Production of $J/\psi$ pairs at the LHC in the $k_T$-factorization approach

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We provide a detailed study of prompt double $J/\psi$ production in proton-proton collisions at the LHC within the framework of non-relativistic QCD (NRQCD) and the $k_T$-factorization approach. We confront our predictions with the recent LHC data. We find that the LHCb data can be fairly described with the theory, while the CMS data are typically underestimated by a factor of 10. The overall situation is similar to what is seen in collinear calculations. We thus conclude that the theory still needs including higher-order contributions.
1. Introduction

Heavy quarkonium production is an interesting and important process as it serves as a complex test of perturbative QCD, parton distributions, and the formation mechanism of the bound states. Production of quarkonium pairs is doubly interesting and is doubly complicated. The differential cross sections recently measured by the LHCb [1] and CMS [2] collaborations provide a field for direct comparisons with the theory.

The theory of double quarkonium production has a long history. First theoretical calculations considering the production of $J/\psi$ pairs in the framework of leading-order (LO) perturbative QCD and nonrelativistic color-singlet model have been made as long as more than 30 years ago [3]. Some later, the consideration was extended to the onium-onium scattering mechanism [4], including both perturbative gluon and non-perturbative Pomeron exchange in the $t$-channel. Some interesting initial gluon polarization effects have been pointed out in Ref. [5]. The role of the color octet production channels have been studied in [6], though, without making a comparison between the predictions and the data for the lack of the latter. The first comparison with the data has been presented in Ref. [7] where all the possible leading-order color-singlet and color-octet contributions were taken into consideration. An extension beyond the leading order in the color-singlet channel was reported in [8], and a crucial importance of the next-to-leading order corrections has been pointed out. Finally, the double parton scattering (DPS) mechanism was taken into consideration in Refs. [9].

![Figure 1: Examples of Feynman diagrams contributing to the prompt double $J/\psi$ production at $O(\alpha_s^3)$. Left panel, color-singlet mechanism; middle and right panels, mixed and color-octet mechanisms.](image)

2. Theoretical framework

In contrast with the papers mentioned above, our present calculation is based on the $k_T$-factorization approach [10]. The collinear and $k_T$-factorization schemes represent different ways of including higher order corrections. These corrections can either be calculated explicitly, within the fixed-order perturbation theory (as contributions to the hard partonic subprocess), or can be taken into account in the form of $k_T$-dependent (unintegrated) parton densities. Strictly speaking, these two types of corrections are not fully identical. In the $k_T$-factorization, summation runs over terms enhanced with "large logarithms" of the type $[\alpha_s \log(1/x) \log(Q^2)]^n$ up to infinitely high order. In the collinear calculation, the corrections are only restricted to a fixed relatively low order, but...
include both enhanced and non-enhanced terms. These two schemes have quite a significant part of higher-order corrections in common, and so, it would be not surprising to find the numerical results close to each other. To avoid double counting, the evaluation of the hard scattering matrix element in the $k_T$-factorization approach should only be done at the leading order.

The partonic subprocesses included in our analysis are the following. The standard leading-order $O(\alpha_s^4)$ color-singlet mechanism $g + g \rightarrow J/\psi + J/\psi$ is represented by the left panel in Fig. 1. The LO color-octet channels $g + g \rightarrow J/\psi + g^*$ and $g + g \rightarrow g^* + g^*$ followed by nonperturbative gluon fragmentation $g^* \rightarrow J/\psi$ are represented by the middle and right panels, respectively.

In general, the latter processes are suppressed by the lower values of the color-octet matrix elements (in comparison with the color-singlet ones), but may take over at large $p_T$ due to different $p_T$ behavior of the differential cross sections: $d\sigma/dp_T \propto 1/p_T^8$ for Fig. 1a,b, versus $\propto 1/p_T^4$ for Fig. 1c. The formally higher-order $O(\alpha_s^6)$ processes of the onium-onium scattering type have also been taken into consideration as they specifically contribute to the events with large rapidity difference between the two mesons.

The evaluation of Feynman diagrams is straightforward and follows the standard QCD rules, with one reservation: in accordance with the $k_T$-factorization prescription [10], the initial gluon spin density matrix is taken in the form $\varepsilon_g^\mu \varepsilon_g^\nu = k_T^\mu k_T^\nu / |k_T|^2$, where $k_T$ is the component of the gluon momentum perpendicular to the beam axis. In the collinear limit, when $k_T \rightarrow 0$, this expression converges to the ordinary $\varepsilon_g^\mu \varepsilon_g^\nu = -\frac{1}{2} g^{\mu\nu}$, while in the case of off-shell gluons it contains an admixture of longitudinal polarization. We have carefully checked that our present results are consistent with earlier calculations made in the collinear limit.

The parameter setting used in numerical calculations is as follows. The charmed quark mass is set to one half of the $J/\psi$ mass, $m_c = m_\psi/2$; the $J/\psi$ radial wave function is supposed to be known from leptonic decay width [11] and set to $|R_\psi(0)|^2 = 0.8 \text{ GeV}^3$; the nonperturbative color-octet matrix elements are taken from Ref. [12], $\langle \mathcal{O}_{\psi}^{J/\psi} \rangle = 1.2 \times 10^{-2} \text{ GeV}^3$, the renormalization

Figure 2: $J/\psi$ pair production at the LHCb conditions, theory versus experimental data of Ref. [1]
and factorization scales are chosen equal to each other and \( \mu_F^2 = \mu_R^2 = \hat{s}/4 \) with \( \hat{s} \) being the invariant energy of the partonic subprocess, and the unintegrated gluon density is given by the A0 set from Ref. [13]. To estimate the theoretical uncertainty band we admit variations in \( \mu_F^2 \) and \( \mu_R^2 \) by a factor of 2 around their default values and use the A+ and A- gluon parametrizations from Ref. [13].

3. Numerical results and discussion

Our numerical results are displayed in Figs. 2 and 3. The LHCb data (see Fig. 2) can be perfectly described with the sole LO color-singlet contribution, while the role of the other considered contributions is really negligible. The agreement with the data looks even better than in the case of collinear approach (though, there are significant uncertainties in the both theoretical calculations and in the experimental points).
At the same time, the theory underestimates the CMS data by a factor of 10. Recall that the CMS conditions require $p_t(J/\psi) > 5$ GeV, while the LHCb kinematics implies no $p_t$ cuts. The dominant theoretical contribution is still due to the leading-order color-singlet mechanism. The onium-onium pseudo-diffractive scattering is suppressed by extra powers of coupling constants and, especially, by the color coefficients, as is explained in detail in the last paper of Ref. [9]. The leading-order color-octet contributions are suppressed by the relatively low values of non-perturbative matrix elements (two orders of magnitude per $J/\psi$, in comparison with the color-singlet wave functions). Color-octet contributions become important in the high $p_t$ region because of their different $p_t$-dependence (already mentioned in the previous section), but are still insufficient to describe the data. Finally, the double parton scattering is suppressed by the relatively narrow CMS rapidity range and neither can fit the data with the conventional choice of $\sigma_{\text{eff}} = 15$ mb.

In general, our calculations turn out to be numerically close to the collinear results and lead us to the same conclusions. We either need to go to higher-order corrections that would provide extra contribution to the high $p_t$ region, or we have to reconsider the double parton scattering mechanism using a significantly different value of $\sigma_{\text{eff}}$, as is proposed in Ref. [14].

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References