FUNDAMENTAL THERMAL ANALYSIS FOR CRYOGENIC SYSTEM DESIGN*

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Abstract
Non-uniform temperature distribution, surface roughness, and superfluid helium level change between 2K dewar and cryomodule are most important thermal analyses in designing cryogenic system. Effective temperature for non-uniform temperature distribution is defined. Thermal radiation property from surface roughness is shown as a function of arbitrary dimension between 2-dimension and 3-dimension. Superfluid helium level change between 2K dewar and cryomodule is shown as a function of vapor pressure. Our research can be useful thermal analyses for cryogenic system design.

INTRODUCTION
Properties of liquid helium were investigated intensively and the known-properties are enough to apply for cryogenic design. Superfluid fog [1] and multi-electron bubble in liquid helium were studied [2]. Size effect of thermal radiation [3, 4] and the effective temperature for non-uniform temperature distribution were investigated [5, 6]. Design of RAON cryogenic system is under development. RAON cryogenic system includes 2 K and 4 K cooling for cavity. In this research, we show fundamental thermal analysis which includes effective temperature, thermal radiation from fractional dimension, vapor pressure, properties of liquid helium, and leak test.

SRF TEST FACILITY
Superconducting Radio Frequency (SRF) test facility in RAON is under construction process. It consists of cryogenic system, clean room for cavity process and assembly, vertical cavity test, cryomodule test, and radiation shield for cavity and cryomodule test. Figure 1 shows the SRF test facility for RAON. The remodeling area for the facility is 1482 m² and the total electric power is 2500 kW. Cryogenic system consists of coldbox, dewar, compressors, pumps, oil removing system, distribution line, etc. Clean room consists of BCP, HPR, high vacuum furnace, cavity assemble place, etc. The power of cryoplant is 330W (4.5K equivalent) which supply 4.5K helium to cavity test and cryomodule test bench. Cavity test can be performed at 2 K since 4.5K liquid helium is supplied to cryostat and the liquid helium is being pumped to cool down.

EFFECTIVE TEMPERATURE
Blackbody radiation shows the temperature of the body for all the range of temperature from lowest temperature to highest temperature. For cryogenic system, temperature sensor is mainly made of semiconductor material such as Si. Temperature calibration of sensor is based on helium vapor pressure between 1.2 and 4.2 K, which has well-known relation between vapor pressure and temperature. The temperature sensor works at constant current mode, in which the resistance of sensor is increased exponentially as temperature decreases. The temperature sensor can be calibrated in the range of 1.2 K to 300 K. The accuracy of temperature is increased as temperature decreases. For non-uniform temperature distribution, the effective temperature of the body can be defined. The effective temperature of the body for segments of different temperature distribution can be generalized as [5]

\[ T_{\text{eff}} = \left( \sqrt[4]{\sum \nu_n \nu_n^4} \right)^{1/4}, \]

where \( \nu = \sum \nu_n \). Eq. (1) represents the definition of the effective temperature of discrete non-uniform temperature distribution. The effective temperature is higher than the average temperature.

EFFECT OF SURFACE ROUGHNESS
Thermal radiation is changed by surface roughness. In reality, the surface roughness shows fractional dimension. Thermal radiation depends on the dimension of the body.

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Energy density of blackbody radiation for arbitrary dimension can be generalized as [7]
\[ u_\beta(D, T) = \frac{4\pi^{D/2} \Gamma(D + 1) \zeta(D + 1)}{\Gamma(D/2)} \left( \frac{\hbar}{k_B T} \right)^{D-1} \]  
(2)
where \( \Gamma \) is the gamma function, \( \zeta \) is the Riemann zeta function, \( \hbar \) is the Planck constant, \( k_B \) is the Boltzmann constant, \( C \) is the speed of light, and \( D \) is the space dimension. From Eq. (2), the energy density can be calculated for arbitrary dimension and temperature. Figure 2 shows the energy density as a function of arbitrary dimension and temperature. Dimension increases as surface roughness increases. Energy density is increased as dimension increases for constant temperature. Energy density is also increased as temperature increases for constant dimension.

![Figure 2: Energy density is shown as a function of dimension and temperature.](image)

Heat conduction depends on the contact area between two bodies. In this case, the surface roughness can be negligible when the surface roughness is much smaller than the wavelength of thermal radiation. However, the surface roughness should be considered when the surface roughness is larger than the wavelength of thermal radiation. For cavity under microwave, the electric field is inversely proportional to the radius of curvature. Electron can mainly be emitted through field emission due to the focused electric field. Generalized electron emission from field and thermionic emission was investigated [8]. The emitted electrons are accelerated and then hit the Nb cavity wall, which generates X-ray through Bremsstrahlung radiation. So, it is important to reduce the surface roughness of Nb cavity by using BCP.

**PROPERTIES OF LIQUID HELIUM**

He I is classical fluid and He II is quantum fluid. Superfluid helium shows the lowest viscosity and the highest thermal conductivity. Superfluid helium can flow for the age of universe. Superfluid fog having negligible viscosity has lower Stokes’ drag compared to normal droplet or hard sphere [1]. The thermal conductivity of superfluid is the highest at 1.9 K. He I has very unstable surface and have lots of bubbles in bulk helium. He II has very stable liquid surface and has no bubbles at all.

Two-phase transfer of superfluid helium is very good technique to supply superfluid helium at the same temperature. Liquid helium at 2 K consists of superfluid and normal fluid according to two-fluid model. Normal fluid flows to lower temperature region and superfluid flows to higher temperature region when there is a temperature difference. Normal fluid has worst thermal conductivity and superfluid has the best thermal conductivity. Therefore, normal fluid contains a lot of bubbles due to non-uniform temperature distribution, but superfluid doesn’t have any bubble due to extremely uniform temperature distribution. Bubbles in liquid helium disappear immediately at 2.172 K when liquid helium cools down.

For normal conductor, electrical conductivity is proportional to thermal conductivity since electrons play important role for electrical conductivity and thermal conductivity. Insulator such as glass has low electrical conductivity as well as low thermal conductivity. However, diamond which is the best electrical insulator shows very high thermal conductivity because phonons in diamond transfer heat wave very effectively.

Cavity made of Nb for QWR will be operated at 4.2 K and cavity for HWR, SSR1, and SSR2 will be operated at 2 K. The operation frequencies for QWR, HWR, SSR1, and SSR2 are 81.25, 162.5, 325, and 325 MHz, respectively. The applied electric field is 35MV/m. Compared to 4.2K operation, the fluctuation of temperature and vapor pressure becomes much lower for 2 K operation.

**VAPOR PRESSURE**

Dynamic equilibrium means that evaporation rate is the same as condensation rate. Vapor pressure of liquid helium is determined by temperature as
\[ P_e(T) = P_o \exp\left(-\frac{L}{k_B T}\right), \]  
(3)
where \( L \) is the latent heat, \( P_o \) is the constant, and \( R \) is the gas constant. Evaporation rate increases as the temperature of liquid helium increase. The flow of liquid helium can be changed by gravity and heat. Gravity makes the level of liquid helium at the same height when two liquid helium reservoirs are connected through a pipe. When there is a level difference, the pressure of liquid helium is built as \( \Delta P = \rho g \Delta T \) where \( g \) is the gravitational constant, \( \rho \) is the density of liquid helium, and \( \Delta T \) is the height of liquid helium level. Liquid helium level is generally measured with two methods. One is to measure the resistance of superconducting wire and the other is to measure the capacitance of liquid helium. When there is temperature difference, superfluid can flow through small porous. Pressure difference due to temperature difference is formed as \( \Delta P = S \rho \Delta T \) where \( S \) represents the entropy. In bulk helium level, the temperature difference does not make pressure difference. Superfluid moves to higher temperature region and normal fluid moves to lower temperature region. So, the temperature difference does not make pressure difference in bulk liquid helium. Think about liquid helium levels at two reservoirs at different
temperature. At higher temperature region, vapor pressure is increased by evaporation. The liquid helium under high vapor pressure moves to lower temperature region. The height difference of liquid helium can be expressed with the vapor pressure:

\[ (h_1 - h_2) = \left( \frac{P_0}{\rho g} \right) \left[ e^{-\frac{L}{RT_2}} - e^{-\frac{L}{RT_1}} \right], \]  

(4)

where \( h_1 \) and \( T_1 \) represents the height of liquid helium and temperature, respectively at region 1, and \( h_2 \) and \( T_2 \) represents the height of liquid helium and temperature, respectively at region 2. Evaporation is high at higher temperature. Because heat conduction is very high at 2K, the temperature of each cavity becomes almost same. If \( T_1 > T_2 \), vapor pressure at \( T_1 \) is higher than that of \( T_2 \) since evaporation rate is high at \( T_1 \). Two temperatures become equalized if pumping rate is much high compared to evaporation rate.

For vertical test, liquid helium is filled to the cryostat. The cryostat has liquid nitrogen shield and vacuum jackets in which multilayer insulations are installed. Once liquid helium is filled to cryostat for cavity test, and then pumps it down to 2K, the level of liquid helium can be reduced by half. We can think isolated liquid helium pumps it down to 2K, the level of liquid helium can be removed from the bulk helium as

\[ \frac{\partial m}{\partial t} = -(m) \frac{\partial T}{\partial t} + \left( \frac{1}{C(T)\tau} \right) \frac{\partial Q}{\partial t}, \]  

(5)

where \( C \) is the specific heat of liquid helium and \( Q \) is the total heat. The specific heat of liquid helium is highest at lambda point, 2.172 K. From 2 K to 2.172 K, it requires higher heat load to destroy superfluid property. The evaporation rate of liquid helium at 1 g/s corresponds to 20W heat removal efficiency. The latent heat of liquid helium is removed by evaporation. Pumping reduces the vapor pressure of liquid helium, which reduces the temperature of liquid helium since heat is removed by forced-evaporation.

**LEAK TEST**

STS 316L is commonly used for cryostat and STS 304 is used for helium gas pipe. Tungsten Inert Gas (TIG) welding techniques is used for the welding for STS 316L and STS 304. Leak should be tested after Tungsten Inert Gas (TIG) welding. Spraying, sniffing, and pressurizing techniques are commonly used for the leak test of cryogenic system. Leak can be tested at room temperature and liquid nitrogen temperature. We can also use thermal shock test with liquid nitrogen for more than three times. There is negligible difference between 77K and 4K for thermal contraction. So, in most cases, the result of leak test is similar between liquid nitrogen test and liquid helium test. However, superfluid helium at 2K can cause super leak since superfluid can flow without viscosity even tiny holes. When there is a leak, we can find the leak in many ways. Here, we mention that leak can be found by minimizing responds time between spraying helium gas and detecting leak, and maximizing leak level in spraying method. For instance, when helium gas is sprayed away from a leak place, the responds time is long and leak detection level is low. However, the responds time becomes short and leak detection level becomes high when helium gas is sprayed at a leak place.

**SUMMARY**

We have shown the effective temperature for non-uniform temperature distribution, thermal radiation property from surface roughness, helium level change between 2K dewar and cryomodule, and leak detection techniques. Effective temperature for non-uniform temperature distribution was introduced and the energy density for thermal radiation was shown as a function of dimension and temperature. Liquid helium level between 2K dewar and cryomodule was shown in terms of vapor pressure difference. Our research can be useful thermal analyses for cryogenic system design.

**REFERENCES**


