that appeared in the Cocconi data.

DRELL : Yes, and if I remember correctly, there was a ratio of roughly 3 to 1.

R. L. WALKER : We are set up at Cal. Tech. for measuring pions at small angles so that we can do this experiment rather easily with better statistics.

DRELL : Lou Hand and Panofsky are also planning to do it at Stanford.

APPENDIX TO SESSION S 1

Papers contributed to the Conference but not presented

MEASUREMENT OF THE PANOFSKY RATIO BY THE METHOD OF GAMMA-GAMMA COINCIDENCES

A. F. Dunaitsev, V. S. Pantuev, Yu. D. Prokoshkin, Tang Syao-wei and M. N. Khachaturyan

Joint Institute for Nuclear Research, Dubna, USSR

The ratio of the probability of two capture processes of stopped negative pions in hydrogen

$$\pi^- + p \rightarrow \pi^0 + n \rightarrow \gamma' + \gamma'' + n\,, \qquad (1)$$

$$\pi^- + p \rightarrow \gamma + n \qquad (2)$$

(the so-called " Panofsky ratio ") was determined earlier in several experiments by investigating the energy spectra of gamma rays emitted in reactions (1) and (2). Despite the fact that experimental methods employed in some of these measurements were identical, the results obtained turned out to have a large spread greatly exceeding the errors given by the authors.

We made use of another experimental possibility of determining the Panofsky ratio (see Fig. 1). Gamma rays arising when π^--mesons stop in liquid hydrogen were detected by the telescope with a convertor (5 mm Pb). The telescope contained scintillation and Čerenkov counters. On the other side of the target there was a large counter of lead glass which detected high-energy gamma rays with efficiency close to 1. The size and the location of the large counter were chosen so that when one of two gamma rays from π^0-meson decay was counted by the telescope, the other necessarily hit the large counter. Thus, comparing the coincidence counting rate of the telescope and that of the large counter to the telescope counting rate we can determine the value of the ratio of reactions (1) and (2).

Preliminary results of measurements carried out by this method with a 170 MeV π^--meson beam were reported at the Kiev Conference in 1959. In the present experiment we used a slower 65 MeV negative pion

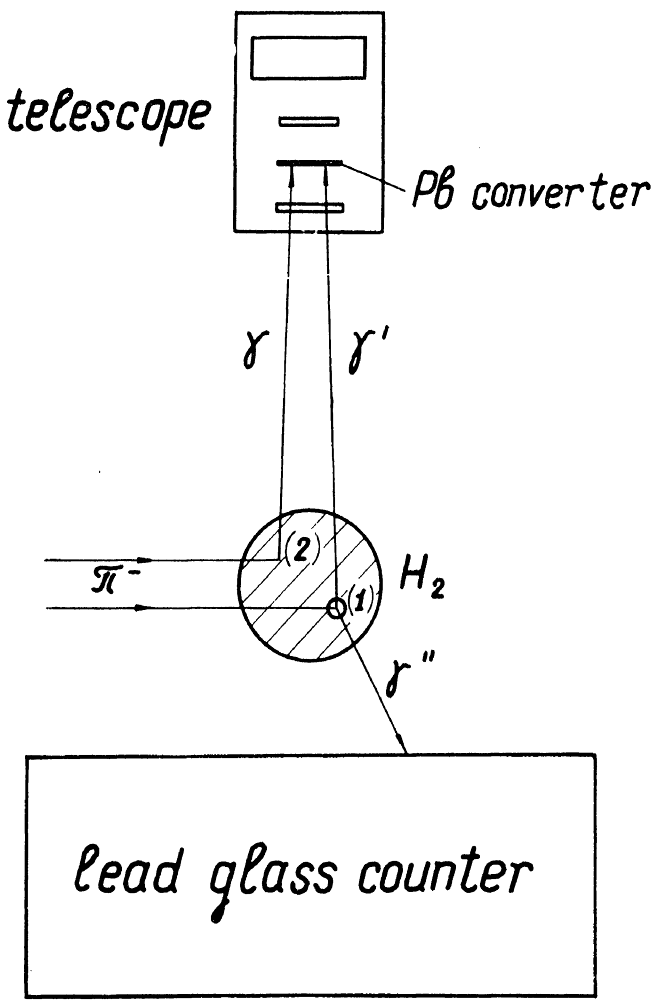

Fig. 1 Experimental arrangement. (1) and (2) represent reactions (1) and (2) respectively.

beam. Negative pions were stopped in a liquid hydrogen target 12 cm in diameter. The π^--meson range was determined with a star detector [1] sensitive only to stopped π^--mesons. Both the telescope and the gamma-gamma coincidence counting rates were caused entirely by hydrogen in the target which may be seen from a comparison of pion range curves measured by various methods (see Fig. 2). On removing hydrogen from the target the counting rate of the telescope decreased more than 500 times and that of gamma-gamma coincidences more than 2000 times. The ratio of the counting rates " convertor in/convertor out " was between 10 and 15.

Relative efficiencies of the telescope and the large counter to gamma rays of different energies were determined experimentally by using an electron beam.

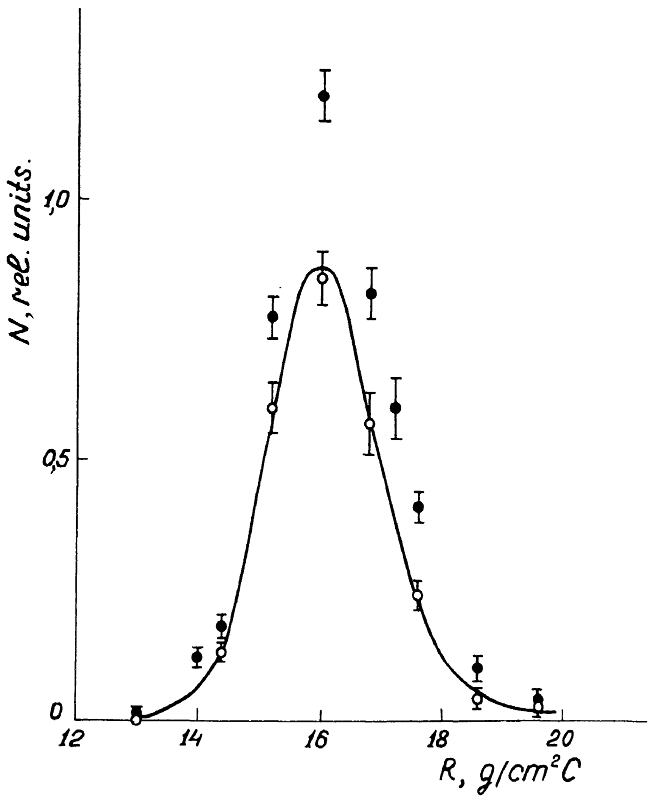

Fig. 2 Range curve of π^- mesons.
○ — gamma-gamma coincidences;
● — telescope counting rate.

The corrections took into account the different character of showers induced by gamma rays and electrons in the counter matter.

The comparison of gamma-gamma coincidence counting rates at various distances between the liquid hydrogen target and the large counter was performed as a test experiment. As is seen in Fig. 3, the counting rate does not change practically on removing the counter from the target. This is in good agreement with calculations.

The following values of the Panofsky ratio (P) were obtained from several runs :

1.38 ± 0.08, 1.45 ± 0.08, 1.33 ± 0.11, and 1.41 ± 0.13.

The above errors show only the statistical accuracy to measurements. Taking into consideration the errors concerned with the determination of the telescope efficiency (2%) and that of the large counter (4%) we have the following final result :

$$P = 1.40 \pm 0.08.$$

This value agrees with the data on photoproduction and scattering of pions.

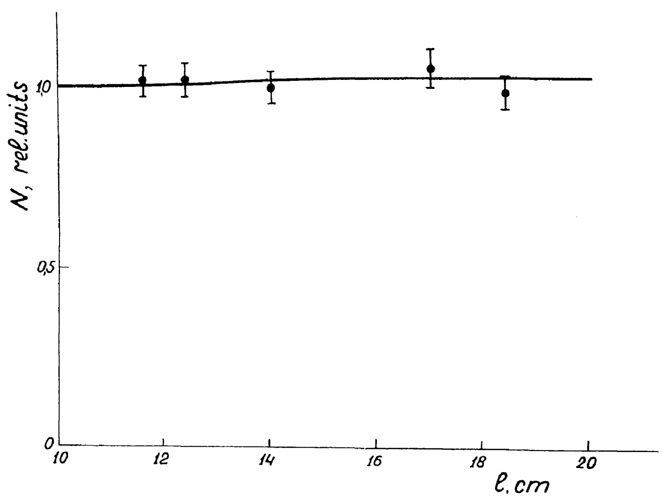

Fig. 3 Counting rate of gamma-gamma coincidences N versus the distance l between the target and the large counter.

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TRIPLE PION PRODUCTION AT 810, 970, AND 1110 MeV (†)

V. P. Kenney, J. G. Dardis and G. Brunhart

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Lindenbaum and Sternheimer have hypothesized [1] that double pion production in pion-nucleon collisions proceeds through the excitation of the nucleon to the $T = J = {}^3/_2$ isobaric level, with subsequent pion emission according to the scheme

$$\pi + N \rightarrow N_0^* \rightarrow I^*_{3/2} + \pi \rightarrow N + \pi + \pi$$

Since it has become evident [2] that there appear to be higher isobaric levels in the $T = \frac{1}{2}$ π-N system, the isobar model has been extended [3] by proposing that the scheme

$$\pi + N \rightarrow N_0^* \rightarrow I^*_{1/2} + \pi \rightarrow N + \pi + \pi$$

also plays an important role in double pion production by pions.

Assuming that the higher $T = \frac{1}{2}$ isobaric levels are indeed important, it is of interest to investigate whether a pion transition between either of the $T = \frac{1}{2}$ levels and the $T = {}^3/_2$ level leads to triple pion production according to the scheme

$$\pi + N \rightarrow N_0^* \rightarrow I^*_{1/2} + \pi \rightarrow I^*_{3/2} + \pi + \pi \rightarrow N + \pi + \pi + \pi$$

Interactions of the type $\pi^- + p \rightarrow p + \pi^+ + \pi^- + \pi^-$ have been identified in data from the Berkeley 10 inch hydrogen bubble chamber operated in pion beams of 810, 970, and 1110 MeV laboratory kinetic energy. The interaction is unique for an inelastic pion process in that all emerging particles are charged and their momenta and directions directly measurable. The momenta, angles between emerging particles, Q values, and scattering angles have been determined for some 55 events at 810 MeV, 70 events at 970 MeV and

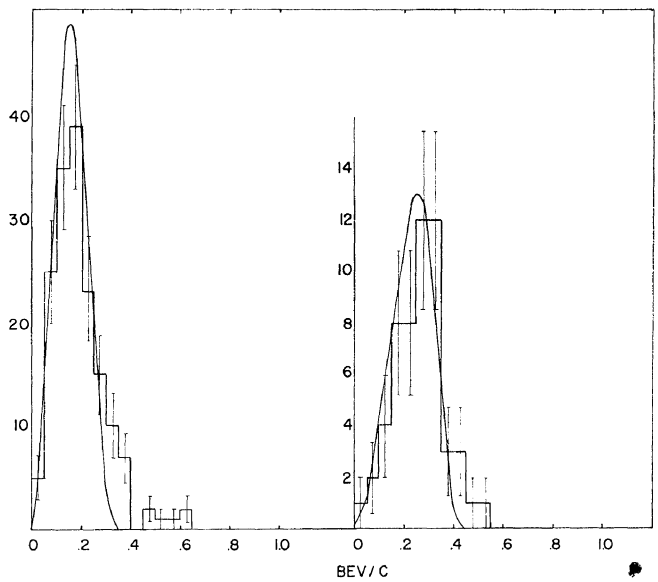

Fig. 1 Left: Combined momentum distributions for all pions, 810 MeV. Right: Proton momentum distributions. Distributions predicted by simple phase space considerations shown as solid curve.

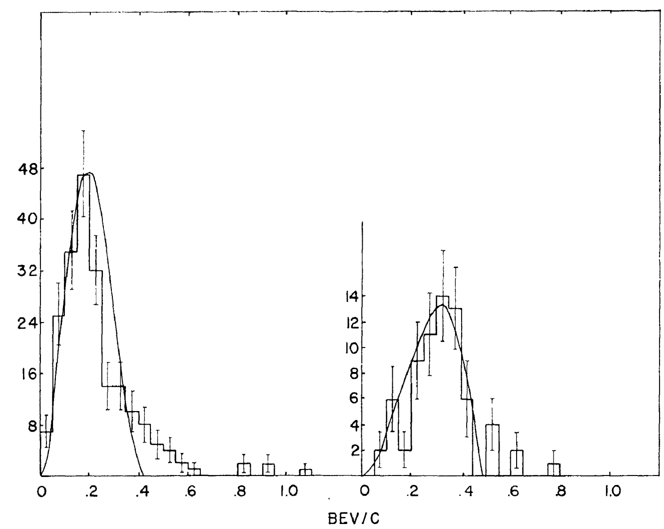

Fig. 2 Left: Combined momentum distributions for all pions, 970 MeV. Right: Proton momentum distributions. Distributions predicted by simple phase space considerations shown as solid curve.

(†) Research supported in part by the National Science Foundation. The cooperation of the Alvarez bubble-chamber group at the Lawrence Radiation Laboratory is acknowledged with appreciation.

99 events at 1110 MeV; the results are shown in Figs. 1-8.

Covariant phase space factors calculated by the method of Hoang and Young[4] have been used to plot momentum distributions for the proton and three pions emerging from each interaction, and to plot distributions for the angles between emitted π, p and π, π pairs.

The agreement of the pion and proton momenta and the angles between emitted pairs with the predictions of simple phase space considerations at all three energies is quite good. This, together with a lack of significant difference in kinematic behavior between positive and negative pions such as might be expected if positive pions were constrained from acting as recoil pions in the first stage of the reaction by isotopic spin requirements, suggests that the isobaric transition $I^*_{1/2} \to I^*_{3/2} + \pi$ is not observed in this interaction.

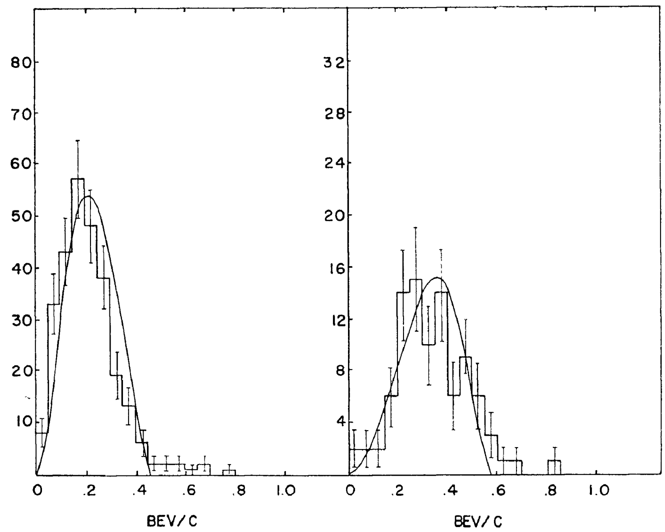

Fig. 3 Left: Combined momentum distributions for all pions, 1110 MeV. Right: proton momentum distributions. Distributions predicted by simple phase space considerations shown as solid curve.

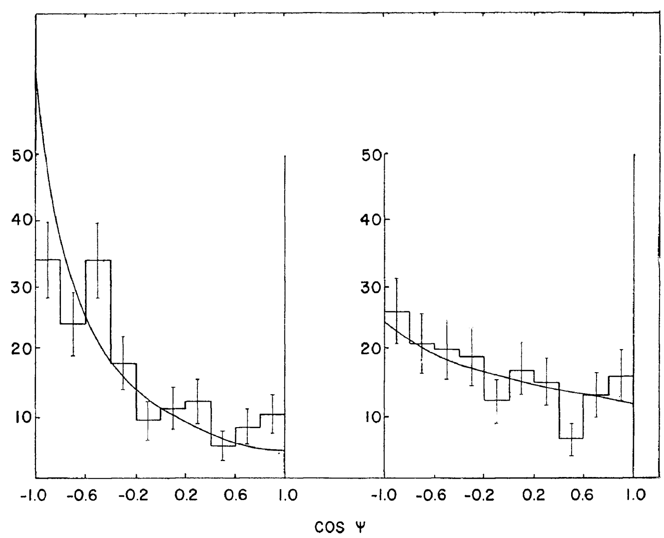

Fig. 4 Left: Distributions of angles between pion-proton pairs, 810 MeV. Right: distributions of angles between pion-pion pairs. Distributions predicted by simple phase space considerations shown as solid curve.

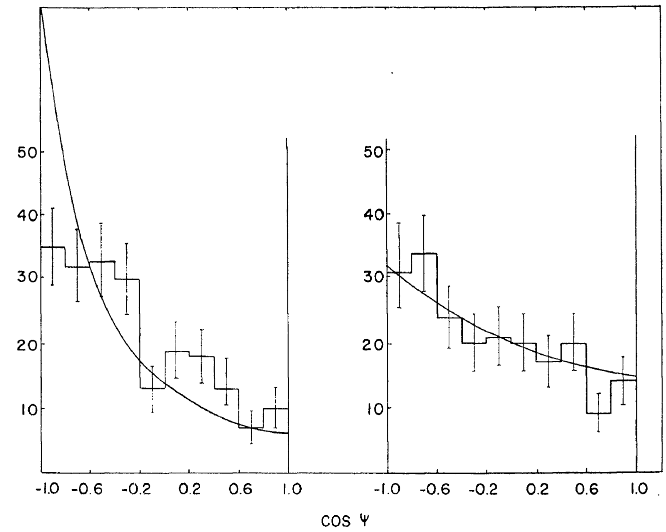

Fig. 5 Left: Distributions of angles between pion-proton pairs, 970 MeV. Right: Distributions of angles between pion-pion pairs. Distributions predicted by simple phase space considerations shown as solid curve.

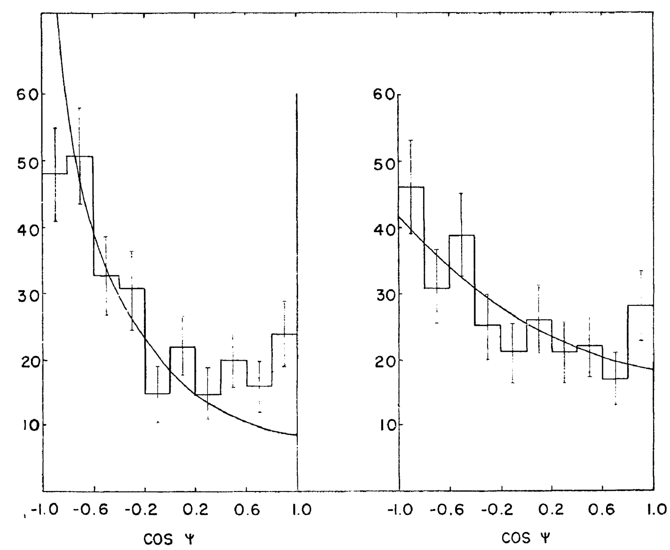

Fig. 6 Left: Distributions of angles between pion-proton pairs, 1110 MeV. Right: Distributions of angles between pion-pion pairs. Distributions predicted by simple phase space considerations shown as solid curve.

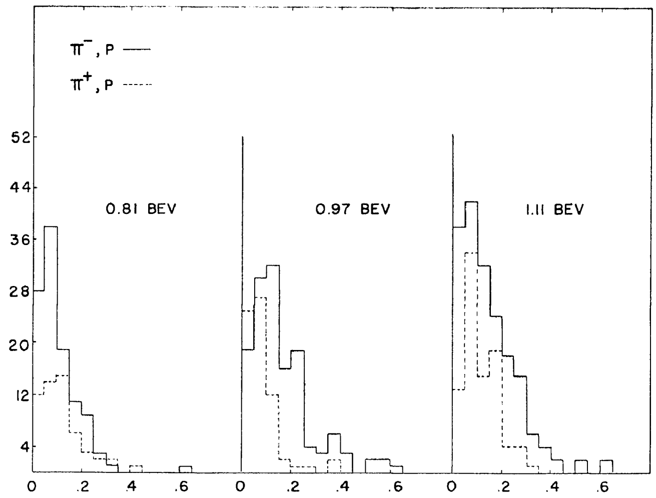

Fig. 7 Q value distributions at 810 MeV, 970 MeV, and 1110 MeV, (left to right) for combined π^-, p pairs (solid histogram) and π^+, p pairs (dashed histogram).

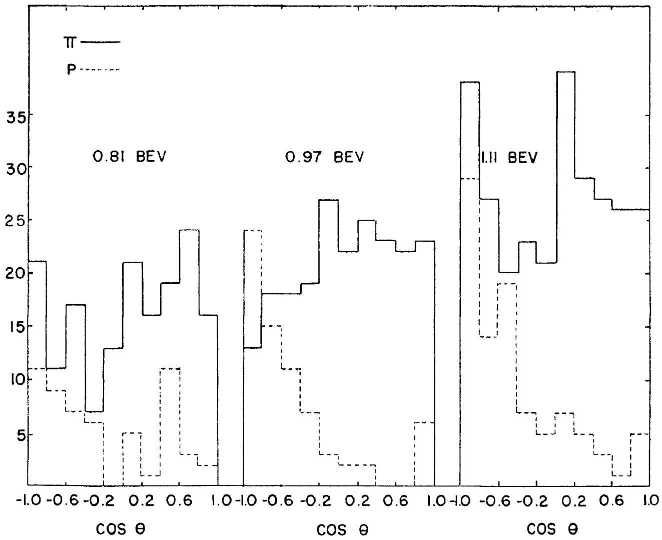

Fig. 8 Scattering angle distributions at 810 MeV, 970 MeV, and 1110 MeV (left to right) for all pions (solid histogram), and protons (dashed histogram).

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π^--p INTERACTIONS AT 225 MeV (*)

J. Deahl, M. Derrick, J. Fetkovich, T. Fields and G. B. Yodh

Carnegie Institute of Technology, Pittsburgh, Pennsylvania

The following reactions have been studied at an incident energy of 225 MeV using a hydrogen bubble chamber:

$$\pi^- + p \rightarrow \pi^- + p \tag{1}$$

$$\pi^- + p \rightarrow \pi^0 + n \tag{2}$$

$$\pi^- + p \rightarrow \pi^- + p + \gamma \tag{3}$$

$$\pi^- + p \rightarrow \pi^- + \pi^+ + n \tag{4}$$

$$\pi^- + p \rightarrow \pi^- + \pi^0 + p \tag{5}$$

The experiment was undertaken in order to obtain pion scattering data of higher accuracy than that previsouly available at this energy, as well as to observe the production of pions in π^--p collisions at an energy close to threshold.

*) Supported in part by the U.S. Atomic Energy Commission.

A cylindrical hydrogen bubble chamber of diameter 15.2 cm and depth 7.6 cm with a 13 kilogauss field was placed in the highest energy negative pion beam of the Carnegie cyclotron. The beam energy was determined from an integral range curve and from curvature measurements on the pictures to be 224±10 MeV. The beam contamination was estimated from the range curve and from counter data in similar beams to be 5±2% muons and 1±1% electrons.

The scanning efficiency for elastic scattering was taken as 99.2±0.6% on the basis of comparison scans and the azimuthal distribution of the scatterings. Comparative scans on the charge exchange data yielded an efficiency of 97±2%. Elastic scattering events were identified by coplanarity and correlation in polar angles of the outgoing particles. Those events for which elastic scattering kinematics did not fit were identified by using momentum balance and range information. The experimental results are listed in Table I. The errors quoted include the statistical error and the uncertainty in the efficiency corrections.

Table I. Results

Total cross sections

Process	Cross section	Number of events
$\pi^-+p\to\pi^-+p$	16.0±0.8 mb	1570
$\pi^-+p\to\pi^-+p+\gamma$	0.050 mb (empirical gamma lab. energy cutoff = 50 MeV)	5
$\pi^-+p\to\pi^0+n$	34.4±1.9 mb	1210 (*)
$\pi^-+p\to\gamma+n$	0.6 mb (from detailed balancing)	17 (estimated from 0.6 mb cross section)
$\pi^-+p\to\pi^-+\pi^++n$	0.03±0.02 mb	3
Total	50.5±1.9 mb	(not including $\pi^-+p\to\gamma+n$ cross section)

Elastic differential cross section (after Coulomb correction)

$$\frac{d\sigma}{d\Omega}=(0.70\pm0.04)+(0.33\pm0.07)\cos\chi_{cm}+(1.69\pm0.13)\cos^2\chi_{cm}$$

where the coefficients are in mb/ster; the angular range $25°<\chi_{cm}<+180°$ was used in obtaining this fit.

Dalitz electron pairs

$\pi^-+p\to\pi^0+n\to e^++e^-+\gamma=n$

Predicted 44, Observed 40 — Includes all film scanned

Dispersion curve

$$\frac{D^b_-(0)}{\lambda_c}=-0.08\begin{matrix}+0.08\\-0.06\end{matrix}\ \text{using}\ \frac{d\sigma}{d\Omega}(0)=(2.72\pm0.15)\times10^{-27}\ \text{cm}^2$$

where λ_c is the Compton wave length of the pion.

(*) Based on 150,000 incident tracks. The other numbers represent 360,000 incident tracks.

We first discuss the scattering results. In Table II are shown the results of several experiments on π^--p scattering in the energy range 220—240 MeV. The total cross section results are in good agreement. One observes that a large discrepancy exists among the experimental results for $\frac{\pi^0}{\pi^-}\equiv\frac{\text{(Charge exchange)}}{\text{(Elastic)}}$; this appears to arise mainly from a scale factor difference for the π^- elastic scattering. The present result is $\frac{\pi^0}{\pi^-}=2.15\pm0.09$, and is independent of the track count and beam contamination. Fig. 1 shows the π^- elastic scattering angular distribution. We found no evidence for a $\cos^3\chi_{cm}$ term in the angular distribution. The result for $D_{-}(0)$ is in good agreement with the curve of Schnitzer and Salzman [5], with $f^2 = 0.08$. An *s,p* phase shift analysis was performed, using the present data as well as the π^+-p angular distribution data [2] and recoil polarization data [6] (π^--p) of Ashkin et al, on an IBM 650 computer. The set of phase shifts yielding the best fit is as follows: $\alpha_1 = 14.8 \pm 3.5^0$, $\alpha_{13} = 0 \pm 2^0$, $\alpha_{11} = 6 \pm 4.5^0$, $\alpha_3 = -15.5 \pm 3.5^0$, $\alpha_{31} = 0 \pm 5.5^0$, $\alpha_{33} = 112.3 \pm 3.0°$. Two other sets of phase shifts were found not to be statistically excludable on the basis of these data.

The observed pion production charge ratio, $\frac{\sigma(\pi^0+\pi^-+p)}{\sigma(\pi^++\pi^-+n)}$ was $\frac{0 \text{ events}}{3 \text{ events}}$. Assuming the two pions to be in a relative *s* state, this ratio would be $^9/_2$ for production from a $T = {}^3/_2$ state ($T_{2\pi} = 2$), and 0 for production from a $T = {}^1/_2$ state ($T_{2\pi} = 0$). These data favor the latter case.

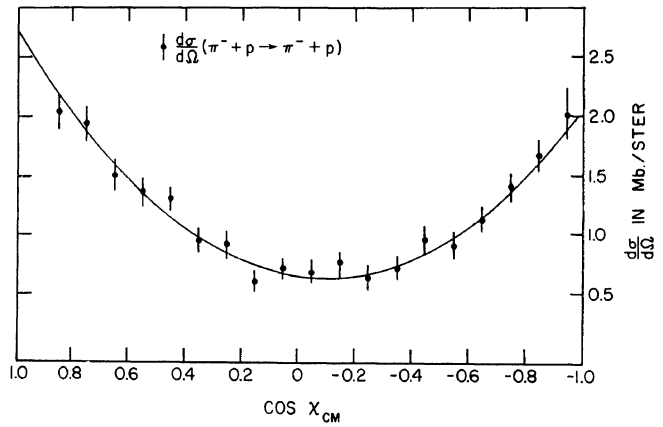

Fig. 1 Angular distribution of elastic scattering.

Table II. Summary of various experimental results

Process	Ashkin [2]	This Experiment	Goodwin [3]	Caris [4]	Zinov [1]	Units
Energy	220 ± 7	224 ± 10	230 ± 8	230 ± 8	240 ± 7	MeV
$\pi^-+p \to \pi^-+p$	19.5 ± 0.6	16.0 ± 0.8	20.8 ± 0.4	—	16.1 ± 0.6	mb
$\pi^-+p \to \pi^0+n$	33.3 ± 0.7	34.4 ± 1.9	—	30.4 ± 1.3	32.2 ± 1.4	mb
$\sigma_{\text{Total}}-\sigma_{\gamma n}$	53.2 ± 1.5	50.5 ± 2.1	48 ± 2 (*)	—	48.3 ± 3.3	mb
a_0 (**)	0.86 ± 0.06	0.70 ± 0.04	0.95 ± 0.04	—	0.81 ± 0.08	mb/ster
a_1 (**)	0.30 ± 0.08	0.33 ± 0.07	0.55 ± 0.06	—	0.23 ± 0.09	mb/ster
a_2 (**)	2.07 ± 0.18	1.69 ± 0.13	2.10 ± 0.09	—	1.41 ± 0.18	mb/ster
$\frac{\sigma(\text{Charge Exch.})}{\sigma(\text{Elastic})}$	1.71 ± 0.06	2.15 ± 0.09	1.31 ± 0.10	1.55 ± 0.07	2.00 ± 0.11	—

(*) From data of Pontecorvo. (**) $\frac{d\sigma\,(\pi^-+p \to \pi^-+p)}{d\Omega} = a_0 + a_1 \cos \chi_{cm} + a_2 \cos^2 \chi_{cm}$.

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π^+-p ELASTIC SCATTERING AT 120 MeV

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We summarize here an investigation of the elastic scattering of 120 MeV kinetic energy positive pions, obtained by exposing a liquid propane bubble chamber to the CERN 600 MeV synchrocyclotron. The bubble chamber has been described in a paper by A. Loria et al [1]. There is no magnetic field. The meson beam was substantially the same as previously used by Ashkin et al [2]. The results refer to 2×10^4 pictures on which a total of 3×10^3 elastic collisions on hydrogen were found, including the 10^3 events which were reported at Kiev [3].

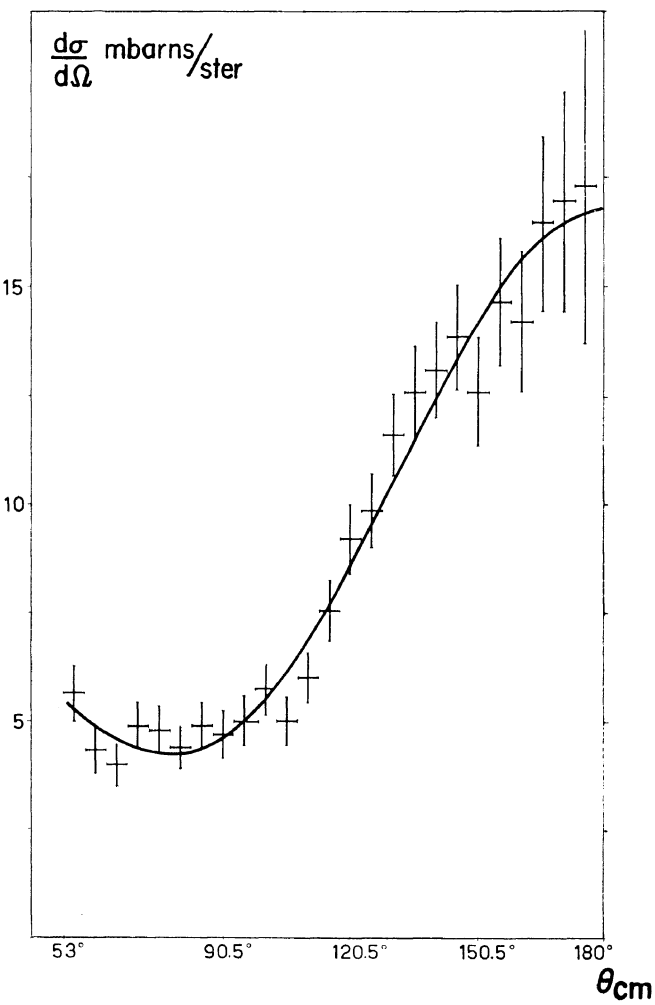

Fig. 1 Differential cross section for elastic scattering of 120 MeV positive pions by protons.

The mean energy was determined very accurately from events in which the proton stopped in the visible region of the chamber: the value obtained was 120.0 ± 0.5 MeV. The contamination from carbon events, as estimated from coplanarity, angular correlation and proton range combined criteria, was found to be less than 2%.

In the fiducial region the absolute scanning efficiency for events parallel to the windows was checked to be greater than 99%. As expected a bias related to the value of the azimuthal angle was found: the experimental angular distribution has been corrected accordingly. The cut-off in the pion scattering angle was 53° in the c.m.s., a limit beyond which the proton track was too short for the event to be analyzed. The angular distribution (Fig. 1) was normalized to a total cross section of 95 ± 6 mb, a figure determined from experimental values at nearby energies.

The phase shift analysis has been performed at Rochester [4]; the results are:

$$\alpha_{31} = -3.2°\pm1.0°$$

$$\alpha_{3} = -11.1°\pm2.1°$$

$$\alpha_{33} = 31.6°\pm1.2°$$

$$\chi^2 = 17.6$$

The phase shifts α_{31} and α_{33} agree well with the values already found and hence do not raise any discussion. α_3 differs instead about 1.5 standard deviations from the value 8.2° which would be expected on the basis of a linear dependence between α_3 and η [4] (Fig. 2). This linear dependence looks now to be highly improbable : our result has in fact confirmed the measurements with nuclear plates in the region of 100 MeV, while more evidence has accumulated both at high and at very low energy.

Some representative points are plotted in Fig. 2, in which a tentative curve is drawn as a solid line :

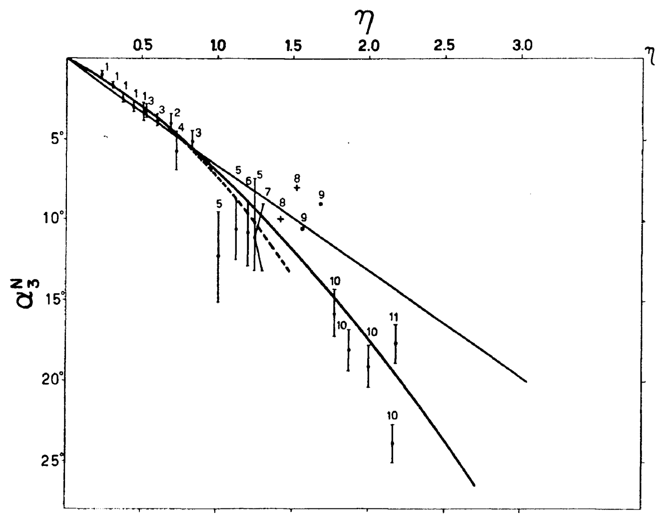

Fig. 2 α_3^N versus momentum.

it is not surprising that the points near the resonance do not fit the curve, because the determination of α_3 is more critical.

The above conclusion is somewhat strengthened by a prediction drawn from the dispersion relations. By simplifying rather drastically for low energy some formulas obtained by Chew et al [5], one easily finds :

$$\sin 2\alpha_3 = -4\eta \frac{M}{\omega+M}\left(\lambda^+ + \lambda^- \frac{\omega}{2M}\right)$$

where η is the momentum in the c.m.s. in units of $m_\pi c$, ω is the total energy in the c.m.s. less the mass of the proton, and M is the mass of the proton.

If one assumes tentatively :

$$\lambda^+ = 0.01 \text{ and } \lambda^- = 0.6$$

so that

$$\left(\frac{\alpha_1(\eta)}{\eta}\right)_{\eta=0} = +0.16 \text{ and } \left(\frac{\alpha_3(\eta)}{\eta}\right)_{\eta=0} = -0.11,$$

i.e. the values given by Orear [6], one obtains the dashed line of Fig. 2, which is satisfactory up to 120 MeV, but is certainly too much bent downward at higher energies.

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THE ELASTIC SCATTERING OF NEGATIVE PIONS BY PROTONS AT 128 AND 162 MeV

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The elastic scattering angular distributions of negative pions on protons has been measured by means of a high pressure hydrogen filled diffusion cloud chamber, with a magnetic field of about 9000 gauss. From 90,000 photographs a total of 344 elastic events with $\theta > 10°$ were found at 120 MeV; 941 elastic events were found at 162 MeV. Total cross sections were determined from the total track length of negative mesons in the diffusion chamber. The total elastic cross section at 120 MeV is 12.8$\pm$1.0 mb and at 162 MeV is 21.4$\pm$1.2 mb. These numbers are in good agreement with other experiments. The angular distributions have been fitted using an *S-P* analysis and are

$$\left.\frac{d\sigma}{d\Omega}\right)_{128\ \mathrm{MeV}} = (1.00 \pm 0.08)[(0.55 \pm 0.07) + $$
$$+ (0.34 \pm 0.12)\cos\theta + (1.30 \pm 0.24)\cos^2\theta]\ \mathrm{mb/ster},$$

and

$$\left.\frac{d\sigma}{d\Omega}\right)_{162\ \mathrm{MeV}} = (1.00 \pm 0.06)[(0.93 \pm 0.07) + $$
$$+ (0.51 \pm 0.12)\cos\theta + (2.28 \pm 0.22)\cos^2\theta]\ \mathrm{mb/ster}.$$

The cross section for forward scattering has been calculated using the angular distribution and the correlation errors. The numbers are 2.20$\pm$0.32 and 3.73$\pm$0.32 mb/ster.

These cross sections give for the real part of the forward scattering amplitude in the center of mass system the values 0.261$\pm$0.031 $\hbar/m_\pi c$ and 0.216$\pm$0.038 $\hbar/m_\pi c$. These values agree with those calculated from dispersion relations with a coupling constant of $f^2 = 0.08$.

MEASUREMENT OF THE TOTAL CROSS SECTION FOR π^-P CHARGE EXCHANGE FROM $T_\pi = 0.4$ GeV TO $T_\pi = 1.5$ GeV

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Very few measurements have been carried out on the total cross section for charge exchange of negative pions on hydrogen ($\pi^- + p \rightarrow \pi^0 + n$).

Measurements of this cross section are of great interest, especially in the region of the new pion-nucleon resonances. From such measurements one would be able to deduce, for instance, the total cross section for elastic pion nucleon scattering in the $T = \frac{1}{2}$ state at all energies at which the total cross section for charged elastic scattering ($\pi^- p \rightarrow \pi^- p$ and $\pi^+ p \rightarrow \pi^+ p$) are known.

In this paper preliminary results are given for a counter experiment being carried out at the Saclay proton-synchrotron.

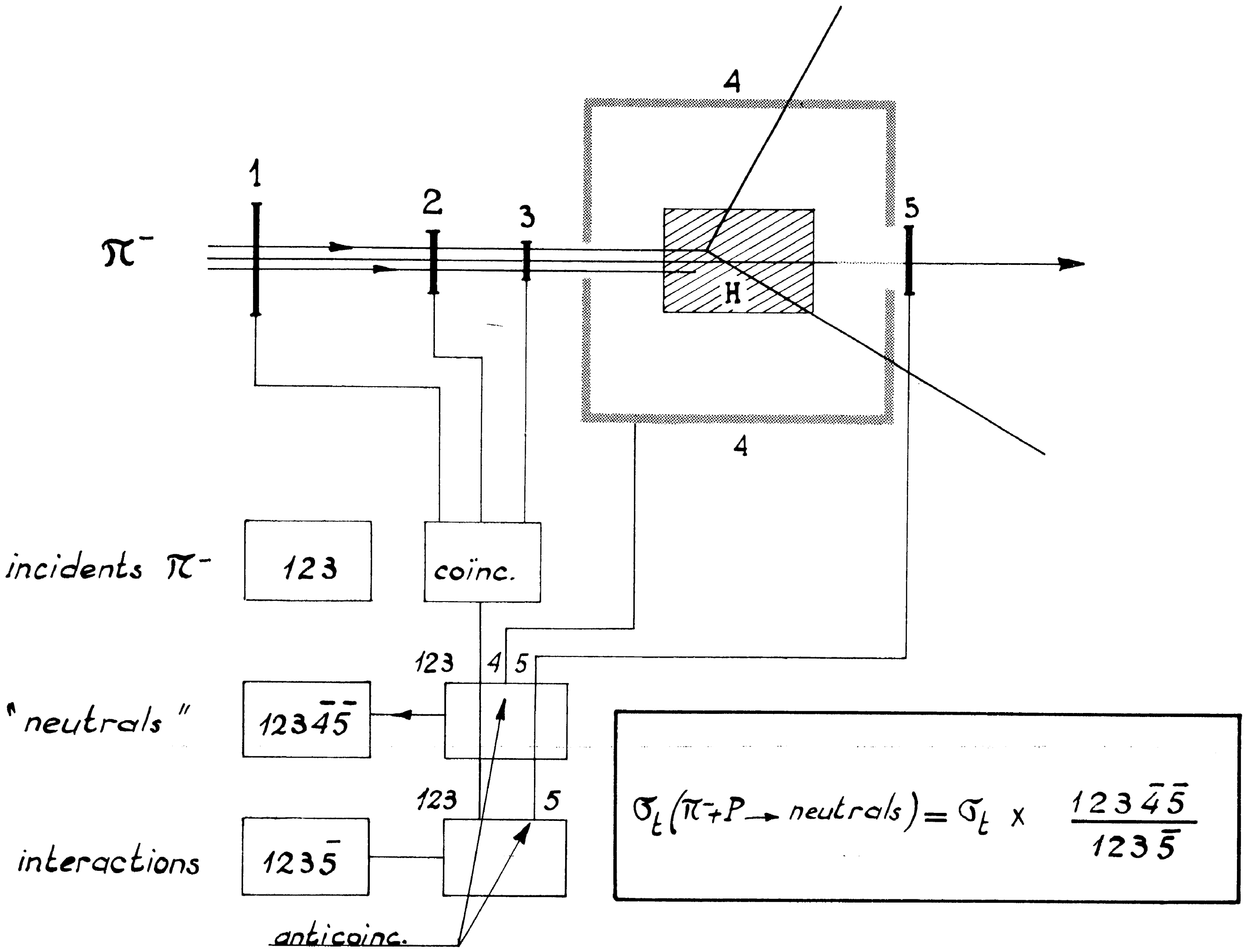

Fig. 1 Schematic drawing of the experimental arrangement.

In the first part of this experiment we have measured the total cross section for all channels resulting in the emission of neutral particles only.

The method consists in measuring the number of π^- which interact in a liquid hydrogen target without triggering a 4π-scintillation counter surrounding the target. (Fig. 1).

This 4π counter is connected in anticoincidence with the counter telescope defining the incident pion beam. It has the form of a regular dodecahedron made of 12 scintillation counters, all connected in parallel. The beam enters and leaves through holes in diametrically opposite scintillators. After leaving the exit hole, the beam passes through a scintillator connected separately in anticoincidence (with only this last counter in anticoincidence one measures the total $\pi^- p$ cross section).

In the second part of this experiment we try to distinguish between elastic and inelastic scatters by a method which consists in placing a lead converter around the hydrogen target. The ratio between the number of " neutral events " measured with and without lead is directly related to the average transmission factor for all γ rays originating from the decay of neutral pions where the appropriate average has been taken over photon angle, photon energy, and π^0 multiplicity. This average transmission factor will be different in the case of 2 γ rays or 4 γ rays. Its value therefore depends on the ratio of elastic charge exchange to charge exchange with production of additional neutral pions.

The results obtained in the first part of this experiment are shown in Fig. 2 (solid line). They were obtained under the following conditions :

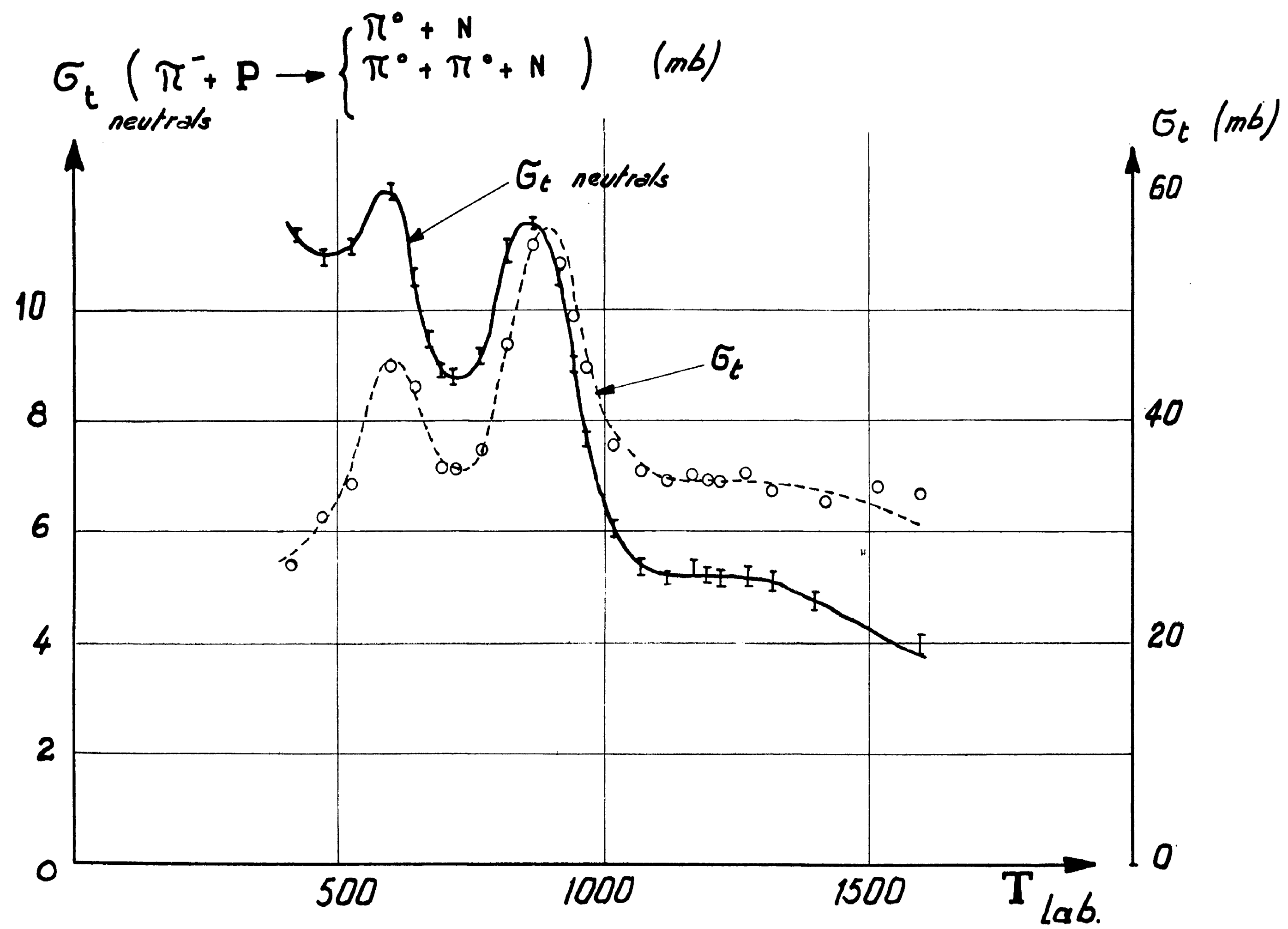

Fig. 2 Total cross section for " neutral events ". Solid line and scale on the left : results of this experiment. Dashed line and scale on the right : $\pi^- p$ total cross section as measured previously [1]. Circles and scale on the right ; $\pi^- p$ total cross section as measured in this experiment.

Energy spread of the pion beam	$\pm 1.5\%$
Beam intensity	1.5×10^3/cycle
Length of H_2 target	20 cm.
Empty to full target ratio . . .	0.1 to 0.2

The errors shown in Fig. 2 are statistical only.

Rough corrections have been made for the following secondary effects which take place in the target walls (mylar and styrofoam) and in the scintillators :

detection efficency for neutrons and γ rays (7%)
delta rays (2%)
Dalitz pairs (1%)

The total cross section for neutral events shows maxima at energies close to the resonances in the $\pi^- p$ total cross section. Fig. 2 also shows the total $\pi^- p$ cross section as published previously [1] (dashed line in Fig. 2) and as measured during this experiment (individual points). It appears that the second maximum is displaced with respect to the 3rd resonance of the total cross section.

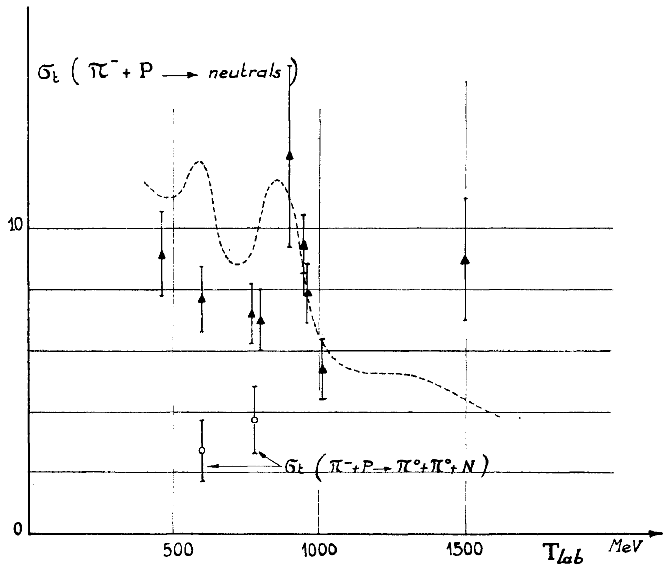

Fig. 3 Dashed line : results of this experiment. Experimental points given in Table I.

Fig. 3 shows that our results (dashed line) are in good agreement with the previously existing data between $T_{\pi^-} = 0.8$ and $T_\pi = 1$ GeV, summarized in Table I.

Table I. Previous measurements of charge exchange total cross section (*)

Energy T_π(GeV)	Reference	Method
0.460	Crittenden et al. Phys. Rev. Letters **2**, p. 121 (1959)	B.P.
0.600	as above	B.P.
0.770	as above	B.P.
0.800	McCormick et al. " Proceedings 1958 Annual International Conference on High Energy Physics at CERN " ; UCRL 8302	B.H.
0.900	Walker et al. Phys. Rev. **104**, p. 526 (1956)	Em
0.950	Erwin et al. Phys. Rev. **109**, p. 1364 (1958)	B.H.
0.960	Walker et al. Phys. Rev. **104**, p. 526 (1956)	D.H.
0.960	Alles-Borelli et al. Nuovo Cimento **14**, p. 211 (1959)	B.H.
1.010	Derado et al. Ann. Physik **4**, p. 103 (1959)	B.H.
1.500	Walker et al. Phys. Rev. **98**, p. 1416 (1955)	Em

(*) Methods : B.P. = propane bubble chamber; B.H. = hydrogen bubble chamber; D.H. = hydrogen diffusion chamber, Em = emulsion.

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ON THE COMPLETE SET OF EXPERIMENTS FOR THE DETERMINATION OF THE AMPLITUDE RATIOS OF PION PRODUCTION IN NUCLEON COLLISIONS WITH DIFFERENT ISOTOPIC SPIN STATES

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A new detailed discussion is given on the total set of experiments for the determination of the amplitude ratios of pion production in nucleon-nucleon collisions in different isotopic spin states. The discussion is connected with Mandelstam's [1] resonance theory of pions produced by nucleons.

The number of experiments which have been carried out up to now in the nucleon energy region of 660 MeV is still inadequate to determine the required ratios between the amplitudes. Experiments should be carried out to measure the differences in meson production caused by the dependence of the production amplitude on the interchange of a neutron and a proton in the initial and final states [2].

If the neutron and the proton are interchanged in the initial state (for example in processes $pn \to nn\pi^+$ and $np \to nn\pi^+$), the difference between these two processes can be determined, when charge symmetry is used, by comparing the cross section of the processes $pn \to pp\pi^-$ and $pn \to nn\pi^+$ at the same π-meson angle.

If the neutron and the proton are interchanged in the final state, the difference between the probabilities of these processes can be determined by simultaneous detection of the particles: a π-meson and one of the nucleons.

If one averages the differential cross section for production over the azimuthal angles of a pion and nucleons, one can introduce the so-called " subtracted total cross sections " measuring the difference of the probabilities of the processes $pp \to np\pi^+$ and $pp \to pn\pi^+$ as well as $np \to np\pi^0$ and $np \to pn\pi^0$. Thus, for example,

$$\Delta\sigma_{10,11} = 4\int_0^{\pi/2}\int_0^{\pi/2}\left[d\sigma^{pp}_{np\pi^+}(\theta_\pi,\theta_{12}) - d\sigma^{pp}_{pn\pi^+}(\theta_\pi,\theta_{12})\right]\times d\Omega(\theta_\pi)d\Omega(\theta_{12}) . \quad (1)$$

The subtracted total cross section of the processes $pn \to nn\pi^+$ and $np \to nn\pi^+$ can be written simply as

$$\Delta\sigma_{01,11} = 2\int_0^{\pi/2}\left[d\sigma^{pn}_{nn\pi^+}(\theta_\pi) - d\sigma^{np}_{nn\pi^+}(\theta_\pi)\right]d\Omega(\theta_\pi) = 2\int_0^{\pi/2}\left[d\sigma^{np}_{pp\pi^-}(\theta_\pi) - d\sigma^{np}_{nn\pi^+}(\theta_\pi)\right]d\Omega(\theta_\pi) . \quad (2)$$

The subtracted total cross section defined in Eq. (2) is the coefficient " b " in the angular distribution of π-mesons

$$d\sigma(np \to \pi^-) = a + b\cos\theta_\pi + c\cos^2\theta_\pi ,$$

the contribution of which to the total cross section is equal, naturally, to zero.

The study of the reactions $np \to pp\pi^-$ and $np \to nn\pi^+$ at 600 MeV [3] neutron energy leads to a conclusion that $|F_{01}| \neq 0$ where F_{ij} denotes three independent amplitudes of π-meson production; the first index corresponds to the isotopic spin of the two nucleons in the initial state and the second index denotes that in the final state. From the same data it follows that $\Delta\sigma_{01,11}$ is close to zero. It is necessary to measure for the neutron beam the subtracted cross section $\Delta\sigma_{01,10}$ which permits one to determine the phase ratio and essentially improve the confidence of the $|F_{01}|$ value. With a proton beam one can determine the ratio between the amplitudes A_{13} and A_{11} by measuring $\Delta\sigma_{10,11}$. Here the first index in A_{ij} shows the isotopic spin of the nucleons in the initial state, and the second index $j = 2T_{\pi N}$; where $T_{\pi N}$ is the isotopic spin of the subsystem consisting of a π-meson and a nucleon.

This experiment can give a very sensitive test of the Mandelstam resonance theory. According to this model $|A_{11}| = 0$, and as its consequence, the well-known equation $\sigma(pp\to\pi^+) = 5\sigma(pp\to\pi^0)$. In fact, experiments carried out at 660 MeV proton energy provide

$$\sigma(pp\to\pi^+) = 3.4\sigma(pp\to\pi^0)$$

which means: $|A_{11}| \neq 0$.

Fig. 1 gives the permissible region of the values $\alpha = \frac{\sigma(pp\to\pi^+)}{\sigma(pp\to\pi^0)}$ and $k = \frac{|A_{13}|}{|A_{11}|}$ which contains a family of curves

$$\alpha = \frac{4+5k^2-\sqrt{8}k\cos\phi_{13}}{2+k^2+\sqrt{8}k\cos\phi_{13}}, \tag{3}$$

depending on the parameter $\cos\phi_{13} \equiv \cos(A_{13},A_{11})$. It is seen from Fig. 1 that if $\alpha = 3.4$, the permissible values of k^2 are in the interval $\frac{1}{20} < k^2 < 64$. When $k = \frac{1}{\sqrt{2}}$ then $\alpha_{min} = 1$ and when $k = \sqrt{2}$ then $\alpha_{max} \to \infty$.

If $\frac{\sigma^+}{\sigma_0} = 5$, the permissible values of k^2 are in the interval $\frac{1}{8} < k^2 < \infty$. Thus, the fact that $\frac{\sigma_+}{\sigma_0} = 5$ cannot be treated absolutely as a proof of the correctness of the resonance theory.

It should be stressed that if the relative phase $\mathbf{A}_{13}\cdot\mathbf{A}_{11}$ changes without change in k, the ratio of the cross sections $\frac{\sigma_+}{\sigma_0} = \alpha$ will change as a result. The authors suppose that such a process should take place in pion production on the coupled nucleons of the nucleus, in particular, on the deuteron.

Fig. 2 gives a diagram plotting permissible values of $|A_{13}|^2$, $|A_{11}|^2$ and $\Omega_{3,1}$ depending on the values of the subtracted cross section $\Delta\sigma_{10,11}$. All the values are given in units of 10^{-27} cm^2. The greatest permissible cross section $\Delta\sigma_{10,11}$ is 9.9×10^{-27} cm^2 at 660 MeV proton energy.

At present we are preparing a concrete scheme to measure experimentally $\Delta\sigma_{10,11}$ at 660 MeV proton energy.

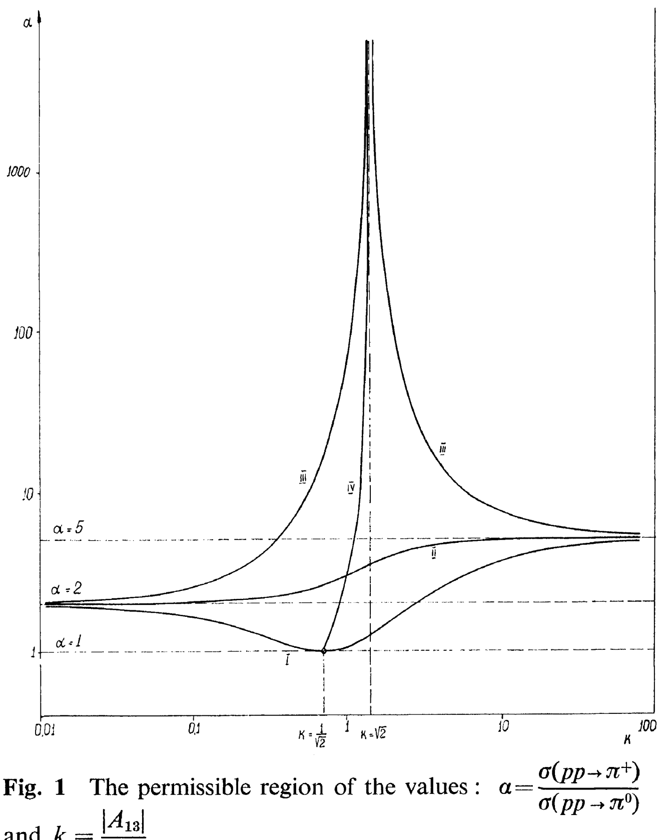

Fig. 1 The permissible region of the values: $\alpha = \frac{\sigma(pp\to\pi^+)}{\sigma(pp\to\pi^0)}$ and $k = \frac{|A_{13}|}{|A_{11}|}$.

Curve I corresponds to: $\varphi_{13} = 0$;
Curve II " $\varphi_{13} = \pi/2$;
Curve III " $\varphi_{13} = \pi$;
Curve IV " $\Delta\sigma_{10,11} = 0$.

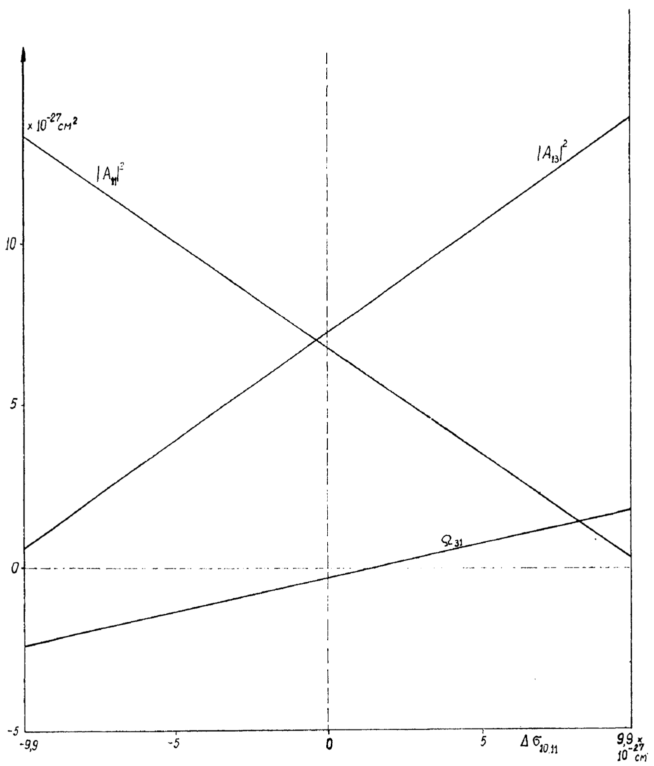

Fig. 2 Dependence of the permissible values $|A_{13}|^2$, $|A_{11}|^2$ and $\Omega_{3,1}$ on the subtracted total cross section $\Delta\sigma_{10,11}$ at 660 MeV proton energy ($\alpha = 3.4$).

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ELASTIC SCATTERING OF 142 MeV NEGATIVE PIONS BY DEUTERONS (*)

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A measurement of elastic π-d scattering at 142 MeV was undertaken in order to provide experimental data to compare with the results of impulse approximation calculations. In particular, the effects of the D wave in the deuteron wave function and the repulsive core in the deuteron potential should be seen at large momentum transfers. The energy of 142 MeV was chosen as a compromise between the desirability on the one hand of scattering with high momentum transfers and on the other hand of having the recoil deuteron come to rest in the chamber volume to aid in the detection of elastic events. At this energy the above-mentioned effects alter the backward cross section by ten to twenty per cent [1]. The statistical accuracy of the previous experimental data [2-5] is inadequate to provide a measure of these effects. In addition, at this energy it is expected that multiple scattering corrections to the impulse approximation are appreciable [6]. Therefore an experiment at this energy could also yield information on such multiple scattering effects.

A 142±7 MeV negative pion beam from the Carnegie Tech. cyclotron was used to bombard a 15 cm deuterium bubble chamber in a 12.7 kilogauss field. Thirty thousand pictures were taken. They were scanned for all two-prong events in a region selected so that a major fraction of the recoil deuterons stopped in the chamber. The following two reactions lead to two-prong events:

$$\pi^- + d \rightarrow \pi^- + d , \quad (1)$$

$$\rightarrow \pi^- + p + n . \quad (2)$$

To distinguish the reactions (1) and (2), angle correlation, coplanarity of the event, and the range of the positive particle were used. The method of selection of elastic π-d scattering was as follows:

(1) All two-pronged events were measured except those which obviously had a neutral outgoing particle (in order to conserve momentum).

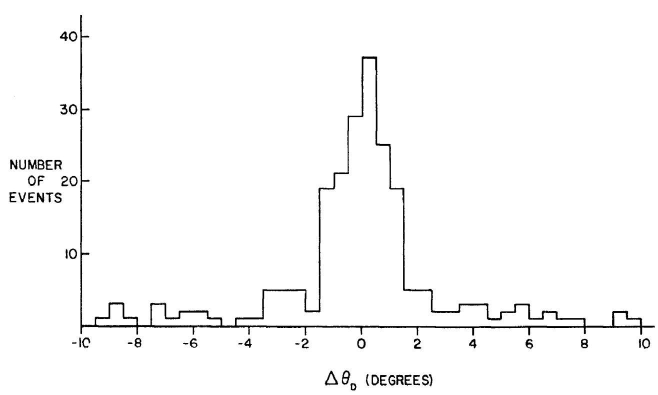

Fig. 1 Number of events *vs.* $\Delta\theta_D$ for the angular range 90° to 120°.

(*) Supported in part by the U.S. Atomic Energy Commission.

(2) The two-pronged events for which the angle of the positive particle (θ_D) was within 10° of the kinematic correlation line were examined for coplanarity and the range of the positive particle. The histogram in Fig. 1 shows the results for the angular range 90° to 120° laboratory pion angle, where $\Delta\theta_D$ represents the deviation from the kinematic correlation line. It is seen that the angular resolution is of the order of 1°.

(3) For coplanar events with $\Delta\theta_D < 5°$ the laboratory energy of the incident pion was calculated from the pion scattering angle and the range of the recoil, assuming it to be a deuteron. Fig. 2 shows a histogram of the events, in the same angular range as Fig. 1, plotted as a function of the calculated incident pion energy.

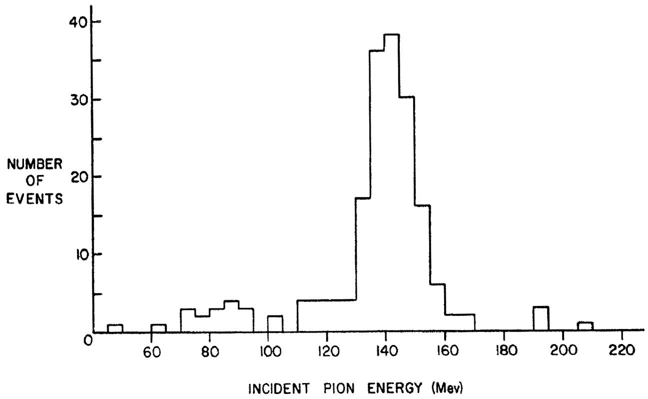

Fig. 2 Number of events *vs.* incident pion energy calculated from the laboratory scattering angle of the pion and the range of the recoil for the angular range 90° to 120°. All events are coplanar within measuring error and have $\Delta\theta_D$ less than 5°.

The number of π-d events in Fig. 2 was determined by comparing it with the beam spectrum obtained from curvature measurements and by carefully investigating the kinematic correlation of the events that make up the tails of the peak.

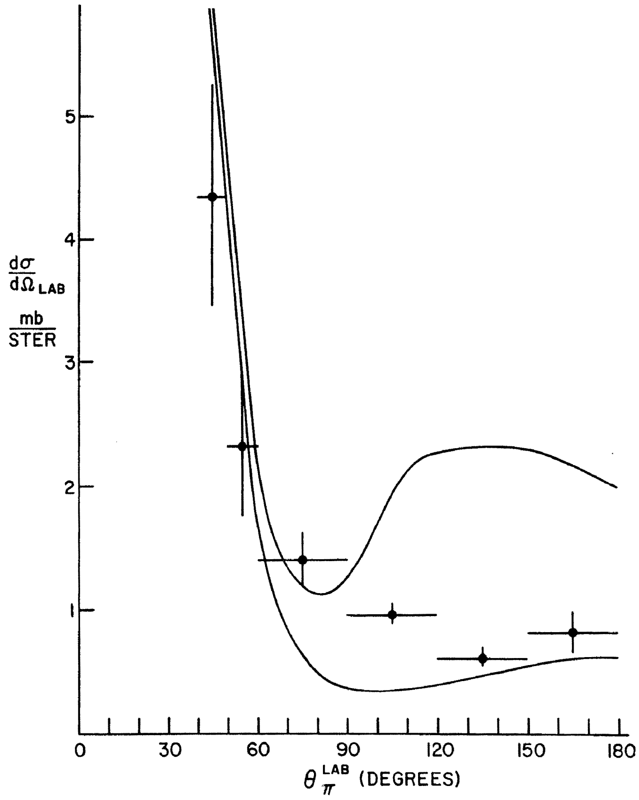

Fig. 3 The angular distribution of elastic $\pi^- - d$ scattering. The theoretical calculation of Bransden and Moorhouse is given by the lower curve. The upper curve gives the result of a calculation by Pendleton.

Table I. Summary of elastic events

	Angular range pion lab angle (degrees)					
	40-50	50-60	60-90	90-120	120-150	150-180
Number of events						
From E_π^{inc} plot (recoil stops inside chamber)	43±10	30±8	56±10	143±14	64±10	20±5
Recoil scatters	1±1	1±1	3±2	25±6	10±4	7±3
Recoil stops outside chamber	0	0	0	4±3	6±3.5	8±4
Efficiency correction	10±5	3±2	12±4	7±3	3±2	5±3
Total	54±11.2	34±8.3	71±11.0	179±15.8	83±11.5	40±8.3
Corrected track length (kilometers)	3.84	3.84	3.84	14.16	14.16	14.16
Differential lab cross section dσ/dΩ mb/sterad	4.34±0.90	2.33±0.57	1.40±0.22	0.96±0.09	0.61±0.08	0.81±0.17

The total number of elastic events was obtained by adding to the above events : (1) those elastic events for which the end point of the recoil lay outside of the chamber; (2) those elastic events for which the recoil deuteron scattered; (3) scanning efficiency correction. The elastic events in (1) and (2) were determined by angular correlation and range requirements. The scanning efficiency correction was deduced from a re-scan of half the film and from the azimuthal distribution of events. The total track length was corrected for beam contamination, π-μ decay, and nuclear interactions. Preliminary results are given in Table I. The density of D_2 in the bubble chamber was 140 grams/liter.

The measured elastic scattering angular distribution is shown in Fig. 3; also shown are curves calculated by Bransden and Moorhouse [7] and by Pendleton [1]. The calculation of Pendleton is based on the impulse approximation and takes into account double scattering, some recoil effects, and the *D* state of the deuteron. The experimental results are at variance with the theory. The experiment is being continued to obtain higher statistical accuracy.

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THE NUCLEON-NUCLEON REPULSIVE CORE AND THE SPIN-ORBIT INTERACTION (*)

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A discussion of recent proposals [1, 2, 3] to explain the phenomenological repulsive core and the spin-orbit term in nucleon-nucleon interactions shows some uncertainties both regarding the large masses, m_{hp}, which correspond to Bryan's V_{LS} and the smaller ones which are suggestive of a connection with proposals of two and three pion bound states. The uncertainty in the values of the larger masses lies in the as yet undisproved possibility of obtaining a representation of *p-p* scattering data by means of a potential with a V_{LS} part of longer range than used by Bryan or Gammel-Thaler. Present evidence is, however, that the short range V_{LS} account for data on polarization better than the long range V_{LS}. The

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derivation of the smaller m_{hp} based on comparison with p-p scattering data at 310 MeV is shown to be subject to a large uncertainty caused by distortion of the wave function by the potential. If one employs the current phenomenological potentials as a guide, a reduction of ~ 45 per cent in the expected contribution of V_{LS} is expected because part of the contribution using a free wave function arises from the region inside the repulsive core. Employing Bryan's potential a total reduction of the expected V_{LS} effect by a factor $\sim {}^1/_4$ or $\sim {}^1/_5$ is expected, the additional factor $\sim {}^1/_2$ arising because of the depression of the wave function close to the core radius.

The examination of the repulsive core which corresponds to the desired magnitude of V_{LS} effects also appears to speak in favor of the larger masses provided it is supposed that the cores used in the phenomenological potentials are approximately correct. The larger m_{hp} give cores which, while appreciably softer than the phenomenological, can conceivably be made to reproduce them when combined with suitable higher order pion effects. For small m_{hp} such combinations appear artificial. Also at larger distances, with $x = h/m_\pi c$ in the range $1.6 < x < 2.0$, the small values of m_{hp} give central potential additions which appear too large, especially since there is evidence [4] that the dominant interaction here is the one pion exchange potential, OPEP.

Since, on the other hand, the value derived by Sakurai [3] agrees approximately with the masses of the supposed two pion resonance [5] and of the hypothesized three pion bound state [6], it appears of interest to consider ways of bringing about agreement using the smaller m_{hp}. A possibility is that the phenomenological core is a substitute for a condition which makes the shape of the wave function similar to that obtained by enforcing a node at the core radius. The region $x < 0.4$ is then not necessarily repulsive. This reinterpretation would be helpful in resolving the difficulty concerning the insufficiently strong binding of H^3 recently brought out by Blatt and Derrick [7] although other ways out of the difficulty are not excluded [1]. This question is closely related to that of the mean life of the vector meson. If this meson should turn out to be the two-pion resonant state and if Fig. 1 of Carruthers and Bethe [8] may be taken as an indication of its mean life, then the mean life $\sim 10^{-13}$ cm/c is too short to consider the meson as a particle except in the more energetic collisions. The low energy interactions would accordingly not be represented by the same potential as those at high energy and the calculation of the long distance tail of the interaction employing the m_{hp} in the Yukawa formula (leading to too large values of the interaction potential) would be unjustifiable.

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ON THE SPIN CORRELATION COEFFICIENT C_{nn} FOR 310 MeV p-p SCATTERING AT 90° (c.m.s.)

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A phase shift analysis has been made on the data of the whole set of experiments on elastic proton interaction at 310 MeV performed at Berkeley. The analysis has not included studies on the spin correlation of scattered protons. This analysis [1], as is known, has led to an ambiguous result. From all possible solutions five independent sets of phase shifts satisfactorily describing experimental data have been chosen. The obtained solutions have led to different values of the coefficient $C_{nn}(90°)$ which determines the correlation between spin components normal to the scattering plane. Thus, for those sets of phase shifts which are denoted by numbers 1, 2, 3, 4 and 6 the value $C_{nn}(90°)$ have been found to be 0.158, 0.711, 0.300, 0.490 and 0.425 [2], respectively.

In this connection the experimental investigation of the spin correlation of scattered protons at an energy of 310 MeV became of great importance and at a number of laboratories (Berkeley, Liverpool and Dubna) preparations were begun for running a complicated experiment on the determination of $C_{nn}(90°)$.

However, an essential decrease of ambiguity of the analysis was achieved by developing and improving the phase shift analysis on nucleon-nucleon scattering itself [3,3a)]. Originally, the performed analysis included 14 phase shifts from states up to the H-wave inclusive. In the paper by Cziffra et al [3] the authors have taken into account the contribution from states with greater orbital momenta on the basis of the one meson approximation developed in the papers by Chew [4] and Okun and Pomeranchuk [5]. This additional contribution is calculated in the first approximation of perturbation theory and adds to the analysis only one new parameter—the pion-nucleon coupling constant g^2. A modified analysis permits one to state that only the first and the second sets of phase shifts satisfactorily describe the experimental data with $g^2 = 14$. Thevalue of the coefficient $C_{nn}(90°)$ becomes equal to 0.38 for the first set and 0.61 for the second in connection with the new values of the phase shifts of the chosen sets.

The first experiments on the determination of $C_{nn}(90°)$ carried out in Liverpool at 320 MeV proton energy and at Dubna at 315 MeV favored the second set of phase shifts, as it has been pointed out at the Kiev Conference on High Energy Physics [6]. Thus, the Liverpool group has found the spin correlation coefficient to be $C_{nn}(90°) = 0.75 \pm 0.11$. Our measurements performed by using preliminary data on the calibration experiment for the determination of the polarization ability of graphite analyzers has led to $C_{nn}(90°) = 0.7 \pm 0.3$.

At present we have finished an experiment on the determination of the analyzing ability of carbon scatterers. The calibration experiment has been performed with the 160 MeV proton beam, the polarization of which has been found to be 0.667 ± 0.027. The polarization ability of the analyzers employed in the measurement of the correlation asymmetry has been obtained to be equal to 0.28 ± 0.02. Taking into account that the value of the coefficient C_{nn} cannot exceed unity, we have

$$C_{nn}(90^0) = +0.84 \begin{matrix} +0.10 \\ -0.22 \end{matrix} .$$

Hence, the obtained large experimental value of the coefficient C_{nn} is not in agreement with the value predicted on the basis of the first set of phase shifts.

The estimates of the contribution of the single interaction b^2 and the triplet interaction c^2 (spin orbital) and h^2 (of the tensor type) have been made on the basis of earlier experimental data on elastic pp-scattering at 310 MeV. Thus, Wolfenstein [7] has found that $15\%<b^2<60\%$, $35\%<c^2<70\%$ and $2\%<h^2<20\%$. The evaluations of Hurushev [8] are the following: $b^2\simeq 25\%$, $c^2\simeq 62\%$ and $h^2\simeq 13\%$.

From the relations

$$b^2 = \tfrac{1}{2}(1-C_{nn})$$
$$c^2 = \tfrac{1}{4}(1+C_{nn}+2D)$$
$$h^2 = \tfrac{1}{4}(1+C_{nn}-2D)$$

on the basis of the obtained value $C_{nn}(90°)$ and the value $D(90°)=0.42$ found by the extrapolation of data [2] the corresponding contributions are obtained to be $b^2\simeq 8\%$, $c^2\simeq 67\%$ and $h^2\simeq 25\%$.

The situation concerning sets of phase shifts describing elastic *p-p* scattering at 310 MeV has somewhat changed as a result of the modified phase shift analysis of previous experimental data [9]. The modification of the analysis consisted in the reduction of the number of the used phase shifts and in the expansion of one meson approximation to states with relatively less orbital momentum. The performed analyses including 5, 7 and 9 phase shifts have shown that if 9 phase shifts instead of the previous 14 ones and the pion-nucleon coupling constant g^2 are taken into account, one obtains quite a satisfactory description of the same experimental data in the case of the second and especially of the first set of phase shifts. The values of the coefficient $C_{nn}(90°)$ calculated from recently found phase shifts have turned out to be ~ 0.41 for the first and the second sets. In connection with this, the authors [9] consider that in order to remove the ambiguity of two sets of phase shifts it is necessary to measure the value C_{kp} at 45°.

However, a new analysis with 9 phase shifts and the constant g^2 has led not only to the disappearance of the difference in the coefficient $C_{nn}(90°)$ for the first and the second sets of phase shifts but also to the value of the coefficient which contradicts the available experimental data. From our point of view this discrepancy should be treated as an indication on the insufficiency of 9 phase shifts taken into account in the last analysis. Introducing into the analysis experimental data on determination of the value $C_{nn}(90°)$ in the case with 9 phase shifts would lead, evidently, to an overstatement of the goodness-of-fit parameter, χ^2, as it occurs in the analysis of experimental data which do not include the values of C_{nn} when only 7 phase shifts are taken into account.

On the basis of the analyses which take into consideration 7 and 9 phase shifts the first set is preferred [9]. But when data obtained from the experiment on the determination of $C_{nn}(90°)$, 14 phase shifts and the constant g^2 are taken into account, as it was stated in [10], both sets give goodness-of-fit parameters just within the range of 50% probability. More accurate measurements of several data used in the analysis will be evidently needed in order to avoid ambiguity in the determination of sets of phase shifts.

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PROTON-PROTON DIFFERENTIAL CROSS SECTION AT 1 BeV

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The *p-p* elastic differential cross section in the angular range 18° to 90° center of mass has been measured at 1010±20 MeV, using a polythene-carbon difference method. The errors resulting from counting statistics are about 3 % on all points. The measurements were performed using the extracted beam of the Birmingham proton-synchrotron, which provides 10^8 protons per pulse, focussed to a 1″ diameter circle at the target position. The pulse duration is 4 milliseconds.

The scattered and recoil protons were detected as triple coincidences between a defining telescope and a single large recoil counter, the resolving time being 8×10^{-9} sec. The angular resolution was about ±1.5° in the c.m. system.

The results, corrected for accidental coincidences, are given in Table 1. Corrections for inelastic *p-p* events and attenuation of the recoil protons are not included, but have been estimated to be less than 1 % on all points.

The results have been fitted with curves of the form

$$\sigma(\theta) = \sum_{i=0}^{l\,\max} \alpha_{2i} P_{2i}(\cos\theta)$$

using a least squares method. The best fit has been obtained for $l_{\max} = 3$, which implies that the major contribution is from partial waves with $l \leq 3$. On the other hand, since the higher order terms give their major contribution at forward angles, the goodness of fit, when these terms are included, is more sensitive to points in this region. This part of the angular distribution is now being investigated in more detail.

The values of the coefficients a_{2i} and their errors are

$$a_0 = +3.477 \pm 0.069$$
$$a_2 = +8.266 \pm 0.257$$
$$a_4 = +4.383 \pm 0.335$$
$$a_6 = +1.137 \pm 0.242$$

The absolute values of the differential cross section have been obtained by normalizing the intercept of the fitted curve at 0^0 to a value of $\sigma(0^0)$ derived from the relation

$$\operatorname{Im} f(0^0) = k\sigma_T/4\pi$$

by using the value of (46.1±0.5) mb for σ_T, found in this laboratory by Law, Hutchinson and White [1]. This assumes a negligible contribution from the real part of the forward scattering amplitude. The elastic cross section, found by integration, is 20.7±1.2 mb.

The error involved in the normalization has been taken into account in the value quoted for the elastic cross section.

Table I. $p-p$ differential cross-section at 1 BeV

c.m. angle	$\sigma(\theta)$ mb/ster	Fitted curve
18°30′	11.52 ±0.28	12.33
24°36′	10.90 ±0.26	9.96
30°42′	7.76 ±0.21	7.60
36°42′	5.54 ±0.10	5.54
41°30′	4.13 ±0.10	4.19
48°30′	2.63 ±0.09	2.74
60°0′	1.51 ±0.04	1.46
71°6′	0.898±0.028	0.921
79°48′	0.719±0.036	0.696
90°0′	0.592±0.029	0.597

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PROTON-PROTON INTERACTIONS AT 2.85 BeV

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I. ELASTIC SCATTERING

The analysis of the proton-proton elastic scattering process has been done as part of an experiment recently completed by the Brookhaven Cloud Chamber group at the Cosmotron. The internal proton beam of the Cosmotron was scattered from a carbon target in the east straight section. Elastically (within 1%) scattered protons at $4.2 \pm 0.3°$ with the beam direction passed through the quadrant and fringing field and out through a thin window into an external shim. After further collimation the beam passed through two bending magnets and then traveled some 95-100 feet to the Brookhaven 20″ liquid hydrogen bubble chamber. Curvature measurements in the bubble chamber magnetic field gave 2.85 BeV as the beam kinetic energy. A portion of the total exposure has been scanned for all two prong events.

For scattering angles less than 3.5° (laboratory) the scanning efficiency is less than 100%. We have therefore computed the elastic scattering cross section for scattering angles greater than 3.5°. Our result is

$$\sigma(\theta \geq 3.5°) = 12.4 \pm 0.7 \text{ mb} .$$

In order to extrapolate the data to 0° we have fitted the measured angular distribution to the shape predicted by an optical model developed by Fernbach, Serber and Taylor [1]. The particular model used here assumes that the incident wave suffers pure diffraction scattering at a circular disk of radius R and is attenuated in amplitude from unity to an amplitude a. The data is shown in Fig. 1 and is seen to fit this shape well at smaller angles for R between 0.9 and 1.0 fermi. Using this shape to extrapolate to 0° we find

$$\sigma_{\text{elastic}} = 17.3 \pm 1.5 \text{ mb}$$

About one third of the quoted error is due to uncertainty in the extrapolation. The total cross section computed from the diffracting disc model and our data agree well with the measurements of Longo et al [2] at this energy.

If the initial scattering from the carbon target produces polarized protons and if the proton-proton elastic scattering has an analyzing power, the observed elastic scatterings will show a right-left asymmetry. To date we obtain

$$\frac{R-L}{R+L} = -5.1 \pm 3.4\,\%$$

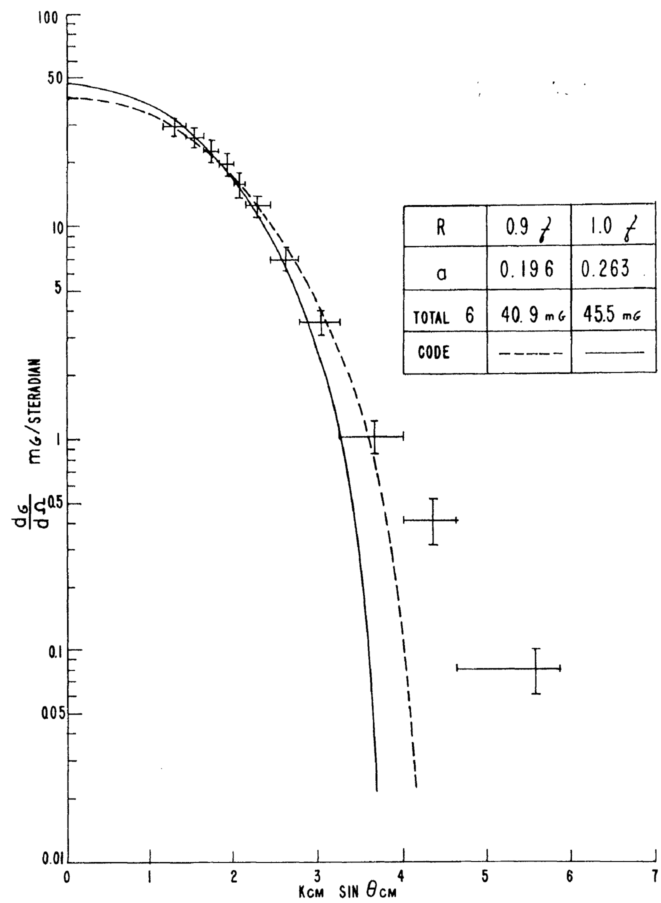

Fig. 1 $p-p$ differential elastic scattering cross section at 2.85 BeV.

for the scattering in hydrogen. The initial scattering on carbon was to the right.

II. SINGLE MESON PRODUCTION

A total of 924 inelastic two prong events have been measured in the same manner as the elastic events. A kinematic analysis of these events has been made using a least squares fit to one of the following four channels:

(1) $p+p \rightarrow p+p$

(2) $p+p \rightarrow p+p+\pi^0$

(3) $p+p \rightarrow p+n+\pi^+$

(4) $p+p \rightarrow d+\pi^+$

Unambigous separation appears to be possible in about 75% of the cases and in ambigous cases the best fit was chosen. Events which did not fit any of the above channels are assumed to be multiple meson production. Using these criteria we find 130 events of type 2, 482 events of type 3, 2 events of type 4 and 310 multiple meson production events.

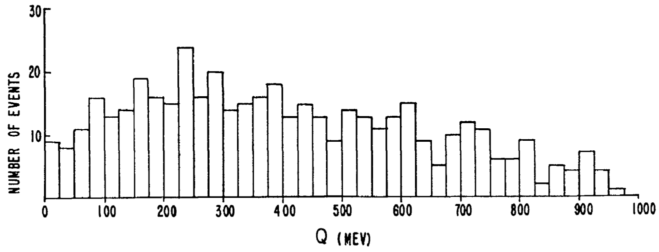

Fig. 3 Pion-neutron Q value distribution for $p+p \rightarrow \pi^+ + p + n$ at $T_{lab} = 2.85$ BeV.

The cross section for single meson production is found to be

$$\sigma_{(\text{single } \pi)} = 15.2 \pm 1.0 \text{ mb}$$

The center of mass angular distribution of the nucleons in type 3 events is shown in Fig. 2. The extremely sharp forward and backward peaking indicates that distant collisions are predominantly responsible for single meson production at this energy. There is no evidence for forward-backward asymmetry, outside of reasonable statistical fluctuations, which might indicate a bias in the identification. The $(\pi^+ - n)$ Q value distribution shown in Fig. 3 does not show any pronounced peaking. However, the $(\pi^+ - p)$ Q value distribution shown in Fig. 4 clearly shows that the single π^+ production is dominated by the $T = 3/2$, $J = 3/2$ resonance at a Q of about 150 MeV. About 40% of the data lies in this resonance region. Some slight evidence for higher resonances is apparent, although more events are required before any definite conclusion may be reached.

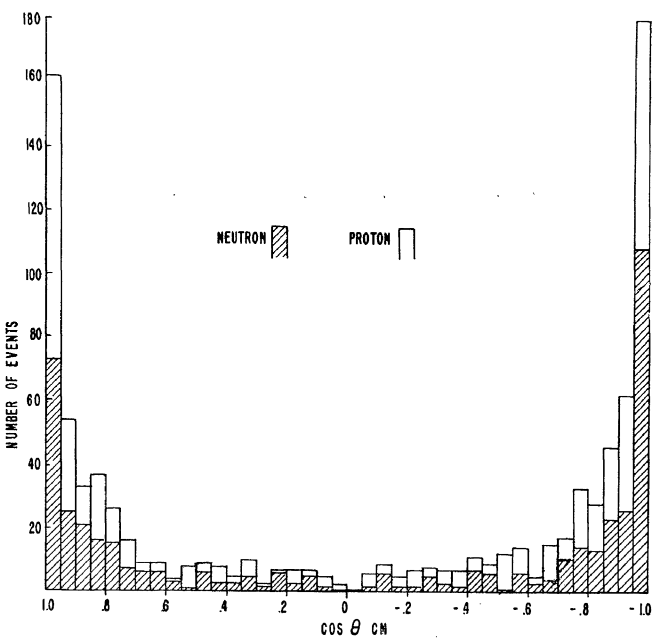

Fig. 2 Center-of-mass angular distribution for $p+p \rightarrow \pi^+ + p + n$ at $T_{lab} = 2.85$ BeV.

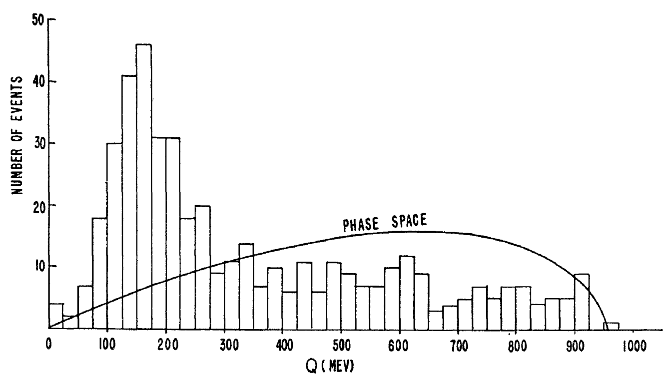

Fig. 4 Pion-proton Q value distribution for $p+p \rightarrow \pi^+ + p + n$ at $T_{lab} = 2.85$ BeV.

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EXTENSION OF THE ISOBARIC NUCLEON MODEL FOR PION PRODUCTION IN π-N, N-N, AND $\bar{\text{N}}$-N COLLISIONS

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We recently reported the method and some preliminary results [1] on our extension of the isobaric nucleon model by explicitly including two higher energy $T = \frac{1}{2}$ isobaric states of the nucleon (with mean masses $N_{2a}^* \approx 1.51$ BeV, $N_{2b}^* \approx 1.68$ BeV) in addition to the $T = J = {}^3/_2$ isobaric state of mean mass $N_1 \approx \approx 1.23$ BeV. N_1 was the only state originally explicitly included [2] although the proposed model was quite generally formulated. The presence of these two higher energy $T = \frac{1}{2}$ states is strongly implied by the behavior of the π-N interaction cross sections as well as the photo-pion production, and also the inelastic π-nucleon interactions [1]. It is then assumed that any of these three isobaric states can be intermediate states in the decay of the original compound state N_0^* formed by the π-N system (note that N_0^* can lie in the continuum above N_{2b}^*). It is then further assumed that transitions from one isobaric state to another are made via single pion emmission.

Although this extension of the model allows us to consider up to four pion final states in π-N collisions, we have at present considered only up to three pion final states in π-N collisions by making the simplifying assumption that if one initially starts in the continuum above N_{2b}^* that only N_{2b}^* or N_{2a}^* are intermediate states but not both in the same event. This simplification does not affect π-N interactions for incident pion energies of $\lesssim 1.1$ BeV, and at higher energies does not affect two pion final states but has an effect on $\sim 10\%$ of the three pion final states.

Table I gives the branching ratios for pion production in $\pi^+ + p$ interactions where the $\sigma_{2T,\alpha,m}$ are the cross sections which are characterized by the total isotopic spin $= T$; $\alpha = 1$ for N_1^*; $\alpha = 2$ for N_2^* where N_2^* represents the complex composed of the two $T = \frac{1}{2}$ isobaric levels N_{2a}^* and N_{2b}^* and their tails.

$m = s$ for production of a single additional pion, and is omitted when only single pion production is possible.

$m = d$ for double pion production, i.e., two additional pions.

Table I. Branching ratios for pion production in $\pi^+ - p$ interactions

Reaction	$N_1^* + \pi$	$N_2^* + \pi$
$(p+0)$	$^{13}/_{15}$	$^1/_3$
$(n++)$	$^2/_{15}$	$^2/_3$
$\bar{\sigma}$	σ_{31}	$\sigma_{32,\,s}$
$(p++-)$		$^5/_9$
$(p+00)$		$^2/_9$
$(n++0)$		$^2/_9$
$\bar{\sigma}$		$\sigma_{32,\,d}$

Table II. Branching ratios for pion production in $\pi^- - p$ interactions. $\bar{A} = \dfrac{A}{1+\varrho_2}$ where, ϱ_2, and a are defined in the paper by Lindenbaum and Sternheimer [1].

Reaction	$N_1^* + \pi$	$N_2^* + \pi$
$(n+-)$	$^5/_9 + {}^{26}/_{45}\varrho_1 + {}^7/_9 a$	$^2/_3 \bar{A}$
$(p0-)$	$^2/_9 + {}^{17}/_{45}\varrho_1 - {}^5/_9 a$	$^2/_3 - {}^1/_3 \bar{A}$
$(n00)$	$^2/_9 + {}^2/_{45}\varrho_1 - {}^2/_9 a$	$^1/_3 - {}^1/_3 \bar{A}$
$\bar{\sigma}$	$(^2/_3)\sigma_{11}$	$(^2/_3)\sigma_{12,\,s}$
$(p+--)$		$^5/_9 \bar{A}$
$(p00-)$		$^2/_9$
$(n+0-)$		$^5/_9 - {}^1/_3 \bar{A}$
$(n000)$		$^2/_9 - {}^2/_9 \bar{A}$
$\bar{\sigma}$		$(^2/_3)\sigma_{12,\,d}$

Table II gives the branching ratios for pion production in $\pi^- + p$ interactions. In this case due to the presence of both $T = \frac{1}{2}$ and $T = \frac{3}{2}$ cross sections it is necessary to introduce the four parameters ρ_1, ρ_2, ϕ_1, and ϕ_2 (see paper [1]) for their definitions).

For nucleon-nucleon interactions the present extension of the model would allow treatment of up to eight-pion final states (i.e., four from each nucleon). However, in order to allow comparisons with the available and future data in the incident proton energy range $\lesssim 3.0$ BeV, we have at present considered the simplifying assumption that only two pions at most are produced from each nucleon which allows up to four pion final states in nucleon-nucleon collisions. This does not affect the two-pion final states. However, the reactions considered are still expected to predominate strongly.

Table III lists the branching ratios for pion production in *p-p* interactions. We have also obtained the branching ratios in *n-p* interactions, but these are not shown as they are more complex due to the presence of both $T = 0$ and $T = 1$ cross sections and interference effects.

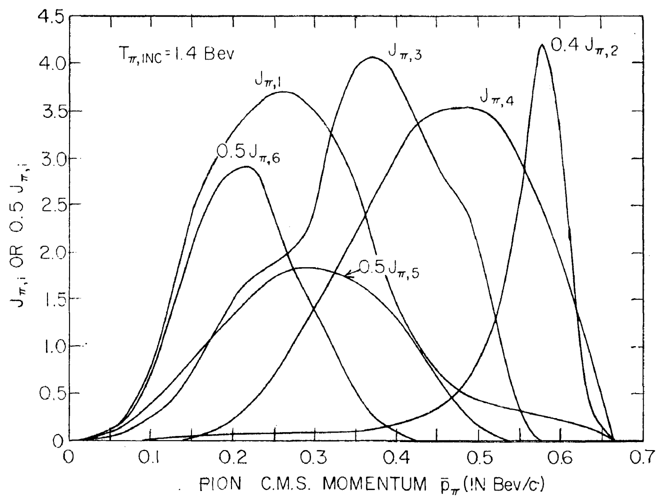

Fig. 1 Basic pion spectra for pion multiplicities of 1 to 6.

Table III. Branching ratios for pion production in $p-p$ interactions

Reaction	$N+N_1^*$	$2N_1^*$	$N+N_2^*$	$N_1^*+N_2^*$	$2N_2^*$
$(pn+)$	$^5/_6$		$^2/_3$		
$(pp0)$	$^1/_6$		$^1/_3$		
$\bar{\sigma}$	$\sigma_1(N+N_1^*)$		$\sigma_{1,s}(N+N_2^*)$		
$(pp+-)$		$^1/_5$	$^5/_9$	$^1/_2$	0
$(pp00)$		$^8/_{45}$	$^2/_9$	$^1/_{18}$	$^1/_9$
$(pn+0)$		$^{26}/_{45}$	$^2/_9$	$^7/_{18}$	$^4/_9$
$(nn++)$		$^2/_{45}$	0	$^1/_{18}$	$^4/_9$
$\bar{\sigma}$		$\sigma_1(2N_1^*)$	$\sigma_{1,d}(N+N_2^*)$	$\sigma_{1,d}(N_1^*+N_2^*)$	$\sigma_{1,d}(2N_2^*)$
$(pp+0-)$				$^7/_{27}$	$^5/_{27}$
$(pp000)$				$^1/_{27}$	$^2/_{27}$
$(pn++-)$				$^{25}/_{54}$	$^{10}/_{27}$
$(pn+00)$				$^2/_9$	$^2/_9$
$(nn++0)$				$^1/_{54}$	$^4/_{27}$
$\bar{\sigma}$				$\sigma_{1,t}(N_1^*+N_2^*)$	$\sigma_{1,t}(2N_2^*)$
$(pp++--)$					$^{25}/_{81}$
$(pp+00-)$					$^{20}/_{81}$
$(pp0000)$					$^4/_{81}$
$(pn++0-)$					$^{20}/_{81}$
$(pn+000)$					$^8/_{81}$
$(nn++00)$					$^4/_{81}$
$\bar{\sigma}$					$\sigma_{1,q}(2N_2^*)$

An analysis of the $\pi^- + p$ inelastic pion production experiments [1] at ~ 1.0 BeV incident pion kinetic energy has been performed and agreement with most of the dominant features has been obtained. Predictions for the momentum spectra of pions and nucleons for the various reactions as well as the branching ratios, etc. have been obtained for π-N collisions at 1.4 BeV. The pion spectra are composed of various sums of the six basic pion spectra $J_{\pi,1} - J_{\pi,6}$ shown in Fig. 1. $J_{\pi,1} - J_{\pi,4}$ are the same as in ref. [1], and determine the two pion final states. $J_{\pi,5}$ and $J_{\pi,6}$ enter into the three pion final states only and represent respectively the decay transitions :

$$N_2^* \to N_1^* + \pi; \quad N_1^* \to N + \pi.$$

Similar predictions for N-N interactions at 2.3 and 3.0 BeV have also been made.

Anti-isobars $\overline{N}_1^*$ have also been considered and this allows a prediction of pion production characteristics in inelastic N-$\overline{N}$ interactions, which do not involve annihilation. Results have been obtained for $\bar{p}$-p and $\bar{p}$-n meson producing interactions which proceed via N_1^* and $\overline{N}_1^*$ involving single or double pion production.

LIST OF REFERENCES AND NOTES

1. Lindenbaum, S. J. and Sternheimer, R. M. Phys. Rev. Letters **5**, p. 24 (1960). This paper contains the references to relevant experiments and other related work.
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