Non-local SFT Tachyon and Cosmology

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Abstract

In this note the recent papers [1] studying cosmological scenarios built upon the generalized non-local String Field Theory and p-adic tachyons are shortly reviewed. A late time behavior of the tachyon and the scale factor of the FRW metric in the presence of the cosmological constant is discussed.

Contemporary cosmological observational data strongly support that the present Universe exhibits an accelerated expansion providing thereby an evidence for a dominating Dark Energy (DE) component. Recent results of WMAP [2] together with the data on Ia supernovae give the following bounds for the DE state parameter:

\[ w_{DE} = -0.97^{+0.07}_{-0.09} \]

or without an a priori assumption that the Universe is flat and together with the data on large-scale structure and supernovae \( w_{DE} = -1.06^{+0.13}_{-0.08} \).

The phantom divide \( w = -1 \) separates the quintessence models, \( w > -1 \), the cosmological constant, \( w = -1 \), and the "phantom" models, \( w < -1 \). In the latter case all natural energy conditions are violated and there are problems of instability both at the classical and quantum levels.

Experimental data, as we see, do not contradict a possibility \( w < -1 \). Studying of such models attracts a lot of attention. Some projects explore whether \( w \) varies with the time or is an exact constant. Varying \( w \) obviously corresponds to a dynamical model of the DE which generally speaking includes a scalar field. An excellent review [3] and references therein may provide the reader with a more detailed discussion of the DE dynamics.

Models with a crossing of the \( w = -1 \) barrier are also a subject of recent studies. Simplest ones include two scalar fields (one phantom and one usual field). General \( \kappa \)-essence models can have both \( w < -1 \) and \( w \geq -1 \) but a dynamical transition between these domains is forbidden under general assumptions and is possible only under special conditions.

In the present note we investigate cosmological applications of a scalar field model coming from the String Field Theory (SFT) tachyon dynamics (see [4] for a review) described by the action

\[ S = \frac{1}{g_0^2} \int d^d x \left( \frac{1}{2} \bar{\Phi} \mathcal{F}(\Box) \Phi - \frac{1}{p + 1} \Phi^{p+1}(x) \right). \]

Here \( g_0 \) is a coupling constant. Function \( \mathcal{F}(\Box) \) is not specified explicitly. Such a theory describes in particular an effective open SFT tachyon as well as a p-adic formulation of

\[ [1] \text{also contains the more comprehensive list of references.} \]
the tachyon dynamics. In both these examples a kinetic operator $\mathcal{F}$ gives a non-local action.

The cosmological model incorporating the non-local dynamics of the open SFT tachyon field was proposed in [5]. This model is based on the SFT formulation of the fermionic NSR string with the GSO-sector. A characteristic feature of this model in the flat background is the presence of a rolling tachyon solution. In the bosonic SFT, however, such a solution does not exist at least in the flat space. It has been shown that the non-locality provides a crossing of the $w = -1$ barrier in spite of the presence of only one scalar field and being a string theory limit the model addresses all stability issues to the string theory.

Below we sketch the main distinguishing cosmological properties of the models coming from the above mentioned action with a general operator $\mathcal{F}$ and an arbitrary parameter $p > 1$. Cosmological scenarios are given by the following action which accounts a minimal coupling of the fermionic SFT tachyon to the gravity

$$S = \int d^nx \sqrt{-g} \left( \frac{R}{2\kappa^2} + \frac{1}{g_0^2} \left( \frac{1}{2} \mathcal{F}(\Box_g)\Phi - \frac{1}{p+1} \Phi^{p+1}(x) - \Lambda_{\Phi} - T \right) \right).$$

Here $\Box_g$ is the Beltrami-Laplace (BL) operator and $\mathcal{F}$ is an analytic function of its argument on the complex plane such that $\mathcal{F}(0) = 1$ and $g$ is the four dimensional spatially flat FRW metric which can be written as $g_{\mu\nu} = \text{diag}(-1,a^2,a^2,a^2)$ with $a = a(t)$ being a space homogeneous scale factor. In this particular case the BL operator is expressed as $\Box_g = -\partial_i^2 + 3H \partial_0 + \frac{1}{a^2} \partial_i^2$, where $H \equiv \dot{a}/a$ is the Hubble parameter and the dot denotes the time derivative. $\kappa$ is a gravitational coupling constant $\kappa^2 = 8\pi G = \frac{1}{M_p^2}$ and we choose such units that it is dimensionless, $T$ encodes perfect fluids which may be considered including the cosmological constant $\Lambda$. $\Lambda_{\Phi}$ is also a constant but we separate it from the cosmological one. It is considered as a part of a scalar field potential so that in the picture where the scalar field potential has a non-perturbative minimum $\Lambda_{\Phi}$ cancels its energy. We define for the sequel $m_p^2 \equiv g_0^2 M_p^2$. Though this is obviously not a full theory which may come form open-closed string interactions it is obvious that such a minimal coupling gives a starting point to have an insight into the problem of an open string modes behavior in a curved space-time.

We discuss only a time-dependent scalar field as well. Thus we can think about the BL operator just as $\mathcal{D} = -\partial_i^2 - 3H \partial_0$. If all the fluids in the latter action are coupled one to each other only through the gravity then independent equations of motion following in this case are

$$\mathcal{F}(\mathcal{D})\Phi = \Phi^p, \quad \frac{\ddot{a}}{a} = -\frac{1}{6m_p^2} \left( \rho_\Phi + 3p_\Phi + \sum_i (1 + 3w_i)\rho_i \right).$$

Here $\rho$-s are energies of fluids and $i$ enumerates perfect fluids coming from the $T$-term in the action. $\rho_\Phi$ and $p_\Phi$ account $\Lambda_{\Phi}$. Also we assume $w_i$ being constants. Note that $w_\Phi$ which is defined through $p_\Phi = w_\Phi \rho_\Phi$ is not a constant.

Under our assumptions there is a non-perturbative vacuum at $\Phi = 1$. In an important case of odd $p$ we also have a symmetry $\Phi \to -\Phi$ so that $\Phi = -1$ also becomes a vacuum. A potential is $V = -\frac{1}{2} \Phi^2 + \frac{1}{p+1} \Phi^{p+1} + \Lambda_{\Phi}$. In cubic super SFT one gets $p = 3$. The zero value of the potential in the minimum is assured by choosing $\Lambda_{\Phi} = \frac{p-1}{2(p+1)}$.

The picture we have in mind is a rolling tachyon which starts rolling from an unstable perturbative vacuum $\Phi = 0$ and approaches a non-perturbative one in an infinite time.
Thus an asymptotic we are going to study is \( \Phi = 1 - \psi \). A linearization around the true vacuum gives the following action

\[
S = \frac{1}{g_s^2} \int dx \sqrt{-g} \left( \frac{1}{2} \psi \mathcal{F}(\mathcal{D}) \psi - \frac{\rho}{2} \psi^2 \right).
\]

This can be considered as an effective action in the true vacuum of the SFT. According to the Sen conjectures we expect that there should not be open string excitations. This is simply translated here imposing that operator \( \mathcal{F}(\mathcal{D}) - \rho \) has no zeros for finite \( \omega^2 \) which are eigenvalues of the BL operator.

An approach we use is based on the Weierstrass product method. Using it one can show that the latter action is equivalent to the following one

\[
S = \frac{1}{g_s^2} \int dx \sqrt{-g} \frac{1}{2} \sum_k (\varepsilon_k \psi_k (\mathcal{D} - \omega_k^2) \psi_k + \bar{\varepsilon}_k \bar{\psi}_k (\mathcal{D} - \omega_k^{2*} \bar{\psi}_k)).
\]

where \( \omega \)'s come as solutions to the algebraic characteristic equation \( \mathcal{F}(\omega^2) = \rho, \varepsilon_k = \mathcal{F}'(\omega_k^2) \) and \( \bar{\varepsilon}_k = \mathcal{F}'(\omega_k^{2*}) \). On-shell \( \psi_k = \alpha_+ \psi_{k+} + \alpha_- \psi_{k-} + \bar{\alpha}_+ \psi_{k+} + \bar{\alpha}_- \psi_{k-} \) where \( \alpha \)'s are integration constants to be adjusted giving real \( \psi_k \).

Computing the energy-momentum tensor for the field \( \psi \) one gets

\[
\rho_\psi = \frac{K + P}{2}, \quad p_\psi = \frac{K - P}{2}, \quad K = \sum_{n=1}^{\infty} c_n \sum_{l=0}^{n-1} \partial_l \mathcal{D}_l \psi \partial_i \mathcal{D}^{n-1-i} \psi, \quad P = \sum_{n=1}^{\infty} c_n \sum_{l=0}^{n-1} \mathcal{D}_l \psi \mathcal{D}^{n-1-l} \psi
\]

where \( c_n \) come from \( \mathcal{F}(z) = c_n z^n \). Taking \( \psi = \sum_k (\psi_{k+} + \psi_{k+}^* + \psi_{k-} + \psi_{k-}^*) \) one yields

\[
K = \sum_k \left( \mathcal{F}'(\omega_k^2) (\psi_{k+} + \psi_{k-})^2 + \mathcal{F}'(\omega_k^{2*}) (\psi_{k+}^* + \psi_{k-}^*)^2 \right),
\]

\[
P = \sum_k \left( \omega_k^2 \mathcal{F}'(\omega_k^2) (\psi_{k+} + \psi_{k-})^2 + \omega_k^{2*} \mathcal{F}'(\omega_k^{2*}) (\psi_{k+}^* + \psi_{k-}^*)^2 \right).
\]

Here prime denotes a derivative with respect to an argument. The main achievement at this stage is that all the information can be extracted by means of solving algebraic characteristic equation and constructing of eigenfunctions of the BL operator. Sum over \( k \) is indefinite until a subset of eigenfunctions of interest is not specified.

There are two remarkable properties worth to mention. First, the coefficients can be expressed entirely in terms of function \( \mathcal{F}(z) \) without annoying summations. This is a great simplification and an opening for a possibility of studying general operators \( \mathcal{F} \). Second, there are no mixed terms involving \( \psi \)-s for different \( k \) as well as \( \psi_i \psi_j^* \) combinations.

We stress that the above analysis does not depend on a background and applicable to a general form of the BL operator.

In the most interesting case of the tachyon field and a cosmological constant \( \Lambda \) the following behavior of the cosmological quantities may be deduced.

\[
\Phi = 1 - \alpha e^{-rt} \cos(\nu t + \varphi)
\]

\[
a(t) = a_0 e^{H_0 t} + \frac{e^{(H_0 - 2r)t}}{24m^2_p (r^2 + \nu^2)((H_0 - r)^2 + \nu^2)} \times
\]

\[
\times ((\nu^2 - r^2 + H_0 r)(2\alpha_K \sin(2\nu t + \varphi_K) - \alpha_P \sin(2\nu t + \varphi_P)) +
\]

\[
+ (H_0 - 2r)\nu(2\alpha_K \cos(2\nu t + \varphi_K) - \alpha_P \cos(2\nu t + \varphi_P))
\]
Here $r$, $\nu$, $\varphi$-$s$, and $\alpha$-$s$ are integration constants. Now one can readily get resulting expressions for $H$ and the state parameter. It is interesting that playing with $r$ and $\nu$ one can observe different behaviors. For instance, for $r = H_0/2$ oscillations in $\alpha(t)$ will not die despite the fact that oscillations in $\Phi$ vanish. On the other hand it is impossible to avoid oscillations completely. These oscillations will be translated to $\omega_p$ and effective total state parameter $\omega$ making possible periodic transition between phantom and quintessence phases.

Thus we have shown explicitly that a periodic crossing of the phantom divide may occur as well as a possibility of non-vanishing oscillations of the cosmological quantities even if the scalar field tends to its vacuum.

It would be very interesting to continue the lines of the present analysis and try to formulate a local action in case of a full non-linear model. If possible, this will open a way to make use of numeric methods in analyzing the model. One natural question to be answered in this way whether the early time dynamics may generate some cosmic fluids which decouple at the late stage of the tachyon evolution. This will justify an appearance of an additional term $T$ in the action at large times.

Also it would be interesting to invert the problem and see how different forms of operator $\mathcal{F}$ and the potential affect the cosmology. This may shade light on a structure of an effective tachyon action if higher excitations are taken into account in the SFT.

Another generalization is an inclusion of closed string scalar fields, the closed string tachyon and dilaton in the analysis to understand a role of closed string excitations and probably explain the $T$ term.

**Acknowledgments.** The author is grateful to the organizers for the warm hospitality and financial support. The work and participation is supported in part by Marie Curie Fellowship MIF1-CT-2005-021982, EU grant MRTN-CT-2004-512194, RFBR grant 05-01-00758, INTAS grant 03-51-6346 and Russian President’s grant NSh-2052.2003.1.

**References**


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