Evaluation of the broadband longitudinal impedance of the CERN PS

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Summary

The CERN Proton Synchrotron (PS) produces beams well beyond its original parameters and especially the high-brightness beams delivered to the Large Hadron Collider are close to present stability limitations. Within the framework of the LHC Injectors Upgrade project, a significant increase in beam intensity is planned. With this background, collective effects triggered by the self-induced electromagnetic fields, which are generated by the interaction of the beam with the surrounding environment, may play an important role in beam stability and machine performance.

In this paper we study the short range wakefields and the corresponding longitudinal broadband coupling impedance of the PS by measuring the incoherent quadrupole synchrotron frequency as a function of beam intensity and comparing the results with the evaluation of the contribution of several machine installations to the total impedance budget.
1 Introduction

The longitudinal broadband impedance is an important parameter to characterize the electromagnetic coupling of a particle beam with the surrounding elements like kickers, radio-frequency (RF) cavities, diameter changes of the beam pipe, bellows and many other installations close to the beam. The measured broadband impedance can be compared to the expected impedance, individually evaluating the contributions of the relevant components. This may show if all major contributing devices are sufficiently well understood. Additionally, in the framework of the LHC Injectors Upgrade (LIU) project, important hardware changes like new injection elements for an increased injection energy are planned [1]. Based on the measurements presented in this paper, it will be possible to assess their impact on the broadband impedance.

First estimations of the coupling impedance in the CERN Proton Synchrotron (PS) date back to the late 1970s. These measurements, based on longitudinal stability during debunching and on quadrupole beam transfer functions [2, 3], were already indicating an impedance in the range of $Z(p)/p \approx 25 \Omega$. A more refined measurement campaign of the broadband impedance [4], employing the same beam transfer function technique as used for the measurements presented in this paper, took place in the framework of the preparation of the PS for its role in the LHC injector chain. A broadband impedance of $Z(p)/p = (21.7 \pm 5.1) \Omega$ was obtained.

With the improvement of the measurement techniques and a better control of the longitudinal beam parameters, notably the capability to produce bunches with constant longitudinal emittance over a wide intensity range, this paper presents the results of the latest measurement campaign. Thereafter the evaluation of the contribution of the main installations to the total longitudinal coupling impedance is shown.

The measurements have been performed by observing the quadrupolar beam transfer function and deducing the zero amplitude synchrotron frequency. This synchrotron frequency is then measured versus bunch intensity. Similar longitudinal broadband impedance measurement campaigns in the SPS were regularly performed, but by injecting a mismatched bunch and observing its quadrupole oscillations [5] instead of exciting the beam with noise.

In the following section we show the measurement set-up and the results of the data analysis, which allowed us to obtain an inductive broadband impedance model for the PS machine. The coupling impedance has been used in a simulation code (section 3) to reproduce the measurement results by means of what we can call a "virtual experiment", which confirmed the validity of the adopted model. Finally, in section 4, we analyze the main machine elements that give an important contribution to the total coupling impedance, and compare the results with those obtained from the measurements.

2 Measurements

The measurements of the quadrupole synchrotron frequency were performed at a fixed momentum of 26 GeV/c and with a single-harmonic RF system at 40 MHz. A single bunch with an intensity of up to $N_p = 4.5 \cdot 10^{11}$ protons was injected from the PS Booster and accelerated in the PS on the 16th harmonic ($h = 16$) of the revolution frequency to a momentum of
26 GeV/c. On the flat-top, the bunch was first synchronized to a fixed revolution frequency of $f_{\text{rev}} = 476.82$ kHz, allowing to pulse a higher-harmonic RF cavity at 40 MHz, the 84th harmonic of the revolution frequency. About 150 ms before extraction the bunch was handed over from $h = 16$ ($f_{\text{RF}} = 7.629$ MHz) to $h = 84$ ($f_{\text{RF}} = 40.052$ MHz). This rebucketing to the 40 MHz RF system was completed 140 ms before extraction. Aside from a 5 ms time window for longitudinal emittance measurements, about 130 ms were left under stationary conditions to perform the beam transfer function (BTF) measurement. During that time, the bunch was held with a constant 40 MHz RF voltage of about 50 kV or 100 kV at $h = 84$. It is worth noting that no beam phase loop was active during the measurements. The length of the magnetic flat-top and hence the measurement duration could not be stretched to avoid a too large heating of the main and auxiliary coils of the magnets, and averaging over many cycles was applied to improve the quality of the measurements.

2.1 Measurement setup

The spectrum of the incoherent quadrupole synchrotron frequency has been obtained by measurement of the longitudinal BTF with the set-up sketched in fig. 1. A bandwidth limited white noise (up to 2 kHz) was generated by the internal source of an Agilent 89410A vector network analyzer. The bandwidth was chosen to fully cover the quadrupole synchrotron frequency and to achieve the best possible resolution within the 130 ms time window for the measurements. The noise signal was gated in order only to affect the beam during the well-defined duration of the measurement and was added as an amplitude modulation to the voltage program of the 40 MHz cavity. Since the peak amplitude of the noise was independent of the voltage in the 40 MHz RF cavity, the amplitude modulation index was about $m = 0.07$ for a peak voltage of 100 kV and $m = 0.13$ for an RF voltage of 50 kV at 40 MHz. A copy of the noise signal used for amplitude modulation was directly fed to the reference channel of the vector network analyzer. Taking the noise signal to the reference

![Figure 1: Set-up to measure the longitudinal quadrupole beam transfer function (BTF).](image-url)
channel directly from the source and not from the gap of the 40 MHz cavity did not introduce a significant error, as the 3 dB bandwidth of the voltage control loop was measured to be of the order of 20 kHz. The delay between noise source and cavity gap does not introduce any significant phase shift in the relevant frequency range up to 2 kHz.

A wall-current monitor (WCM95) was used to pick-up the longitudinal beam signal, followed by a peak detector with a time constant of several turns, but well below the quadrupole synchrotron tune. The attenuation of the beam signal was chosen to get optimum signal amplitude to the peak detection circuit and special care was taken to avoid saturation. The peak-detected beam signal was then fed to the second channel of the vector network analyzer (see fig. 1). The gating switch in front of the second input protects the measurement instrument from over-voltage which may occur outside the measurement time window.

Amplitude and phase of the quadrupole BTF are then determined by the analyzer by calculating the vectorial ratio of peak-detected beam signal and noise excitation and subsequent averaging of measurements on many individual acceleration cycles. A typical quadrupole synchrotron frequency spectrum is shown in fig. 2.

![Figure 2: Example screen-shot of amplitude and phase of the quadrupole BTF for a bunch of about $9 \cdot 10^{10}$ particles kept by 95 kV at 40 MHz.](image)

The zero-amplitude quadrupole synchrotron frequency is given by the discontinuity of the phase curve following the 180° phase advance [6, 7]. This well-defined point of the phase of the transfer function, when the beam response vanishes, is readily determined from the measurements. The small peak in both amplitude and phase, at half that frequency, is a direct observation of the synchrotron frequency due to residual oscillations.
2.2 Data analysis

Two series of single bunch measurements were made during two different machine development sessions. The momentum of $p_0 = 26$ GeV/c was chosen to minimize the effects of space charge as will be shown in section 4. We repeated the measurements at different RF peak voltages. In table 1 we show the resulting incoherent quadrupole frequency shift $f_{2s}$. Each measurement was taken by averaging over 16 acceleration cycles with approximately the same intensity and longitudinal emittance. Even with control of the intensity by transverse shaving in the PS Booster, a slight increase of the longitudinal emittance at higher intensities is difficult to avoid. For some measurements we then decided to use an additional controlled longitudinal emittance blow-up with a phase-modulated RF system at 200 MHz [8].

During the measurements we recorded the longitudinal bunch profile in order to obtain the bunch length at each intensity, a parameter necessary for determining the machine broadband impedance. Depending on the kind of fit used for the bunch shape, Gaussian or parabolic line density, the bunch length is shown in the same table: for the Gaussian model we quote the standard deviation $\sigma_G$, and for the parabolic one the total length $\tau_b$. Both models fit well the measured bunch profile, as shown in fig. 3.

![Figure 3: Example of Gaussian and parabolic fit of the longitudinal measured bunch shape.](image)

The results of the measurements have been used to obtain the low-frequency longitudinal impedance of the machine. To do that, we first write the longitudinal equation of motion of a single particle in presence of the self-induced wakefields as [9]

$$\ddot{\tau} + \omega_{s0}^2 \tau = \frac{eN_p\omega_{s0}^2}{2\pi V_{RF}h \cos \phi_s} \sum_{p=-\infty}^{\infty} Z(p\omega_0)\sigma_0(p\omega_0)e^{ip\omega_{s0}t},$$  \hspace{1cm} (1)

with $\tau$ the position of the particle with respect to the synchronous one, $\omega_{s0}$ the natural synchrotron frequency, $h$ the harmonic number, $\phi_s$ the synchronous phase ($\cos \phi_s < 0$ above transition), $\omega_0$ the revolution frequency, $p\omega_0 = \omega$, $Z(\omega)$ the longitudinal broadband impedance,
Table 1: Measured incoherent quadrupole frequency shift and bunch length (standard deviation $\sigma_G$ for Gaussian fit, total length $\tau_b$ for parabolic line density fit) at different beam intensities and RF peak voltages.

<table>
<thead>
<tr>
<th>$N_p \times 10^{11}$</th>
<th>$V_{RF} \text{ (kV)} \pm 5%$</th>
<th>$f_{2s} \pm 12 \text{ (Hz)}$</th>
<th>$\sigma_G \text{ (ns)}$</th>
<th>$\tau_b \text{ (ns)}$</th>
</tr>
</thead>
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<tr>
<td>1.40 ± 0.03</td>
<td>80</td>
<td>960</td>
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<td>9.07 ± 0.04</td>
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<td>2.73 ± 0.008</td>
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<td>0.90 ± 0.02</td>
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<td>1040</td>
<td>1.97 ± 0.02</td>
<td>7.71 ± 0.09</td>
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</tbody>
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and $\sigma_0(\omega)$ the bunch spectrum of the stationary distribution. For a Gaussian distribution with standard deviation $\sigma_G$, the bunch spectrum is

$$
\sigma_0(\omega) = e^{-\omega^2 \sigma^2_G},
$$

(2)

whereas, for a parabolic line density of total length $\tau_b$, it is given by

$$
\sigma_0(\omega) = \frac{3\sin(\omega \tau_b/2) - \omega \tau_b/2 \cos(\omega \tau_b/2)}{(\omega \tau_b/2)^3}.
$$

(3)

In the absence of wakefields, linear synchrotron motion has been assumed. For the theory, the RF voltage is considered linear within the bunch duration and all the particles oscillate at the same frequency $\omega_0$ independently of their amplitude. The effect of stationary wakefields in eq. (1) is to introduce a change in the synchronous phase, an incoherent synchrotron frequency shift, and other non-linear terms which make the frequency shift amplitude dependent, or, equivalently, they produce a synchrotron frequency spread.

This can be easily seen if we make a series expansion of the exponential $e^{ip\omega_0\tau}$:

$$
\dot{\tau} + \omega_{d0}^2 \tau = \frac{eN_p \omega_0^2}{2\pi V_{RF} h \cos \phi_s} \sum_{p=-\infty}^{\infty} Z(p\omega_0) \sigma_0(p\omega_0) \left(1 + ip\omega_0\tau - \frac{(p\omega_0\tau)^2}{2} + \ldots\right). \tag{4}
$$

The constant term (independent of $\tau$) on the right hand side of eq. (4) produces a phase shift in combination with the real part of the impedance, which is an even function of the frequency, and it is given by

$$
\Delta \phi = \hbar \omega_0 \Delta \tau = \frac{eN_p \omega_0}{2\pi V_{RF} h \cos \phi_s} \sum_{p=-\infty}^{\infty} \text{Re} \left[Z(p\omega_0)\sigma_0(p\omega_0)\right]. \tag{5}
$$

We recall that the phase shift given by the above expression is based on the linear expansion of the equation of motion and has to be considered as an approximation. In fact, if we use, for example, a broadband resonator characterized by a shunt resistance $R_s$, a quality factor $Q$ and a resonant frequency $\omega_r$, with a Gaussian distribution, by replacing the summation with an integral, we get

$$
\Delta \phi = \frac{\pi eN_p R_s}{V_{RF} \cos \phi_s \sqrt{Q^2 - 1/4}} \text{Re} \left[\omega_1 e^{-\omega^2 \sigma^2_G} \text{Erf}(-i\omega_r \sigma_G)\right]. \tag{6}
$$

with

$$
\omega_1 = \frac{\omega_r}{Q} \left(\frac{i}{2} + \sqrt{Q^2 - \frac{1}{4}}\right). \tag{7}
$$

By using instead the loss factor of a Gaussian bunch coupled with a resonator impedance, the correct synchronous phase shift is slightly modified as

$$
\Delta \phi = \frac{eN_p R_s}{2V_{RF} \cos \phi_s \sqrt{Q^2 - 1/4}} \text{Re} \left[\omega_1 e^{-\omega^2 \sigma^2_G} \text{Erf}(-i\omega_r \sigma_G)\right]. \tag{8}
$$
Eq. (4) can also be used to obtain an approximate expression of the incoherent syn-
chrontron frequency shift. In fact, if we consider the first order term in \( \tau \) of eq. (4), the
oscillation frequency of a particle in the bunch becomes

\[
\omega_s^2 - \omega_s_0^2 = \frac{e N_p \omega_0}{2 \pi V_{RF} h} \sum_{p=-\infty}^{\infty} \text{Im}[Z(p \omega_0)] p \sigma_0(p \omega_0) ,
\]

that is

\[
\omega_s^2 - \omega_s_0^2 = \frac{e N_p \omega_0}{2 \pi V_{RF} h \cos \phi_s} \sum_{p=-\infty}^{\infty} \text{Im}[Z(p \omega_0)] p \sigma_0(p \omega_0) . \tag{10}
\]

If the incoherent frequency shift is small compared to the unperturbed synchrotron fre-
quency (\( \Delta \omega_s \ll \omega_s_0 \)), we can approximate by

\[
\frac{\omega_s - \omega_s_0}{\omega_s_0} = \frac{\Delta \omega_s}{f_s_0} \simeq \frac{e N_p \omega_0}{4 \pi V_{RF} h \cos \phi_s} \sum_{p=-\infty}^{\infty} \text{Im}[Z(p \omega_0)] p \sigma_0(p \omega_0) . \tag{11}
\]

In the case of a pure inductive broadband impedance, we finally obtain

\[
\frac{\Delta f_s}{f_s_0} = \frac{e N_p \omega_0}{4 \pi V_{RF} h \cos \phi_s} \frac{\text{Im}[Z(p)]}{p} \sum_{p=-\infty}^{\infty} p^2 \sigma_0(p \omega_0) , \tag{12}
\]

which relates the incoherent dipole synchrotron frequency to the inductive machine impedance
\( \text{Im}[Z(p)]/p \). For the quadrupole frequency we can multiply by 2 the above expression so
that finally

\[
\frac{\Delta f_{2s}}{f_{s0}} = \frac{e N_p \omega_0}{2 \pi V_{RF} h \cos \phi_s} \frac{\text{Im}[Z(p)]}{p} \sum_{p=-\infty}^{\infty} p^2 \sigma_0(p \omega_0) . \tag{13}
\]

As for the synchronous frequency shift, if we consider a Gaussian distribution, and ap-
proximate the summation with an integral, we get

\[
\frac{f_{2s}}{f_{s0}} = 2 + \frac{e N_p}{\sqrt{2 \pi V_{RF} h \cos \phi_s \omega_0^2 \sigma_s^2}} \frac{\text{Im}[Z(p)]}{p} = 2 - \tilde{X} \frac{\text{Im}[Z(p)]}{p} . \tag{14}
\]

By using the measurement results and the above expression, we can plot the normalized
incoherent quadrupole synchrotron frequency as a function of \( \tilde{X} \), as shown in fig. 4 for the
first set of measurements (10 May 2012) and in fig. 5 for the second one (13 June 2012).
As can easily be seen, the two independent sets are in a very good agreement. The error
bars have been obtained by using the uncertainty propagation from the data of table 1.
The slope of the linear regression, obtained with the method of least squares, gives directly
the broadband longitudinal impedance of the machine \( \text{Im}[Z(p)]/p = (9.1 \pm 2.1) \) \( \Omega \) and
\( \text{Im}[Z(p)]/p = (11.3 \pm 1.9) \) \( \Omega \), respectively.

This analysis however, as for the synchronous phase shift, is valid only if we can neglect
higher order terms in \( \tau \) in the expansion of the exponential term \( e^{ip \omega_0 \tau} \) in eq. (4). In
order to solve the equation of motion (1) exactly without any approximation, instead of
the Gaussian distribution, we can consider a parabolic line density interacting with a pure
Figure 4: Quadrupole frequency shift and linear fit with Gaussian distribution function for the first set of measurements.

Figure 5: Quadrupole frequency shift and linear fit with Gaussian distribution function for the second set of measurements.

inductive impedance, i.e. with constant \( Im[Z(p)]/p \). In this case the infinite summation on the right side of eq. (1) can be expressed in closed form

\[
\sum_{p=-\infty}^{\infty} p \sigma_0(p \omega_0) e^{ip\omega_0 \tau} = i \frac{3 \pi \tau}{\omega_0^2 (\tau_b/2)^3},
\]

and it gives a coherent force linear with \( \tau \), such that the single particle equation of motion
can be reduced to
\[
\ddot{\tau} + \omega_s^2 \tau = - \frac{3eN_p\omega_s^2}{2V_{RF} h \cos \phi_s \omega_0^2 (\tau_b/2)^3} \frac{\text{Im}[Z(p)]}{p} \tau ,
\]
(16)
which gives a quadrupole synchrotron frequency shift of
\[
\frac{f_{2s}}{f_{s0}} = 2 + \frac{12eN_p}{V_{RF} h \cos \phi_s \omega_0^2 \tau_b^3} \frac{\text{Im}[Z(p)]}{p} = 2 - \hat{\chi} \frac{\text{Im}[Z(p)]}{p}.
\]
(17)

If we compare eq. (14) with eq. (17), by considering \(\tau_b \simeq 4\sigma_G\), we observe that the parabolic line density, with the exact solution of the equation of motion, predicts a frequency shift about two times less than that obtained with the first order expansion and a Gaussian distribution. This factor of two has also been found in [9], and it does not depend much on the kind of considered longitudinal distribution function, rather on the truncation in the expansion of the exponential term of eq. (1).

Applying parabolic fits to the measured bunch profiles, and plotting the normalized synchrotron frequency versus \(\hat{\chi}\) of eq. (17), we obtain the results shown in figs. 6 and 7. From a linear fit, the longitudinal broadband impedance becomes \(\text{Im}[Z(p)]/p = (17.6 \pm 3.6) \Omega\) and \(\text{Im}[Z(p)]/p = (21.0 \pm 4.0) \Omega\). In both cases, these values are very close to the results obtained in previous measurements [4, 11].

![Figure 6: Quadrupole frequency shift and linear fit with parabolic line density for the first set of measurements.](image)

The difference in the values of \(\text{Im}[Z(p)]/p\), derived by the exact solution with parabolic line density and by the first order approximation with Gaussian distribution, is confirmed by the results of simulations, which indicate that the parabolic line density is more suited to the PS case, as we will show in the next section, where the same analysis technique as for the measurements has been applied to results from simulations.
To conclude the data analysis, we observe that, during our measurements, most probably the PS machine operated below the microwave instability threshold. In fact, by applying the Boussard criterion [12], the single-bunch intensity threshold is

$$N_p = \frac{(2\pi)^{3/2} \eta (E_0/e) \sigma_G \sigma_p^2}{\epsilon |Z(p)/p|},$$

(18)

with $\eta$ the slippage factor, and $\sigma_p$ the RMS relative momentum spread. With the typical beam parameters of the measurements, that is $\eta = 2.53 \cdot 10^{-2}$, $\sigma_G = 2.3$ ns and $\sigma_p = 3.6 \cdot 10^{-4}$, we get $N_p \simeq 10^{12}$, which is more than a factor of two higher than the maximum intensity (table 1). Moreover, previous observations of bunch profiles with a sampling rate of 20 Gs/s during the first turn in the SPS did not show any evidence of microwave instability [13].

3 Simulations and analytical results

Simulations of longitudinal beam dynamics, taking into account the collective effects due to wakefields, are a powerful tool to better understand the beam behavior. We have therefore performed a series of simulations by using a tracking code initially developed to study the longitudinal beam dynamics in the electron storage ring DAΦNE at LNF-INFN [14, 15] and then adapted to the beam parameters of the CERN PS. The results gave us also indications on which expression, between eq. (14) and eq. (17), is more suited for the PS to evaluate the broadband impedance.

Indeed, with the simulation code we can track the synchrotron oscillations of each macro-particle and, by means of the FFT, obtain the corresponding frequency spectra. By including the collective effects due to the wakefields, an incoherent quadrupole frequency shift as a function of beam intensity, with constant longitudinal emittance, can be extracted. In fig. 8
an example of the FFT phase of the incoherent frequency spectrum around \( \omega = 2\omega_0 \) is shown at different bunch intensities. Cf., the phase of the measured beam signal shown in fig. 2. A quadrupole synchrotron frequency decreasing with bunch intensity is clearly visible in the zoomed portion.

![Figure 8: Phase of the incoherent frequency spectrum at different bunch intensities obtained with the simulation code with an imaginary impedance of \( Z(p)/p = 18.4i \ \Omega \).](image)

The resulting frequency shift is shown in table 2, as well as the standard deviation \( \sigma_G \) for Gaussian distribution and total length \( \tau_b \) for parabolic line density obtained with a pure inductive impedance of \( Z(p)/p = 18.4i \ \Omega \) at different bunch intensities, and with \( V_{RF} = 100 \) kV. The other machine parameters are the same as during the measurements. The natural synchrotron frequency is here 547 Hz.

<table>
<thead>
<tr>
<th>( N_p \times 10^{11} )</th>
<th>( f_{2s} ) (Hz)</th>
<th>( \sigma_G ) (ns)</th>
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<tbody>
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<td>9.89</td>
</tr>
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<td>2.60</td>
<td>10.01</td>
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<td>1025</td>
<td>2.64</td>
<td>10.16</td>
</tr>
<tr>
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<td>2.68</td>
<td>10.32</td>
</tr>
<tr>
<td>6</td>
<td>976</td>
<td>2.74</td>
<td>10.55</td>
</tr>
</tbody>
</table>

With the values of table 2 it is possible to perform the same analysis we did with the measured data. Of course, in this case, we already know that the broadband machine impedance must turn out to be the one that we used as input value, that is \( Z(p)/p = 18.4i \ \Omega \).

If we use a Gaussian distribution with eq. (14) we obtain the red line of the plot in fig. 9 and an impedance \( Z(p)/p \) of only about \((8.5 \pm 0.6)i \ \Omega\), whereas, if we consider a parabolic
line density with eq. (17), the result (blue line) is twice that value, giving an impedance
\( Z(p)/p = (18.2 \pm 1.2)i \Omega \) very close to the impedance initially assumed for the simulations. This again confirms the factor 2 in the determination of the broadband impedance for the PS case, indicating that the correct result of the analysis of the measurements is obtained by using eq. (17).

Figure 9: Quadrupole frequency shift and linear fit with Gaussian distribution and parabolic line density by using the simulation results.

In fig. 10 we show the results of the simulations compared with the measured ones, for the case of parabolic line density. The agreement is very good, indicating that, with a simple model of inductive broadband impedance, we are able to reproduce the measurement results. There is a small displacement between the two fit curves, the intercept of the measurements being a bit too high (it should not be greater than 2). However, this overestimation is within the measurement uncertainty of ±3σ. Some comments about the RF peak voltage evaluation that can be the cause of this effect can be found in appendix A.

Another effect that can be evaluated with the impedance model obtained is the contribution of the wakefields to the bunch length. This contribution is shown in fig. 11, where we have plotted the bunch length as a function of intensity for an impedance \( Z(p)/p = 18.4i \Omega \).

The red line represents the results of the simulations, which have been compared with the analytical expression (blue line) \([9, 16]\)

\[
\left( \frac{\sigma_z}{\sigma_{z0}} \right)^3 - \left( \frac{\sigma_{z0}}{\sigma_z} \right) - \frac{3}{16} \frac{eN_p c^3}{\sigma_{z0}^3 \omega_0^3 \hbar V_{RF}} \frac{Im[Z(p)]}{p} = 0, \tag{19}
\]
obtained by considering proton bunches with a constant longitudinal emittance and parabolic line density.

The lengthening of the bunch with intensity is the effect of the potential well distortion due to wakefields. However, it represents only about 10% of the total bunch length in the full range of our measurements. This means that it can not be easily evidenced by
Figure 10: Comparison between the incoherent quadrupole frequency shift obtained with the simulations and the results of the two sets of measurements.

Figure 11: Bunch length as a function of intensity obtained with simulations and theory, by using $Z(p)/p = 18.4i \, \Omega$ and $V_{RF} = 100$ kV.

measurements, which, moreover, should be performed at constant longitudinal emittance. This requirement was hard to achieve, especially at higher intensities, as shown in fig. 12 where we have reported the RMS longitudinal emittance as a function of beam intensity obtained in the second series of measurements with and without a controlled longitudinal emittance blow-up by means of the phase-modulated RF system at 200 MHz.

Different emittance curves are shown for measurements at 50 kV and 100 kV, while the physical emittance should be independent from the bunching voltage. This effect, in the range of about 10%, can be attributed to the emittance measurement based on longitudinal
tomography. The bucket area, and hence the area of reconstruction, in the 100 kV is larger compared to the one with 50 kV, increasing the contribution of the reconstruction noise to the RMS emittance. It is worth noting that reflections from the pick-up, generating unphysical tail after the bunch (fig. 3), enhance the reconstruction noise.

Figure 12: RMS longitudinal emittance as a function of intensity under different conditions.

4 The PS broadband impedance model

In this section we evaluate the broadband longitudinal coupling impedance we expect in the PS by taking into account important contributions of several machine installations. The measured impedance value is very close to the budget obtained up to now.

The choice of performing the measurements at the flat-top energy of about 26 GeV has been made to reduce the contribution of the direct space charge to the broadband impedance, which, above transition, is capacitive. The space charge impedance due to the non relativistic velocity of the charges \((v = \beta c)\) in a circular pipe of radius \(b\) can be written in the form \([17, 18, 19]\)

\[
Z(p) = -i \frac{Z_0}{\beta \gamma^2} g_l
\]

with \(Z_0 = 377 \, \Omega\) the impedance of the free space, \(\gamma\) the relativistic Lorentz factor, and \(g_l\) a geometric factor depending on the transverse bunch distribution. In particular, for a uniform disk distribution of radius \(a\) we obtain the widely used expression

\[
g_l = \ln \frac{b}{a} + \frac{1}{2}
\]

In case of elliptic vacuum chamber, it is possible to substitute \(b\) with an equivalent radius related to elliptic functions \([21]\), which, for the PS case (semi-axes 35×73 mm), is \(b = 43\) mm. The absolute value of the imaginary part of the space charge impedance is shown in
fig. 13 as a function of $\gamma$ by assuming an injection beam radius of $4\sqrt{2}$ mm and a high energy beam radius of $1\sqrt{2}$ mm [20]. The $\sqrt{2}$ term has been used to take into account a Gaussian transverse profile instead of the uniform disk distribution.

From fig. 13, we can see that the contribution to the total broadband impedance due to the direct space charge at high energy is about 2 $\Omega$, the same order of magnitude as the uncertainty of the measurement results.

A very important contribution to the total machine impedance is given by the several ferrite loaded kickers of the PS [22]. The longitudinal impedance has been evaluated by using the field matching technique [23], which was shown to be in good agreement with measurements [24] and CST Microwave Studio [25] simulations [26]. The total longitudinal impedance of all the kickers is shown in fig. 14.

Also the connection regions between the beam pipe and the vacuum pumps give an important contribution to the geometrical impedance, especially since about 100 of these pump ports are present. Indeed the connection is not a simple hole, and there is no RF shielding in the port of the pumps. In fig. 15 we show a sketch of two of these structures.

The length of the cylindrical pipe connecting the beam pipe to the vacuum pump does not affect, at first order, the coupling impedance. Simulations have been performed with CST and the results are shown in fig. 16. The impedance is only inductive and, for a single pump, its value is $Z(p)/p = 2.8 \cdot 10^{-2}$ $\Omega$.

We have also evaluated the impedance due to the resistive wall. For a circular pipe of radius $b$ with high conductivity $\sigma_c$, such that $\varepsilon^2/\omega^2 b$ and $b$ are much bigger than the skin depth $\delta$, the coupling impedance is given by

$$ \frac{Z(p)}{p} = \frac{Z_0\delta}{2b} [1 + i \cdot \text{sgn}(\omega)] $$ (22)

Figure 13: Absolute value of the imaginary part of the space charge impedance for the PS as a function of the relativistic factor $\gamma$.

Values measured with the wire scanned on October-November 2009.
Figure 14: PS kickers longitudinal impedance.

Figure 15: Sketch of two vacuum pump connections with the beam pipe.

Figure 16: Impedance of a vacuum pump connection to the beam pipe, obtained with CST.

In case of an elliptic vacuum chamber, as in the PS, $b$ represents the minor semi-axis,
and the impedance has to be multiplied by a form factor that depends on the ellipticity of the beam pipe [27, 28, 29]. For the PS vacuum chamber, this form factor is about 0.96. The skin depth depends on the pipe material, which, in our case, is stainless steel 316 LN (about 70% of the machine, with conductivity $\sigma_c = 1.3 \times 10^6$ S/m) and Inconel X750 alloy (about 20% of the machine, with conductivity $\sigma_c = 8.3 \times 10^5$ S/m) [30]. Both these conductivities for the PS satisfy the approximations leading to eq. (22). The impedance contribution due the resistive wall is shown in fig. 17 for both materials as a function of frequency. The impedance at the revolution frequency [31] is $Z(p)/p = 2.2 (1 + i)\Omega$ for the stainless steel 316 LN, and $Z(p)/p = 0.8 (1 + i)\Omega$ for the Inconel X750 alloy, while at the bunch spectrum cut-off [32] it is $Z(p)/p = 0.07 (1 + i)\Omega$ for the stainless steel 316 LN and $Z(p)/p = 0.02 (1 + i)\Omega$ for the Inconel X750 alloy.

Concerning the RF cavities, we have taken into account the effects of the resonant modes due to the 10 MHz cavities (for a total of 10 cavities), one 20 MHz cavity, short circuited during our measurements, one 40 MHz and two 80 MHz cavities. All of them can be approximated by single resonance oscillators, as their gaps are short compared to the bunch length and their first higher order modes are well above the fundamental resonance. Their contribution to the total machine broadband impedance is mainly resistive. As an example, in fig. 18, we show the geometry of the 80 MHz cavity, and the wake potential of a $\sigma_G = 2.3$ ns Gaussian bunch, obtained with ABCI [33] (red line), compared to the wake potential obtained by accounting only for the contribution of the fundamental mode (blue line) [34].

An important source of geometrical impedance is due to the many step transitions existing in the PS vacuum chamber. In fig. 19 we report the vertical apertures (horizontal and vertical) along the machine, showing many discontinuities [35]. When a bunch passes through a step transition, two kinds of electromagnetic fields are excited [36]: the field scattered by the sharp edges and the one necessary to restore the boundary conditions at the pipe walls. The scattered field gives an important contribution to the resistive part of the impedance and represents energy lost both for a step-out (particle entering in a pipe with a larger

Figure 17: Resistive wall impedance for the PS vacuum chamber.
radius) and a step-in transition. On the other hand, the energy lost by the self field in a step-out transition for restoring the boundary conditions is regained in the step-in, such that the total energy lost is zero. This means that if a bunch is not able to excite the scattered field, the overall impedance seen for a pair step-out/in transitions is purely inductive.

Scattered fields exist only if the bunch can excite the modes propagating in the vacuum chamber. In case of the PS, the elliptic vacuum chamber of $35 \times 73$ mm has a cut-off frequency of the first TM mode at $f_c = 2.54$ GHz. The spectra of the bunches of table 1 have a characteristic frequency $1/\sigma_G$ less than 0.5 GHz and they are not able to excite propagating waveguide modes. Hence this kind of bunches sees only the inductive part of the impedance due to the electromagnetic fields that restore the boundary conditions. A simplified expres-
sion for the low frequency impedance of a step transition in a circular beam pipe can be obtained by solving two quasistatic problems, electrostatic and magnetostatic [37], and it can be written as

$$\frac{Z(p)}{p} = \frac{i\omega_0 Z_0 h^2}{4\pi^2 bc} \left( 2 \ln \frac{2\pi b}{h} + 1 \right), \quad (23)$$

with $h$ the height of the step. By considering a circular pipe from 4 cm to 8 cm, we obtain $Z(p)/p = 1.8 \cdot 10^{-2} \text{ } \Omega$. The above expression is however approximate for the PS where the beam pipe is elliptic and, as can be seen in fig. 19, the steps are not the same in the horizontal and vertical planes. With CST, a simulation of a step out from an elliptic pipe of $35 \times 73$ mm to a circular one of $73 \times 73$ mm gives an imaginary impedance of $Z(p)/p = 8 \cdot 10^{-3} \text{ } \Omega$, about half of the value predicted by eq. (23), as it should be, due to the fact that, in this case, the step in the horizontal plane is zero.

Other methods to calculate the impedance of a step in a circular pipe take into account the propagating modes of two semi-infinite waveguides into which the vacuum chamber can be divided, and use boundary conditions at the step transition [38, 39]. In all cases, the resulting impedance is purely inductive giving, for the couple step out/in configuration, a value of about $Im[Z(p)]/p = 1.6 \cdot 10^{-2} \text{ } \Omega$.

Another important inductive element is represented by the bellows. The impedance at low frequency can be obtained from that of a short pillbox with a width $w$ much lower that the height $h$, and it is given by [31]

$$\frac{Z(p)}{p} = \frac{i n_c \omega_0 Z_0}{2\pi bc} \left( w h - \frac{w^2}{2\pi} \right), \quad (24)$$

with $n_c$ the number of corrugations per bellow.

For the PS case we have assumed $w = 3 \text{ mm}$, $h = 14 \text{ mm}$, 8 corrugations per bellow, and two bellows for each of the 100 dipoles. The total impedance, by considering a circular pipe cross section, is about $Z(p)/p = 1.1 \text{ } \Omega$. However, the chamber of the bellows is not circular, so we expect a bit lower value. Indeed, CST simulations, the results of which are shown in fig. 20, give a total inductive impedance of $Z(p)/p = 0.85 \text{ } \Omega$, anyway very close to the one evaluated by using the circular cross section formula.

![Figure 20: Impedance of a bellow obtained with CST simulations.](image)

There are other sources of geometrical impedance, such as discontinuities of different kinds, shapes and sizes. We expect that they mainly contribute to the inductive part of the
impedance since the bunch spectrum is not able to excite diffracted fields propagating in the vacuum chamber, and we are in similar conditions as for step transitions. In table 3 we summarize the contributions to the machine impedance due to the above reported installations.

Table 3: Impedance contribution of important machine elements evaluated at the bunch spectrum cut-off. For the RF cavities the fundamental mode parameters are reported, including fast RF feedback.

<table>
<thead>
<tr>
<th>Machine element</th>
<th>$Z(p)/p$ at $\omega = 1/\sigma_G$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Space charge</td>
<td>-1.9$i$Ω</td>
</tr>
<tr>
<td>Magnetic kickers</td>
<td>$(1.6+i\cdot13.8)$Ω</td>
</tr>
<tr>
<td>Pumping ports</td>
<td>$2.8i$Ω</td>
</tr>
<tr>
<td>Resistive wall</td>
<td>$0.09(1+i)$Ω</td>
</tr>
<tr>
<td>Steps</td>
<td>$0.96i$Ω</td>
</tr>
<tr>
<td>Bellows</td>
<td>$0.85i$Ω</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>f (MHz)</th>
<th>Q</th>
<th>$R/Q$ (Ω)</th>
<th>Number of cavities</th>
<th>comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.6</td>
<td>5</td>
<td>30</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>4.6</td>
<td>43.5</td>
<td>1</td>
<td>short-circuited</td>
</tr>
<tr>
<td>40</td>
<td>70</td>
<td>33</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>80</td>
<td>100</td>
<td>56</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

This study is still in progress, however, the total longitudinal broadband impedance estimated so far is close to the measured one. In order to compare the results, since for some elements the impedance $Z(p)/p$ is not constant but a function of frequency, we have evaluated the wake potential of a Gaussian distribution with $\sigma_G = 2.3$ ns in both cases, that is for the measured inductive part of the impedance ($\pm$ its uncertainty) and for the one given by the sum of the several contributions. Fig. 21 shows that the two wake potentials are very close each other, indicating that an inductive impedance is a fairly good model for studying the longitudinal single bunch beam dynamics of the PS.

Instead of a purely inductive impedance, we observe that we could also use, a broadband resonator model with an impedance of the kind

$$Z(\omega) = \frac{R_s}{1 + iQ\left(\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega}\right)}$$  

with $R_s$ the shunt impedance, $Q$ the quality factor, and $\omega_r$ the resonant frequency. At low frequency $\omega \ll \omega_r$, this can be approximated with

$$Z(\omega \to 0) \simeq i\frac{R_s\omega}{Q\omega_r}.$$

(26)

If we consider a quality factor equal to 1 [40] and a resonant frequency equal to the frequency cut-off of the elliptic beam pipe [36], which in the PS is 2.54 GHz, with $Z(p)/p =$
18.4\,\Omega we get $R_s \simeq 98\,\text{k}\Omega$. The wake potential of this broadband resonator impedance for a Gaussian bunch of 2.3 ns, is exactly the same of that of a pure inductive impedance, as can be seen in fig. 22, where a comparison between the two is shown.

An improved model of the machine impedance can be obtained by observing that there is a small asymmetry in the wake potential obtained with the impedance budget, which is mainly due to the resistive contribution to the impedance of the RF cavities and the ferrite loaded kickers. More precisely the impedance can then be derived from the Heifets-Bane
model [36, 41], of which we maintain only the first two terms, the inductive and the resistive one, which best describe our particular impedance-generating elements. In fig. 23, we show, with the red curve, the wake potential given by this improved impedance model, with an inductance of $L = 5.67 \mu H$, which corresponds to $Z(p)/p = 17 \Omega$, and a resistance of $R = 294 \Omega$, which corresponds to $Z(p)/p = 2 \Omega$ if evaluated at the bunch spectrum cut-off. Finally we observe that the wake potential can also be well fitted by a broadband resonator with a resonant frequency of $f_r = 0.5$ GHz, and a shunt impedance of $R_s = 17.5$ k$\Omega$, as shown with the black curve in the same figure. It is interesting to observe that this resonant frequency corresponds approximately to the frequency at which the real part of the kickers longitudinal impedance has its maximum.

![Wake potential graph](image)

Figure 23: Wake potentials of a 2.3 ns Gaussian bunch given by the Heifets-Bane model $Z(\omega) = (i5.67 \cdot 10^{-6} \omega + 294) \Omega$ and a broadband resonator with $f_r = 0.5$ GHz, and $R_s = 17.5$ k$\Omega$, compared to the total wake budget.

5 Conclusions and outlook

The inductive broadband longitudinal coupling impedance of the CERN Proton Synchrotron has been measured from the incoherent quadrupole frequency shift as a function of single bunch intensity. Its value of $Z(p)/p = (18.4 \pm 2.2)i \Omega$ is in good agreement with measurements made more than 10 years ago. The procedure and the diagnostics now available can be used to monitor the impedance over the coming years, for example if new elements are added or others removed from the machine, also with a view to the LHC Injectors Upgrade project.

Moreover, an analysis of the contributions to the total broadband impedance of several installations, such as ferrite loaded kickers, RF cavities, resistive wall, space charge, and many geometrical impedances, resulted in excellent agreement with the measurements. The inductive impedance model, or the other improved models presented that also take into
account a small resistive contribution to the total impedance, can be used to study with the help of a simulation code the beam dynamics under the effects of wakefields.

Acknowledgements

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A Beam-based voltage calibration

During the two machine development sessions, different 40 MHz cavities have been operated (C40-78 on 10 May 2012 and C40-77 on 13 June 2012). The choice of changing the 40 MHz for the second set of measurements was made since the axis intercept of the linear fits in section 2.2 should represent an indirect measure of the RF voltage. For a wrong assumption of the RF voltage during the measurement, this axis intercept of eqs. (14) and (17) may deviate systematically from two. Additionally, previous calibrations indicated a difference of the detected voltage for the same programmed voltage of the order of 15%.

A beam-based technique to check the actual voltage in the 40 MHz cavities has been applied. The convergence of the iterative tomographic reconstruction of the longitudinal bunch distribution in a bucket with a given RF voltage assumption can be used as a measure for the error in that voltage assumption [42, 43]. It is important to point out that a strongly mismatched bunch is preferred for such a tomographic voltage calibration since the capability to resolve the evolution of tails, etc. is then most affected by any incorrectly chosen RF voltage. For an ideally matched bunch the tomographic voltage calibration method would not work; the longitudinal emittance would scale with the RF voltage assumed, but in the absence of unmatched parts of the distribution, no discrepancy minimization can be achieved as a function of the voltage.

A.1 Voltage step calibration

During the accelerator start-up 2012, a beam-based voltage measurement has been performed for both 40 MHz cavities in the PS. To maximize the mismatch of the longitudinal distribution, hence the precision of the calibration, a non-adiabatic voltage step from 40 kV to 100 kV has been introduced in the voltage program for the initially matched bunch. The evolution of the bunch profile versus time (measurement with C40-77) is illustrated in fig. 24.

Despite various machine parameters like beam energy or phase-slip factor, all of which are known with excellent accuracy, two main parameters have to be chosen for the reconstruction: the location in time of the synchronous phase with respect to the measured bunch profiles and the RF voltage. Fig. 25 shows the result of a two-dimensional scan, changing the assumptions for RF voltage and time of the stable fixed point.
To derive a numerical result for the RF voltage from the blue island of optimum convergence, a two-dimensional second-order polynomial fit was applied to the data shown in fig. 25. This fit, shown in fig. 26, represents the original convergence data very well. The best assumption for the RF voltage in C40-77 during the voltage calibration becomes 95.7 kV with an uncertainty below 5%. The optimum time of the stable fixed point is irrelevant. The same technique applied to C40-78 results in an RF voltage of 79.6 kV. For both cavities...
the programmed voltage was 100 kV.

A.2 Voltage calibration from dipole oscillations

The longitudinal emittance measurement data recorded during the broadband impedance measurement session do not appear to be well suited for a beam-based tomographic voltage estimate at first sight. However, significant dipole oscillations, irrelevant to the quadrupole synchrotron frequency, have been observed in a few cases. The same reconstruction technique as outlined above has thus been applied to such datasets, with a programmed voltage of 50 kV and 100 kV for both 40 MHz cavities.

As an example, the bunch profile evolution used as input data for the tomographic voltage calibration of C40-77 with a programmed voltage of 100 kV is illustrated in fig. 27. The corresponding convergence plot is shown in fig. 28 and the corresponding polynomial fit in fig. 29. The actual RF voltage in C40-77 thus becomes 92.6 kV with an error of about 5%. Clearly the island of optimum convergence is significantly larger compared to the voltage step calibration, hence the larger uncertainty.

A.3 Summary

Table 4 summarizes the results of the RF voltage calibrations. At a programmed voltage level of 50 kV, the amplitude of the dipole oscillations is less significant, which explains the larger uncertainty (larger island of optimum convergence).

The results of the beam-based voltage calibration obtained from data recorded during the broadband impedance measurements in May and June 2012 are fully compatible with the results of the voltage step calibrations in March 2012. Hence the actual voltages in both cavities for a given programmed voltage remained as expected constant within the measurement uncertainty over this period of several weeks.
Figure 28: Final convergence (40 iterations) of the tomographic reconstruction versus time of stable fixed point and RF voltage (C40-77) for the bunches profiles shown in fig. 27. The red dot indicates the parameter set with best convergence from the polynomial fit.

Figure 29: Two-dimensional second-order polynomial fit of the data presented in fig. 28. The red dot again indicates best convergence.

Table 4: RF voltages from tomographic reconstructions for both 40 MHz cavities measured at programmed voltages of 50 kV and 100 kV.

<table>
<thead>
<tr>
<th>Voltage step calibration (March 2012)</th>
<th>Cavity</th>
<th>$V_{prog}$ [kV]</th>
<th>$V_{meas}$ [kV]</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>C40-77</td>
<td>100</td>
<td>95.7</td>
<td>&lt; ±5 %</td>
<td></td>
</tr>
<tr>
<td>C40-78</td>
<td>100</td>
<td>79.6</td>
<td>&lt; ±5 %</td>
<td></td>
</tr>
<tr>
<td>Dipole oscillations (10 May 2012)</td>
<td>C40-78</td>
<td>50</td>
<td>40.1</td>
<td>≃ ±10 %</td>
</tr>
<tr>
<td></td>
<td>C40-78</td>
<td>100</td>
<td>83.2</td>
<td>≃ ±5 %</td>
</tr>
<tr>
<td>Dipole oscillations (13 June 2012)</td>
<td>C40-77</td>
<td>50</td>
<td>48.1</td>
<td>≃ ±15 %</td>
</tr>
<tr>
<td></td>
<td>C40-77</td>
<td>100</td>
<td>92.6</td>
<td>≃ ±5 %</td>
</tr>
</tbody>
</table>

References


[34] R. Losito, PS/RF/Note 96-14, CERN, CERN, Geneva, Switzerland (1996).


